Corporate governance, tax evasion and business cycles

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\textbf{Abstract:} We develop an agency model of corporate tax evasion and auditing by a residual claimant government and embed it to a macroeconomic environment characterised by credit constraints. In our economy, tax auditing by the government reduces the information asymmetry between lenders and entrepreneurs with an investment opportunity. Corporate governance quality consequently affects macroeconomic variables; with changes in tax rates, auditing and quality of corporate governance having aggregate effects. We show that changes in the revenue system; tax and audit rates, can directly affect asset prices and inflate the effects of exogenous shocks to the economy.

\textbf{Keywords:} corporate governance, credit constraints, taxation, asset pricing, tax evasion, agency problem

\textbf{JEL codes:} H2, H26, H3, E13, E26, J81

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1 Introduction

This paper presents a model that links the quality of corporate governance and the tax system to macroeconomic outcomes. We do this within a framework of an agency model of credit rationing. Our analysis is inspired by the tax claim a government has to firm profits. As noted by Desai, Dyck and Zingales (2007), the state can be thought of as the largest minority shareholder of most corporations due to its tax claim on profits. When there is an asymmetric information problem between lenders and borrowers, then the state, by occasionally auditing borrowing firms, provides the services of a typical “monitor” in the sense of models of managerial incentives or compensation (Hanlon, Hoopes and Shroff, 2014; Holmstrom and Tirole, 1997).

The classic principal-agent problem in corporate finance focuses on how to design contracts that align owners and managers interests. One way of overcoming the agency problem is to have some shareholders engage in the monitoring of management. In the credit rationing models of Holmstrom and Tirole (1997), the monitor can be a large shareholder who solves the asymmetric information problem at a cost. However, in the presence of taxation with the possibility of auditing, the asymmetric information problem can also be resolved by the government or tax authority that audits firms to determine if profit has been truthfully reported. We show in this paper that the possibility of auditing by the tax authority helps to reduce moral hazard in a setting where “entrepreneurs” are credit constrained.

We embed our agency problem into the DSGE model of financial frictions developed by Kiyotaki and Moore (2012, hereafter KM). However, unlike the KM model where the financial friction in the form of a credit constraint is calibrated from the data, our model is designed so that the level of the constraint is an equilibrium outcome. In our version of the KM model, the fraction of future earnings that an entrepreneur can pledge to lenders, the mortgageable fraction of equity, is a function of the quality of corporate governance, the tax system and the equilibrium equity price. We perform a detailed analysis of the principal agent problem that gives rise to the credit constraint and show how changes in governance institutions and tax rates have both a level and amplification effect on output, investment and asset prices.

In our economy, ex-ante identical entrepreneurs have access to two types of risky capital production technologies, a safer and riskier one. The riskier technology is less costly but provides an opportunity to extract private benefits: an entrepreneur who has used the riskier technology can claim the higher cost associated with the safer one when declaring profits. The quality of corporate governance determines what fraction of the cost difference between the two technologies can be diverted as a pure private benefit to the entrepreneur. However, since using the higher cost in profit declaration is tax evasion, the entrepreneur faces a risk of discovery upon auditing by a tax authority.
authority. The audit would reveal her true cost and technology, both to the tax authority and lenders. Consequently, tax rates, audit frequencies and surcharges for evasion, together with corporate governance quality engendered in the level of private benefits will determine an entrepreneur’s incentive to choose between the two technologies. These choices determine how much funds can be raised to invest and the aggregate level of investment, capital and output. High quality corporate governance accompanied by high tax rates increase investment, output and consumption while lowering asset prices while reducing evasion. Low taxes and a poor corporate governance environment have the opposite effect.

The mechanism driving our results is as follows. When taxes and the penalties for tax evasion are low, entrepreneurs have a higher incentive to use riskier capital production technologies and enjoy the associated private benefits. Since investment is now more risky, entrepreneurs need to raise a higher value of internal funds in order to credibly signal their commitment to the safe technology, so the mortgageable fraction of equity falls. This results into a lower equilibrium aggregate capital level and consequently lower investment, output and consumption. When an economic shock occurs under low taxes, the fall in investment is deeper and the recovery slower as riskier capital production technologies imply additions to the aggregate stock of capital can only occur gradually. The converse is true under high taxes and higher quality governance institutions, which discourage the use of risky technologies. We now provide a brief survey of the literature that relates our work to empirical findings on corporate governance, tax auditing and the cost of credit.

