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Innovation and endogenous growth over business cycle with frictional labor markets

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Abstract

This paper proposes a microfounded model featuring frictional labor markets that generates procyclical R&D expenditures as a result of optimizing behavior by heterogeneous monopolistically competitive firms. This allows to show that business cycle fluctuations affect the aggregate endogenous growth rate of the economy. Consequently, transitory shocks leave lasting level effects. This mechanism is responsible for economically significant hysteresis effects that increase the welfare cost of business cycles by two orders of magnitude relative to the exogenous growth model. I show that this has serious policy implications and creates ample space for policy intervention. I find that several static and countercyclical subsidy schemes are welfare improving.

Keywords:

business cycles, firm dynamics, search and matching, innovation, endogenous growth

JEL:

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1 Introduction

Recent economic literature has started to pay significant attention to the links between firm-level heterogeneity and dynamics, and macroeconomic outcomes. This paper presents a model of heterogeneous, monopolistically competitive establishments who endogenously choose the intensity of research and development. The model features also endogenous entry and exit, and incorporates search and matching frictions in the labor market. The paper brings together several strands of literature on business cycles and growth and carries important policy implications on industrial policy over the business cycle.

The two main mechanisms that generate volatile and procyclical R&D expenditures are increased willingness of incumbents to invest in R&D in good times, as well as procyclical entry rates. This translates to the endogenous growth rate of the economy to be also procyclical, and gives rise to hysteresis effects, as in response to transitory shocks the balanced growth path permanently shifts. As a consequence, welfare effects of business cycle are much higher than for the exogenous growth models, as consumption is not only volatile but also subject to level effects.

The results from the model indicate that around 7% of a temporary shock is translated to the permanent level shift in the balanced growth path. This has significant welfare consequences, as the cost of business cycle fluctuations is of two orders of magnitude higher than in the exogenous growth variant of the model. The presence of large welfare effects and the ability to potentially affect the growth rates and volatility of the economy through appropriate industrial policy creates space for policy intervention via static and countercyclical subsidies. Of those the most positive welfare effect is achieved through countercyclical subsidies to incumbents' operating cost, as it prevents excessive exits and encourages more R&D spending. Moreover, I find that accounting for frictions in the labor market results in welfare gains from static subsidies to incumbents' operating cost, a result at odds with the endogenous growth models that abstract from this friction.

The paper is based on the neo-Schumpeterian endogenous growth paradigm, pioneered by Grossman and Helpman (1991) and Aghion and Howitt (1992), and it is grounded in works in the second generation of this literature by Dinopoulos and Thompson (1998), Peretto (1998), Young (1998) and Howitt (1999) by focusing on operations of individual establishments. Those works show that while in the aggregate the population of R&D scientists may rise, the important statistic is the R&D labor per establishment, which remains constant under mild assumptions regarding the market structure. Indeed, Laincz and Peretto (2006) show that since 1964 the number of full-time equivalent R&D employees per establishment has been almost constant and does not trend over time.

The paper also belongs to the growing body of the literature concerned with firm level heterogeneity and dynamics. Bartelsman and Doms (2000) provide a review of the early literature focused on documenting productivity differences and growth across firms and linking those phenomena to aggregate outcomes. Foster et al. (2001) emphasize the role of cyclical entry for aggregate productivity growth. The role of entry and exit channels for macroeconomic dynamics has been recognized and studied by Hopenhayn (1992), Devereux et al. (1996), Campbell (1998), Jaimovich and Floetotto (2008), Bilbiie et al. (2012), Chatterjee and Cooper (2014) and Lee and Mukoyama (2015), although none of those works incorporate the full set of firm dynamics considered here. Recently, the experience of the Great Recession and the subsequent slow recovery motivated researchers to investigate possible links between cyclical changes in firm and establishment dynamics and other macroeconomic variables, a phenomenon dubbed missing generation of firms. Siemer (2014) finds that tight financial constraints during the Great Recession were responsible for both low employment growth and firm entry rates. Messer et al. (2016) show using regional US data that low entry rates in the 2007-2009 period contributed significantly to low employment and labor productivity growth. Clementi and Palazzo (2016) study full firm dynamics over business cycle, although their analysis focuses on the firm-level investment in physical capital, rather than innovation, which is the core mechanism of this paper.

Following the seminal contribution by Klette and Kortum (2004), there is a frugal literature on the relationship between innovation and firm dynamics. This paper is close in spirit to work by Acemoglu et al. (2013) who study the consequences of subsidy schemes for R&D expenditures and growth, and related works include Akcigit and Kerr (2010) and Acemoglu and Cao (2015). The common assumption in those papers is that the incumbent firms innovate on their own products in a neo-Schumpeterian quality-ladder setup. I contribute to that literature by considering similar underlying mechanisms in a stochastic setup, and I am able to analyze the effect of countercyclical subsidies.

The model also features frictional labor market, subject to the search and matching friction in the tradition of Diamond (1982) and Mortensen and Pissarides (1994). I follow an approach proposed by Gertler and Trigari (2009) that assumes nonlinear vacancy posting costs and is remarkably successful in replicating the labor market dynamics. Therefore this paper is also related to the literature focusing on the impact of labor market frictions, such as the presence and level of firing costs, on reallocation and productivity growth. In a seminal paper Hopenhayn and Rogerson (1993) assess the impact of firing costs on reallocation and productivity, and find non-negligible negative effects. Similar conclusions are reached by the works reviewed and systematized in Hopenhayn (2014). Bassanini et al. (2009) find that firing costs tend to reduce growth in industries where firing costs are more likely to be binding. Davis and Haltiwanger (2014) argue that a recent decrease in labor market fluidity in the United States negatively impacted job reallocation rates and harmed productivity growth. Da-Rocha et al. (2016) find much bigger static and dynamic losses in aggregate total factor productivity when the presence of firing costs alters the establishment-level productivity distribution. Mukoyama and Osotimehin (2017) analyzes the effects of firing taxes in a model with rich firm dynamics, although the model does not incorporate aggregate shocks. Although the analysis of the impact of firing costs is not possible in the setup chosen for this paper, the fluidity of the labor market is affected by the level of hiring costs, and some parallel conclusions can be drawn.

The remainder of the paper is organized as follows. The next section describes the model, deriving the problem of incumbents and potential entrants, and describing the details of labor market frictions. The third section discusses the data sources and parameter values, including those that are estimated. This section also documents stochastic properties of the model economy in comparison to the data. The fourth section is devoted to a discussion of policy implications, providing an estimate of the welfare cost of business cycles for the US economy and a comparison of the effects of several subsidy schemes. The last section concludes.

2 Model

The model is mostly inspired by a closed economy version of the model sketched in Endogenous Firm Productivity section of Melitz and Redding (2014), as well as by Acemoglu et al. (2013). It features monopolistically competitive, single-establishment firms, heterogeneous with respect to their products' quality, that endogenously decide on their expenditures on R&D in order to raise their products' quality.

The model is based on the previous work by Bielecki (2017), although it features two major changes. First, I introduce physical capital as another factor of production. Second, instead of modeling the labor market as Walrasian, I assume that labor market is subject to the search and matching friction as in Gertler and Trigari (2009). Following Christiano et al. (2011) I assume that the hiring and wage bargaining processes are managed by employment agencies who then supply firms with labor services at a common price.

2.1 Households

There is a unit mass of representative households. Each representative household consists of a large family of workers, giving rise to within-household insurance, as in Merz (1995) and Andolfatto (1996). Any individual worker may be within a given time period employed and receiving wage income or unemployed and receiving unemployment benefits[1]. As in Acemoglu et al. (2013), there are two types of workers: skilled of mass s and unskilled of mass $1 - s$. Regardless of the labor market status or skill category each individual enjoys the same level of consumption.

The representative household aims to maximize expected lifetime utility of its members:

$$U_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta} \tag{1}$$

where, β is the discount factor c_t is the per capita consumption and θ is the inverse of the elasticity of intertemporal substitution. The household is subject to the following budget constraint:

$$c_t + k_{t+1} = (1 + r_t - dp)\, k_t + s\left[w_t^s n_t^s + b_t^s\left(1 - n_t^s\right)\right] + (1-s)\left[w_t^u n_t^u + b_t^u\left(1 - n_t^u\right)\right] + t_t$$

where k_t is the per capita stock of physical capital which yields interest rate r_t, dp is the rate of capital depreciation, w_t^s and w_t^u are real wage rates for skilled and unskilled labor, respectively, n_t^s and n_t^u are the shares of skilled and unskilled workers that are currently

[1]Note that I abstract from the real-world possibility that an individual is not active on the labor market.

employed, b_t^s and b_t^u denote unemployment benefits, and t_t denotes any lump sup net transfers that households receive, including all profits.

The first order conditions of the households result in the following Euler equation:

$$c_t^{-\theta} = \mathrm{E}_t \left[\beta c_{t+1}^{-\theta} \left(1 + r_{t+1} - dp\right)\right] \tag{2}$$

As all firms in the economy are ultimately owned by households, I assume that their managers discount future profit streams consistent with the stochastic discounting kernel of the households:

$$\Lambda_{t,t+1} = \mathrm{E}_t \left[\left(\frac{c_{t+1}}{c_t}\right)^{-\theta}\right] \tag{3}$$

2.2 Final goods producer

The final goods producing sector is modeled as a single representative perfectly competitive firm that transforms a continuum of mass $M_t \in (0,1)$ of intermediate good varieties[2] into final goods using the CES aggregator:

$$Y_t = \left[\int_0^{M_t} y_t\left(i\right)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i\right]^{\frac{\sigma}{\sigma-1}}$$

where $y_t\left(i\right)$ denotes the output of i-th variety and $\sigma \in (1,\infty)$ is the elasticity of substitution between any two varieties. The standard solution of the cost minimization problem yields the price index of the final good as a function of the varieties' prices $P_t\left(i\right)$:

$$P_t = \left[\int_0^{M_t} P_t\left(i\right)^{1-\sigma} \mathrm{d}i\right]^{\frac{1}{1-\sigma}}$$

as well as the Hicksian demand function for the i-th variety:

$$y_t\left(i\right) = Y_t p_t\left(i\right)^{-\sigma} \tag{4}$$

where $p_t\left(i\right) = P_t(i)\,/P_t$ is the variety's price relative to the price index.

2.3 Intermediate goods producers

The intermediate goods producing sector is modeled as a single industry sector populated by monopolistically competitive continuum of mass M_t of active single-establishment firms[3], each producing a distinct variety. To produce an establishment needs to incur fixed costs

[2]The condition that the mass of intermediate goods varieties is bounded between 0 and 1 is supported by assuming that each individual possesses an idea for a product, but only a subset of those individuals are entrepreneurs and only a fraction of possible goods is actively produced.

[3]Klette and Kortum (2004) in a relatively similar setting show that the behavior of multi-product firms can be summarized as if they consisted of a number of independent product lines.

f_t, representing expenditures on management and other non-production activities. The production function of an establishment is of a Cobb-Douglas functional form:

$$y_t(i) = Z_t k_t^p(i)^\alpha \left[q_t(i)\, n_t^p(i)\right]^{1-\alpha}$$

where Z_t is the stochastic aggregate productivity parameter, $k_t^p(i)$ and $n_t^p(i)$ denote, respectively, the employment of capital services and unskilled labor, $q_t(i)$ is the quality level of i-th variety at time period t, and α is the elasticity of output with respect to capital.

The solution of the cost minimization problem yields the following expression for the marginal cost, depending on the idiosyncratic quality level of an establishment:

$$mc_t^p(i) = \frac{1}{Z_t}\left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1-\alpha}\right)^{1-\alpha}$$

where $\tilde{w}_t^u$ denotes the unskilled wage paid to the employment agency.

It is straightforward to show that the optimal pricing strategy given flexible prices and the demand for an individual variety given by Equation 4 follows the standard constant mark-up pricing formula:

$$p_t(i) = \frac{\sigma}{\sigma-1} mc_t(i)$$

Following Melitz (2003), I assume that the distribution of idiosyncratic quality levels at time t is described by some probability density function $\mu_t(q)$ with support on a subset of $(0,\infty)$. It is convenient to define an aggregate quality index Q_t such that the aggregate state of the intermediate goods producing sector can be summarized as if it was populated by mass M_t of establishments all with quality level Q_t. The index is given by the following formula:

$$Q_t^{1-\alpha} = \left[\int_0^\infty \left(q^{1-\alpha}\right)^{\sigma-1} \mu_t(q)\, \mathrm{d}q\right]^{\frac{1}{\sigma-1}}$$

As the aggregate quality level grows over time, the idiosyncratic quality levels of individual establishments are best expressed in relative terms. Therefore, I construct the following measure of relative quality:

$$\phi_t(i) \equiv \left(q_t(i)/Q_t\right)^{(1-\alpha)(\sigma-1)}$$

The aggregate final goods output can be then expressed as:

$$Y_t = M_t^{\frac{1}{\sigma-1}} Z_t \left(K_t^p\right)^\alpha \left(Q_t N_t^p\right)^{1-\alpha} \tag{5}$$

where K_t^p and N_t^p denote, respectively, aggregate capital stock and employment in the production sector and the dependence of output on M_t reflects the love-for-variety phenomenon.

