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Machine learning in algorithmic trading strategy optimization - implementation and efficiency

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Abstract: The main aim of this paper was to formulate and analyze the machine learning methods, fitted to the strategy parameters optimization specificity. The most important problems are the sensitivity of a strategy performance to little parameter changes and numerous local extrema distributed over the solution space in an irregular way. The methods were designed for the purpose of significant shortening of the computation time, without a substantial loss of a strategy quality. The efficiency of methods was compared for three different pairs of assets in case of moving averages crossover system. The methods operated on the in sample data, containing 20 years of daily prices between 1998 and 2017. The problem was presented for three sets of two assets portfolios. In the first case, a strategy was trading on the SPX and DAX index futures, in the second on the AAPL and MSFT stocks and finally, in the third case on the HGF and CBF commodities futures. The major hypothesis verified in this thesis is that machine learning methods select strategies with evaluation criterion near to the highest one, but in significantly lower execution time than the Exhaustive Search.

Keywords: machine learning, algorithm, trading, investment, automatization, strategy, optimization, differential evolutionary method, cross-validation, overfitting

JEL codes: C4, C45, C61, C15, G14, G17

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Introduction

The last years witnessed a huge growth of the machine learning popularity and its quick development. The newly established algorithms were used to solve many difficult problems from various fields of science and to produce solutions facilitating many areas of life. Therefore, the application of such methods to improve the process of strategy adjustment seemed to be a natural choice.

The main aim of this study was to formulate and analyze the machine learning methods, fitted to the strategy parameters optimization s pecificity. The most important problems are the sensitivity of a strategy performance to little parameter changes and numerous local extrema distributed over the solution space in an irregular way. The methods were designed for the purpose of significant shortening of the computation time, without a substantial loss of a strategy quality. The efficiency of methods was compared for three different pairs of assets in case of moving averages crossover system. Considered algorithms - the Extended Hill Climbing, Grid Method and Differential Evolution Method are based on the well-known machine learning methods or intuitive ideas based on observation of previous steps in order to improve the next ones.

The machine learning methods, discussed in this paper were designed to select the strategy parameters in order to maximize strategy performance, measured by the specified optimization criterion. The methods operated on the in-sample data, containing 16 years of daily prices and their results were verified on 4 years of out of sample d ata. The problem was presented for three sets of two assets portfolios. In the first case, a strategy was trading on the *SPX* and *DAX* index futures, in the second on the *AAPL* and *MSFT* stocks and finally, in the third case on the *HGF* and *CBF* commodities futures.

The major hypothesis verified in this paper is that results of the ML methods are the same or only slightly worse than the ones the highest evaluation criterion, obtained by the Exhaustive Search (brute force approach), but the time required to their execution is significantly lower than computation time of checking all points from the solution space. The additional research question is that the strategies obtained by the machine learning methods are associated with a lower risk of overfitting than the strategies resulted from the Exhaustive Search procedure.

The distributions of optimization criteria and the computation time of 1000 executions of different methods were compared and presented along with the Exhaustive Search results. The adjustment quality was assessed on in-sample data and additional out of sample data from the next 4 years in order to test the overfitting tendency. The trading performance for different sets of assets was calculated in the same framework of the simulated trading implemented for purposes of this research.

The basic machine learning methods have serious disadvantages. For instance, the well-known Hill Climbing returns the local extremum, without guarantee of reaching the global one. That algorithm is inadequate for the global search problem, but it could be used as a main component of the more complex and efficient method of global optimization. The purpose of this paper is not to design the most profitable strategy, but rather to compare the efficiency of different machine learning methods and the Exhaustive Search. Tests in the out-of-sample period were performed to assess overfitting problem.

Since the machine learning methods proved their value, by solving plenty of complicated problems, hence it was reasonable to expect the satisfying results of such methods used for the strategy optimization. The initial intuition was that the machine learning methods would return the results a bit worse than the optimal one, but in disproportionately shorter time, than checking all possibilities in order to get the best ones (the Exhaustive Search).

Moreover it was expected, that the machine learning methods were less likely to overfit strategy than the Exhaustive Search. The discussed methods were based on an assumption, that conditional expected value of the optimization criterion is usually higher for the points surrounded by those with high value of this criterion. Therefore, the low regularity of the solution space could be a real obstacle for methods performance. There was no reason to assume even a moderate level of the space regularity, so the machine learning methods probably could not find the optimal points, if they were not in the high-valued neighbourhood. That property could lead to reducing overfitting risk, because usually, the parameter vector surrounded by those with similar strategy performance have bigger chance to be profitable in the future, than those from a less stable place.

The structure of this paper was composed as folllows. The first part contained the literature review. The second part was devoted to data description. In the third one machine learning methods used in this paper were explained, as well as the trading assumptions and basic terms used in the paper in order to clarify. The fourth part contains efficiency tests of the considered machine learning methods, with special focus on the optimization criterion and computation time distributions for different methods. The summary of paper and conclusions are included in the last part.

1. Literature review

The machine learning methods have been developed for decades, even before that term was coined in the fifties (Samuel, 1959). Nevertheless, the increased interest in that field was observed in recent years due to the technical possibility to apply the artificial intelligence in the various fields of science and life. The phenomenon of learning from the computational viewpoint was discussed by Valiant (1984). The human's natural ability to learn and adapt was presented in terms of the information's selection and automatic adjustment process, resulting in the algorithm's modifications.

This approach is followed by plenty of the modern machine learning methods and it is close to the general ideas of the classic statistical modeling, where including new dataset leads to changes in the model properties. The traditional statistical and econometric models usually assume that data is produced by the stochastic process from the specified class. The fitting procedure is aimed at finding the process accurate to actual data when the machine learning methods are often based on the iterated improvements without specified model form. The differences between these two approaches called respectively data models and algorithmic models, are widely discussed in Breiman (2001). The field of machine learning contains plenty of various algorithms and methods, used to solve a wide range of problems. Some methods have strong mathematical foundations, for instance, methods based on Markov Chain Monte Carlo (Neal, 1993), when others, such as the hill climbing or evolutionary methods, are based on heuristic approach (Juels and Wattenbergy, 1994). The commonly used methods and algorithms with application in scientific problems are discussed in Hastie et. al (2013) and Hastie et. al (2001).

The algorithmic strategies are widely used in the financial markets, but most of them are not discussed in papers, due to exclusive character. Nevertheless, some types of the quantitative strategies are widely known and therefore discussed in books and papers. The strategy based on the technical analysis indicators, such as the simple moving average crossover method considered in this paper is analyzed for specified cases in Gunasekarage and Power (2001). Since machine learning methods have started to gain popularity, as a tool to solve problems in various fields, numerous attempts to use it for trading strategies occurred. Beyond the commercial usage, many academic papers describing strategies, with logic based on a machine learning have been published. For example group of researchers at Sanković et. al (2015), presented the strategy, based on the technical analysis and Least Squares Support Vector Machines. In contrast to this paper, they used machine learning methods as a part of a system generating trading signals, not as a part of system optimization process.

The more recent research was conducted by Ritter (2017), who used Q-learning with the proper rewarding function to handle the risk-averse case and tested strategy in the simulated trading. Dunis and Nathani (2007) presented the quantitative strategies, based on the neural networks such as the Multilayer Perceptron (MLP), Higher Order Neural Networks (HONN) and on the K Nearest Neighbors method. The authors proved that methods can be effectively used for generating excess returns from trading on gold and silver. The comparison between the performance of machine learning methods and the linear models of ARMA type not only lead to construct a better strategy but additionally showed the presence of nonlinearities in the considered time series.

The application of the machine learning methods in order to predict future prices nowadays becomes more and more popular. Shen, Jiang and Zhang (2012) presented the forecasting model for stock market indexes, based on Support Vector Machines, and tested the trading system based on the produced predictions. The similar approach was followed by Choundhry and Kumkum (2008), where they introduced the hybrid machine learning system, combining Support Vector Machines with Genetic Algorithm in order to predict the stock prices. The machine learning methods were used for predicting by Patel et al. (2015) in more recent research as well. The paper is focused on methods of data pre-processing for purposes of further forecasting. Therefore, many books and papers discuss the general aspects and methods, such as walk-forward optimization (Kirkpatrick and Dahlquist, 2011 or Pardo, 2011).

Differential Evolution, which is one of the method considered in this paper was designed by Storn and Price (1997) and discussed in further papers, such as Price et al. (2006). The algorithm was proposed for solving complicated problems with irregular solution space. It was used to solve non-convex portfolio optimization problems in Ardia et. al (2010) and the problem of minimizing CVAR for the large-scale portfolio in Ardia et.al. (2011a). The method proved to be an efficient and effective way to optimize complex problems.

2. Machine learning methods and their benchmark

2.1. Basic terms and methodological issues

All statistics used in the optimization criterion were determined by the equity line and could be easily calculated based on it. Therefore, the net profits and losses (PnLs) calculations were the most complex component of the strategy evaluation procedure. The system was based on the technology called *Rcpp*, allowing to use efficient *C++* programs inside *R* project. More precisely, the main function for calculating net PnL was implemented in *Rcpp* framework in order to accelerate computations based on loops. Other parts of system were designed in R due to vectorization possibility and high-performance of build in functions.

2.1.1. The basic concepts and terms of the automatic trading

- **annualized rate of return** - relative change of an asset value, normalized according to time. The annualized rate of return, calculated for the asset of value process V_t in specified period (t_1, t_2) is defined by the following formula:

$$ARC(V)_{t_1}^{t_2} = \left(\frac{V_{t_2}}{V_{t_1}}\right)^{\frac{1}{D(t_1,t_2)}} - 1, \tag{1}$$

  where $D(t_1, t_2)$ stands for the time between t_1 and t_2 in years.
- **maximum drawdown** - the maximum percentage loss of value of the equity line. For price process S_t and period $[t_1, t_2]$, the maximum drawdown is defined by the following formula:

$$MDD(S)_{t_1}^{t_2} = sup_{(x,y)\in\{[t_1,t_2]^2\,:\,x\leq y\}} \frac{S_x - S_y}{S_x}. \tag{2}$$

- **annualized standard deviation** - the empirical standard deviation normalized, according to the time. For specified time series R_t, the annualized standard deviation in the period $[t_1, t_n]$ is calculated by using the formula:

$$ASD(V)_{t_1}^{t_n} = \sqrt{\frac{1}{n}\sum_{t=t_1}^{t_n}(R_t - \bar{R})^2 * \frac{1}{D(t_1, t_2)}}, \tag{3}$$

  where $\bar{R} = \frac{1}{n}\sum_{t=t_1}^{t_2} R_t$ and $D(t_1, t_2)$ is the time between t_1 and t_2 in years.
- **moving average** - for the specified time series S and the wages vector $w = (w_0, w_1, ...)$ the moving average is a time series given by the formula:

$$MA(S)_t^w = \sum_{i=0}^{t} S_{t-i} w_i. \tag{4}$$

2.1.2. Assumptions

The main problem was to find the best investment strategy in the specified class of strategies, following the two simple moving averages crossover approach. The behavior of each strategy was fully determined by a vector of parameters from four-dimensional space ψ, each standing for different moving average window width. More specifically, every strategy from ψ was parametrized by a vector $p = (p_1, p_2, p_3, p_4) \in \psi$, such that the trading signals U_t^i for $i \in \{1, 2\}$ were given by the following formula:

$$U_t^i = sign\big(MA_1^i(S^i)_{t-1} - MA_2^i(S^i)_{t-1}\big), \tag{5}$$

where $MA_j^i(S^i)$ denoted the simple moving average of length p_{2i+j-2} for price series S^i, what means that p_1 and p_2 stands for lengths of moving averages used for first asset, and analogously p_3 and p_4 refer to the second asset. Additionally, we took convention, that strategy on one asset is called *momentum* if the first moving average length is smaller than the second, and analogously *contrarian* if the first moving average length is higher.