Relation to the literature  Our paper is based on the agency model of Holmstrom and Tirole (1997). Since tax auditing is necessary to reveal evasion, the costly state verification models of corporate finance provide a natural point of analysis. In the literature, various authors provide analyses of the role of taxes in management or entrepreneurship decisions. Chetty and Saez (2010) show that taxes on dividends create a dead weight loss in an agency model where managers and shareholders have conflicts of interest. Dividend taxation distorts the managers payout decision which causes over investment in unproductive projects from retained earnings. Cullen and Gordon (2007) show that taxes have an effect on entrepreneurial risk taking through the differential treatment of investment and labour income in the tax code. Frydman and Hilt (2017) study the historical role of investment banks as “corporate monitors” and show that investment and leverage declined following the Clayton Antitrust Act that prohibited banks from serving in the boards of US railroad companies for which they underwrote securities.

Within our main focus of tax evasion, two papers have some bearing on our analysis. Chen and Chu (2005) consider the principal-agent problem of shareholder sanctioned corporate tax evasion
with delegated management. The optimal contract between owners and managers is incomplete as
the manager must be offered inducement to not cooperate with tax authorities. This creates a loss
of internal control and increases retained earnings. Bayer (2006) develops a model of competition
between a taxpayer and the tax authority in detection of concealed income. Similar to the game
in subsection 2.2 of our paper, there is a tax-authority (principal) and a taxpayer (agent) with
strategies of auditing and truthful reporting respectively. However, the game here is sequential
with the taxpayer moving first and then the authority deciding whether to audit or not. Their
model, like ours, predicts more tax evasion if tax rates are raised and that the tax payer always
evades at least with a very small probability. Finally, our results are related to the findings of Li,
Rocheteau and Weill (2012) on liquidity and fraudulent practices. In their model, assets such as
equity or capital face different resaleability constraints due to the potential for fraud. In the context
of our model with equity being the claim to lender’s share of the outcome of an investment project,
the possibility of tax evasion or low tax audits, makes it more difficult to predispose of equity
holdings.

Our paper is also related to Artavanis, Morse and Tsoutsoura (2016) who use a bank-underwriting
model to infer the level of tax evasion based on lending. The key innovation of their paper
is the use of a bank’s loan to income ratio; which should not be exceeded if the bank were to
manage default risk. The loan-to-income ratio is similar to a leverage ratio that would be found
in standard macroeconomic models of the credit constraint (see e.g. Kiyotaki and Moore, 1997a).
Our macroeconomic model, based on Kiyotaki and Moore (2012), has the concept of a leverage
ratio; an investing entrepreneur can only credibly pledge a fraction of the future returns of their
project. However, unlike their work and many papers that analyse credit constraints, we provide
micro-foundations that can explain why the leverage ratio may vary across firms, industries and or
countries. In our model, this ratio will be affected by parameters of the economy such as the level
of taxes, the ease of evasion and/or the success of audits.

There is empirical evidence to support the notion that the ease of financing is related to the level
of tax evasion or quality of corporate governance. In a cross country analysis, Beck, Lin and Ma
(2014) find that firms with better credit information sharing systems evade taxes to a lesser degree.
Our work also provides a theoretical foundation to the findings of Guedhami and Pittman (2008)
and Graham, Li and Qiu (2008) on how, respectively, tax auditing and corporate misreporting
affect yield spreads. Guedhami and Pittman (2008) find that debt financing is cheaper when a
firm faces a higher probability of an IRS audit; with the impact larger for firms with concentrated
ownership. The lower cost of funds for high audit probability firms arises from the reduction in
information asymmetry between investors and the management (or controlling shareholders). The
size of the economic effect is large: increasing IRS audit probability from 19% to 35% reduces
the interest rate of a firm by on average 25 basis points. Graham et al. (2008) study the effect of financial restatement as a result of an audit on bank loan contracting. They find that loans initiated after a restatement have higher spreads, shorter maturity and a higher likelihood of being secured. Similar findings have been made by Karjalainen (2011) and Hasan, Hoi, Wu and Zhang (2014). Bolton, Freixas, Gambacorta and Mistrulli (2016) provide results within the monitoring framework through their findings on relationship banking and lending over the business cycle. In their work, relationship banks by gathering information about their borrowers are able to funnel funds to their clients at lower rates compared to transaction banks during a crisis.