2.4 Incumbents

I assume that each incumbent establishment can direct resources to R&D activities in attempt to improve their varieties' quality. The success probability function is taken from

Pakes and McGuire (1994) and Ericson and Pakes (1995):

$$\chi_t(i) = \frac{a x_t(i)}{1 + a x_t(i)}$$

where $\chi_t(i)$ denotes the probability of making a quality improvement and a is a parameter that describes the efficacy of R&D input $x_t(i)$ in generating improvements. R&D input requires a combination of skilled labor and capital:

$$x_t(i) = \frac{k_t^x(i)^\alpha \left[Q_t n_t^x(i)\right]^{1-\alpha}}{Q_t \phi_t(i)}$$

where $k_t^x(i)$ and $n_t^x(i)$ denote, respectively, the employment of capital services and skilled labor.

The presence of aggregate and relative quality levels in the expression lends itself to an intuitive interpretation. Aggregate quality level in the numerator multiplies with R&D laborers as they have access to a pool of common knowledge. However, over time it is harder to come up with new ideas unless more resources are committed to R&D activities, which is captured by aggregate quality level in the denominator. Finally, the presence of relative quality level in the denominator represents the catch-up and headwind effects, depending on establishments' position in the quality distribution.

In the absence of the last channel, establishments with higher quality product would have comparative advantage over their competitors and the success probability would be an increasing function of establishment size. This however is at odds with the empirically observed regularity known as Gibrat's law, according to which firm growth rates and firm size are uncorrelated. Empirical evidence on the evolution of firms shows that either the Gibrat's law cannot be rejected for large enough firms (see e.g. Hall (1987)) or that the larger firms have slower rates of growth (see e.g. Evans (1987), Dunne et al. (1989) or Rossi-Hansberg and Wright (2007)).

The solution of the cost minimization problem results in the following expression for the marginal cost in the R&D sector:

$$mc_t^x(i) = Q_t^\alpha \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^s}{1-\alpha}\right)^{1-\alpha} \phi_t(i) \equiv \bar{mc}_t^x \phi_t(i)$$

where $\tilde{w}_t^s$ denotes the skilled wage paid to the employment agency, and $\bar{mc}_t^x$ is the skilled marginal cost component common to all establishments.

I also assume that the managerial activities require the same combination of physical capital and skilled labor as R&D activities. Therefore, the fixed cost can be expressed as a product of the common skilled marginal cost and a constant f. Accordingly, the real profit can be expressed as the following function, which is affine in terms of $\phi_t(i)$:

$$\pi_t(i) = Y_t \left[\left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \right) \phi_t(i) - \omega_t f \right] \tag{6}$$

and where $\omega_t \equiv \bar{mc}_t^x / Y_t$ is the ratio of common skilled marginal cost and aggregate output.

The dynamic problem of the incumbents can be cast in the recursive form. Since all establishments with the same relative quality levels will make identical decisions, I drop the subscript i. Additionally, for establishments with low enough ϕ_t the expected stream of future profits turns negative and they decide to exit at the end of the current period.

The value of an establishment with relative quality level ϕ_t is given by the following expression:

$$V_t(\phi_t) = \max_{\chi_t \in [0,1)} \left\{ \pi_t(\phi_t, \chi_t) + \max\left\{0, \mathrm{E}_t\left[\beta \Lambda_{t,t+1}(1-\delta_t) V_{t+1}(\phi_{t+1} | \phi_t, \chi_t)\right]\right\}\right\}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor consistent with the households' valuation of current and future marginal utility from consumption (Equation 3), δ_t denotes endogenous establishment death shock probability, which will be described in detail later, and the relative quality of a variety in the next period is subject to the following lottery[4]:

$$\phi_{t+1} = \begin{cases} \iota \phi_t / \eta_t & \text{with probability } \chi_t \\ \phi_t / \eta_t & \text{with probability } 1 - \chi_t \end{cases}$$

where ι denotes the size of the innovative step and η_t is the rate of growth of the aggregate quality index (raised to a certain power), taken as given by the individual establishments:

$$\eta_t \equiv \left(\frac{Q_{t+1}}{Q_t} \right)^{(1-\alpha)(\sigma-1)}$$

Since the aggregate quality index is trending upwards over time, it is useful to consider the following stationarization. Define $v_t(\phi_t) \equiv V_t(\phi_t) / Y_t$ to be the ratio of the value function and current aggregate output. For the problem rewritten in relative terms the level of aggregate quality becomes irrelevant, and its rate of growth is a function of the current state only.

Moreover, for large enough ϕ_t the probability that an establishment will want to exit in the foreseeable future is very small, and the $\max\{0, \cdot\}$ operator can be disregarded. As the real profit function is affine in ϕ_t and the value function is a weighted sum of present and future profit streams, it is also affine in ϕ_t. Therefore, I impose the affine functional form on $v_t(\phi_t) \equiv A_t + B_t \phi_t$:

$$A_t + B_t \phi_t = \max_{\chi_t \in [0,1)} \left\{ \begin{array}{c} \left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a} \frac{\chi_t}{1-\chi_t} \right) \phi_t - \omega_t f \\ + \mathrm{E}_t \left[\beta \Lambda_{t,t+1} (1-\delta_t) \left(\frac{Y_{t+1}}{Y_t} \right) (A_{t+1} + B_{t+1} \phi_{t+1}) \right] \end{array} \right\} \tag{7}$$

[4]The underlying absolute quality levels evolve according to the lottery:

$$q_{t+1} = \begin{cases} \iota^{1/[(1-\alpha)(\sigma-1)]} q_t & \text{with probability } \chi_t \\ q_t & \text{with probability } 1 - \chi_t \end{cases}$$

The solution to the incumbents' problem must then satisfy the following first order and envelope conditions:

$$0 = -\frac{\omega_t}{a}\frac{1}{\left(1-\chi_t\right)^2} + \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(B_{t+1}\frac{(\iota-1)}{\eta_t}\right)\right] \tag{8}$$

$$B_t = \left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\right) + \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)B_{t+1}\frac{\chi_t\left(\iota-1\right)+1}{\eta_t}\right] \tag{9}$$

Note that the relative quality level does not impact the optimal innovative success probability χ_t, as long as ϕ_t is high enough, in line with Gibrat's law.

Obviously, one needs to specify the decisions of establishments with lower levels of ϕ_t. For sufficiently low levels of ϕ_t the establishment exits and thus does not engage in R&D activities at all. Therefore, its value function is given by:

$$A_t + B_t\phi_t = \frac{1}{\sigma M_t}\phi_t - \omega_t f \tag{10}$$

It now remains to specify what happens in the intermediate range of relative quality levels. For the sake of tractability I opt to represent the true value function with its piecewise linear approximation, namely, I extend the functions given by Equations 7 and 10 until they intersect for the relative quality level ϕ_t^*, given implicitly by the following condition:

$$\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\phi_t^* = \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(A_{t+1} + B_{t+1}\frac{\chi_t\left(\iota-1\right)+1}{\eta_t}\phi_t^*\right)\right] \tag{11}$$

All establishments with relative quality levels no higher than ϕ_t^* exit, and all continuators choose the same level of χ_t. By assuming that the quality is distributed according to the Pareto distribution with power parameter equal to one[5], I am able to provide a closed form expression for the mass of establishment exits:

$$M_t^x = M_t\left(1-\alpha_{t-1}\right)\left(1 - \frac{\phi_{t-1}^*}{\phi_t^*\eta_{t-1}}\right) \tag{12}$$

2.5 Entrants

The mass of prospective entrants is assumed to be a priori unbounded. Similar to active establishments, they can engage in R&D activities. In contrast to incumbents, the successful outcome of their innovation effort is not an improvement in an existing product, but rather creating a new one, which may or may not replace an existing variety.

To attempt entry, prospective entrants hire physical capital and skilled labor just as incumbents do, including also the necessity to cover fixed costs. Successful entrants begin their production in the next period. The stationarized expected value of entry is given by:

$$v_t^e = \max_{\chi_t^e\in[0,1)}\left\{-\omega_t\left(f^e + \frac{1}{a^e}\frac{\chi_t^e}{1-\chi_t^e}\right) + \chi_t^e\mathrm{E}_t\left[\Lambda_{t,t+1}\left(\frac{Y_{t+1}}{Y_t}\right)v_{t+1}\left(\phi_{t+1}^e\right)\right]\right\} \tag{13}$$

[5] This assumption is ubiquitous in the firm size distribution literature. For empirical support see e.g. Axtell (2001).

where χ_t^e is the probability of entering the market next period, a^e is a parameter that describes the efficacy of R&D input and ϕ_{t+1}^e denotes the relative quality draw upon entry. Since entrants tend to perform more radical innovations than incumbents, as emphasized by e. g. Acemoglu and Cao (2015) and Garcia-Macia et al. (2016), I assume that they draw from the incumbents' distribution of quality levels, upscaled by a factor which precludes the need to resort to limit pricing.

The first order condition of the entrants' problem can be expressed as:

$$\frac{\omega_t}{a^e} \frac{1}{\left(1-\chi_t^e\right)^2} = \mathrm{E}_t \left[\Lambda_{t,t+1} \left(\frac{Y_{t+1}}{Y_t}\right) v_{t+1}\left(\phi_{t+1}^e\right)\right] \tag{14}$$

Additionally, since the mass of prospective entrants is unbounded, the following free entry condition holds in every period:

$$v_t^e = 0 \tag{15}$$

Hence, if the mass of successful entrants is denoted by M_t^e and the chosen success probability is χ_t^e, then the mass of agents attempting entry has to equal M_t^e/χ_t^e.

The modeled entry process is undirected, but allows for the possibility that a successful entrant leapfrogs over an incumbent. To capture that feature, I assume that the space of all possible varieties occupies a unit interval, while active establishments occupy its subset M_t. This represents the notion that every individual in an economy possesses a potential business idea but only a fraction of them become entrepreneurs and their varieties are produced. Since the mass of households equals unity, it is natural to assume that the mass of potential ideas also equals unity.

Upon successful entry the new establishment randomly draws its "location" from the unit interval and a fraction of them replaces active establishments. To ensure no limit pricing in equilibrium, I assume that entrants enjoy a relative quality advantage over the incumbents. Therefore, if an entrant replaces an incumbent that has innovated successfully, the product line will be characterized by $\left[\iota\sigma/\left(\sigma-1\right)\right]^{1/[(1-\alpha)(\sigma-1)]}$ times higher quality level than in the previous period, and in case of replacing an incumbent that has not succeeded in innovating the product line's quality increases by a factor of $\left[\sigma/\left(\sigma-1\right)\right]^{1/[(1-\alpha)(\sigma-1)]}$. Accordingly, the expected relative quality level of entrants is equal to:

$$E_t\left[\phi_{t+1}^e\right] = \frac{\sigma}{\sigma-1}$$

I can now specify the process for the endogenous probability of an incumbent receiving an exit shock. There are three conditions under which an active establishment exits, and I assume that at the end of each period the events follow a specific order. First, the incumbents with relative quality level below ϕ_t^* exit "voluntarily" as their varieties become obsolete. Second, incumbents receive exogenous exit shocks. Finally, a fraction of incumbents are leapfrogged by entrants and thus creatively destroyed. Therefore, the mass of active establishments in the next period is given by:

$$M_{t+1} = M_t - M_t^x - \delta^{exo}\left(M_t - M_t^x\right) + \left[1 - \left(1-\delta^{exo}\right)\left(M_t - M_t^x\right)\right] M_t^e$$

where δ^{exo} is the exogenous exit shock probability and the mass of successful entrants M_t^e is multiplied by the probability that an entrant draws an "unoccupied" location. As by definition creative destruction replaces an incumbent with an entrant, it does not directly affect the mass of active establishments. The expression for active establishment mass can be also written as:

$$M_{t+1} = M_t - M_t^x - \delta_t \left(M_t - M_t^x\right) + M_t^e \tag{16}$$

Then by comparing the two formulations one gets the following expression for endogenous exit shock probability:

$$\delta_t = 1 - \left(1 - \delta^{exo}\right)\left(1 - M_t^e\right) \tag{17}$$

Intuitively, the probability of not receiving an exit shock is a product of the probabilities of not receiving an exogenous shock and not being creatively destroyed, as the two are independent from each other.

