CROSS-SECTIONAL RETURNS WITH VOLATILITY REGIMES FROM DIVERSE PORTFOLIO OF EMERGING AND DEVELOPED EQUITY INDICES
Cross-Sectional Returns With Volatility Regimes
From Diverse Portfolio of Emerging and Developed Equity Indices

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Abstract
This article aims to extend evaluation of classic multifactor model of Carhart (1997) for the case of global equity indices and to expand analysis performed in Sakowski et. al.(2015). Our intention is to test several modifications of these models to take into account different dynamics of equity excess returns between emerging and developed equity indices. Proposed extensions include volatility regime switching mechanism (using dummy variables and the Markov approach) and the fifth risk factor based on realized volatility of index returns. Moreover, instead of using data for stocks of a particular market (which is a common approach in the literature), we check performance of these models for weekly data of 81 world investable equity indices in the period of 2000-2015. Such approach is proposed to estimate equity risk premium for a single country.

Empirical evidence reveals important differences between results for classical models estimated on single stocks (either in international or US-only framework) and models evaluated for equity indices. Additionally, we observe substantial discrepancies between results for developed countries and emerging markets. Finally, using weekly data for the last 15 years we illustrate importance of model risk and data overfitting effects when drawing conclusions upon results of multifactor models.

Keywords:
cross-sectional models, asset pricing models, equity risk premia, equity indices, new risk factors, sensitivity analysis, book to market, momentum, market price of risk, emerging and developed equity indices, data overfitting, model risk

JEL:
C15, G11, F30, G12, G13, G14, G15

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1 Introduction

Studies which try to estimate equity risk premium and explain variability of stock market returns are numerous in financial literature. The discussion started with the seminal papers introducing the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965) and Black, Jensen, and Scholes (1972). Then, it has greatly evolved with the three-factor model of Fama and French (1992) and four-factor model of Carhart (1997). Nowadays, studies concentrate around other modifications which propose numerous set of risk factors to better explain variability of stock market returns. This paper aims to introduce several new ideas to this debate.

During the last three decades, many studies empirically verified validity of the CAPM. They showed that CAPM alone is not able to explain cross-sectional variation of average stock returns and that it can be easily extended. The results of these studies revealed other important risk factors which explain outperformance of the given groups of stocks:

- value investing strategy effects, i.e. investing in stocks that have high book to market, dividends yield, earnings ratio, etc. produce higher risk adjusted returns (Fama and French (1992), Lakonishok, Shleifer, and Vishny (1994), Arshanapalli, Coggin, and Doukas (1998), Bondt and Thaler (1985) and Bondt and Thaler (1987)),
- size effects (i.e. small minus big stocks effect) (Fama and French (2012)),
- momentum and reversal effect (i.e. winners minus loosers effect) captured for many different time frames (Wu (2002), Jegadeesh and Titman (1993), and Asness (1995)),
- liquidity effect (Rahim and Noor (2006), Liu (2004)),
- investment factor and return on equity factor (Chen, Novy-Marx, and Zhang (2011)),
- profitability factor and investment factor (Fama and French (2015)),
- betting against beta effect (Frazzini and Pedersen (2014))\(^1\)
- accounting manipulation factor (Foye, Mramor, and Pahor (2013)),
- cash-flow-to-price factor (Hou, Karolyi, and Kho (2011)),

At the same time many authors claim that CAPM still works, arguing that deviations due to missing factors are difficult to detect and it is relatively difficult to remove data-snooping bias in case of multifactor models (MacKinlay (1995)). Other sources of errors that can be encountered while performing stock returns analysis include among others look ahead bias (Lo and MacKinlay (1990)).

\(^1\)The BAB factor captures the phenomenon that long leveraged low beta assets and short high-beta assets produce significant positive risk-adjusted returns.
Based on the current state of the art for stock returns and the fact that very few papers covered the problem for equity indices returns so far, we want to better explain the diversity of equity indices returns and hence follow the conclusion of Griffin (2002) who stated that Fama-French factors are country-specific rather than global.

Therefore, the main aim of this paper is to present a cross-sectional analysis for global indices with a special attention to differences between developed and emerging market indices. We want to find an answer to a question whether by introducing modifications of the well-known asset pricing models we are able to identify these equity indices which are relatively cheap (or expensive), at the same time taking into account all other important risk factors.