Optimization criterion was based on the typical descriptive statistics used by traders - the annualized returns (ARC), the annualized standard deviation (ASD) and maximum drawdown (MDD). The criterion was determined by the following formula:

$$OC = \frac{ARC * |ARC|}{ASD * MDD}. \tag{6}$$

The construction of the optimization criterion OC reflected the preference of moderately profitable strategies with low risk, over the high-profitable ones, associated with much higher risk. That approach was designed to find more safe and stable strategies, in order to generate profits in the future. This criterion additionally punished strategies with high *MDD*, what means that the strategies with lower returns and constant increasing trend are more likely to be selected than the more profitable ones, associated with periods of significant losses.

Conditions on the financial markets were different during the tested time period, from

1998 to *2017*. For the sake of simplicity, the constant transactional percentage costs were assumed. In the simulated trading, every trade required bearing the fee equal to 0.25% of its value. Additionally, we asumed leverage on the level of *40%*, what means that every considered strategy invested 20% of the total account balance on each asset (40% in total). The rebalancing took place once each 5 trading days.

The available strategies were fully determined by four parameters, standing for moving averages widths. Consequently, the strategy was optimized on the parameters (solution) space Ψ composed of vectors of four numbers from the set $\{1, 5, 10, ..., 100\}$ (i.e $\Psi = \{1, 5, 10, ..., 100\}^4$).

2.2. Specificity of the problem

The problem of selecting the best parameters of a trading strategy could be parametrized and reformulated in terms of optimization. The optimization criterion (*OC*) is, as specified before, calculated based on annualized returns and risk measures. The parameter space and the reward function had some important properties, that need to be included in machine learning methods design in order to meet problem specific requirements and reach better efficiency.

Solution space (Φ) is discrete, thus the application of algorithms, based on the steps of decreasing size was strictly limited. Moreover, the function being optimized had no simple analytical formula. In consequence, there was no way to apply gradient-based methods. The *OC* could be obtained from the statistics of the equity line for the specified strategy parameters. The calculations were not very complicated, but they required relatively long time to be executed. The high time complexity was caused by a need to calculate profits and losses for every trading day. Therefore, the main difficulty was caused rather by the time expensive criterion calculations, than by the big number of possible parameter combinations (*194 481*).

Additionally, the performance of automatic strategies is usually sensitive to the parameters change, therefore even subtle difference could severely affect the results. In consequence, one can expect multiple local extrema scattered over the parameter space and big differences in criterion value of the points near each other. High sensivity of the optimization criterion (objective function) to parameters was crucial for the machine learning efficiency and led to selection of more complex methods, adjusted to the problem specificity. A lthough the optimization criterion was unstable, some level of regularity was necessary for machine learning methods to work. Machine learning algorithms selected the points (candidate solutions), surrounded by other with the high criterion value, what could positively affect results stability and reduce overfitting risk.

The machine learning methods presented in this paper were based on well-known concepts. The main effort was to design methods based on these algorithms, but able to run on an atypical problem, hard to be solved by the basic ones. Although the presented methods could result in lower overfitting risk, the paper was focused on the improvement of the strategy selection procedure in terms of time and hence no features, aimed at reducing the overfitting risk would be discussed.

2.3. Extended Hill Climbing (EHC)

The basic **Hill Climbing** is a local search method, based on a very intuitive approach - going always in the way that improves the situation. The method operates on a graph, composed of nodes (points) with optimization criterion value and edges, which determine the relation of being neighbours. The basic Hill Climbing algorithm is a simple loop, starting at the specified point and repeatedly changing current point to its neighbour with higher optimization criterion value, as long as an improvement was possible by making the step forward. The classic Hill Climbing procedure checked all neighbours and selected the one with the highest value. The method traverses the parameter space with only one rule of always going up, using no information from the past, except the current position. The accurate parallel for that algorithm is "trying to find the top of Mount Everest in a thick fog while suffering from amnesia" (Russell and Nowig 2003). The method can find only the local extremum, thus it is local search. That is the significant limitation of this method's use, because there was no reason to expect that method will end search in one of the best solutions. Neighbours are often defined as points with the specified distance between them. In this case, the algorithm traverses the parameter space using steps of a specified size. The Hill Climbing is well-adjusted for problems with great regularity, such as convex ones, where exactly one local maximum exists (Skiena 2008). It is no reason to assume that the considered problem have that property, therefore the main effort in this method improvement was made by adjusting method to spaces with many local maxima.

The Extended Hill Climbing (EHC) is composed of the independent Hill Climbing executions, called walks. These walks are starting at different random points and the best result among them is returned at the end. Every single walk procedure checks the neighbours of the current point and goes ahead when the first improvement is found. It is substantially different approach than followed by the classic one, reducing computation time, because it does not require calculating optimization criteria for all neighbours. Another new feature is the use of a few different neighbours structures. More specifically, algorithm checks neighbours differing by exactly one parameter, what implied that a walk is on the perpendicular multidimensional grid. At the very beginning and after making every move algorithm checks points with distance at specified, relatively high level. When no improvement is possible, the method checks points with lower distance from the current one. This procedure is repeated until no improvement is possible either the next move is made or the specified minimum stepsize is exceeded. The stepsize series is defined as $\{F, \lceil \frac{F}{k} \rceil, \lceil \frac{F}{k^2} \rceil, ..., 1\}$, where F is a starting step, in this paper equal to 5 and k is equal to 2. The initial preference of big steps resulted in fast crossing the space and the possibility of walking by small steps allowed the algorithm to finally search the small neighbourhood in order to find solution with optimization criterion as high as possible. This feature is crucial element of method- algorithm uses steps of various sizes, thus a supervisor do not need to select step size as cautiously as for method with fixed step size. What is more, method always uses step of minimal size at the end, so there is no risk that some points cannot be reached by the walk.

The number of walks required to get satisfying results is random. Thus, declaring the fixed number could result in low stability of results- the difference between optimization criteria obtained in independent optimizations could be significant. The extended method set the number of walks in a dynamic way, dependent on the efficiency of previous walks. The algorithm starts twice as many new walks, if the previous set of walks improved the optimization criterion, what suggest that there is still possibility to improve results. That solution guarantes the higher results stability, at the expense of the time stability. The time required for execution could be much higher, when method starts in the different starting point, but on the other hand, “bad” starting points should not affect the final results. In this paper initial number of walks is equal to 5.

2.4. Grid Method (GM)

The second machine learning method, called *the Grid Method (GM)* is designed to operate on a limited space of discrete parameters, called grids. The method is composed of simple exhaustive searches, finding the best points from the parameters subgrids. The subgrids with a decreasing interspace are considered in the consecutive steps of the method. Firstly, the subgrids of full range and the relatively high interspace between parameters are considered and some of the best feasible solutions are used as a starting point for new independent procedures. Every starting point becomes the center of a new subgrid, with a lower interspace between parameters. After predefined number of iterations the interspace between parameters is minimal and then solution with the biggest criterion among all subgrids is returned. This method is purely deterministic and need the initial subgrid of parameters space to reflect the properties of whole space, such that the best global solutions will be around the best solutions from the initial subgrid. Otherwise, the method cannot return satisfactory results. Therefore, the high-value solution, surrounded by the worse ones might not be found by this method.

The search could be improved by setting different meta-parameters, such as the number of starting points or the interspace between parameters in the initial grid. There is a natural trade-off between method's accuracy and the computation time due to the fact that the computation time was approximately proportional to the number of evaluations. Setting the meta-parameters allows to balance between method precision and time in easy and effective way. Another big advantage is deterministic nature of the method. There is no uncertainty about the method results or computation time, which could be observed for random methods, such as the *Extended Hill Climbing*. Moreover, the computation time could be estimated before execution due to the predefined number of evaluated points. The method is easy to parallelize, as well as the previous one- the procedures for separated grids could be executed at the same time on different CPUs. The discussed method is prepared for the purpose of this paper - it is not an extension of existing machine learning methods, however, it follows the basic idea, so it could be method in use, following the similar approach.

Throughout that paper, the number of starting points is always six and the interspace between initial subgrid parameters is equal to the initial step of the previous method - *5*. The grid size is always $5x5$, centered at the best point from the previous search procedure step.

2.5. Differential Evolution Method (DEM)

The Differential Evolution is the optimization method inspired by the biological phenomenon of evolution. The considered machine learning method follows that approach, by taking the random sample (population) from the solution space (Φ), disturbing the parameters (mutation of the population characteristics) and creating a new sample from the most profitable strategies (reproduction). The steps are repeated - the new population is created, with the disturbed characteristics similar to the best ones from previous step. This approach is similar to the rule of *the Grid Method*. Both methods repeatedly check all strategies from a specified sample and create the next one, using the information learned before.

The biological evolution never stops, although the method must stop at some point in order to return the final results. The specification of the stopping rule is crucial both for time efficiency and fitting level of the method. It should be designed in order to make sure that new iterations are called when it leads to better results, but stopped when further results improvement is more likely impossible.

The R implementation of the procedure *JDEoptim* from *DEoptim* package of version 2.2-4 with default meta parameters (Mullen et al. 2016) is used as a main component of the method. The differential evolution operates on the continuous spaces of real numbers, therefore it is inadequate for selection of the integer parameters. However, the discrete space can be intrapolated on the continuous one by several methods. The optimization criterion function $OC : \Psi = \{1, 5, ..., 100\}^4 \rightarrow \mathbb{R}$ was extended to $\tilde{\Psi}$ operating on the continuous real space $[0, 100]^4$ in the following way: $\tilde{\Psi}(x_1, x_2, x_3, x_4) = \Psi(5\lfloor\frac{x_1}{5}\rfloor, 5\lfloor\frac{x_2}{5}\rfloor, ..., 5\lfloor\frac{x_4}{5}\rfloor)$ with additional assumption that parameters equal to 0 are changed to 1. The extended function $\tilde{\Psi}$ simply returns the value of Ψ for rounded values of parameters.

2.6. The Exhaustive Search (ES)

The strategies selected by different methods were analyzed and compared with the optimal strategy, maximizing the optimization criterion in the in-sample period. The optimal strategy was found in every case by the Exhaustive Search- algorithm checking all possible combinations of parameters in order to select one with the highest criterion value. Following this approach always leads to get the highest possible criterion value, but it requires plenty of time. The main purpose of using machine learning methods instead of the Exhaustive Search was to get significantly lower computation time without loss of the results quality. Therefore, the difference in computation time reflects the value of information learned in previous steps for further search procedure efficiency. Moreover, the Exhaustive Search will be treated as a benchmark due to its intuitive character and widespread use.

3. Data description

The main goal of machine learning methods was to find parameters vector $\psi \in \Psi =$
$= \{1, 5, ..., 100\}^4$ (i.e vector of four parameters, each from the set $\{1, 5, ..., 100\}$) in order to select the self-financing strategy ϕ maximizing OC within the framework of the assumptions.

Every considered portfolio was composed of two securities of the same kind. The first pair contained the futures contracts on two important and highly correlated market indexes- American *S&P500 Index (SPX)* and German *Deutscher Aktienindex (DAX)*. The next considered pair was composed of two big American high-tech companies stocks - *Apple Inc. (AAPL)* and *Microsoft Corp. (MSFT)*. These companies are major representatives of the IT sector and American economy, but there was a real difference, between their dynamics of growth. The last considered assets were two commodities futures contracts - *High-Grade Copper Futures (HG.F)* and *Crude Oil Brent Futures (CB.F)*.

The machine learning methods searched for strategy optimal in the in-sample period from the beginning of 1998 to the end of 2013. Strategies were validated on the out-of-sample data from the beginning of 2014 to the end od 2017. All strategies operated on daily data, taking position each trading day. The length of in-sample period was big enough to make sure, that the different market trends were included for all time series. On the other hand, the out-of-sample length allowed to properly validate strategies and assess overfitting level.