It is now possible to characterize the process governing the evolution of the aggregate quality index. First, by the law of large numbers, a fraction χ_t of incumbents with relative quality levels above ϕ_t^* manage to improve their varieties, while the incumbents with obsolete varieties exit. Second, incumbents receive death shocks which are uncorrelated with their quality levels and thus leave the distribution unchanged. Finally, entrants draw their quality from the distribution of incumbents' qualities, rescaled upwards. By assuming Pareto distribution of quality levels it is possible to derive the exact closed form expression for the rate of growth of the aggregate quality index:

$$\eta_t = \left(1 - \chi_t + \chi_t \iota\right)\left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma - 1}\right) \tag{18}$$

2.6 Frictional labor markets

I assume that labor markets are subject to the search and matching friction. At the end of each period a constant fraction of workers randomly separates from their previously held job positions and enter the pool of unemployed. The transition from the unemployed to employed state depends on the endogenously determined job finding probability, which is influenced by the intensity of hiring. The assumption of constant separation rate and fluctuating hiring rate is consistent with the US data, as argued by Shimer (2005, 2012), although Petrongolo and Pissarides (2008) point out that it might not be an appropriate assumption for other countries.

I also assume that the unskilled and skilled labor markets are separated, with differing unemployment rates, vacancy rates, and so on. To facilitate exposition, and since both markets operate based on the same principles, I present the workings of the representative labor market, omitting the superscript.

2.6.1 Aggregate labor market dynamics

By excluding the possibility that an agent can be inactive on the labor market, the mass of unemployed workers is given by:

$$u_t = 1 - n_t \tag{19}$$

The mass of new matches m_t is a function of the mass of unemployed and the aggregate mass of vacancies v_t:

$$m_t = \sigma_m u_t^{\psi} v_t^{1-\psi} \tag{20}$$

where the parameter σ_m describes the efficiency of the matching process and ψ is the elasticity of matches with respect to the mass of unemployed.

The job finding probability p_t and job filling probability q_t can be obtained via the following transformation:

$$p_t = m_t / u_t \tag{21}$$

$$q_t = m_t / v_t \tag{22}$$

Following Gertler and Trigari (2009) and Gertler et al. (2008), and in contrast to the standard modeling approach by Mortensen and Pissarides (1994), I assume convex costs with respect to the hiring rate[6]:

$$x_t = \frac{q_t v_t}{n_t} \tag{23}$$

The process for mass of employed workers is given by the following relationship:

$$n_{t+1} = (\rho + x_t)\, n_t \tag{24}$$

where $1 - \rho$ is a constant separation rate.

2.6.2 Employment agencies and workers

Since the problem of the individual establishments is already quite complex and adding idiosyncratic employment and wage levels would make the model intractable, I follow Christiano et al. (2011) in assuming that both hiring and wage bargaining is managed by employment agencies. The agencies then supply labor services to establishments at uniform cost determined on the agencies-establishments side of the labor market, although the wages individual workers receive will differ due to the assumption of staggered real wage contracts.

Each employment agency chooses its desired hiring rate to maximize the value of contracting an extra worker, conditional on the agency-specific wage level:

$$J_t(j) = \max_{x_t(j)} \left\{ \begin{array}{c} \tilde{w}_t - w_t(j) - \frac{\kappa}{2} x_t^2(j) \\ + (\rho + x_t(j)) \, \mathrm{E}_t \left[\beta \Lambda_{t,t+1} J_{t+1}(j) \right] \end{array} \right\}$$

[6] As noticed by Fujita (2004), the standard search and matching model generates counterfactual shape of the impulse response function of vacancies to labor productivity shocks. The setup proposed by Gertler and Trigari (2009) and Gertler et al. (2008) alleviates this issue.

The first order condition of the agency can be expressed in the following two forms:

$$\kappa x_t(j) = \mathrm{E}_t\left[\beta\Lambda_{t,t+1}J_{t+1}(j)\right]$$
$$\kappa x_t(j) = \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left[\tilde{w}_{t+1} - w_{t+1}(j) + \frac{\kappa}{2}x^2_{t+1}(j) + \rho\kappa x_{t+1}(j)\right]\right]$$

and all agencies with the same level of offered wages will choose the same hiring rate.

The workers can be either employed or unemployed, and I denote the values of those states by $\mathcal{E}$ and $\mathcal{U}$, respectively. The value of being employed by j-th agency is given by:

$$\mathcal{E}_t(j) = w_t(j) + \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left[\rho\mathcal{E}_{t+1}(j) + (1-\rho)\mathcal{U}_{t+1}\right]\right]$$

An unemployed worker is a priori uncertain about the wage offer she will receive upon creating a successful match with an agency. By denoting with G the cumulative distribution of wages the expected value of being newly hired is approximated by:

$$\mathcal{E}_t \approx \int \mathcal{E}_t(w_t)\,\mathrm{d}G(w_t)$$

where the approximation is valid up to a first order conditional on wage distribution along the balanced growth path to be degenerate[7]. The value of being unemployed then follows:

$$\mathcal{U}_t = b_t + \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left[p_t\mathcal{E}_{t+1} + (1-p_t)\mathcal{U}_{t+1}\right]\right]$$

Accordingly, the surplus of a worker employed by agency j and the average surplus of newly hired workers equal:

$$H_t(j) = \mathcal{E}_t(j) - \mathcal{U}_t$$
$$H_t = \mathcal{E}_t - \mathcal{U}_t$$

And the individual worker's surplus can be rewritten as:

$$H_t(j) = w_t(j) - b_t + \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left[\rho H_{t+1}(j) - p_t H_{t+1}\right]\right]$$

2.6.3 Staggered wage bargaining

The wages are subject to the Calvo-like staggered wage contract friction at the employment agency level, with the average contract duration of $1/(1-\lambda)$. Therefore, the wage offered by an employment agency is given by:

$$w_t(j) = \begin{cases} w_t(r) & \text{with probability } 1-\lambda \\ w_{t-1}(j)\cdot Q_t/Q_{t-1} & \text{with probability } \lambda \end{cases}$$

where $w_t(r)$ denotes the wage bargained when employment agencies are allowed to renegotiate. I assume that in the case of being unable to renegotiate wages are indexed with

[7] See Gertler and Trigari (2009) for the full argument.

aggregate quality growth. This assumption is necessary for the balanced growth path distribution of wages to collapse to a single point. As a consequence, the average wage will follow the standard Calvo assumption:

$$w_t = \lambda \frac{Q_t}{Q_{t-1}} w_{t-1} + (1-\lambda) w_t(r) \tag{25}$$

An agency that receives a signal to renegotiate in the current period bargains with the marginal worker over the surplus. The bargained contract wage maximizes the following Nash product:

$$w_t(r) = \arg\max H_t(r)^{\psi} J_t(r)^{1-\psi}$$

where I already impose the Hosios (1990) condition that both sides' bargaining power correspond to matching function elasticities. The first order condition for the Nash bargaining problem is given by:

$$\psi \frac{\partial H_t(r)}{\partial w_t(r)} J_t(r) = (1-\psi) \frac{\partial J_t(r)}{\partial w_t(r)} H_t(r)$$

While Gertler and Trigari (2009) consider a case where the above formula gives rise to the horizon effect of the agency, the effect disappears under assumption that the wage bargaining and hiring decisions are simultaneous, i.e. internalizing the first order condition of the employment agency[8]. Then the solution of the Nash bargaining problem is of the conventional surplus sharing form:

$$\psi J_t(r) = (1-\psi) H_t(r)$$

If the wages were renegotiated on the period-by-period basis, then the contract wage would be equal to:

$$w_t^f = \psi \left(\tilde{w}_t + \frac{\kappa}{2} x_t^2 + p_t \kappa x_t \right) + (1-\psi) b_t \tag{26}$$

However, the problem is more involved in the case of staggered contracts. Denote by $W_t(j)$ the expected discounted sum of future wages received over the duration of the relationship with the employment agency:

$$W_t(j) = \Delta_t w_t(j) + (1-\lambda) \mathrm{E}_t \sum_{s=1}^{\infty} (\beta\rho)^s \Lambda_{t,t+s} \Delta_{t+s} w_{t+s}(r)$$

where the first part represents contract that is not renegotiated and the wage is only indexed, while the second part represents future, renegotiated contracts at the same employment agency, and:

$$\Delta_t = \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\rho\lambda)^s \Lambda_{t,t+s} \frac{Q_{t+s}}{Q_t} \tag{27}$$

[8] In any case, the quantitative impact of the horizon effect is negligible.

The surplus of workers at renegotiating agency can be then rewritten as:

$$\begin{aligned} H_t(r) &= w_t(r) + \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\rho H_{t+1}(r)\right] - b_t - \mathrm{E}_t\left[\beta\Lambda_{t,t+1}p_t H_{t+1}\right] \\ &= W_t(r) - \mathrm{E}_t\sum_{s=0}^{\infty}(\beta\rho)^s\Lambda_{t,t+s}\left(b_{t+s} + p_{t+s}H_{t+s+1}\right) \end{aligned}$$

Similarly, the surplus value of employed worker from the point of view of the employment agency can be rewritten as:

$$\begin{aligned} J_t(r) &= \tilde{w}_t + \frac{\kappa}{2}x_t^2(r) + \rho\mathrm{E}_t\left[\beta\Lambda_{t,t+1}J_{t+1}(r)\right] - w_t(r) \\ &= \mathrm{E}_t\sum_{s=0}^{\infty}(\beta\rho)^s\Lambda_{t,t+s}\left(\tilde{w}_{t+s} + \frac{\kappa}{2}x_{t+s}^2(r)\right) - W_t(r) \end{aligned}$$

By substituting the above expressions in the surplus sharing equation one can obtain:

$$\begin{aligned} W_t(r) = {} & \psi\mathrm{E}_t\sum_{s=0}^{\infty}(\beta\rho)^s\Lambda_{t,t+s}\left(\tilde{w}_{t+s} + \frac{\kappa}{2}x_{t+s}^2(r)\right) \\ & + (1-\psi)\,\mathrm{E}_t\sum_{s=0}^{\infty}(\beta\rho)^s\Lambda_{t,t+s}\left(b_{t+s} + p_{t+s}H_{t+s+1}\right) \end{aligned}$$

or, after simplifying, in the following recursive form:

$$\begin{aligned} \Delta_t w_t(r) = {} & \psi\left(\tilde{w}_t + \frac{\kappa}{2}x_t^2(r)\right) + (1-\psi)\left(b_t + p_t\mathrm{E}_t\left[\beta\Lambda_{t,t+1}H_{t+1}\right]\right) \\ & + \rho\lambda\mathrm{E}_t\left[\beta\Lambda_{t,t+1}\Delta_{t+1}w_{t+1}(r)\right] \end{aligned} \tag{28}$$

where the first two terms comprise the target wage w_t^o, which in turn can be expressed in relation to the flexible contract wage:

$$\begin{aligned} w_t^o &= \psi\left(\tilde{w}_t + \frac{\kappa}{2}x_t^2(r)\right) + (1-\psi)\left(b_t + p_t\mathrm{E}_t\left[\beta\Lambda_{t,t+1}H_{t+1}\right]\right) \\ &= w_t^f + \psi\left(\frac{\kappa}{2}\left(x_t^2(r) - x_t^2\right) + p_t\kappa\left(x_t(r) - x_t\right)\right) \\ &\quad + (1-\psi)\,p_t\mathrm{E}_t\left[\beta\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right] \end{aligned} \tag{29}$$

The above equation emphasizes the presence of spillovers of economy-wide wages on the bargaining wage. Intuitively, more intensive hiring by an agency requires also higher bargained wages, which are also upwardly pressured by the future average wage.

Finally, let x_t denote the average hiring rate:

$$x_t = \int_0^1 x_t(j)\,\frac{n_t(j)}{n_t}\mathrm{d}j$$

Then the job creation condition can be used to express x_t as:

$$\begin{aligned} \kappa x_t = {} & \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(\tilde{w}_{t+1} - w_{t+1} + \frac{\kappa}{2}x_{t+1}^2 + \rho\kappa x_{t+1}\right)\right] \\ & + \mathrm{E}_t\left[\begin{array}{c} \beta\Lambda_{t,t+1}\int_0^1\left(\frac{\kappa}{2}x_{t+1}^2(j) + \rho\kappa x_{t+1}(j) - w_{t+1}(j)\right)\frac{n_t(j)}{n_t}\mathrm{d}j \\ -\left(\frac{\kappa}{2}x_{t+1}^2 + \rho\kappa x_{t+1} - w_{t+1}\right) \end{array}\right] \end{aligned} \tag{30}$$

Note that along the balanced growth path the deviations of individual employment agencies' decisions from average disappear and as a first order approximation one can take only the first line of the above equation.

2.7 Market clearing

Factor markets are assumed to clear at each period:

$$N_t^p = (1-s)\, n_t^u \quad \text{and} \quad N_t^s = s n_t^s \tag{31}$$

$$K_t = K_t^p + K_t^s \tag{32}$$

Supply and demand for skilled inputs are equal:

$$\left(\hat{K}_t^s\right)^{\alpha} (N_t^s)^{1-\alpha} = M_t f + (M_t - M_t^x) \left(\frac{1}{a}\frac{\chi_t}{1-\chi_t}\right) + \frac{M_t^e}{\chi_t^e}\left(f^e + \frac{1}{a^e}\frac{\chi_t^e}{1-\chi_t^e}\right) \tag{33}$$

where the three sources of demand are: fixed costs of active establishments, R&D activities of incumbents with non-obsolete varieties and fixed costs and R&D activities of prospective entrants.