Our main research questions can be stated as follows:

1. Can multifactor models be used for explanation of equity risk premium for global indices? Our intention is to answer this question on single equity indices basis and on aggregated level as well.
2. Are sensitivities to risk factors stable across countries? Do they differ during various phases of economic cycles?
3. Can we include volatility risk factor to better explain variability of risk premia?
4. Does volatility regime switching mechanism (using dummy variables or the Markov approach) enable us to explain equity risk premium for global indices?
5. Can we build a zero investment portfolio based on analysed risk factors?
6. Are signs of beta coefficients coherent with the results or single stocks?
7. Is it possible to distinguish countries with consistently high beta sensitivities?
8. Which risk factor was the most important in portfolio construction?

The structure of the paper is as follows. Section 2 presents methodology of our study. Equity risk premium, functional forms of the alternative models and econometric issues are discussed in this part. Section 3 provides description of both the data and the procedure we used to build risk factors. We also analyse dynamics of risk factors in time here. Section 4 presents results. The last section concludes.

2 Methodology

2.1 Motivation

The methodology is based on the seminal paper of Carhart (1997), who proposed the four-factor model for analysis of mutual funds performance. One of the reason why we prefer the model of Carhart (1997) over the methodology of Fama and French (1992) (the three-factor model for stocks return analysis) are the results of Fama and French (2012) and comprehensive results obtained for emerging markets by Cakici, Fabozzi, and Tan (2013). They focused on 18 emerging markets treating each of them separately. Their results revealed significance of value and momentum everywhere except Eastern Europe and additionally showed that momentum and value factors were negatively correlated.
At this moment it is important to explain rationale for choosing equity indices instead of single stocks. The main reason behind this is that from the global investment perspective single countries may be treated as an asset class. This issue is very important from the global portfolio selection problem, where asset allocation approach seems to gain more popularity. This is confirmed by the dynamic development of ETFs and derivatives providing country exposure. This approach seems to better reveal global factors than regressions on single stocks and enables equity risk premia for countries to be assessed separately. What is also important, the literature on this subject is very limited.

Taking into account that our research is intended for equity indices with special attention to emerging and developed markets, we propose several amendments to the initial methodology of Carhart (1997). Neccessary modifications include:

• converting monthly to weekly data in order to reveal dynamics during shorter time intervals,
• including new risk factors that explain the diversity of returns more deeply, i.e. realized volatility as fifth factor,
• necessary conversion of well-known risk factors from the single country level to the worldwide level,
• creating an adequate zero investment portfolio that fully reflects the influence of particular risk factor on equity risk premia,
• introducing volatility switching mechanism to take into account different dynamics of equity indices during high and low volatility periods.

2.2 Equity risk premium

It is also important to define the equity risk premium as the expected excess return of equities over the risk free rate. The point here is that current literature (Duarte, Rosa, et al. (2015)) proposes many alternative ways to measure it.

Equity risk premium can be defined using:

• historical returns approach:

\[ ERP = \sum_{t=t_0}^{n} R_t - R_f \]  

where \( R_t - R_f \) is excess return at time \( t \) over risk-free rate.

• earnings yield approach:

\[ ERP = \frac{E}{P} - R_f \]  

where \( \frac{E}{P} \) is earnings to price ratio

• dividend yield approach:

\[ ERP = \frac{D}{P} + g - R_f \]  

where \( \frac{D}{P} \) is dividend to price ratio and \( g \) is dividend growth rate.
• regression- and factor-based approach which can be characterised by point-in-time estimates instead of long-term estimates only, not dependent on e.g. tax policy, and which allows dynamic forecasts:

\[ ERP_t = \alpha + \sum_{i=1}^{n} \beta_i X_{i,t} + \epsilon_t \]

where \( X_{i,t} \) are the \( i \)-th risk factor at moment \( t \) and and \( \beta_i \) is sensitivity to this factor.

• survey-based approach which is often systematically biased, negatively correlated with future returns, and positively with previous returns.

In this article, when we talk about equity risk premium we refer to the definition based on historical returns. Selection of the particular definition of equity risk premium can certainly affect final conclusions. Before we focus on this issue, we describe alternative factor models used in this research.