Table 1: The descriptive statistics of the considered assets

	In-sample						Out-of-sample					
	SPX	DAX	AAPL	MSFT	HGF	CBF	SPX	DAX	AAPL	MSFT	HGF	CBF
%ARC	3.92	4.79	35.22	6.32	9.41	12.14	9.67	8.07	22.68	25.70	-0.62	-11.01
%ASD	20.39	24.97	46.69	33.06	28.48	34.54	11.94	18.37	22.27	21.43	19.25	33.07
IR	0.19	0.19	0.75	0.19	0.33	0.35	0.81	0.44	1.02	1.20	-0.03	-0.33
%MDD	56.78	72.68	43.80	71.65	68.37	73.48	14.16	29.27	30.45	18.05	42.47	75.83

%ARC - annualized rate of return (%), %ASD - annualized standard deviation (%), %MDD- maximum drawdown of capital (%), IR- information ratio calculated as %ARC / %ASD, OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD), SPX - S&P500 Index, DAX - Deutscher Aktienindex, AAPL - Apple Inc. stock, MSFT - Microsoft Corp. stock, HGF - High Grade Copper Futures, CBF - Crude Oil Brent Futures. The statistics have been calculated for in-sample period from the beginning of 1998 to the end of 2013 and for out-of-sample period from the beginning of 2013 to the end of 2017, on daily data.

The most rapid growth of value was observed for *AAPL* stock, whose price increased on average by around *35%* annually. The disproportion between the profitability of *AAPL* and other assets did not diminish, after adjusting for risk and applying terms of IR measure. The standard deviation of *AAPL* returns was higher than for any other asset, however in a way disproportionately lower than the returns. Therefore, one can say that risk was fully compensated by enormously high returns. Additionally, *AAPL* had the lowest maximum drawdown (*43.80%*)

among all considered price series (around *60-70%*).

The returns of *AAPL* was still high, but noticeably lower in the out-of-sample period, although the IR measure was higher than in the in-sample period. The returns of *AAPL* was lower than in earlier period but associated with much lower risk. Microsoft shares price increased faster than other asset prices in the out-of-sample period. The IR was higher than for any time series in the out-of-sample. It is worth noting, that the commodities (*HG.F* and *CB.F*) return was negative in the out-of-sample period. The graphs of the prices, normalized by the initial value would be presented together with strategies equity lines in the next part of this paper.

4. Efficiency tests for different methods

Methods described before were tested on the three pairs of assets by running whole optimization process on the data from in-sample period. Extended Hill Climbing and Differential Evolution Method were executed *1000* times for every pair due to their random nature. The strategies with median optimization criterion were treated as a final results for comparison purposes and called *median strategies*.

4.1. S&P500 Index (SPX) and Deutscher Aktienindex (DAX)

All statistics and graphs refering to the methods performance on that pair of assets were denoted by acronym **SPXDAX**.

4.2. In-sample methods efficiency

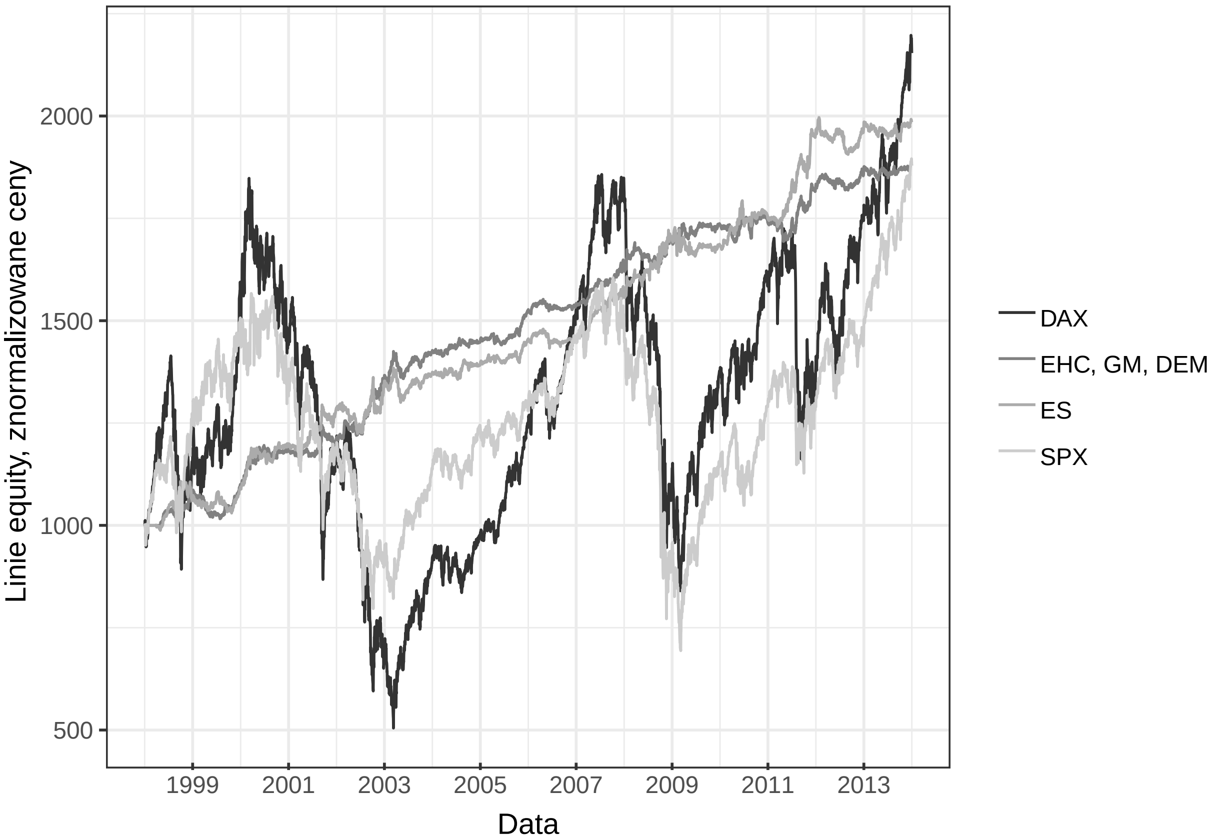
All machine learning methods had the same selected median strategy, different than the one resulted from Exhaustive Search procedure. Nevertheless, all methods used contrarian approach on *SPX* and momentum component operating on *DAX*. Both the optimal strategy resulted from *ES* and median strategy of machine learning methods generated only moderate profits during the whole in-sample period. On the other hand, the strategy met the requirements of stability and safeness. Strategies of that kind were preferred over the more profitable ones due to using low leverage and including two risk measures in the construction of the optimization criterion. The maximum drawdown was especially low, despite the relatively long time horizon. The resulting strategies never lost more than *5%* of the available money during whole 16 years of the in-sample period, comparing with more than *50%* on the basis instruments (SPX and DAX indexes).

Table 2: The median strategies parameters and statistics resulted from all methods SPXDAX

	In-sample				Out-of-sample			
	ES	EHC	GM	DEM	ES	EHC	GM	DEM
k1	60.00	100.00	100.00	100.00	60.00	100.00	100.00	100.00
k2	45.00	35.00	35.00	35.00	45.00	35.00	35.00	35.00
k1.2	65.00	45.00	45.00	45.00	65.00	45.00	45.00	45.00
k2.2	75.00	85.00	85.00	85.00	75.00	85.00	85.00	85.00
%ARC	4.27	3.92	3.92	3.92	-0.03	-0.62	-0.62	-0.62
%ASD	5.17	4.63	4.63	4.63	4.02	3.74	3.74	3.74
IR	0.83	0.85	0.85	0.85	-0.01	-0.17	-0.17	-0.17
%MDD	4.53	4.30	4.30	4.30	7.20	6.34	6.34	6.34
OC	77.79	77.16	77.16	77.16	0.00	-1.62	-1.62	-1.62

%ARC - annualized rate of return (%), %ASD - annualized standard deviation (%), %MDD- maximum drawdown of capital (%), IR-information ratio calculated as %ARC / %ASD, OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD), k1, k2, k1.2, k2.2 - strategy parameters, width of the moving averages' windows. The statistics of the equity lines have been calculated for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

Figure 1: The equity lines of the strategies selected by all methods for SPXDAX - in-sample

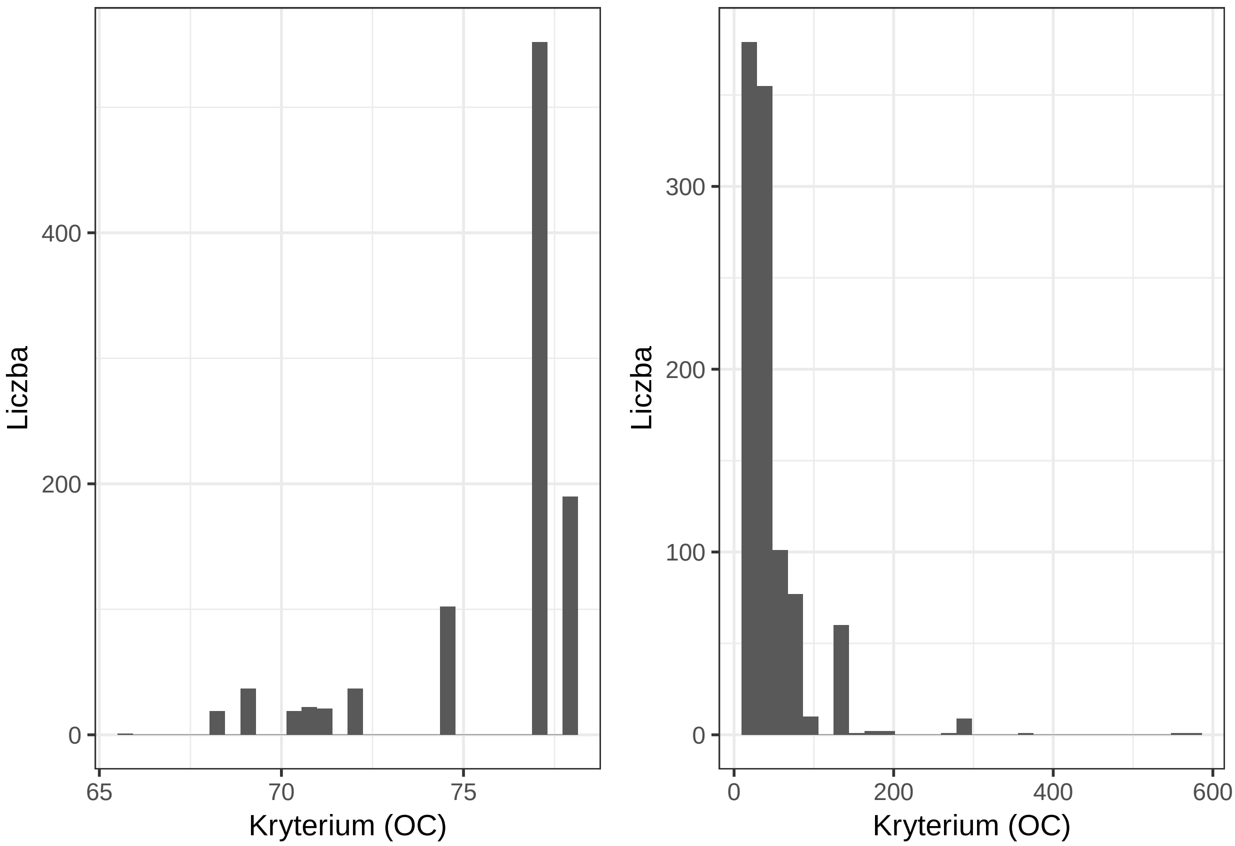


SPX - S&P500 Index, DAX - Deutscher Aktienindex, ES, EHC, GD, DEM - equity line of the median strategy resulted from respectively Exhaustive Search, Extended Hill Climbing, Grid Method and Differential Evolution Method. Prices of the both assets have been normalized in order to have initial value equal to 1000. The equity line has been calculated for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

The strategy components could hedge each other in order to reduce the portfolio risk and obtain more smooth equity line (stable profits). Both strategies followed two opposite approaches in trading on two similar assets. The strategy contained the contrarian part, operating on *SPX* and the momentum one trading on *DAX*. The Exhaustive Search reached the global maxima, but the total calculation time was equal to 609.37 minutes.

The empirical distributions of the reached criterion and computation time of *1000* independent *EHC* and respectively *DEM* executions are presented on Figure 2 and Figure 3. Most of the independent procedures for both methods returned the same strategy with the second best optimization criterion. No selected solution had significantly worse performance and considerable number of them reached the highest criterion as well. Therefore, the results proved both the high efficiency and stability of the methods (Table 3). The median of the Extended Hill Climbing procedure computation time is equal to *30.97* seconds, when the Exhaustive Search took more than *10* hours. The computation time improvement is indisputable. Due to the dynamic stopping rule the execution time was highly varied across the sample. Some runs lasted around *11* seconds, when the others took around *9* minutes. Nevertheless, the observed low level of stability did not affect the time advantage over the Exhaustive Search, because all procedures lasted incomparably less.