Finally, the final goods output is spent on consumption, investment and covering hiring costs:

$$Y_t = C_t + K_{t+1} - (1-dp)\, K_t + \kappa^u \left(x_t^u\right)^2 N_t^p + \kappa^s \left(x_t^s\right)^2 N_t^s \tag{34}$$

3 Data and results

3.1 Data, calibration and estimation

The data used in this paper come from several major sources. The primary source of data on establishment dynamics comes from the US Bureau of Labor Statistics (BLS) Business Employment Dynamics (BDM) database. The BDM, based on the Quarterly Census on Employment and Wages (QCEW) records changes in the employment level of more than 98% of economic entities in the US. Unfortunately, the data series is relatively short, starting as late as of 1992q3. Data on GDP, its components and R&D expenditures are provided by the US Bureau of Economic Analyses (BEA), while data on R&D employment come from the National Science Foundation (NSF). Historical establishment employment data are taken from County Business Patterns (CBP). Data on hours and wages are taken from the Nonfarm Business Sector statistics provided by the BLS. Data on unemployment rate and vacancy rate are also taken from the BLS, although for years 1951-2000 the data on vacancies are based on the composite help-wanted index by Barnichon (2010).

In the BDM, an establishment is defined as an economic unit that produces goods or services, usually at a single physical location, and engages in one, or predominantly one, activity for which a single industrial classification may be applied[9]. Thus an establishment,

[9]This and the following definitions are quoted from the Business Employment Dynamics Technical Note, available at http://www.bls.gov/news.release/cewbd.tn.htm.

as measured by the BLS, corresponds quite closely to the theoretical concept of establishment considered in the model.

Expansions (contractions) are defined as units with positive employment in the third month in both the previous and current quarters, with a net increase (decrease) in employment over this period. Viewed through the lens of the model, expansions are the result of a successful innovation, while contractions are a consequence of being unable to innovate an thus declining relative quality level. Openings are defined as establishments with either positive third month employment for the first time in the current quarter, with no links to the prior quarter, or with positive third month employment in the current quarter following zero employment in the previous quarter. Closings are defined as establishments either with positive third month employment in the previous quarter, with no employment or zero employment reported in the current quarter.

The problem with using these statistics directly is that both openings and closings are an upward biased measure of "true" entry and exit patters, as they are very sensitive to seasonal employment patterns. To correct for this issue, BLS produces data on establishment births and deaths, which are a subset of openings and closings, controlled for re-openings and temporary shutdowns via "waiting" for three quarters for status confirmation. While this correction introduces some discrepancies in the aggregate data, the gains from using data closer to the model objects should significantly outweigh the associated cost.

The parameters that influence the balanced growth path of the economy are calibrated to reflect the long-run averages in the US data and are summarized in Table 1. The values of parameters governing the behavior of the labor markets were taken from previous literature. Differentiated separation rates for unskilled and skilled workers are taken from Cairo and Cajner (2017) and adapted to the quarterly model setup[10]. The adjustment cost parameters were chosen to match the average job finding probability in the US, which Shimer (2005) reports to be equal to 0.45 at monthly frequency and Cairo and Cajner (2017) document that the job finding probabilities differ only slightly among the workers' education groups. As in Shimer (2005) the unemployment benefits are assumed to be equal to 40% of the steady state wage. Following Gertler and Trigari (2009) I set the elasticity of matches to unemployment to 0.5 and impose the Hosios (1990) condition that the bargaining power parameters correspond to matching elasticities. Finally, I set the matching efficiency parameter to match the observed average vacancy to unemployment ratio to 0.54, although Shimer (2005) emphasizes that the value of this parameter is virtually irrelevant as beside influencing the average labor market tightness it has no impact on other variables.

Both the capital share of income and quarterly depreciation rate are set to values ubiquitous in the business cycle literature. The discount factor, which in the calibration process depends on the value of elasticity of intertemporal substitution, is chosen so that the average annual net interest rate is equal to 5%. The share of skilled workers is picked to be in the middle of the plausible range of values proposed by Acemoglu et al. (2013) and corresponds to

[10] Cairo and Cajner (2017) document statistics for workers differentiated by their education level. I treat skilled workers to be analogous to holders of college degree and unskilled to be analogous to high school graduates.

the value used by Bielecki (2017) and adjusted to account for the presence of unemployment in the model.

Finally, the set of parameters governing the establishment dynamics is calibrated to match specific moments reported in Table 2. As I have 6 moments to match with 8 free parameters, I impose a constraint that the R&D efficacy parameter and fixed cost are equal for both incumbents and entrants.

Table 1: Calibrated parameters affecting the steady state

Parameter	Description	Value	Justification
ρ^u	Unskilled retention rate	0.9725^3	Cairo and Cajner (2017)
ρ^s	Skilled retention rate	0.99^3	Cairo and Cajner (2017)
κ^u	Unskilled hiring cost	2	Unskilled job finding probability
κ^s	Skilled hiring cost	15.8	Skilled job finding probability
b^u	Unskilled unemp. benefit	0.14	40% of steady state unskilled wage
b^s	Skilled unemp. benefit	0.41	40% of steady state skilled wage
ψ	Elasticity of matches	0.5	Gertler and Trigari (2009)
σ_m	Matching efficiency	1.7	Average tightness = 0.54
α	Capital share of income	0.3	Standard
dp	Capital depreciation rate	0.025	Standard
β	Discount factor	0.9996	Annual net interest rate of 5%
s	Share of skilled workers	0.1039	Bielecki (2017)
ι	Innovative step size	1.016	Annual pc. GDP growth
δ^{exo}	Exog. exit shock prob.	0.0174	Exit rate
a, a^e	R&D efficiency	7.96	Expansions = contractions
f, f^e	Fixed cost	0.94	Share of R&D in GDP
θ	Inverse of IES	2.3	Share of investment in GDP
σ	Elasticity of substitution	4.9	Share of R&D employment

Table 2: Long-run moments: comparison of model and data

Description	Model	Data	Source
Annual pc. GDP growth	2.07%	2.08%	BEA, 1948q1-2016q2
Exit rate[a]	3.07%	3.07%	BDM, 1992q3-2016q2
Relative share of expanding estabs.	1.01	1.01	BDM, 1992q3-2016q2
Share of R&D in GDP	2.21%	2.23%	BEA, 1948q1-2016q2
Share of investment in GDP	16.91%	17.17%	BEA, 1948q1-2016q2
Share of R&D employment	1.25%	0.98%	NSF & CBP, 1964-2008

[a]Calculated from the data as the average between death and birth rates.

To obtain the values of parameters that do not affect the steady state but govern the cyclical behavior of the model, I employ the estimation procedure. The prior distributions were chosen to be relatively uninformative, and in particular the prior distribution for the renegotiation frequency parameter was set to uniform on the unit interval. Table 9 in the Appendix contains full information on the priors used.

The observable variable used in the estimation is the quarterly growth rate of Real Gross Domestic Product divided by the Labor Force, observed in periods 1948q2-2017q2. An advantage of the model with explicitly modeled long-run growth is that there is no need to detrend the data and valuable information is retained. The model was estimated using standard Bayesian procedures with help of Dynare 4.5 and results were generated using two random walk Metropolis-Hastings chains with 200,000 draws each with an acceptance ratio of 0.23.

Table 3 presents the estimation results. The data were clearly informative about the estimated parameters, as the posterior and prior means differ significantly and the highest posterior density (HPD) intervals are relatively tight. This observation can be also confirmed by comparing the plots of prior and posterior densities displayed in Figure 3.

The most interesting parameter is λ that determines contract renegotiation probability, and its value implies that wage contracts last on average for 5 quarters. This value is slightly higher than assumed by Gertler and Trigari (2009) in their calibrated model, where they consider average durations of 9 and 12 months, and also higher than estimated by Gertler et al. (2008) where contracts last for 3.5 quarters[11]. However, assuming this value of the parameter yields excellent performance in case of labor market variables, which were not observed directly during the estimation procedure.

Table 3: Prior and posterior means of parameters affecting cyclical behavior

Parameter	Description	Prior mean	Post. mean	90% HPD interval
λ	Calvo parameter (wages)	0.5	0.796	$[0.691, 0.909]$
ρ_Z	Autocorr. of TFP process	0.7	0.946	$[0.905, 0.990]$
σ_Z	Std. dev. of TFP shock	0.01	0.012	$[0.011, 0.013]$

3.2 Model performance and impulse response functions

Table 4 presents the comparison of the Hodrick-Prescott filtered moments between the model and data. Data for the variables presented in the upper and middle parts of the table are based on the 1951q1- 2016q4 sample. Output is based on the Gross Domestic Product by BEA, consumption on the sum of Personal Consumption Expenditures on Nondurable Goods and Services, investment on the sum of Personal Consumption Expenditures on Durable Goods and Fixed Private Investment, and R&D expenditures on Gross Domestic Product: Research and Development. Wages are based on Nonfarm Business Sector: Compensation

[11] Note however that Gertler et al. (2008) impose a relatively tight prior on this parameter.

Per Hour by BLS, and hours on Hours of All Persons. Unemployment rate is taken from the BLS, and vacancy rate is taken from JOLTS by BLS and spliced with composite help-wanted index by Barnichon (2010). Data for variables presented in the lower part of the table are based on the 1992q3- 2016q4 sample, covering 99 periods, and come from the BDM. All variables trending with population size were divided by the Civilian Labor Force by BLS, and variables in nominal terms were deflated by the Gross Domestic Product: Implicit Price Deflator by BEA.

The upper section of the table is concerned with output and its components, as well as R&D expenditures. The model fits the data very well for output and its components, and only fails to account for much weaker correlation of R&D expenditures with output.

The middle section of the table focuses on variables pertaining to the operations of the labor market. The model wages are stronger correlated with output and have higher autocorrelation than in the data, and model hours are not as volatile as in the data. However, the model is very successful in matching the cyclical behavior of unemployment, vacancies and tightness, achieving nearly perfect fit. Additionally, Table 5 presents correlations between key labor market variables and confirms that the model is able to replicate the Beveridge curve comovements.

The final section presents the moments related to the establishment dynamics. Although the fit is a bit worse than in the case of previously discussed variables, most of the model moments remain close to their data counterparts, with the exception that the model predicts much smaller volatility of establishment dynamics. The model also predicts that the establishment mass is slightly negatively correlated with output, even though the correlation of net entry with output is almost exactly the same as in the data. A brief look at the impulse response functions in Figure 1 reveals that this result is most likely driven by a small and short-lived decrease in the mass of establishments immediately after the shock hits, and for the subsequent periods the mass of active establishments moves in tandem with output.

To sum up, although the model is not able to match the data perfectly, the fit is more than satisfactory and provides a solid foundation for further analysis.

Table 4: Business cycle moments: comparison of model and data

	Standard deviation		Correlation with Y		Autocorrelation	
Variable	Data	Model	Data	Model	Data	Model
Output	1.58	1.58	1.00	1.00	0.82	0.83
Consumption	0.87	0.80	0.78	0.98	0.82	0.75
Investment	4.54	5.55	0.76	0.99	0.87	0.87
R&D	2.36	2.07	0.32	0.94	0.89	0.92
Wages	0.95	0.82	0.10	0.54	0.68	0.96
Hours	1.36	0.65	0.86	0.93	0.89	0.91
Unemployment	12.76	10.80	-0.77	-0.80	0.89	0.91
Vacancies	13.78	12.80	0.83	0.94	0.91	0.88
Tightness	26.00	22.57	0.82	0.92	0.91	0.92
Establishments	0.62	0.22	0.71	-0.14	0.87	0.92
Expansions	2.84	0.47	0.82	0.65	0.75	0.95
Contractions	2.38	0.42	-0.11	-0.89	0.69	0.91
Net Entry	0.31	0.09	0.31	0.35	0.24	0.51

Table 5: Correlations between labor market variables

Correlation	Data	Model
Unemployment, Vacancies	-0.92	-0.82
Tightness, Unemployment	-0.98	-0.95
Tightness, Vacancies	0.98	0.96

Figure 1 displays the impulse response functions to a 1% productivity shock. An increase in productivity raises output directly, but also induces higher investment which raises the stock of physical capital and more intensive hiring, which reduces unemployment and increases hours worked in the economy. The response of output to the shock is highly persistent, both due to labor market frictions and the endogenous quality component which permanently shifts output upwards. Expenditures on R&D are also procyclical and persistent.

Due to staggered wage contracts average wages respond on impact quite modestly as a large fraction of labor agencies are unable to renegotiate the wages. The impulse response of wages displays a hump-shaped pattern, reaching its peak around 3 years after the shock hits. Increased productivity of labor induces the employment agencies to post vacancies, increasing labor market tightness, which subsequently increases employment and thus hours worked.