### 2.3 Multi factor models

We start with the classic Carhart four-factor model:

\[
(R_i - R_f)_t = \alpha_i + \beta_{MKT,i}(Rm - R_f)_t + \beta_{HML,i}HML_t + \beta_{SMB,i}SMB_t + \beta_{WML,i}WML_t + \epsilon_{i,t}
\]

where:
- \((R_i - R_f)_t\) is weekly return of equity index in excess to weekly risk free rate,
- \((Rm - R_f)_t\) is equally weighted equity index less than risk free rate,
- \(HML_t\) is the monthly premium on the book-to-market factor,
- \(SMB_t\) is the monthly premium on the size factor
- \(WML_t\) is the monthly premium on winners-minus-losers factor.

The WML factor is calculated by subtracting the equal weighted average of the highest performing equity indices from the equal weighed average of the lowest performing equity indices (Carhart (1997)).

Next, we add to the model an additional factor based on realized volatility (VMC - volatile minus calm):

\[
(R_i - R_f)_{i,t} = \alpha + \beta_{MKT,i}(Rm - R_f)_t + \beta_{HML,i}HML_t + \beta_{SMB,i}SMB_t + \beta_{WML,i}WML_t + \beta_{VMC,i}VMC_t + \epsilon_{i,t}
\]

The VMC factor is the monthly premium on volatile minus calm (VMC) equity indices and is obtained by subtracting the equal weighted average return of the highest volatility equity indices from the equal weighted average return of the lowest volatility equity indices. The definition of high or low volatility is based on 63 days realized volatility calculated separately for each equity index.

The detailed procedure of calculating HML, SMB, VML and VMC risk factors and definitions of zero-investment portfolios based on them is summarised in Section 3.1.
Since we want to take into account impact of different market environments on the factor sensitivities, we consider adding to the model a regime switching mechanism. The first attempt to capture a change in the volatility regime focuses on including different dynamics of equity risk premia in: 1) high and low volatility environment and 2) during upward and downward movements of the market. We add dummy variables with appropriate interactions and the functional form of the regression is then as follows:

\[(R_i - R_f)_{i,t} = \alpha_i + \beta_{MKT,i}(Rm - Rf)_t + \beta_{HML,i}HML_t + \beta_{SMB,i}SMB_t + \beta_{WML,i}WML_t + \beta_{VMC,i}VMC_t + \gamma_iD_t + \gamma_{MKT,i}(Rm - Rf)_tD_t + \gamma_{HML,i}HML_tD_t + \gamma_{SMB,i}SMB_tD_t + \gamma_{WML,i}WML_tD_t + \gamma_{VMC,i}VMC_tD_t + \varepsilon_{i,t}\]  

We consider two alternative definitions of the dummy variable used in equation (6):

- \(D_t = 1\) for high volatility periods, for low volatility periods, where the division is based on realized volatility calculated in USD for the market index and the periods brackets were defined \(ex-ante\),

- \(ex-post\) identification of upward (\(D_t = 1\)) and downward (\(D_t = 0\)) movements of the market\(^2\). Such division is especially important when we perform analysis between 2000 and 2015 since this period of equity markets was characterized by two strong bull and two strong bear markets which was not observable before within such relatively short data span.

The last form of the multi-factor model tested in this paper is a simple form of Markov switching model with two states of the world. For this purpose we used five-factor model in the form presented in equation (6). The rationale for the selection of the last model was based on several studies which showed among other things that various types of regime switching models can be useful in explaining equity risk premia Tan (2013). Ammann and Verhofen (2006) revealed that value investing seems to be a rational strategy in the High-Variance Regime, while momentum investing in the Low-Variance Regime. They additionally presented an empirical out-of-sample backtest indicating that this switching strategy can be profitable. Moreover, Angelidis and Tessarakomatis (2014) show that there are significant costs to investors who fail to take into account the existence of regimes in portfolio construction and asset allocation. Hammerschmid and Lohre (2014) showed regime shifts is preserved in the presence of fundamental variables known to predict equity risk premia.

Hence, the complete list of models used in this study is presented below:

1. Carhart.4F.localcurncy (four-factor model, factors based on local currency),
2. Carhart.4F.USD (four-factor model, factors based on USD),

\(^2\)In the course of our analysis, periods of high/low volatility occurred to coexist with periods of upward and downward market movements.
3. Carhart+VMC.5F.USD (five-factor model, VMC added, factors based on USD),
4. Carhart+VMC.5F.dummyRVinteract.USD (five-factor model, VMC and volatility
dummies added, factors based on USD),
5. Carhart+VMC.5F.dummyUp&Downinteract.USD (five-factor model, VMC and mar-
et trend dummies added, factors based on USD),
6. Carhart+VMC.5F.MarkovSwitch2Reg.USD (five-factor model, VMC and added, fac-
tors based on USD)

2.4 Methodological and diagnostic issues

In the process of estimation of multi-factor models using time-series data, we can potentially
suffer from several econometric problems or issues which should be solved in the process of
estimation (possible ARCH effect, autocorrelation, heteroscedasticity of the error term or
differences between various methods of estimation of our models). Surprisingly, this issue
is barely ever discussed in the literature of multi-factor models.