Figure 2: The histograms of the reached optimization criterion and the execution time of EHC for SPXDAX - in-sample



OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD). The optimization criterion have been calculated from the sample of 1000 independent algorithm executions. The strategies have been working on the daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

Table 3: The summary of the reached optimization criterion and the execution time of methods for SPX-DAX - in-sample

	ES		EHC		GM		DEM	
	OC	Time [sec]	OC	Time [sec]	OC	Time [sec]	OC	Time [sec]
Minimum	77.79	36562.17	65.58	11.87	77.16	128.04	71.93	13.11
1st Quantile	77.79	36562.17	74.39	13.93	77.16	128.04	77.16	24.84
Median	77.79	36562.17	77.16	30.97	77.16	128.04	77.16	31.15
Mean	77.79	36562.17	75.94	43.1	77.16	128.04	77.34	42.73
3rd Quantile	77.79	36562.17	77.16	65.32	77.16	128.04	77.79	61.5
Max	77.79	36562.17	77.79	569.39	77.16	128.04	77.79	141.08
Standard deviation	0.000	0.00	2.61	48.77	0.00	0.00	0.36	24.06

OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD). The equity lines have been calculated for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

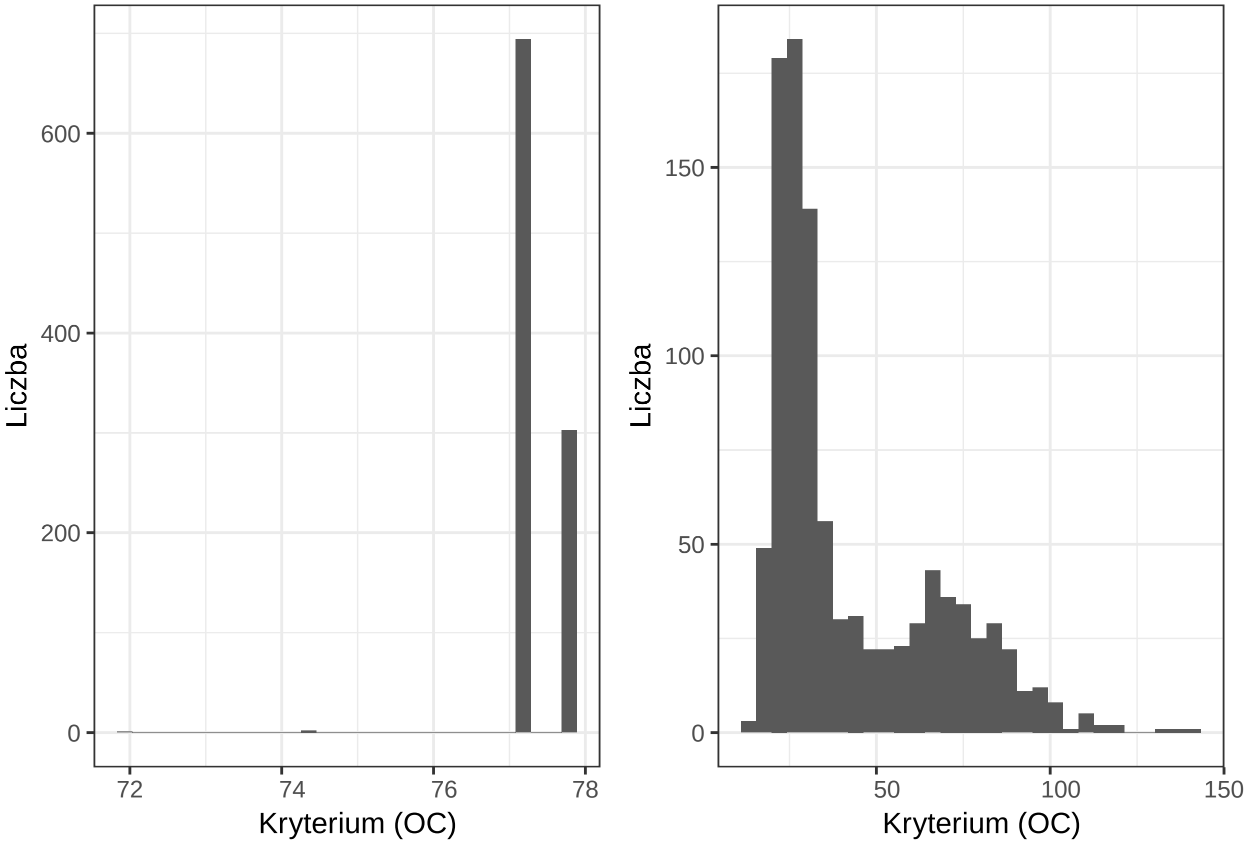
The Grid Method resulted in the second best strategy, exactly the same as median strategy from *the Extended Hill Climbing*. The calculation time was longer than for the previous method. On the other hand, the execution lasted *128.04* seconds, what was still much less than for *the Exhaustive Search*. The method had some advantages over the previous, machine learning method as well. The optimization criterion was similar, but there was no uncertainity about neither results nor time, while the *EHC* and *DEM* results was random.

Most of the *DEM* executions selected exactly the same strategy, as two previous methods. The median execution time for *DEM*, equaled to *31.15* seconds and time required to execute procedure had lower standard deviation than for *EHC*. The *Differential Evolution method* gave strategies similar to the optimal ones but in relatively short and stable time.

4.2.1. Out-of-sample results

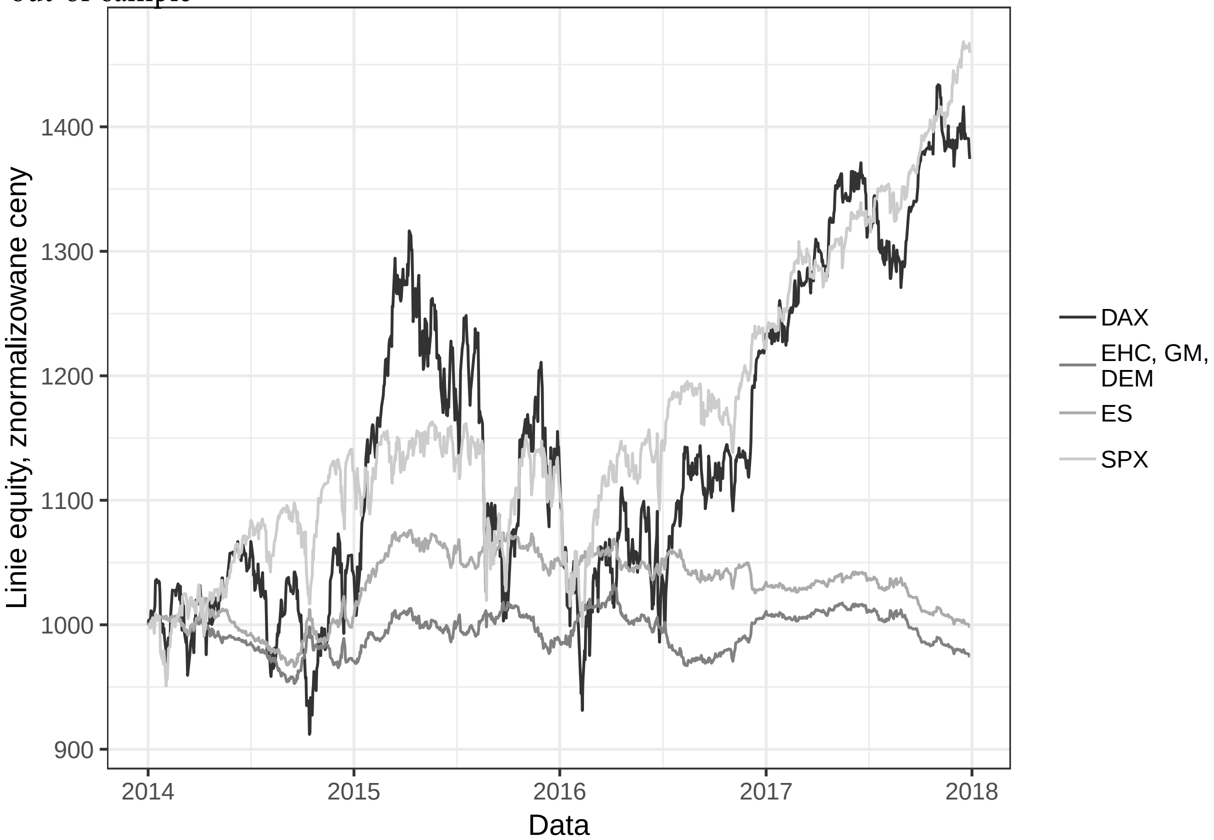
As expected, the out-of-sample strategy performance was worse than in the in-sample period. The strategies obtained by the Exhaustive Search and all considered machine learning methods were ineffective in the out-of-sample period and resulted in the return close to zero at the end of a time horizon.

Figure 3: The histograms of the reached optimization criterion and the execution time of DEM for SPXDAX - in-sample



OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD). The optimization criterion have been calculated for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

Figure 4: The equity lines of the strategies selected by the all methods for SPXDAX - out-of-sample



ES, EHC, GD, DEM - equity line of the median strategy resulted from respectively Exhaustive Search, Extended Hill Climbing, Grid Method and Differential Evolution Method. Prices of the both assets have been normalized in order to have initial value equal to 1000. The equity line has been calculated for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 2014 to the end of 2017 has been simulated, with assumption of fee equal to 0.25% of the position value.

4.3. Apple Inc. (AAPL) and Microsoft Corp. stock (MSFT)

The strategy was optimized for stocks of high-tech companies *Apple Inc.* and *Microsoft Corp.* by different methods. The dynamic growth of prices was a great trading opportunity, therefore the strategies were able to generate high profit in both periods. Similarly to the previous pair, the high correlation between prices was observed both in the in-sample and in the out-of-sample periods, what gave posibility to design strategies with hedging elements and obtain results associated with a lower risk. All statistics and graphs referring to that case were denoted by *AAPL MSFT*.

4.3.1. In-sample methods efficiency

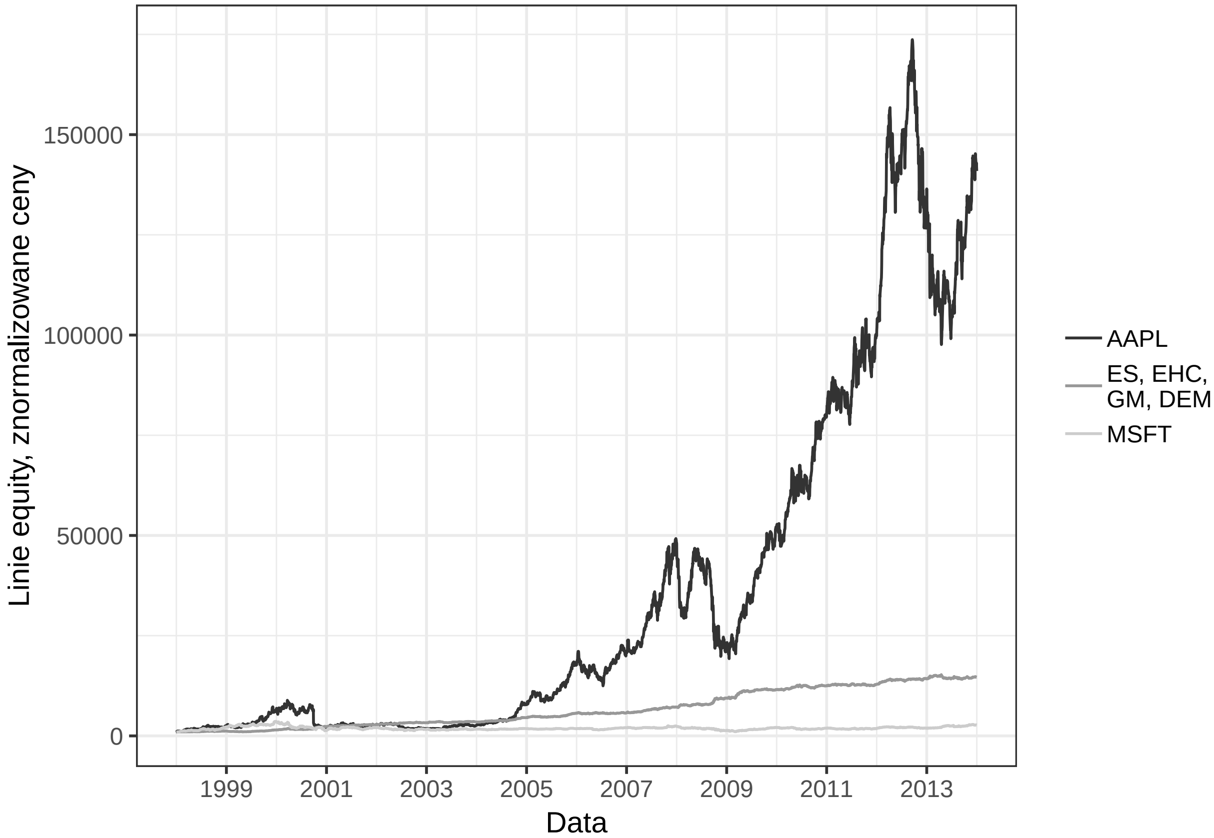
%ARC - annualized rate of return (%), %ASD - annualized standard deviation (%), %MDD- maximum drawdown of capital (%), IR- information ratio calculated as %ARC / %ASD, OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD), k1, k2, k1.2, k2.2 - strategy parameters, width of the moving averages' windows. The statistics of the equity line have been calculated for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