Figure 1: Impulse response functions to 1% productivity shock (%)

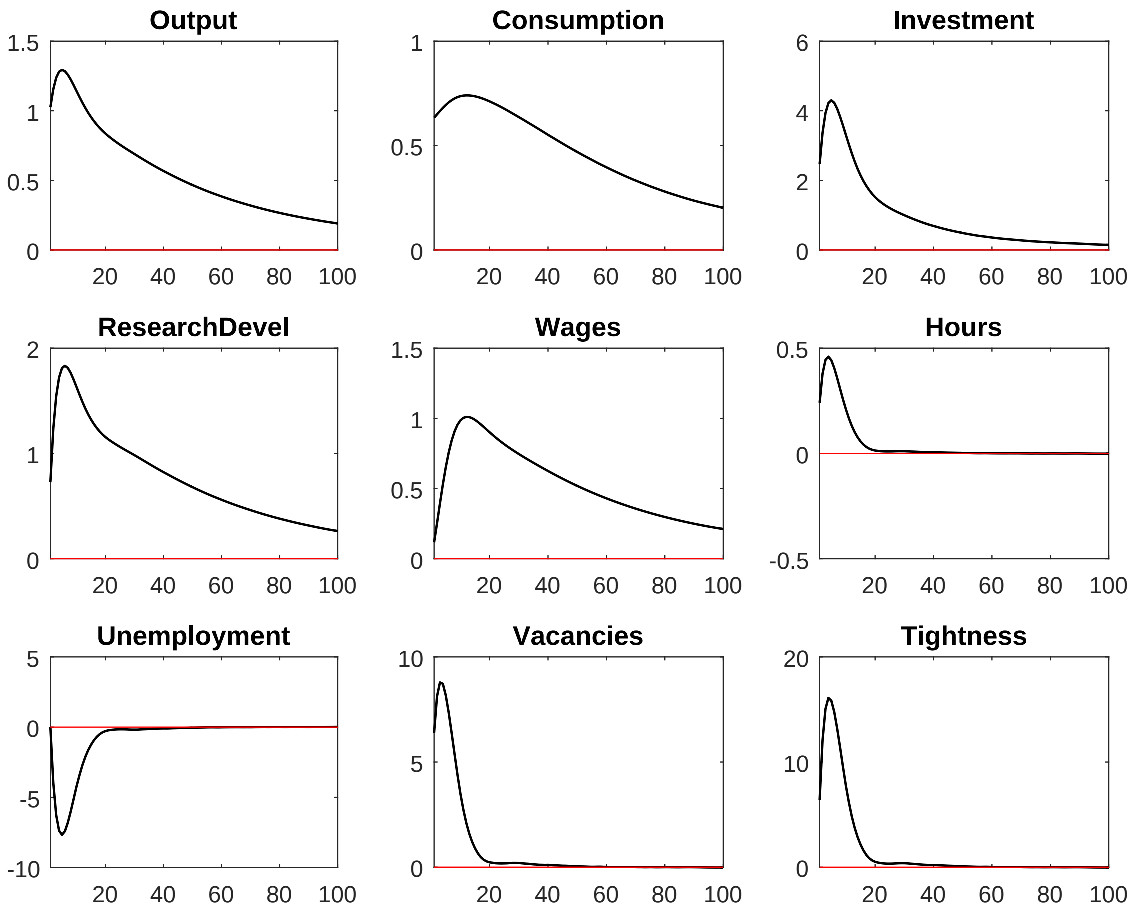
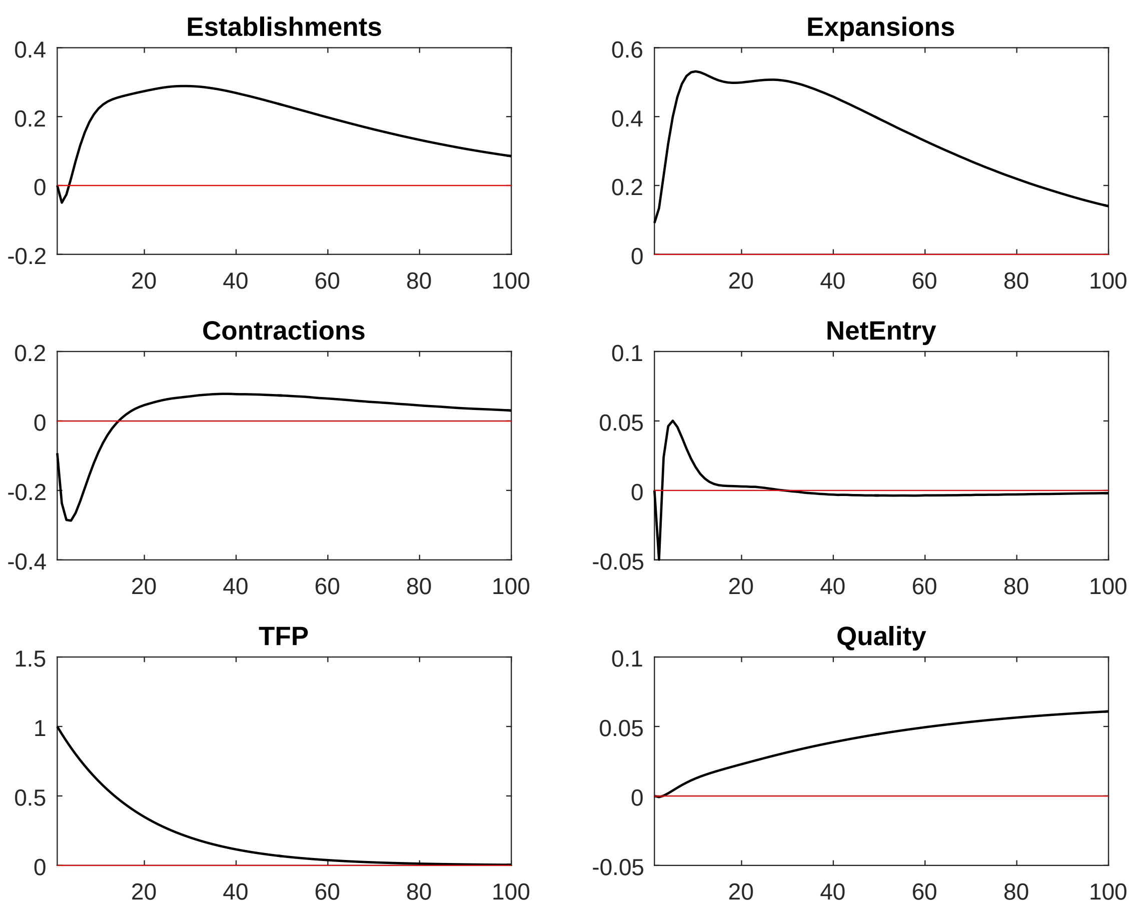


Figure 2 displays the impulse response functions of establishment dynamics. Following the productivity shock incumbents increase their R&D intensity, and the mass of expanding establishments increases while the mass of contracting establishment decreases. The increased demand from incumbents for scarce skilled labor results in a brief reduction in net entry rates, which translates to a small decrease in the mass of establishments. As the mass of employed skilled workers increases due to elevated hiring, net entry becomes positive and the mass of establishments increases substantially. Both elevated intensity of R&D by the incumbents and higher entry lead to an increase in the rate of growth of the aggregate quality index. For the first 5 years after the shock the increase in quality is fueled both by higher employment of skilled workers and bigger stock of physical capital, afterwards only more abundant physical capital maintains faster growth in quality level. The level of quality flattens out gradually and stabilizes at a level around 7% higher than it would be absent the shock.

As a robustness check, Figure 4 in the Appendix presents the Bayesian impulse response functions taking into account parameter uncertainty. All of the results remain unchanged.

Figure 2: Impulse response functions to 1% productivity shock, continued (%)



4 Policy implications

The previous section documents the hysteresis effect of temporary shocks on the level of the balanced growth path of the economy. This implies that business cycle fluctuations bear additional welfare costs which are unaccounted for in the models where growth results from exogenous processes.

To quantify the welfare comparisons across different states of the world, I employ the consumption equivalent transformation. The consumption equivalent is equal to the lifetime percentage change in the path of households' consumption that make them indifferent across "living" in two distinct states of the world. The consumption equivalent-adjusted lifetime utility is given by

$$W_0 (eq) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{((1+eq)\, c_t)^{1-\theta}}{1-\theta} = (1+eq)^{1-\theta}\, \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$$

The consumption equivalent across two different worlds can be then computed as follows:

$$eq_{a,b} = \left(\frac{U_0^b}{U_0^a} \right)^{\frac{1}{1-\theta}} - 1$$

where U_0^a and U_0^b denote expected lifetime utilities in worlds a and b, respectively. Then $eq_{a,b}$ has the interpretation of which proportion of consumption the agent living in world a would we willing to forfeit in order to "move" to world b.

Table 6 presents the comparison of expected lifetime utilities in three distinct worlds: nonstochastic, where the economy is not subject to shocks and always remains on its balanced growth path, and two stochastic worlds. In the first of them growth is fully exogenous and the quality index does not react in response to stochastic shocks. The second stochastic world represents the model economy.

The welfare effect of business cycles in the stochastic world with exogenous growth is very small in magnitude and actually indicates welfare gain. The reason for that is that an economy with physical capital has on average higher stock of capital when subject to stochastic shocks, as agents engage in precautionary saving to better smooth their consumption. This in turn implies that the average level of output, and also consumption, are also higher. As the welfare costs of volatility around an invariant trend are minuscule, the level effect dominates. This is a standard result in the business cycle literature.

On the other hand the welfare costs of business cycles under endogenous growth are substantial. Since the transitory shocks leave lasting impacts on the level of BGP, it increases dramatically the uncertainty about future consumption paths. As a result, agents would require a compensation of 5.8% of their consumption in order to be indifferent between living in the stochastic and nonstochastic worlds.

Table 6: Welfare cost of business cycles

State of the world	Welfare	Consumption equivalent
Non-stochastic (BGP)	-177.55	–
Stochastic with exogenous growth	-177.46	-0.04%
Stochastic with endogenous growth	-191.04	5.79%

Due to the presence of significant welfare costs of business cycles and the potential ability to affect the growth rate of the economy, ample space for policy intervention arises. I analyze the effects of employing two types of subsidy schemes: static and countercyclical, financed through a lump-sum tax/transfer scheme.

In the static case the subsidy acts as if a certain parameter was lowered or raised by 10%. Accordingly, a subsidy to operation cost acts as if the costs themselves were 10% lower, and subsidies to R&D act as if the research efficiency was 10% higher. Table 7 presents the results of subsidizing operation cost of incumbents and prospective entrants, their R&D expenditures, and the costs of hiring. Lastly, although it cannot be treated as a subsidy, I analyze the effects of increasing the labor contract renegotiation probability by 10%. In the last column I report the consumption equivalent multiplied by negative one, so that a positive value of the statistic indicates welfare gain.

The results indicate that subsidizing both operating cost and R&D expenditures of incumbent establishments is strongly welfare improving. This result may be surprising in the perspective of existing endogenous growth literature that almost unanimously generates result that subsidizing operating costs of incumbents is welfare deteriorating, as in e.g. Acemoglu et al. (2013). The reason I obtain the opposite results stems from the fact that my model features a frictional labor market. As can be seen in Table 7, subsidizing incumbents' operational cost leads to much lower rate of unemployment, as an effect of decreased churning in the labor market and higher establishment mass. This results in a higher level of aggregate output, as both the employment and love-for-variety effects move in the same direction. The static level gain dominates the effects that stem from lower rate of growth of the economy.

The remaining results have a very intuitive interpretation. In general, households prefer to live in worlds with ceteris paribus higher growth rates, lower volatility and lower unemployment rates. The subsidy to entrants' operating cost helps in lowering the unemployment rate and generates welfare gain even though the growth rate is slightly lower and the economy is slightly more volatile. As already discussed, subsidies to incumbents' R&D expenditures give rise to significant welfare gains, as despite slightly elevated unemployment rates the rate of growth of economy is much higher and it is less volatile. The small positive welfare effect from subsidizing entrants' R&D stems from lower unemployment rate. Decreasing the hiring costs in the labor market, both for the unskilled and skilled workers, generates welfare improvement, mostly stemming from decreased unemployment rates. What is important, subsidizing the hiring in the unskilled labor market where the majority of workers operate, yields also smaller volatility of the economy. Finally, increasing contract renegotiation frequency is welfare improving, although the consumption equivalent is rather small.