An attempt to estimate our regressions correctly with full econometric diagnostics takes
us to the point where we should go one of the below paths:

1. To estimate models with the same functional forms and compare their results across
   all markets, ignoring any diagnostic issues, as it is presented in financial literature for
   years.

2. To perform all diagnostics concerning time-series issues and correct the first esti-
   mations, which will most probably result in different model functional forms across
   investigated markets and hence make it difficult to compare results for them.

Taking into account that we do intend to compare single alpha, beta coefficients and R
squared coefficient among equity indices, we decided to choose the first approach. We believe
that this allows to analyze explaining power of models estimated for different markets.
Nevertheless, the issue of performing model diagnostics seems to be important and definitely
should be addressed in future research.

3 Data

We gathered the data for the most comprehensive set of investable equity indices\textsuperscript{3} covering
the period between 1990 and 2015. However the study was intentially limited to 2000-2015
because of unavailability of longer time series for some of risk factors, especially for emerging
markets countries.

The analysis was performed on weekly data for 81 most representative and investable
equity indices, covering all continents. We include data for 27 developed and 54 emerging
markets indices. The detailed list of all equity indices and their descriptive statistics can
be obtained upon request.

The reason behind selection of weekly instead of monthly data was the intention to
evaluate theoretical value of excess returns for the given equity index more frequently. All
returns and risk factors, with exception of the four-factor model of Carhart with factors

\textsuperscript{3}For practical purposes we used only these indices which can be easilly invested through options, futures
or ETFs.
based on local currency (Carhart.4F.localcurncy), were calculated after converting local prices to USD. Surprisingly however, results did not differ significantly between the same model calculated in local currency and in USD.

3.1 Description of risk factors

Detailed analysis of dynamics of standard four factors from the Carhart model helped us to define the final specification of the five factor model. Below we present the detailed description of procedure of calculating HML, SMB, VML and VMC risk factors, definitions of zero-investment portfolios based on them and then our observation concerning these factors dynamics.

The \((Rm - Rf)\) factor represents weekly excess return of the market portfolio over the risk-free rate. The market portfolio consists of equally weighted all 81 equity indices.

The HML is a zero-investment portfolio that is long on the highest decile group of book-to-market (B/M) equity indices and short on the lowest decile group. The difference of returns of these extreme decile groups is calculated in each weekly interval, which finally constitutes HML factor. Based on these returns we created cumulative returns for HML and then LMH zero investment portfolio.

The SMB is a zero-investment portfolio that is long on the highest decile group of small capitalization (cap) equity indices and short on the lowest decile group. The difference of returns of these extreme decile groups is calculated in weekly interval as well. Similarly, based on these returns we created cumulative returns for SMB and then BMS zero investment portfolio.

The WML is a zero-investment portfolio that is long on the highest decile group of previous 1-year return winner equity indices and short on its lowest decile group (loser equity indices). The difference of returns of these extreme decile groups is calculated again for each weekly interval and based on that we create cumulative returns for WML and then LMH zero investment portfolio.

Finally, the VMC is a zero-investment portfolio that is long on the highest decile group of high volatility equity indices and short on its lowest decile group (low volatility equity indices). The difference of returns of these extreme decile groups is calculated again for each weekly interval and based on that we create cumulative returns for VMC and then CMV zero investment portfolio.

3.2 Analysis of risk factors dynamics

Figure 1 shows the dynamics of the market index factor \((Rm - Rf)\) and market index returns \((Rm)\). We cannot observe any substantial differences between them. This actually informs us that we analyzed the period of exceptionally low rates, and that interest rates had only marginal impact on the value of this factor.
Figure 1: Dynamics of cumulative Rm-Rf factor and separately for market index (Rm).

Rm-Rf factor was calculated on weekly data in USD between 2000-2015. Rm represents equally weighted market index based in USD. Lines present cumulative returns for Rm-Rf and Rm factors.