Table 4: The median strategies parameters and statistics resulted from all methods AAPLMSFT

	In-sample				Out-of-sample			
	ES	EHC	GM	DEM	ES	EHC	GM	DEM
k1	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
k2	60.00	60.00	60.00	60.00	60.00	60.00	60.00	60.00
k1.2	75.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00
k2.2	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00
%ARC	17.79	17.79	17.79	17.79	1.37	1.37	1.37	1.37
%ASD	11.13	11.13	11.13	11.13	5.68	5.68	5.68	5.68
IR	1.60	1.60	1.60	1.60	0.24	0.24	0.24	0.24
%MDD	7.71	7.71	7.71	7.71	9.54	9.54	9.54	9.54
OC	368.83	368.83	368.83	368.83	3.49	3.49	3.49	3.49

All the considered methods selected exactly the same strategy. Simple moving averages crossover approach was highly effective due to the enormously high growth of the Apple stock. That strategy had a large return in the in-sample period, *17.79%* annually, and low risk measures as well. The annualized standard deviation of returns was equal to *11.13%*when the maximum drawdown was lower than *8%*. The return generated by the strategy was high but incomparably lower than percentage growth of *AAPL*. Nevertheless, buy-and-hold strategy following the specified rebalancing rule with leverage at level *40%* generated a return of *13.59%* with annualized standard deviation equal to *9.24%* and the maximum drawdown of *9.71%*. Therefore, the selected strategy is both highly profitable and associated with relatively low risk as well. The Exhaustive Search lasted 547.02 minutes in that case.

Figure 5: The equity line of the strategy selected by all methods for AAPLMSFT - in-sample



AAPL - Apple Inc. stock, MSFT - Microsoft Corp. stock, ES, EHC, GD, DEM - equity line of the median strategy resulted from respectively Exhaustive Search, Extended Hill Climbing, Grid Method and Differential Evolution Method. Prices of the both assets have been normalized in order to have initial value equal to 1000. The equity line has been calculated for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

The Extended Hill Climbing method returned exactly the same results as the Exhaustive Search, but in significantly lower time. There were a number of similarities between the method performance in the current and the previous case. The Extended Hill Climbing had the high stability of results, but uncertain computation time. Despite the low time stability, the method proved to be a far more efficient than the Exhaustive Search. The median of execution time was equal to *18.18* seconds (Table 5).

Figure 6: The histograms of the reached optimization criterion and the execution time of EHC for AAPLMSFT - in-sample

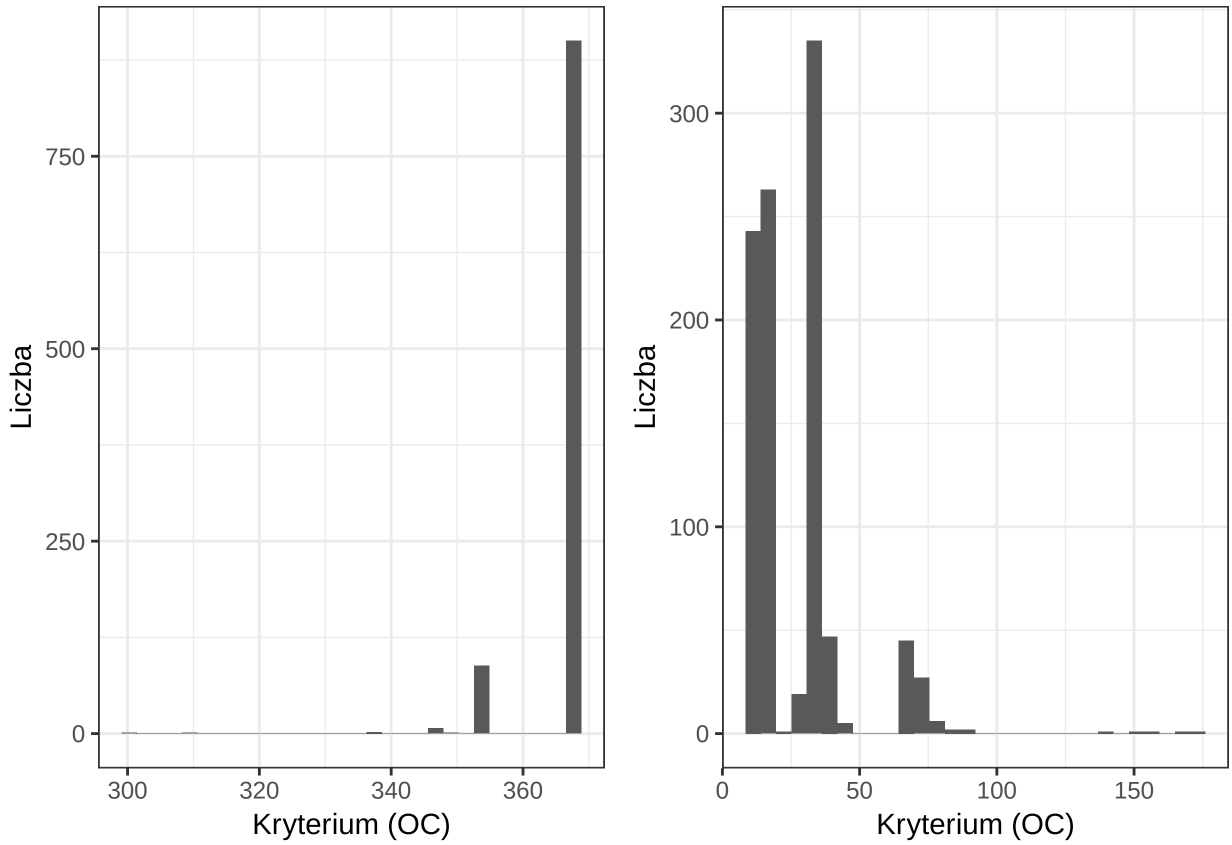


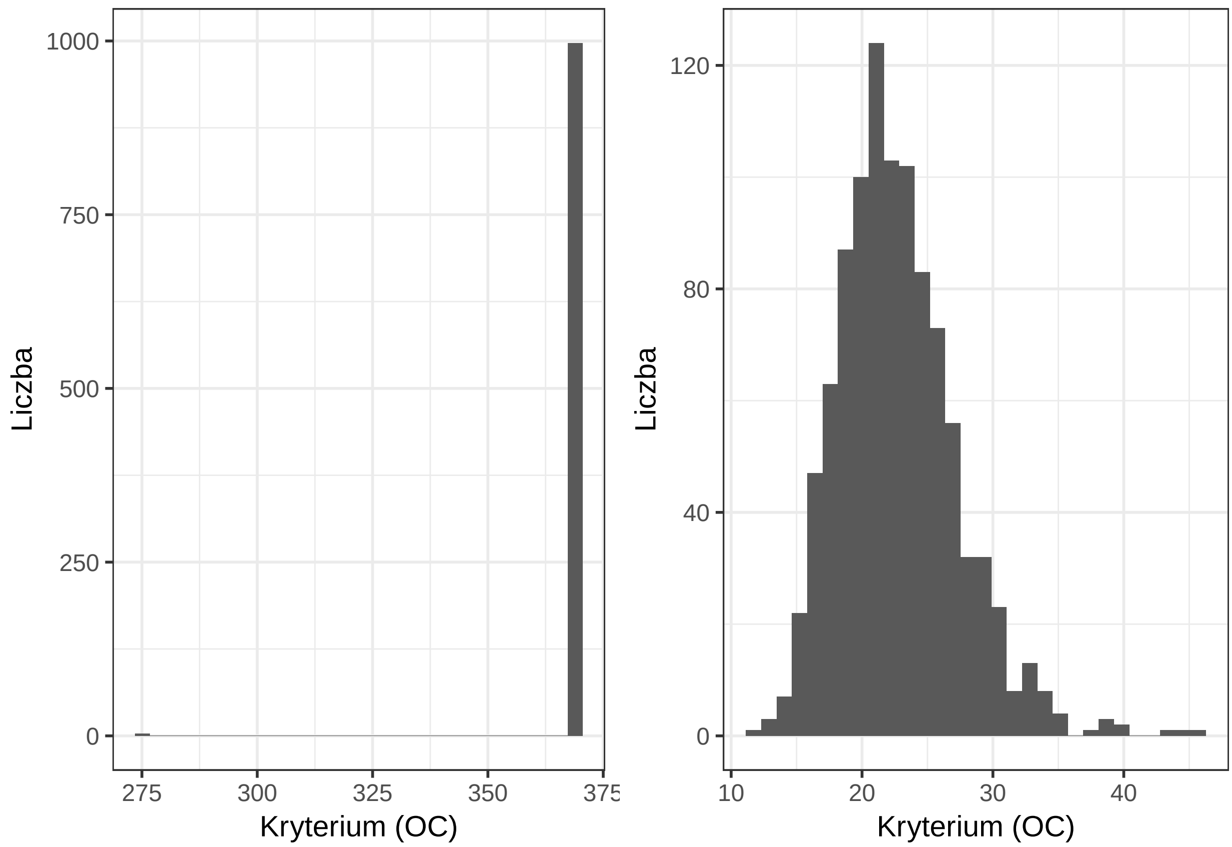
Table 5: The summary of the reached optimization criterion and the execution time of methods for AAPLMSFT - in-sample

	ES		EHC		GM		DEM	
	OC	Time [sec]	OC	Time [sec]	OC	Time [sec]	OC	Time [sec]
Minimum	368.83	32821.18	301.34	11.96	368.83	150.66	274.97	11.82
1st Quantile	368.83	32821.18	368.83	14	368.83	150.66	368.83	19.67
Median	368.83	32821.18	368.83	18.18	368.83	150.66	368.83	22.19
Mean	368.83	32821.18	367.16	27.4	368.83	150.66	368.55	22.71
3rd Quantile	368.83	32821.18	368.83	32.97	368.83	150.66	368.83	25.27
Max	368.83	32821.18	368.83	174.06	368.83	150.66	368.83	45.8
Standard deviation	0.00	0.00	5.51	18.71	0.00	0.00	5.14	4.52

OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD). The optimization criterion have been calculated from the sample of 1000 independent algorithm executions. The strategies have been working on the daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

The Grid Method resulted in exactly the same strategy as both previous methods. The computation time was equal to 150.66 seconds.