Table 7: Effects of static subsidies

	γ^{BGP}	γ	ΔQ_{20}	ΔQ_{100}	U^{BGP}	U	u^{BGP}	u	-eq
Baseline	2.07	2.09	2.23	5.75	-177.55	-191.04	5.65	5.70	–
f	2.03	2.05	2.35	5.98	-167.63	-180.89	5.18	5.24	4.11%
f^e	2.06	2.08	2.25	5.78	-177.16	-190.63	5.63	5.68	0.16%
a	2.12	2.14	2.18	5.57	-174.51	-186.74	5.68	5.73	1.74%
a^e	2.07	2.08	2.24	5.76	-177.41	-190.89	5.64	5.69	0.06%
κ^u	2.07	2.09	2.20	5.67	-175.57	-189.22	5.13	5.19	0.73%
κ^s	2.07	2.09	2.29	5.83	-177.29	-190.90	5.62	5.68	0.06%
λ	2.07	2.08	1.80	5.15	-177.55	-190.94	5.65	5.64	0.04%

Table 8 reports the welfare effects of applying countercyclical subsidies. The subsidy scheme works as follows: if output is 1% below trend, the subsidy increases by 0.5%. As such, it is actually a tax in the boom periods. The results fall in line with ones obtained in the simpler model by Bielecki (2017). Countercyclical subsidies to operating costs of both incumbents and entrants are welfare enhancing. On the other hand, subsidizing incumbents' R&D expenditures takes away precious resources from entrants when they need them most, and it generates a significant welfare loss. Finally, countercyclical hiring subsidies generate a negligible positive welfare effect.

Table 8: Effects of countercyclical subsidies

	ΔQ_{20}	ΔQ_{100}	U	u	-eq
Baseline	2.23	5.75	-191.04	5.698	–
f	2.89	7.50	-190.33	5.687	0.28%
f^e	2.27	5.86	-190.99	5.698	0.02%
a	0.96	2.64	-195.16	5.700	-1.66%
a^e	2.25	5.79	-191.00	5.698	0.02%
κ^u	2.22	5.73	-191.07	5.695	0.01%
κ^s	2.23	5.75	-191.03	5.698	0.00%

To sum up, the most welfare improving subsidies are static subsidies to incumbents' operating cost and R&D expenditures, and countercyclical subsidies to incumbents' operating cost. This provides justification for policies aiming to decrease firm exit during recessions.

5 Conclusions

In this paper I have presented an endogenous growth model, featuring monopolistically competitive, heterogeneous establishments that endogenously decide on the intensity of R&D, and subject to the search and matching friction on the labor markets. The model is able to generate volatile and procyclical R&D expenditure patterns and is consistent with the business cycle dynamics of GDP and its components, labor market variables, as well as establishment dynamics.

The model makes predictions on the strength of the impact of business cycle fluctuations on the endogenous growth rates of the economy. The results suggest that the mechanism governing innovation dynamics generates hysteresis effects of temporary shocks on the BGP level, translating around 7% of the strength of a shock to the level shift of the BGP, impacting significantly the assessment of welfare costs of business cycles.

I find that the welfare effects of business cycles are nontrivial and of two orders of magnitude higher than in the models with exogenous growth. Considerable welfare effects and the potential to influence endogenous growth rates creates ample scope for policy intervention. I examine the welfare effects of both static and countercyclical subsidy schemes.

In line with the extant endogenous growth literature, I find that static subsidies to R&D, as well as to the entrants, are welfare improving. In opposition to the previous results in the literature, I find that subsidizing incumbent firms generates large and positive welfare effects, as the static gains of bigger number of firms active in the market, leading to lower unemployment and love-for-variety effects dwarf dynamic losses of lowered entry rates. I also confirm that decreasing frictions in labor markets is welfare improving.

In the case of countercyclical subsidies I find that subsidizing incumbents' R&D expenditures is welfare deteriorating, while subsidizing their operating costs is welfare enhancing. This gives further support for policies designed to subsidize existing firms during recessions.

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A Appendix

A.1 Additional derivations

A.1.1 Solutions of cost minimization problems

Intermediate goods production sector

$$\begin{aligned} \min \quad & tc_t^p(i) = \tilde{w}_t^u n_t^p(i) + r_t k_t^p(i) \\ \text{subject to} \quad & y_t(i) = Z_t k_t^p(i)^\alpha \left[q_t(i) n_t^p(i)\right]^{1-\alpha} \end{aligned}$$

FOCs

$$\begin{aligned} n_t(i) &: \quad \tilde{w}_t^u = \lambda^p (1-\alpha) Z_t k_t^p(i)^\alpha q_t(i)^{1-\alpha} n_t^p(i)^{-\alpha} \\ k_t(i) &: \quad r_t = \lambda^p \alpha Z_t k_t^p(i)^{\alpha-1} q_t(i)^{1-\alpha} n_t^p(i)^{1-\alpha} \end{aligned}$$

Divide

$$\begin{aligned} \frac{\tilde{w}_t^u}{r_t} &= \frac{1-\alpha}{\alpha} \frac{k_t^p(i)}{n_t^p(i)} \\ k_t^p(i) &= \frac{\alpha}{1-\alpha} \frac{\tilde{w}_t^u}{r_t} n_t^p(i) \\ n_t^p(i) &= \frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} k_t^p(i) \end{aligned}$$

Production function

$$y_t(i) = Z_t k_t^p(i)^\alpha \left[q_t(i) n_t^p(i)\right]^{1-\alpha} = Z_t k_t^p(i)^\alpha \left[q_t(i) \frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} k_t^p(i)\right]^{1-\alpha} = Z_t k_t^p(i) \left[q_t(i) \frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u}\right]^{1-\alpha}$$

$$k_t^p(i) = \frac{y_t(i)}{Z_t} \left[q_t(i) \frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u}\right]^{\alpha-1}$$

Total cost

$$\begin{aligned} tc_t^p(i) &= \tilde{w}_t^u n_t^p(i) + r_t k_t^p(i) = \tilde{w}_t^u \frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u} k_t^p(i) + r_t k_t^p(i) = \left(\frac{1-\alpha}{\alpha} + 1\right) r_t k_t^p(i) = \frac{r_t}{\alpha} k_t^p(i) \\ &= \frac{r_t}{\alpha} \frac{y_t(i)}{Z_t} \left[q_t(i) \frac{1-\alpha}{\alpha} \frac{r_t}{\tilde{w}_t^u}\right]^{\alpha-1} = \frac{y_t(i)}{Z_t} \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1-\alpha}\right)^{1-\alpha} \end{aligned}$$

Real marginal cost

$$mc_t^p(i) = \frac{1}{Z_t} \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1-\alpha}\right)^{1-\alpha}$$

Research and development sector

$$\begin{aligned} \min \quad & tc_t^x(i) = \tilde{w}_t^s n_t^x(i) + r_t k_t^x(i) \\ \text{subject to} \quad & x_t(i) = \frac{k_t^x(i)^\alpha \left[Q_t n_t^x(i)\right]^{1-\alpha}}{Q_t \phi_t(i)} \end{aligned}$$

FOCs

$$n_t^x(i) \quad : \quad \tilde{w}_t^s = \lambda(1-\alpha)\frac{Z_t k_t^x(i)^\alpha Q_t^{1-\alpha} n_t^x(i)^{-\alpha}}{Q_t \phi_t(i)}$$
$$k_t^x(i) \quad : \quad r_t = \lambda\alpha\frac{Z_t k_t^x(i)^{\alpha-1} Q_t^{1-\alpha} n_t^x(i)^{1-\alpha}}{Q_t \phi_t(i)}$$

Divide

$$\frac{\tilde{w}_t^s}{r_t} = \frac{1-\alpha}{\alpha}\frac{k_t^x(i)}{n_t^x(i)}$$
$$k_t^x(i) = \frac{\alpha}{1-\alpha}\frac{\tilde{w}_t^s}{r_t}n_t^x(i)$$
$$n_t^x(i) = \frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s}k_t^x(i)$$

R&D production function

$$x_t(i) = \frac{k_t^x(i)^\alpha [Q_t n_t^x(i)]^{1-\alpha}}{Q_t \phi_t(i)} = Q_t^{-\alpha} k_t^x(i)\left(\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s}\right)^{1-\alpha} / \phi_t(i)$$

$$k_t^x(i) = x_t(i) Q_t^\alpha \left(\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s}\right)^{\alpha-1}\phi_t(i)$$

Total cost

$$tc_t^x(i) = \frac{r_t}{\alpha}k_t^x(i) = \frac{r_t}{\alpha}x_t(i) Q_t^\alpha \left(\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s}\right)^{\alpha-1}\phi_t(i)$$
$$= x_t(i) Q_t^\alpha \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^s}{1-\alpha}\right)^{1-\alpha}\phi_t(i)$$

Real marginal cost

$$mc_t^x(i) = Q_t^\alpha \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^s}{1-\alpha}\right)^{1-\alpha}\phi_t(i) \equiv \bar{m}c_t^x \phi_t(i)$$

Total cost as function of desired innovative success probability

$$\chi_t(i) = \frac{a x_t(i)}{1 + a x_t(i)}$$
$$x_t(i) = \frac{1}{a}\frac{\chi_t(i)}{1-\chi_t(i)}$$

$$tc_t^x(i) = \frac{\bar{m}c_t^x}{a}\frac{\chi_t(i)}{1-\chi_t(i)}\phi_t(i)$$

A.1.2 Aggregate production function

Relative inputs

$$\frac{y_t(i)}{y_t(j)} = \frac{Y_t p_t(i)^{-\sigma}}{Y_t p_t(j)^{-\sigma}} = \left[\frac{\frac{\sigma}{\sigma-1}\frac{1}{Z_t}\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha}}{\frac{\sigma}{\sigma-1}\frac{1}{Z_t}\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u/q_t(j)}{1-\alpha}\right)^{1-\alpha}}\right]^{-\sigma} = \left(\frac{q_t(i)^{\alpha-1}}{q_t(j)^{\alpha-1}}\right)^{-\sigma} = \left(\frac{q_t(i)^{1-\alpha}}{q_t(j)^{1-\alpha}}\right)^{\sigma}$$

$$\frac{y_t(i)}{y_t(j)} = \frac{Z_t k_t^p(i)\left[q_t(i)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{1-\alpha}}{Z_t k_t^p(j)\left[q_t(j)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{1-\alpha}}$$

$$\frac{k_t^p(i)\,q_t(i)^{1-\alpha}}{k_t^p(j)\,q_t(j)^{1-\alpha}} = \left(\frac{q_t(i)^{1-\alpha}}{q_t(j)^{1-\alpha}}\right)^{\sigma}$$

$$\frac{k_t^p(i)}{k_t^p(j)} = \left(\frac{q_t(i)}{q_t(j)}\right)^{(1-\alpha)(\sigma-1)}$$

$$k_t^p(i) = \left(\frac{q_t(i)}{q_t(j)}\right)^{(1-\alpha)(\sigma-1)} k_t^p(j)$$

$$k_t^p(i) = \left(\frac{q_t(i)}{Q_t}\right)^{(1-\alpha)(\sigma-1)} \bar{k}_t^p$$

$$n_t^p(i) = \left(\frac{q_t(i)}{Q_t}\right)^{(1-\alpha)(\sigma-1)} \bar{n}_t^p$$

where $\bar{k}_t^p \equiv K_t^p/M_t$ and $\bar{n}_t^p \equiv N_t^p/M_t$.

Final goods output

$$\begin{aligned}
Y_t &= \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}}\,\mathrm{d}i\right]^{\frac{\sigma}{\sigma-1}} = \left[M_t\int_0^{\infty} y_t(q)^{\frac{\sigma-1}{\sigma}}\mu_t(q)\,\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}}\left[\int_0^{\infty}\left[Z_t k_t^p(q)^{\alpha} q^{1-\alpha} n_t^p(q)^{1-\alpha}\right]^{\frac{\sigma-1}{\sigma}}\mu_t(q)\,\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} Z_t\left[\int_0^{\infty}\left[\left(\frac{q}{Q_t}\right)^{(1-\alpha)(\sigma-1)}\left(\bar{k}_t^p\right)^{\alpha}\left(\bar{n}_t^p\right)^{1-\alpha} q^{1-\alpha}\right]^{\frac{\sigma-1}{\sigma}}\mu_t(q)\,\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} Z_t\left(\bar{k}_t^p\right)^{\alpha}\left(\bar{n}_t^p\right)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)}\left[\int_0^{\infty}\left[\left(q^{1-\alpha}\right)^{\sigma}\right]^{\frac{\sigma-1}{\sigma}}\mu_t(q)\,\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}-1} Z_t\left(K_t^p\right)^{\alpha}\left(N_t^p\right)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)}\left[\left[\int_0^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}\right]^{\sigma} \\
&= M_t^{\frac{1}{\sigma-1}} Z_t\left(K_t^p\right)^{\alpha}\left(N_t^p\right)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)}\left(Q_t^{1-\alpha}\right)^{\sigma} \\
&= M_t^{\frac{1}{\sigma-1}} Z_t\left(K_t^p\right)^{\alpha}\left(Q_t N_t^p\right)^{1-\alpha}
\end{aligned}$$

A.1.3 Real profit function

Real operating profit

$$
\begin{aligned}
\pi_t^o(i) &= p_t(i)\,y_t(i) - mc_t^p(i)\,y_t(i) - f_t = p_t(i)\,y_t(i) - p_t(i)\frac{\sigma-1}{\sigma}y_t(i) - f_t \\
&= \left(1-\frac{\sigma-1}{\sigma}\right)Y_t p_t(i)^{1-\sigma} - f_t = \frac{1}{\sigma}Y_t\left[\frac{\sigma}{\sigma-1}\frac{1}{Z_t}\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma} - f_t \\
&= \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}}Y_t Z_t^{\sigma-1}\left[\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u/q_t(i)}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma} - f_t
\end{aligned}
$$