Finally, our paper is related to the large macroeconomic literature on the role credit constraints and liquidity shocks have in amplifying aggregate fluctuations. This literature includes the works of Chen (2001); Minetti (2007), Bolton and Freixas (2006), Kiyotaki and Moore (2012) and Rocheteau and Wright (2013) amongst others. The first two authors use the continuous investment environment of Holmstrom and Tirole in dynamic general equilibrium models with a banking sector to show how credit constraints interact with lending to enhance the effects of negative productivity shocks or technology shocks. In Bolton and Freixas, asymmetric information and information dilutions costs as in Myers and Majluf (1984) make it expensive for banks to raise capital and extend credit. The last two papers, examine the interaction of liquidity and asset prices in monetary economies. Since the Great Recession of 2007, there has been a renewed interest in macroeconomic models that link liquidity to the severity of downturns. Our work is related to this literature. Bigio (2015) studies an economy where asymmetric information about the quality of capital determines liquidity. Like in our case, his model involves an interaction between asymmetric information and limited enforcement. Benhabib, Dong and Wang (2018) add a simple asymmetric information problem to a standard RBC type model. Their economy feature a fixed and endogenous measure of dishonest and honest borrowers, respectively. Their endogenous measure of honest borrowers is conceptually similar to our fraction of risky, possibly tax evading entrepreneurs.

The remainder of the paper is structured as follows. In Section 2 we describe the economic environment: the agency problem (2.2) and macroeconomic equilibrium (2.3). We then calibrate the model and perform quantitative analysis in Section 3. Section 4 concludes.

## 2 The Economic Environment

Our modelling approach is based on the continuous investment environments of Holmstrom and Tirole (1997); Tirole (2006, Chapter 3). We begin the analysis in an environment without taxes
then introduce taxation with costly state verification (audits). We first describe the agency problem in the market for loanable funds and how the introduction of taxation with the possibilities of evasion and auditing affects how much financing an entrepreneur can raise. We formalize this credit constraint as a multiplier to the entrepreneur’s initial assets and show how this multiplier depends on the parameters of our economy.

Having described the agency problem, we embed it to the macroeconomic environment of Kiyotaki and Moore (2012) with liquidity shocks and credit constraints but stripped of money (this aspect of the model is not relevant to our purposes). In the Kiyotaki and Moore model, entrepreneurs use their own skill and capital stock to produce output with inelastic labour supply. Capital depreciates and is replenished through investment, but the investment technology, for producing new capital from output, is not commonly available: in each period, only some of the entrepreneurs are able to invest, and the arrival of investment opportunities is randomly distributed across entrepreneurs through time. As a result, there is need to shift savings from those without investment opportunities (savers/lenders/investors) to those who do (entrepreneurs/borrowers). Unlike in the Kiyotaki and Moore model, where the credit constraint is exogenously specified and parametrized as a number smaller than unity, our agency problem in subsection 2.2 describes how the credit constraint is related to aspects of corporate governance and the taxation system. The macroeconomic equilibrium is then described in subsection 2.3.