Figure 7: The histograms of the reached optimization criterion and the execution time of DEM for AAPLMSFT - in-sample



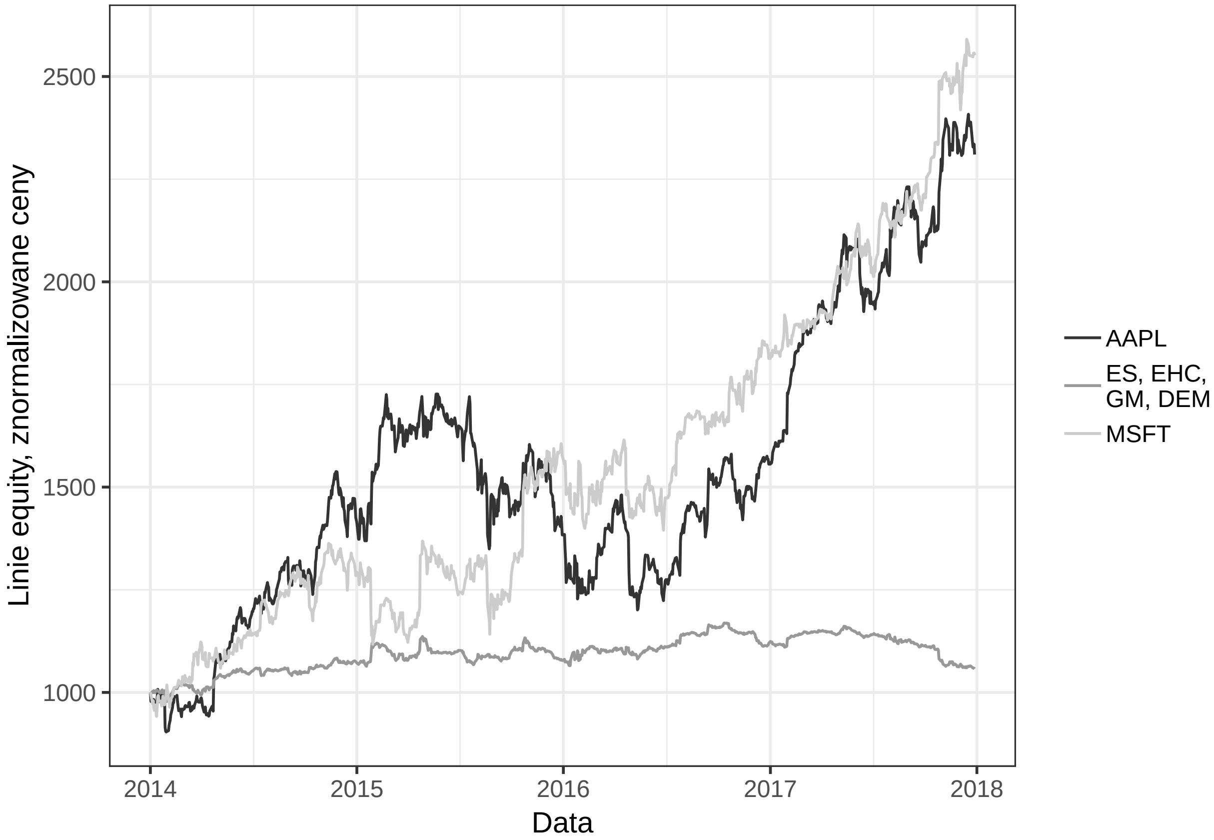
OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD). The optimization criterion have been calculated from the sample of 1000 independent algorithm executions. The strategies have been working on the daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

The Differential Evolution reached the global maxima in almost every attempt. Moreover, the median of the execution time was higher (*22.19*) than for *the Extended Hill Climbing*, but the execution time had a lower standard deviation. Therefore, the differential evolution proved to be efficient and stable method in that case. The median strategy was the optimal one, the same as for all other considered methods.

4.3.2. The out-of-sample results

All considered machine learning methods selected the same strategy as the exhaustive search procedure. That strategy was optimal on the in-sample period in terms of optimization criterion, resulted in annualized returns at the level of *17.79%* and relatively low both annualized standard deviation (*ASD*) and maximum drawdown (*MDD*). Performance of the selected strategy on the out-of-sample period was substantially worse than on the in-sample period. Strategy had more than *10* times lower *ARC*, similar *MDD* and a little lower *ASD* (Table 4). Big difference between performance on the consecutive periods suggest high level of strategy overfitting, however strategy generated positive profits on the out-of-sample period.

Figure 8: The equity lines of the strategies selected by the all methods for AAPLMSFT - out-of-sample



AAPL - Apple Inc. stock, MSFT - Microsoft Corp. stock, ES, EHC, GD, DEM - equity line of the median strategy resulted from respectively Exhaustive Search, Extended Hill Climbing, Grid Method and Differential Evolution Method. Prices of the both assets have been normalized in order to have initial value equal to 1000. The equity line has been calculated for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 2014 to the end of 2017 has been simulated, with assumption of fee equal to 0.25% of the position value.

4.4. High Grade Copper Futures (HG.F) and Crude Oil Brent Futures (CB.F)

The last considered pair of assets was composed of the two commodities futures contracts. The problem of finding the optimal strategy was harder, than for the previous ones. The difference between commodities behavior in both periods and a weaker statistical relationship between them were the main reasons of the difficulties. The statistics and graphs from that case were always denoted by *HGFCBF*.

Table 6: The median strategies parameters and statistics resulted from all methods HGFCBF

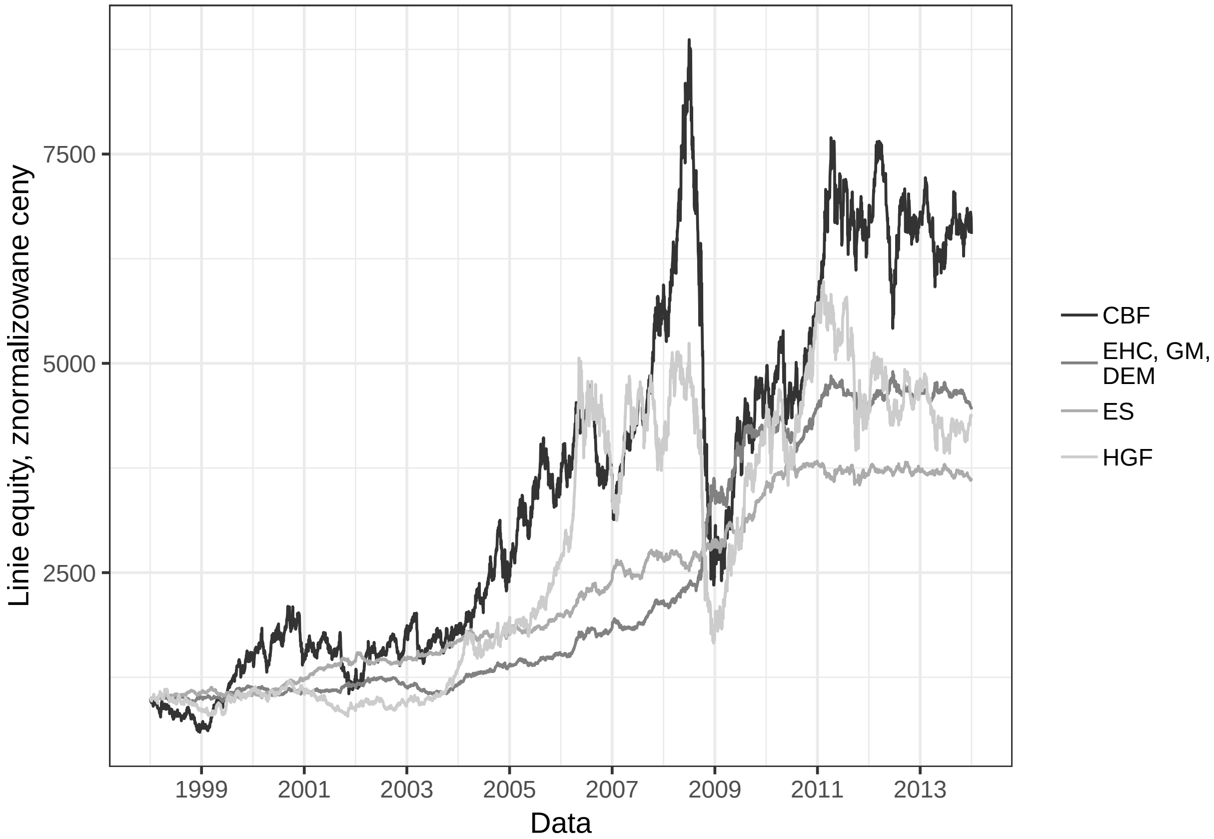
	In-sample				Out-of-sample			
	ES	EHC	GM	DEM	ES	EHC	GM	DEM
k1	60.00	60.00	60.00	60.00	60.00	60.00	60.00	60.00
k2	75.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00
k1.2	50.00	30.00	30.00	30.00	50.00	30.00	30.00	30.00
k2.2	25.00	95.00	95.00	95.00	25.00	95.00	95.00	95.00
%ARC	8.18	9.53	9.53	9.53	-1.59	6.60	6.60	7.38
%ASD	8.16	9.83	9.83	9.83	7.09	8.17	8.17	8.07
IR	1.00	0.97	0.97	0.97	-0.22	0.81	0.81	0.91
%MDD	7.51	9.52	9.52	9.52	15.86	12.16	12.16	12.16
OC	109.11	97.04	97.04	97.04	-2.24	43.81	43.81	55.49

OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD). The empirical statistics have been calculated from the sample of 1000 independent algorithm executions. The strategies have been working on the daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

4.4.1. In-sample methods efficiency

The Exhaustive Search selected the strategy with an annualized return equal to *8.18%*, when the all machine learning methods selected median strategy with returns at level of *9.53*. Nevertheless, that strategy was optimal in terms of optimization criterion, depending on the returns and risk measures as well. The Exhaustive Search process took 703.226 minutes.

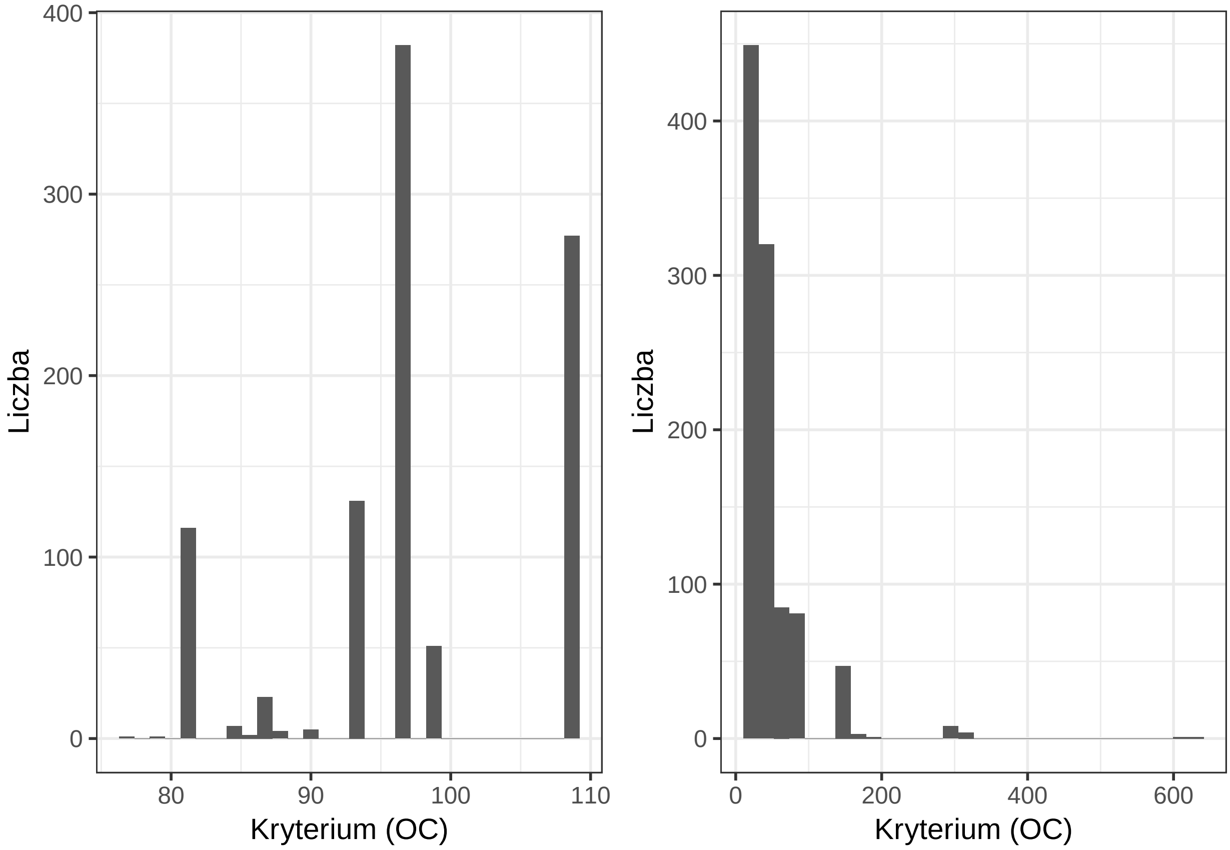
Figure 9: The equity lines of the strategy selected by all methods for HGFCBF - in-sample



HGF - High Grade Copper Futures, CBF - Crude Oil Brent Futures, ES, EHC, GD, DEM - equity line of the median strategy resulted from respectively Exhaustive Search, Extended Hill Climbing, Grid Method and Differential Evolution Method. Prices of the both assets have been normalized in order to have initial value equal to 1000. The equity line has been calculated for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

The conclusions from computing *1000* procedures of *EHC* were the same as for the previous cases (Table 7). The reached optimization criterion was rather stable, in contrast to an unstable computation time. Once again, all learning procedures took a far less time than the Exhaustive Search (median of execution time was equalled *33.07* seconds).