Price index (where $R_t \equiv P_t r_t$ and $W_t^u \equiv P_t \tilde{w}_t^u$)

$$
\begin{aligned}
P_t &= \left[\int_0^{M_t} P_t(i)^{1-\sigma}\,\mathrm{d}i\right]^{\frac{1}{1-\sigma}} = \left[M_t\int_0^{\infty} P_t(q)^{1-\sigma}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{1-\sigma}} \\
&= M_t^{\frac{1}{1-\sigma}}\left[\int_0^{\infty}\left[\frac{\sigma}{\sigma-1}\frac{1}{Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u/q}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1}M_t^{\frac{1}{1-\sigma}}\frac{1}{Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha}\left[\int_0^{\infty}\left(q^{\alpha-1}\right)^{1-\sigma}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1}M_t^{\frac{1}{1-\sigma}}\frac{1}{Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha}\left[\left[\int_0^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}\right]^{-1} \\
&= \frac{\sigma}{\sigma-1}M_t^{\frac{1}{1-\sigma}}\frac{1}{Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha}\left(Q_t^{1-\alpha}\right)^{-1} \\
&= \frac{\sigma}{\sigma-1}M_t^{\frac{1}{1-\sigma}}\frac{1}{Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u/Q_t}{1-\alpha}\right)^{1-\alpha}
\end{aligned}
$$

Real input cost index

$$
\begin{aligned}
\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t^u/Q_t}{1-\alpha}\right)^{1-\alpha} &= \frac{\sigma-1}{\sigma}P_t M_t^{\frac{1}{\sigma-1}}Z_t \\
\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u}{1-\alpha}\right)^{1-\alpha} &= \frac{\sigma-1}{\sigma}M_t^{\frac{1}{\sigma-1}}Z_t Q_t^{1-\alpha} \\
\frac{1}{Z_t}\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u}{1-\alpha}\right)^{1-\alpha} &= \frac{\sigma-1}{\sigma}M_t^{\frac{1}{\sigma-1}}Q_t^{1-\alpha}
\end{aligned}
$$

Real operating profit

$$\pi_t^o(i) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1} \left[\left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{w_t/q_t(i)}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma} - f_t$$
$$= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t Z_t^{\sigma-1} \left[\frac{\sigma-1}{\sigma} M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\alpha} q_t(i)^{\alpha-1}\right]^{1-\sigma} - f_t$$
$$= \frac{Y_t}{\sigma M_t} \left[\left(\frac{q_t(i)}{Q_t}\right)^{1-\alpha}\right]^{\sigma-1} - f_t$$
$$= \frac{Y_t}{\sigma M_t} \phi_t(i) - f_t$$

Real profit

$$\pi_t(i) = \pi_t^o(i) - \frac{\bar{mc}_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \phi_t(i)$$
$$= \left(\frac{Y_t}{\sigma M_t} - \frac{\bar{mc}_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)}\right) \phi_t(i) - f_t$$
$$= \left(\frac{Y_t}{\sigma M_t} - \frac{\bar{mc}_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)}\right) \phi_t(i) - \bar{mc}_t^x f$$
$$= Y_t \left[\left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a} \frac{\chi_t(i)}{1-\chi_t(i)}\right) \phi_t(i) - \omega_t f\right]$$

A.1.4 Evolution of aggregate quality index

Following Melitz (2003), I consider the current period distribution of quality levels $\mu_t(q)$ to be a truncated part of an underlying distribution $g_t(q)$, so that:

$$\mu_t(q) = \begin{cases} 1/\left[1 - G_t\left(q_{t-1}^*\right)\right] g_t(q) & \text{if } q \geq q_{t-1}^* \\ 0 & \text{otherwise} \end{cases}$$

where $q_t^* = (\phi_t^*)^{1/[(1-\alpha)(\sigma-1)]} Q_t$.

Aggregate quality index at the end of period t:

$$Q_t^{1-\alpha} = \left[\int_0^\infty \left(q^{1-\alpha}\right)^{\sigma-1} \mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}} = \left[\frac{1}{1-G_t(q_{t-1}^*)} \int_{q_{t-1}^*}^\infty \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}$$

The aggregate quality level after exits and innovation resolution but before entry:

$$Q_t^* = \left\{\frac{1}{1-G_t(q_t^*)} \left[(1-\chi_t) \int_{q_t^*}^\infty \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q)\,\mathrm{d}q + \chi_t \int_{q_t^*}^\infty \left(\iota^{\frac{1}{(1-\alpha)(\sigma-1)}} q\right)^{(1-\alpha)(\sigma-1)} g_t(q)\,\mathrm{d}q\right]\right\}^{\frac{1}{\sigma-1}}$$
$$= \left[(1-\chi_t+\chi_t\iota) \frac{1}{1-G_t(q_t^*)} \int_{q_t^*}^\infty \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}$$

Aggregate quality index in $t+1$ after entry:

$$Q_{t+1} = \left\{ \frac{1-\chi_t+\chi_t\iota}{1-G_t\left(q_t^*\right)} \left[\begin{array}{c} \left(1-\frac{M_t^e}{M_{t+1}}\right)\int_{q_t^*}^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1} g_t(q)\mathrm{d}q \\ +\frac{M_t^e}{M_{t+1}}\int_{q_t^*}^{\infty}\left(\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{(1-\alpha)(\sigma-1)}} q\right)^{(1-\alpha)(\sigma-1)} g_t(q)\mathrm{d}q \end{array} \right] \right\}^{\frac{1}{\sigma-1}}$$

$$= \left[(1-\chi_t+\chi_t\iota)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right)\frac{1}{1-G_t\left(q_t^*\right)}\int_{q_t^*}^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1} g_t\left(q\right)\mathrm{d}q \right]^{\frac{1}{\sigma-1}}$$

Transformed aggregate growth rate η_t:

$$\begin{aligned}
\eta_t &= \left(\frac{Q_{t+1}}{Q_t}\right)^{(1-\alpha)(\sigma-1)} \\
&= \left\{ \frac{\left[(1-\chi_t+\chi_t\iota)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right)\frac{1}{1-G_t\left(q_t^*\right)}\int_{q_t^*}^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1} g_t\left(q\right)\mathrm{d}q\right]^{\frac{1}{\sigma-1}}}{\left[\frac{1}{1-G_t\left(q_{t-1}^*\right)}\int_{q_{t-1}^*}^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1} g_t\left(q\right)\mathrm{d}q\right]^{\frac{1}{\sigma-1}}} \right\}^{\sigma-1} \\
&\approx (1-\chi_t+\chi_t\iota)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right)
\end{aligned}$$

where if the distribution is invariant with respect to the cutoff points q_{t-1}^* and q_t^* (as is the case with Pareto and other power-law distributions) then the above relationship holds with equality.

A.1.5 Target wage

Expression for target wage

$$w_t^o = w_t^f + \psi\left(\frac{\kappa}{2}\left(x_t^2\left(r\right)-x_t^2\right)-p_t\kappa x_t\right)+\left(1-\psi\right)p_t E_t\left[\beta\Lambda_{t,t+1}H_{t+1}\right]$$

Average vs conditional on renegotiation worker surplus

$$H_t = H_t\left(r\right)+\Delta_t\left(w_t-w_t\left(r\right)\right)$$

Therefore

$$\begin{aligned}
&\left(1-\psi\right)p_t E_t\left[\beta\Lambda_{t,t+1}H_{t+1}\right] = \\
&= \left(1-\psi\right)p_t E_t\left[\beta\Lambda_{t,t+1}\left[H_{t+1}\left(r\right)+\lambda\Delta_{t+1}\left(w_{t+1}-w_{t+1}\left(r\right)\right)\right]\right] \\
&= \left(1-\psi\right)p_t E_t\left[\beta\Lambda_{t,t+1}H_{t+1}\left(r\right)\right]+\left(1-\psi\right)p_t E_t\left[\beta\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_{t+1}-w_{t+1}\left(r\right)\right)\right] \\
&= \psi p_t E_t\left[\beta\Lambda_{t,t+1}J_{t+1}\left(r\right)\right]+\left(1-\psi\right)p_t E_t\left[\beta\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_{t+1}-w_{t+1}\left(r\right)\right)\right] \\
&= \psi p_t\kappa x_t\left(r\right)+\left(1-\psi\right)p_t E_t\left[\beta\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_{t+1}-w_{t+1}\left(r\right)\right)\right]
\end{aligned}$$

Resulting target wage

$$\begin{aligned}
w_t^o = w_t^f &+ \psi\left(\frac{\kappa}{2}\left(x_t^2\left(r\right)-x_t^2\right)+p_t\kappa\left(x_t\left(r\right)-x_t\right)\right) \\
&+ \left(1-\psi\right)p_t \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_{t+1}-w_{t+1}\left(r\right)\right)\right]
\end{aligned}$$

A.2 Full set of model equations

Stationarized variables notation

$$\hat{X}_t \equiv X_t/Q_t$$

Stationarizing variables

$$g_t^Q \equiv Q_{t+1}/Q_t = \eta_t^{\frac{1}{(1-\alpha)(\sigma-1)}} \tag{A.1}$$

$$\gamma_{t,t+1} \equiv Y_{t+1}/Y_t = g_t^Q \cdot \hat{Y}_{t+1}/\hat{Y}_t \tag{A.2}$$

Incumbents' problem

$$\phi_t = 1 \tag{A.3}$$

$$v_t = A_t + B_t\phi_t \tag{A.4}$$

$$\pi_t = \left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\right)\phi_t - \omega_t f \tag{A.5}$$

$$A_t + B_t\phi_t = \pi_t + \mathrm{E_t}\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\gamma_{t,t+1}\left(A_{t+1} + B_{t+1}\frac{\chi_t\left(\iota-1\right)+1}{\eta_t}\phi_t\right)\right] \tag{A.6}$$

$$0 = -\frac{\omega_t}{a}\frac{1}{\left(1-\chi_t\right)^2} + \mathrm{E_t}\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\gamma_{t,t+1}B_{t+1}\frac{\left(\iota-1\right)\phi_t}{\eta_t}\right] \tag{A.7}$$

$$B_t = \frac{1}{\sigma M_t} - \frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t} + \mathrm{E_t}\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\gamma_{t,t+1}B_{t+1}\frac{\chi_t\left(\iota-1\right)+1}{\eta_t}\right] \tag{A.8}$$

Entrants' problem

$$v_t^e = -\omega_t\left(f^e + \frac{1}{a^e}\frac{\chi_t^e}{1-\chi_t^e}\right) + \chi_t^e\mathrm{E_t}\left[\beta\Lambda_{t,t+1}\gamma_{t,t+1}\left(A_{t+1} + B_{t+1}\frac{\sigma}{\sigma-1}\phi_{t+1}\right)\right] \tag{A.9}$$

$$0 = -\frac{\omega_t}{a^e}\frac{1}{\left(1-\chi_t^e\right)^2} + \mathrm{E_t}\left[\beta\Lambda_{t,t+1}\gamma_{t,t+1}\left(A_{t+1} + B_{t+1}\frac{\sigma}{\sigma-1}\phi_{t+1}\right)\right] \tag{A.10}$$

$$v_t^e = 0 \tag{A.11}$$

Establishment dynamics

$$\delta_t = 1 - \left(1-\delta^{exo}\right)\left(1-M_t^e\right) \tag{A.12}$$

$$\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\phi_t^* = \mathrm{E_t}\left[\beta\Lambda_{t,t+1}\left(1-\delta_t\right)\gamma_{t,t+1}\left(A_{t+1} + B_{t+1}\frac{\chi_t\left(\iota-1\right)+1}{\eta_t}\phi_t^*\right)\right] \tag{A.13}$$

$$M_t^x = M_t\left(1-\chi_{t-1}\right)\left(1 - \frac{\phi_{t-1}^*}{\phi_t^*\eta_{t-1}}\right) \tag{A.14}$$

$$M_{t+1} = \left(1-\delta_t\right)\left(M_t - M_t^x\right) + M_t^e \tag{A.15}$$

$$\eta_t = \left(1-\chi_t+\chi_t\iota\right)\left(1 - \frac{M_t^e}{M_{t+1}} + \frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right) \tag{A.16}$$

Skilled sector

$$\omega_t \hat{Y}_t = \left(\frac{r_t^k}{\alpha}\right)^{\alpha} \left(\frac{\hat{\hat{w}}_t^s}{1-\alpha}\right)^{1-\alpha} \tag{A.17}$$

$$\left(\hat{K}_t^s\right)^{\alpha} \left(N_t^s\right)^{1-\alpha} = M_t f + \left(M_t - M_t^x\right)\left(\frac{1}{a}\frac{\chi_t}{1-\chi_t}\right) + \frac{M_t^e}{\chi_t^e}\left(f^e + \frac{1}{a^e}\frac{\chi_t^e}{1-\chi_t^e}\right) \tag{A.18}$$

$$\frac{r_t^k}{\hat{\hat{w}}_t^s} = \frac{\alpha}{1-\alpha}\frac{N_t^s}{\hat{K}_t^s} \tag{A.19}$$