Figure 2 presents fluctuations of the second factor ($HML_t$). It reveals two distinct periods. The first one (2000-2011) shows a strong HML effect showing much better performance of equity indices with high book-to-market characteristics. Similar phenomenon has been heavily presented in the literature for stock returns. In the second period, starting from 2012, the HML effect disappeared and has been entirely transformed into the LMH effect which is quite surprising and requires additional research.

Figure 2: Cumulative returns of HML factor with top/bottom 10% percentiles

HML factor was calculated on weekly data between 2000-2015. Lines present cumulative returns for, respectively, HML, LMH, top and bottom book-to-market values decile portfolios.

Figure 3: Fluctuations of the third risk factor ($SMB_t$) are presented in the Figure 3. Again, it can be divided into two periods. The first one (2000-2006) is characterised by
outperformance of small capitalisation equity indices what was revealed in the literature for single stocks. In the second period (2006-2015) this effect is quite reversed and we can observe high outperformance of big capitalisation equity indices.

Figure 3: Cumulative returns of SMB factor with top/bottom 10% percentiles.

SMB factor was calculated on weekly data in USD between 2000-2015. Lines present cumulative returns for, respectively, SMB, BMS, top and bottom capitalization decile portfolios.

The fourth risk factor \(WML_t\) shows that WML effect is the strongest one (Figure 4) and that it is relatively stable during the whole period and exactly confirms the short-term momentum effect observed in financial literature.

Figure 4: Cumulative returns of WML factor with top/bottom 10% percentiles.

WML factor was calculated on weekly data in USD between 2000-2015, with returns based on last 1 year. Lines present cumulative returns for, respectively, WML, MLW, top and bottom momentum decile portfolios.

Finally, the fifth risk factor \(VMC_t\) reveals similar dynamics to HML and SMB effect
VMC factor was calculated on weekly data in USD between 2000-2015. Lines present cumulative returns for, respectively, VMC, CMV, top and bottom 63 days realized volatility decile portfolios.

(Figure 5) dividing the period into two different subperiods. The first one ends exactly before bear market in 2008 and is characterised by outperformance of high volatility equity indices. In the second period (2008-2015) this effect is exactly reversed and we can observe high outperformance of low volatility equity indices.

Presented dynamics of five risk factors suggest that their explanatory power with respect to excess returns can be rather limited with exception of the first factor. The analysis of fluctuations of portfolios based on our risk factors in USD in comparison to their dynamics in local currency (Sakowski, Slepaczuk, and Wywial (2015)) reveals very similar dynamics. This informs us that currency effect is not the main driver which can be used in order to explain these effects.

4 Results

4.1 Comparison of various models’ results

After estimation of six multi-factor models for 81 different countries it is really hard to present results in a form that is understandable to the reader. For clarity purposes, we start with comparison of densities of Rsquared coefficients for all models separately for developed and emerging markets (Figure 6).
Densities of R Squared coefficients for emerging markets are marked with red colors palette, while densities of R Squared for developed countries are marked with blue colors palette.

Such comparison reveals our first conclusion that the results of six multi-factor models (including regime switching mechanism as well) do not differ significantly for all countries. This conclusion does not change when we analyse results separately for emerging and developed countries.

However, focusing on the results between two groups of emerging and developed countries we come to two observations.

First, the highest explanatory power of the five-factor model can be observed for developed equity indices. In this group almost all Rsquared values are higher that 50%. On the other hand, for emerging markets they get much lower values. This conclusion does not differ when we analyse the results of different models tested.

The second issue noticed here is that multifactor models have correct functional form for developed countries while they could be misspecified for the emerging markets subgroup. The reason for this difference could be that majority of all models proposed during the last 30 years were prepared for developed countries on the basis of empirical investigations of developed market data while emerging market data were practically unavailable. In the final part of results section we try to present rationale for this phenomenon.

Taking into account the fact that one of our main outcomes is that results do not differ significantly between tested models we decided to focus in detail on interpretation of the five-factor model (3. Carhart+VMC.5F.USD). Our results for equity indices are in many ways similar to well know studies for stock returns (Lieksnis (2010), Davis, Fama, and French (2000)), however they do not reveal so strong effects connected with our risk factors as was presented in the literature before.

In order to draw more conclusions with regard to different results for developed and emerging markets, we analysed the densities of parameters estimates and Rsquared coefficients separately for these two types of equity indices (Figure 7).