Figure 10: The histograms of the reached optimization criterion and the execution time of EHC for HGFCBF - in-sample



OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD). The optimization criterion have been calculated from the sample of 1000 independent algorithm executions. The strategies have been working on the daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

Table 7: The summary of the reached optimization criterion and the execution time of methods for HGFCBF - in-sample

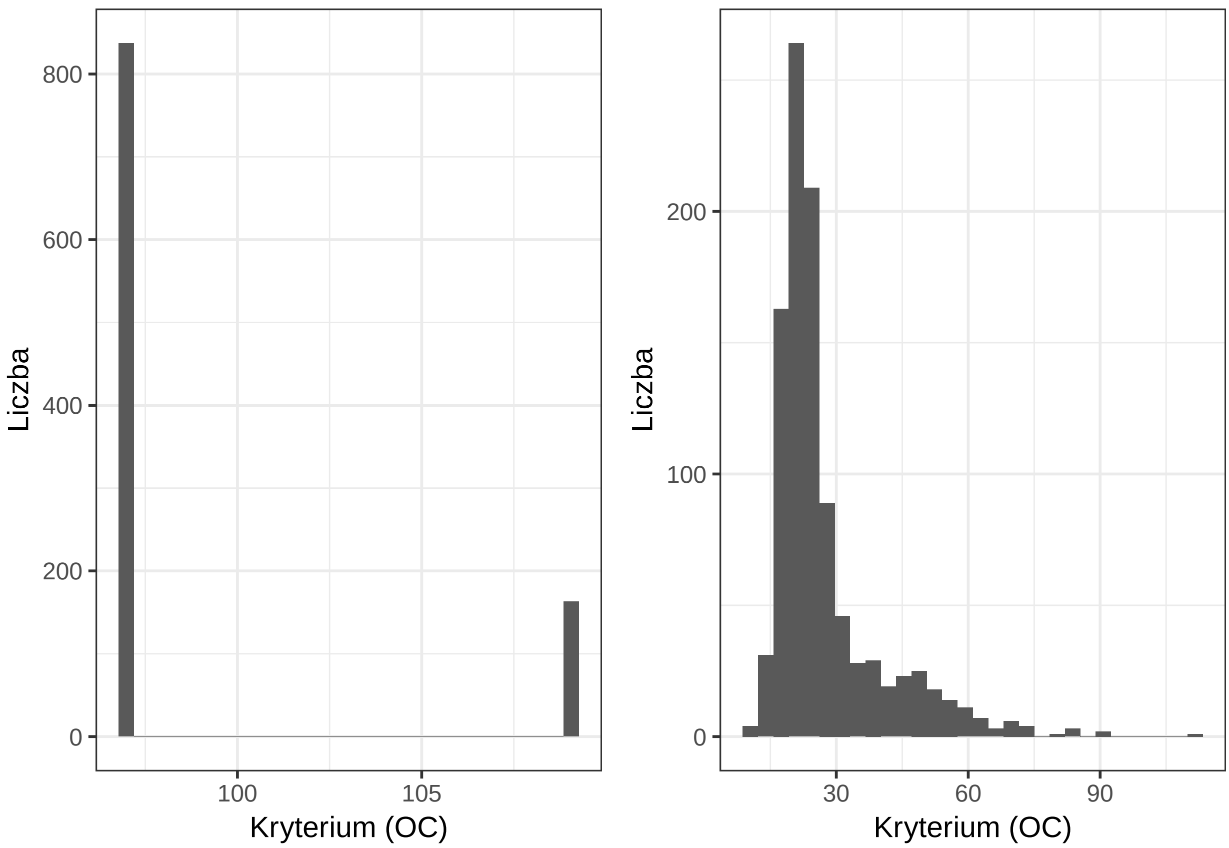
	ES		EHC		GM		DEM	
	OC	Time [sec]	OC	Time [sec]	OC	Time [sec]	OC	Time [sec]
Minimum	109.11	42193.57	77.29	12.29	97.04	113.75	97.04	9.76
1st Quantile	109.11	42193.57	93.62	14.73	97.04	113.75	97.04	19.92
Median	109.11	42193.57	97.04	33.09	97.04	113.75	97.04	23.08
Mean	109.11	42193.57	97.82	42.69	97.04	113.75	99.01	27.34
3rd Quantile	109.11	42193.57	109.11	36.85	97.04	113.75	97.04	29.15
Max	109.11	42193.57	109.11	622.17	97.04	113.75	109.11	110.93
Standard deviation	0.00	0.00	8.59	51.28	0.00	0.00	4.46	12.5

OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD). The optimization criterion have been calculated from the sample of 1000 independent algorithm executions. The strategies have been working on the daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

The *GM* method returned the median strategy of *EHC* and its computation time lasted 113.752 seconds. The conclusions were consistent with those discussed in previous cases. The method obtained good results in the fixed time when the *Extended Hill Climbing* optimization gave similar results in shorter, but more random time. The Grid Method resulted in a strategy near to optimal with a reasonable time of execution.

The median of *DEM* criteria was the same as for *the Extended Hill Climbing*. The computation time of the *Differential Evolution* had the higher stability than *EHC* and the lowest median value among all methods (*23.08* seconds).

Figure 11: The histograms of the reached optimization criterion and the execution time of DEM for HGFCBF - in-sample



OC - optimization criterion calculated as 100 * (%ARC * |%ARC|) / (%ASD * %MDD). The optimization criterion have been calculated from the sample of 1000 independent algorithm executions. The strategies have been working on the daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

4.4.2. The out-of-sample results

All the considered machine learning methods selected finally the same strategy, slightly worse than the optimal one in the in-sample period but significantly better in the out-of-sample period. The annualized returns were about *6.60%* with reasonable standard deviation and maximum drawdown. It is worth noticing, that the strategy with the highest optimization criterion was omitted by all machine learning methods, probably because of the low stability of criterion around that point. The parameters vector was probably surrounded by the low-value ones, and therefore machine learning methods could not find it. In consequence, the method selected the point from the more stable neighbourhood, what resulted in better performance in the out-of-sample period, what showed a potential of machine learning in reducing overfitting risk.

Figure 12: The equity lines of the strategies selected by the different methods for HGFCBF - out-of-sample

SPX - S&P500 Index, DAX - Deutscher Aktienindex, ES, EHC, GD, DEM - equity line of the median strategy resulted from respectively Exhaustive Search, Extended Hill Climbing, Grid Method and Differential Evolution Method. Prices of the both assets have been normalized in order to have initial value equal to 1000. The equity line has been calculated for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 2014 to the end of 2017 has been simulated, with assumption of fee equal to 0.25% of the position value.

4.5. The comparison of tested methods with the Exhaustive Search

Throughout the paper, three machine learning optimization methods, adjusted to the problem specificity were discussed. The performance of each method was tested by solving three problems of selection of trading strategy parameters on period from the beginning od 1998 to the end of 2017. The machine learning algorithms solved the problem in significantly shorter time than the Exhaustive Search procedure with no significant difference in results quality.

As noted before, the machine learning methods gave results similar to the optimal ones obtained by the Exhaustive Search procedure. The critical difference was in the computation time. Checking all possible parameters required a plenty of time. It lasted a few hours, whereas the machine learning methods produced the comparable results in a fraction of a minute. The advantage in time eciency would be critical for complex problems, for instance considering a larger parameter space. The relative time difference was significant- for instance *DEM* took over *1650* times less time than the full exhaustive procedure in *SPXDAX* and *AAPLMSFT* cases. Assuming the same proportion, the *DEM* optimization, requiring less than one hour could replace the *ES* lasting two months. The results obtained in the paper suggested that machine learning methods introduced before could be an effective replacement for the Exhaustive Search, reducing the computation time without affecting the quality of results (Table 8 and Table 9).

Table 8: Mean and median optimization criterion reached by the different methods, refered to the ES method in percent - in-sample

	ES	Grid	EHC median	DEM median	EHC mean	DEM mean
SPXDAX	100	99.19	99.19	99.19	97.62	99.42
AAPLMSFT	100	100.00	100.00	100.00	99.55	99.92
HGFCBF	100	88.94	88.94	88.94	89.65	90.74

SPXDAX - case of trading on S&P500 Index and Deutscher Aktienindex, AAPLMSFT - case of trading on Apple Inc. and Microsoft Corp. stocks, HGFCBF - case of trading on High-Grade Copper and Crude Oil futures. ES - the Exhaustive Search, EHC - the Extended Hill Climbing, DEM - the Differential Evolution. The simulations has been performed for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

Table 9: Mean and median computation time of the methods, refered to the ES method in percent

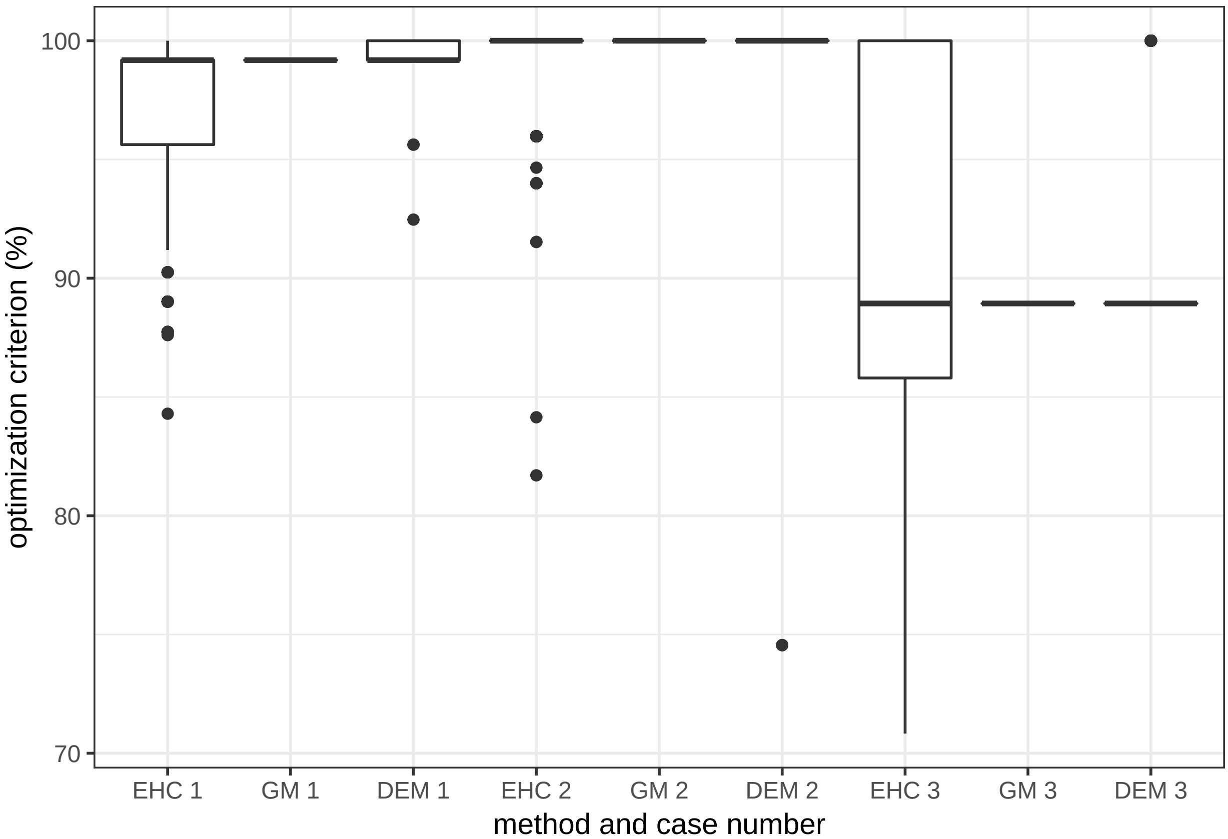
	ES	Grid	EHC median	DEM median	EHC mean	DEM mean
SPXDAX	100	0.35	0.08	0.09	0.12	0.12
AAPLMSFT	100	0.46	0.06	0.07	0.08	0.07
HGFCBF	100	0.27	0.08	0.05	0.10	0.06