Unskilled sector

$$\hat{Y}_t = Z_t M_t^{\frac{1}{\sigma-1}} \left(\hat{K}_t^p\right)^{\alpha} \left(N_t^p\right)^{1-\alpha} \tag{A.20}$$

$$\hat{\hat{w}}_t^u = (1-\alpha)\frac{\sigma-1}{\sigma} Z_t M_t^{\frac{1}{\sigma-1}} \left(\hat{K}_t^p\right)^{\alpha} \left(N_t^p\right)^{-\alpha} \tag{A.21}$$

$$r_t^k = \alpha\frac{\sigma-1}{\sigma} Z_t M_t^{\frac{1}{\sigma-1}} \left(\hat{K}_t^p\right)^{\alpha-1} \left(N_t^p\right)^{1-\alpha} \tag{A.22}$$

Households

$$1 = \mathrm{E}_t\left[\beta\left(g_t^Q \cdot \hat{C}_{t+1}/\hat{C}_t\right)^{-\theta}\left(1 + r_t^k - dp\right)\right] \tag{A.23}$$

$$\Lambda_{t,t+1} = \mathrm{E}_t\left[\left(g_t^Q \cdot \hat{C}_{t+1}/\hat{C}_t\right)^{-\theta}\right] \tag{A.24}$$

Frictional labor markets (notation $w_t^* \equiv w_t(r)$)

$$m_t^u = \sigma_m \left(u_t^u\right)^{\psi} \left(v_t^u\right)^{1-\psi} \tag{A.25}$$

$$m_t^s = \sigma_m \left(u_t^s\right)^{\psi} \left(v_t^s\right)^{1-\psi} \tag{A.26}$$

$$n_{t+1}^u = \left(\rho^u + x_t^u\right) n_t^u \tag{A.27}$$

$$n_{t+1}^s = \left(\rho^s + x_t^s\right) n_t^s \tag{A.28}$$

$$u_t^u = 1 - n_t^u \tag{A.29}$$

$$u_t^s = 1 - n_t^s \tag{A.30}$$

$$q_t^u = m_t^u / v_t^u \tag{A.31}$$

$$q_t^s = m_t^s / v_t^s \tag{A.32}$$

$$p_t^u = m_t^u / u_t^u \tag{A.33}$$

$$p_t^s = m_t^s / u_t^s \tag{A.34}$$

$$x_t^u = q_t^u v_t^u / n_t^u \tag{A.35}$$

$$x_t^s = q_t^s v_t^s / n_t^s \tag{A.36}$$

$$\kappa^u x_t^u = \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(\hat{\tilde{w}}_{t+1}^u - \hat{w}_t^u + \frac{\kappa^u}{2}\left(x_{t+1}^u\right)^2 + \rho^u\kappa^u x_{t+1}^u\right)\right] \tag{A.37}$$

$$\kappa^s x_t^s = \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(\hat{\tilde{w}}_{t+1}^s - \hat{w}_t^s + \frac{\kappa^s}{2}\left(x_{t+1}^s\right)^2 + \rho^s\kappa^s x_{t+1}^s\right)\right] \tag{A.38}$$

$$\kappa^u x_t^{u*} = \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(\hat{\tilde{w}}_{t+1}^u - \hat{w}_t^{u*} + \frac{\kappa^u}{2}\left(x_{t+1}^{u*}\right)^2 + \rho^u\kappa^u x_{t+1}^{u*}\right)\right] \tag{A.39}$$

$$\kappa^s x_t^{s*} = \mathrm{E}_t\left[\beta\Lambda_{t,t+1}\left(\hat{\tilde{w}}_{t+1}^s - \hat{w}_t^{s*} + \frac{\kappa^s}{2}\left(x_{t+1}^{s*}\right)^2 + \rho^s\kappa^s x_{t+1}^{s*}\right)\right] \tag{A.40}$$

$$\Delta_t^u = 1 + \beta\rho^u\lambda\mathrm{E}_t\left[\Lambda_{t,t+1}g_t^Q\Delta_{t+1}^u\right] \tag{A.41}$$

$$\Delta_t^s = 1 + \beta\rho^s\lambda\mathrm{E}_t\left[\Lambda_{t,t+1}g_t^Q\Delta_{t+1}^s\right] \tag{A.42}$$

$$\Delta_t^u\hat{w}_t^{u*} = \hat{w}_t^{uo} + \rho^u\lambda\mathrm{E}_t\left[\beta\Lambda_{t,t+1}\Delta_{t+1}^u\hat{w}_{t+1}^{u*}\right] \tag{A.43}$$

$$\Delta_t^s\hat{w}_t^{s*} = \hat{w}_t^{so} + \rho^s\lambda\mathrm{E}_t\left[\beta\Lambda_{t,t+1}\Delta_{t+1}^s\hat{w}_{t+1}^{s*}\right] \tag{A.44}$$

$$\hat{w}_t^{uf} = \psi\left(\hat{\tilde{w}}_t^u + \frac{\kappa^u}{2}\left(x_t^u\right)^2 + p_t^u\kappa^u x_t^u\right) + (1-\psi)\,b_t^u \tag{A.45}$$

$$\hat{w}_t^{sf} = \psi\left(\hat{\tilde{w}}_t^s + \frac{\kappa^s}{2}\left(x_t^s\right)^2 + p_t^s\kappa^s x_t^s\right) + (1-\psi)\,b_t^s \tag{A.46}$$

$$\begin{aligned}\hat{w}_t^{uo} &= \hat{w}_t^{uf} + \psi\left(\frac{\kappa^u}{2}\left(\left(x_t^{u*}\right)^2 - \left(x_t^u\right)^2\right) + p_t^u\kappa^u\left(x_t^{u*} - x_t^u\right)\right) \\ &\quad + (1-\psi)\,p_t^u\mathrm{E}_t\left[\beta\Lambda_{t,t+1}\lambda\Delta_{t+1}^u g_t^Q\left(\hat{w}_t^u - \hat{w}_t^{u*}\right)\right]\end{aligned} \tag{A.47}$$

$$\begin{aligned}\hat{w}_t^{so} &= \hat{w}_t^{sf} + \psi\left(\frac{\kappa^s}{2}\left(\left(x_t^{s*}\right)^2 - \left(x_t^s\right)^2\right) + p_t^s\kappa^s\left(x_t^{s*} - x_t^s\right)\right) \\ &\quad + (1-\psi)\,p_t^s\mathrm{E}_t\left[\beta\Lambda_{t,t+1}\lambda\Delta_{t+1}^s g_t^Q\left(\hat{w}_t^s - \hat{w}_t^{s*}\right)\right]\end{aligned} \tag{A.48}$$

$$\hat{w}_t^u = \lambda\hat{w}_{t-1}^u + (1-\lambda)\,\hat{w}_t^{u*} \tag{A.49}$$

$$\hat{w}_t^s = \lambda\hat{w}_{t-1}^s + (1-\lambda)\,\hat{w}_t^{s*} \tag{A.50}$$

$$\hat{b}_t^u = 0.4\hat{w}_{ss}^u \tag{A.51}$$

$$\hat{b}_t^s = 0.4\hat{w}_{ss}^s \tag{A.52}$$

Market clearing

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t + \kappa^u\left(x_t^u\right)^2 N_t^p + \kappa^s\left(x_t^s\right)^2 N_t^s \tag{A.53}$$

$$g_t^Q\hat{K}_{t+1} = (1-dp)\,\hat{K}_t + \hat{I}_t \tag{A.54}$$

$$\hat{K}_t = \hat{K}_t^p + \hat{K}_t^s \tag{A.55}$$

$$N_t^p = (1-s)\,n_t^u \tag{A.56}$$

$$N_t^s = sn_t^s \tag{A.57}$$

Shock

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t} \tag{A.58}$$

Welfare

$$U_t = \frac{\left(\hat{C}_t Q_t\right)^{1-\theta}}{1-\theta} + \beta \mathrm{E}_t \left[U_{t+1}\right] \tag{A.59}$$

A.3 Additional tables and figures

Table 9: Prior distributions of parameters

Parameter	Description	Distribution shape	Mean	Std. dev.
λ	Average contract duration	Uniform $[0, 1]$	0.5	0.289
ρ_Z	Autocorr. of TFP process	Beta	0.7	0.175
σ_Z	Std. dev. of TFP shock	Inverse Gamma	0.01	∞

Figure 3: Prior and posterior distributions

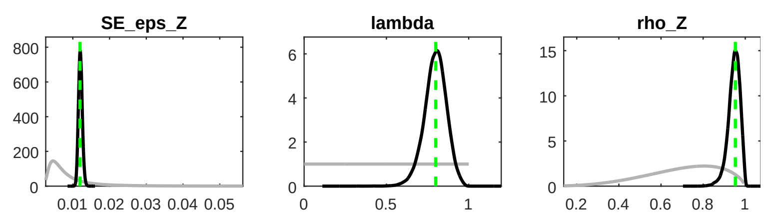
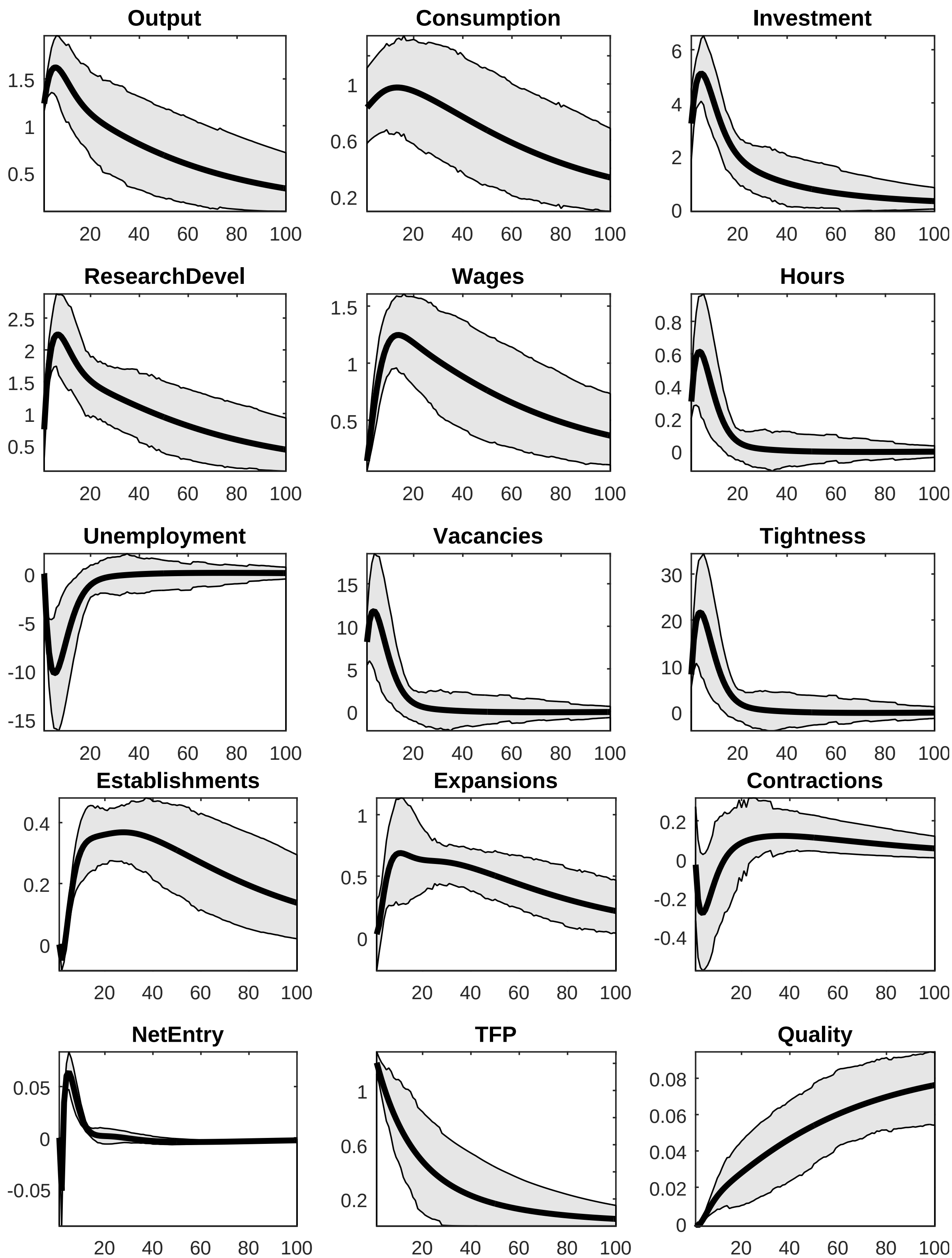


Figure 4: Bayesian impulse response functions



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