### 2.1 Macroeconomic Environment

The basic continuous investment model has been extended to a macroeconomic setting to study asset prices, liquidity and aggregate economic activity in the papers of Chen (2001) and Minetti (2007). However, similar extensions using the more elegant exposition of Kiyotaki and Moore (1997b) have been made by Tirole (2006, Chapter 14). The interaction of aggregate liquidity with aspects of corporate governance has also been investigated by Holmstrom and Tirole (2013) using the Diamond-Dybvig model and the model of Woodford (1990). In a series of papers, Eisfeldt and Rampini (2006,0,0) investigate how the moral hazard problems between managers and investors affect capital reallocation over the business cycle. We use a macroeconomic environment similar to that of Kiyotaki and Moore (2012) with slight modifications.\(^1\) Their model fits our conceptual framework since as noted by John Moore in his Clarendon Lectures, a variety of moral hazard

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\(^1\)Many papers investigate how credit constraints and/or liquidity shocks affect investment and the business cycle. Models with some form of asymmetric information in the economic environment include the early work of Carlstrom and Fuerst (1997) and more recently Eggertsson and Krugman (2012); Li, Rocheteau and Weill (2012); He and Krishnamurthy (2013); Malherbe (2014); Atolia, Einarsson and Marquis (2015). All these could suit our purposes. For a more comprehensive list, see the survey papers by Gertler and Kiyotaki (2010) and Goldstein and Razin (2015, Sec. 3).
assumptions are needed to justify the existence of a credit constraint.

Consider an infinite horizon discrete time economy with two objects traded: non-durable output and equity. At each time period, there are two types of agents, entrepreneurs and investors/savers. At date \( t \), a typical entrepreneur’s discounted utility is:

\[
\sum_{j=0}^{\infty} \beta^j \log c_{t+j}
\]

where \( \{c_t, c_{t+1}, \ldots \} \) is a consumption path and \( 0 < \beta < 1 \). All entrepreneurs have access to a decreasing returns to scale technology of producing output from capital. An entrepreneur holding \( k_{j,t-1} \) units of capital can produce \( \chi_t k_{j,t-1}^\phi \) units of output where \( \phi \in (0, 1) \) and the productivity parameter \( \chi_t > 0 \) is common across all entrepreneurs and follows a stationary stochastic process. There is a continuum of unit measure capital holders and aggregate output is given by:

\[
Y_t = \left( \int_0^1 \chi_t k_{j,t-1}^\phi d j \right)^{\frac{\alpha}{\phi}} = \chi_t^\alpha K_t^{\alpha}
\]

where \( \alpha \in (0, 1) \) and \( K_t = \int_0^1 k_{j,t} d j \) is aggregate capital. Production is completed within the period \( t \) during which capital depreciates to \( (1 - \delta)k_{t-1} \) for \( \delta \in (0, 1) \). The return to capital \( r_t \) depends upon the productivity \( \chi_t \) and the aggregate amount of capital.

The entrepreneur also has an opportunity to produce new capital. At each date \( t \) with probability \( \pi \), she has an opportunity to produce \( i_t \) units of capital from some units of output. The arrival of such an opportunity is independently distributed across entrepreneurs and through time while also being independent of the productivity. Newly produced capital is available for production the next period:

\[
k_t = (1 - \delta)k_{t-1} + i_t
\]

In order to finance the cost of investment, the entrepreneur who has an investment opportunity may issue equity claims to the future return of newly produced capital. One unit of equity is the claim to the return of one unit of investment: it produces \( r_{t+1} \) units of consumption goods at date \( t + 1 \), \( r_{t+2}(1 - \delta) \) units at date \( t + 2 \), \( r_{t+3}(1 - \delta)^2 \) at date \( t + 3 \) and so on.

We make two critical assumptions. First, we will assume that the entrepreneur with an investment opportunity faces a liquidity constraint: she can only sell a fraction \( \phi_t < 1 \) of her current equity holdings to raise funds for financing the investment. This assumption means the entrepreneur cannot self finance and will need to borrow. Second, the entrepreneur has access to two risky technologies of producing capital, a “safer” and “riskier” technology. As we describe in the next subsection, the safer technology uses a unit of the consumption good to produce at
least a unit of the capital with a high success probability while the riskier technology uses less than a unit of the consumption good to produce at least a unit of capital but with a lower success probability. We further assume that investing has positive NPV only when the safer technology is used and that outsiders cannot freely observe the type of technology that has been chosen. Since the entrepreneur cannot pre-commit to use the safer technology, she must be given an incentive to do so. This “incentive compatibility” constraint together with a potential lender’s “breakeven” condition creates a credit constraint: the entrepreneur can credibly pledge only a fraction $\theta_t < 1$ of future returns.