Our additional conclusions can be summarized as follows:

1. The results of regressions for developed countries with highest Rsquared coefficients
Figure 7: Density of parameters estimates and Rsquared separately for developed and emerging equity indices.

The data cover the period between 2000-2015. Five-factor model. Factors based on USD.
have negative (but close to zero) alpha coefficients (significant in 50% of cases) which informs us that there was no any excess returns which were not explained by our five-factor model. On the other hand, on average alpha coefficients for emerging equity indices are positive but still rather insignificant.

2. Beta for \((R_m - R_f)\) factor is on average higher for developed countries and additionally less diversified across countries in comparison to emerging markets,

3. The sensitivity to HML factor is similar for developed and emerging markets, however again it is much more diversified for emerging equity indices.

4. The average values of SMB beta are negative for developed countries and lower in comparison to emerging markets, however their diversity is much higher for emerging markets as well.

5. Characteristics of WML beta estimates is very similar to HML but the difference between average values of beta and the diversity of betas is even larger.

6. The only important observation concerning VMC beta is that one more time dispersion is much higher among emerging markets,

7. Separate densities for Rsquared for developed and emerging markets confirmed previous observations that regression for developed markets have higher explanatory power than these for emerging markets.

These observations suggest that the five-factor model can be a quite robust approach for developed markets and with high explanatory power. However, it should be amended and enhanced with additional risk factors and probably some state variables for emerging markets.

4.2 Explanation of results

Having analysed our results we can ask a natural question why they are different than those presented in the literature for single stocks. We’d like to investigate why our multi-factor models are not able to explain variability of excess returns for emerging markets.

We see four possible explanations for these observations.

The first one is the different time span. In most of previous studies the data covered more than 80% of bull markets - from late 1960 until the beginning of 2000s (Figure 8). Contrary to that, in our research we had two distinct bull markets and two distinct bear markets what results in rather horizontal long-term trend during the last 15 years. These could be the reason for substantially lower R-squared in our research when compared with results of studies for equity stock returns.
The second reason could be different explanatory power of risk factors in a strong upward trend vs up and down movements. This is well illustrated on Figure 9.

Figure 9: Risk factors from the five-factor model versus market index returns (Rm).

This chart presents risk factors dynamics in the period between 2000-2015.

The third explanation is that most of previous studies used the data for the developed markets. Moreover, various modifications of multifactor models (i.e., additional factors, functional form) were introduced based on the analysis of such data, what in our opinion illustrates overfitting bias.

Lastly, the reason could be also different time frequency. We used weekly data instead of monthly data as we want to explain variability of excess returns on more frequent basis.
The interpretation presented above is only a possible explanation of different results in our research. However, if it turns out to be the correct one, then it is a very convincing example of data overfitting problem and model risk.

5 Summary

It is important to underline that these results are only the first part of rather larger attempt to fully understand reasons behind cross-section variability of world equity indices excess returns.

The most surprising issue concerning our results is that the difference between various multi-factor models are not significant and that we observe substantial differences between model explanatory power for developed and emerging markets. Therefore, to summarize our results we can note that the results of alternative multi-factor models do not differ significantly for all countries. This conclusion does not change when we analyze results separately for emerging and developed countries.

Our second observation is that the highest explanatory power of the five-factor model is observed for developed equity indices. In this group almost all R-squared values are higher than 50%. On the other hand, for emerging markets they get much lower values. This conclusion does not differ when we analyse the results of different models tested.

These two points lead us to the third observation that: multi-factor models do have a correct functional form for developed countries, while they could be misspecified for the emerging markets subgroup. Therefore, we claim that results for emerging market equity indices require further investigation and future research should be focused mainly on two issues.

The first one are additional factors, where we can investigate liquidity risk (Rahim and Noor 2006, Liu 2004), return-on-equity effects, earning surprises or macro surprises, systemic risk, liquidity risk or betting-against-beta effects (Frazzini, Pedersen 2014)). Secondly, researchers should concentrate on the novel model implementations concerning its functional form and introducing state variables.

To conclude, we believe that further research should additionally address the questions whether:

- sensitivities to risk factors are stable during various phases of economic cycles,
- correlations among international equity markets differ between high and low volatility periods,
- we can build a zero investment portfolio with positive alpha based on analysed risk factors,
- we can identify risk factors that are important in a process of portfolio construction.
References