SPXDAX - case of trading on S&P500 Index and Deutscher Aktienindex, AAPLMSFT - case of trading on Apple Inc. and Microsoft Corp. stocks, HGFCBF - case of trading on High-Grade Copper and Crude Oil futures. ES - the Exhaustive Search, EHC - the Extended Hill Climbing, DEM - the Differential Evolution. The simulations has been performed for the strategy working on daily frequency, investing 20% of capital in position on each asset with rebalancing every 5 trading days. Trading from the beginning of 1998 to the end of 2013 has been simulated, with assumption of fee equal to 0.25% of the position value.

The first box plot (Figure 13) presents the optimization criterion across the samples. There are almost no significant differences between the results of tested methods. The second box plot (Figure 14) presents the computation time across the samples. The *GM* had substantially higher median time, compared to the other machine learning methods, but without uncertainty.

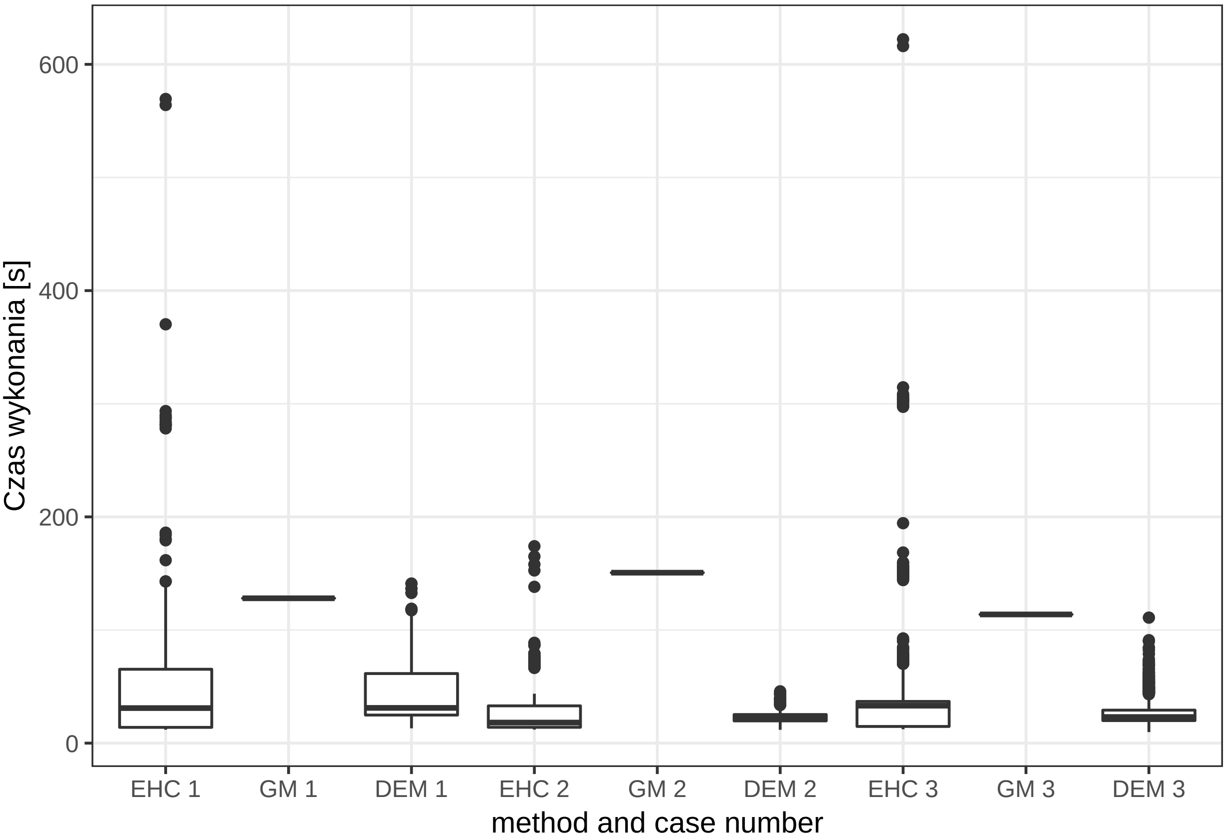
The time required for *DEM* execution was relatively low and stable, especially compared with *EHC*, which had lower time stability, what was illustrated on Figure 14 by the box size and plenty of outliers.

Figure 13: The boxplot of the optimization criterion of strategies selected by the machine learning methods, as percentage of the global maxima found by the Exhaustive Search



The samples were denoted by the algorithm acronym and the number of trading case, so *1, 2* and *3* stands for respectively *SPXDAX*, *AAPLMSFT* and *HGFCBF*. The box plots presents the empirical distribution quartiles and highlight the outliers. Half of the observations are inside the corresponding box, when the line inside marks the median. The observation was considered as an outlier and marked by a circle if the distance from both first and third quartile (from the nearest side of the box) was higher than *1.5* interquartile r ange. The range of observations, without outliers was marked by the whiskers. That type of box plot was often called *the Turkey Box Plot*. It was worth to notice, that the box plots of *Grid Method* results were just a line because the results of that method were deterministic.

Figure 14: The boxplot of machine learnings methods computation time empirical distribution



The samples were denoted by the algorithm acronym and the number of trading case, so *1, 2* and *3* stands for respectively *SPXDAX*, *AAPLMSFT* and *HGFCBF*. The box plots presents the empirical distribution quartiles and highlight the outliers. A half of the observations are inside the corresponding box, when the line inside marks the median. The observation was considered as an outlier and marked by a circle if the distance from both first and third quartile (from the nearest side of the box) was higher than *1.5* interquartile range. The range of observations, without outliers was marked by the whiskers. That type of box plot was often called *the Turkey Box Plot*. It was worth to notice, that the box plots of *Grid Method* results were just a line because the results of that method were deterministic.

5. Conclusions

Three machine learning methods (EHC, GM and DEM) were implemented and tested on simple moving averages crossover strategy optimization problem. Machine learning methods were a heuristic searches, based on simple algorithms, commonly used for similar problems. The methods were adjusted to the considered problem specificity, such as discreteness of parameters or low regularity of the solution space.

Machine learning methods were compared based on the value of optimization criterion, including annualized rate of return from strategy and two risk measures - the annualized standard deviation and the maximum drawdown. All statistics were calculated for the simulated trading on the period from the beginning of 1998 to the end of 2013. The optimization criterion calculated for the strategies and the computation time, required to proceed the whole search process, were compared with the Exhaustive Search. The considered strategies traded on the specified pairs of assets and were tested separately on *SPX* and *DAX* indexes futures, *AAPL* and *MSFT* stocks and finally on the pair composed of two commodity futures - *HG.F* and *CB.F*.

The strategies were compared, in terms of the optimization criterion, based on the annualized returns and including the risk metrics, such as the annualized standard deviation of returns and the maximum drawdown of the equity line. Applying such approach in the optimization process led to selection of more stable strategies. Using maximum drawdown component eliminated the strategies generating all profits in one short period of time. That approach significantly reduced the risk of overfitting, caused by adjustment strategy to few past extreme market situations.

The first method, called *Extended Hill Climbing* was composed of the independ local search walks, starting in the randomly drawn points with specified stopping rule, based on the level of optimization criterion improvement in the previous steps. That method generated a stable results, what means that the strategies returned by the different program executions should be similar to each other. The method produced results comparable to the optimal one in relatively short time, but the stability of the execution time was low. On average, method was quick and efficient, but time of the whole process was varied. Therefore, usually method returned the satisfying solution, but the execution time in different iterations significantly differ.

The second implemented ML method was purely deterministic algorithm, called *Grid Method*. The main idea of the search was to use denser and denser subgrids, centered at the points with high optimization criterion value. The method returned the strategy parameters, with optimization criterion similar to the optimal one with time a few times longer than two other ML methods, but still a few times shorter than full exhaustive procedure. The big advantage of that method is the stable computation time and results, which came from its deterministic nature. This property of the optimization procedure could be appreciated especially for usage in more complex, automatic systems.

The last method, called *the Differential Evolution*, was in fact one of the most popular heuristic algorithm to solve irregular continuously parameterized problems, adjusted to the specificity of integer parameter spaces. The adjustment was based on the transformation of the discrete solution space into the continuous one, in a way saving the space specificity. Additionally, the stopping time feature was added to improve computation time in cases, when further search steps would be probably useless. That method generated very stable results and had a low execution time with a low variance.

The performance of strategies in the in-sample period was better than in the out-of-sample. Although the main goal was to introduce and compare optimization methods, it is worth to point out the difference between in-sample and out-of-sample strategies accuracy. The strategies optimized by different methods in the in-sample periods bear losses in the out-of-sample period for two out of three cases (*SPXDAX* and *AAPLMSFT*). The unsatisfactory results during the second period led us to the conclusion, that the selected strategies were not supposed to generate profit in the future trading. The considered models had relatively few parameters, but it was enough to produce an overfitted strategy, too well-adjusted to the training data and in consequence ineffective on the test set.

Slightly different situation was in case of out-of-sample results for commodity futures trading *(HG.F CB.F)*. In that case, all tested machine learning methods omitted the strategy with the higher optimization criterion in the in-sample period, probably because of the weak performance of neighbouring similar strategies. In consequence, the strategy selected by all methods (in fact strategy with median optimization criterion across the sample) performed well in the out-of-sample period, generating profits, while the one with the highest optimization criterion was bearing losses. It seems to confirm basic intuition - the model avoided overfitting to the training dataset, what caused worse performance there, but also gave a chance to get better results in the future. Selected simple moving averages crossover strategies were generally not profitable on the price time series from outside the training set, but there was a significant premise, machine learning methods developed in this paper, could be used to optimize trading systems, based on another logic and significantly improve its computation time. The optimization time is crucial, because the shorter time, the faster the results of the optimization are available for a supervisor or the wider space of parameters and more sophisticated systems can be fitted in an efficient way.

To sum up, the presented results seems to be consistent with the main hypothesis. The machine learning methods required much less time than the Exhaustive Search and produced similar results in the considered cases. In consequence, the main hypothesis was not rejected. The machine learning methods reached only slightly worse in-sample optimization criterion but in the significantly lower execution time. The additional research question, that the machine learning methods leads to lower overfitting risk could not be answered, based on the results presented in this paper. In two scenarios, the machine learning methods selected very similar strategy to the optimal one. Nevertheless, the methods selected worse strategies in the in-sample period in the last case, the final strategy generated profit in the out-of-sample period, while the one obtained by the ES resulted in the loss of the invested capital. The property of the overfitting reduction was observed only in one case, so it cannot lead to certain conclusions.

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