Let $a_t$ be the investing entrepreneur’s holding of equity (which may be of her own or others’ capital) at the end of period $t$. To finance investment at the beginning of the period, she can dispose of a fraction $\phi_t$ of her depreciated holdings from the previous period: $(1 - \delta)a_{t-1}$. She can pledge only a fraction $\theta_t$ of the gross return from investing, so at the end of the period, she must hold a fraction $(1 - \theta_t)$ of claims to the newly created capital $i_t$. These two “liquidity constraints” imply that

$$a_t \geq (1 - \theta_t)i_t + (1 - \phi_t)(1 - \delta)a_{t-1}$$

In Kiyotaki and Moore (2012), the parameters $\theta_t$ and $\phi_t$ are exogenous. In our model, the assumptions on capital production technologies create an agency problem between the entrepreneur and potential lenders. The value of $\theta_t$ will consequently depend on aspects of the corporate governance and taxation system of the economy. We develop this relationship with the tax system in the next subsection where we introduce taxation and auditing by the tax authority after discussing the basic agency problem. The tax system enters into the agency problem because an entrepreneur who has used the riskier technology has an incentive to report the higher cost associated with the safer technology i.e. evade taxes. But auditing can reveal the true investment technology used by the entrepreneur and as such ease the borrowing constraint. In other words, the tax authority’s acts as a typical “monitor” in standard models contracting with asymmetric information (e.g. Holmstrom and Tirole, 1997). Thinking of the tax authority as a monitor is not controversial and has been previously discussed in the literature (Desai et al., 2007).

### 2.2 The Agency Problem

**Entrepreneur** An entrepreneur has access to two types technologies for producing capital: safe and risky. The safe technology uses $i_t$ units of the consumption good to produce $Ri_t$ units of capital with probability $p_H$ or 0 with probability $(1 - p_H)$. The risky technology uses $i_{L,t} < i_t$ units of the consumption good to produce $Ri_t$ units of capital with probability $p_L < p_H$ or 0 with probability $(1 - p_L)$. The subscripts $\{H, L\}$ denote high and low respectively. If investment occurs at the
beginning of a period \( t \) and is successful, \( R_i \) units of new capital are available at the end of the period. Let \( q_t \) be the price of a unit of capital at the end of date \( t \). The gross return to investment over the period is:

\[
\text{Gross Return} \equiv q_t R_i
\]

The rate of return to investment is greater than the rate of return to capital (\( R > 1 + r_t - \delta \)), so the entrepreneur will always want to invest as much as possible: \( i_t \in [0, \infty) \). Let \( a_t \) denote her initial assets. These assets can either be invested in a technology or used for consumption. In order to invest, the entrepreneur needs to liquidate her assets into consumption goods. However, she faces the following liquidity constraint: only a fraction \( \phi_t \) of the assets can be immediately sold to raise funds for investment. Then the investor can raise \( A_t = [r_t + q_t \phi_t (1 - \delta)] a_{t-1} \) units of the consumption–investment good. In order to invest \( i_t > A_t \), the entrepreneur will need to borrow the amount:

\[
\text{Borrowing} = i_t - [r_t + q_t \phi_t(1-\delta)] a_{t-1} = i_t - A_t
\]

where \( A_t \) is the entrepreneur’s “net worth” or “cash-in-hand”. Whenever investment is undertaken, the entrepreneur has private information about the technology she has used. If she chooses the safe technology, then the net return is \((q_t R - 1)i_t \). If she chooses the risky technology, then the net return is \((q_t R_i - i_{L,t}) = (q_t R - 1)i_t + B_{i_t} \), where \( B_{i_t} = B(i_t - i_{L,t}) \) and \( 0 < B < 1 \). The private benefit \( B_{i_t} > 0 \) is not observable to any potential lender.

**Lenders** Both the entrepreneur and lenders (or investors) are risk neutral. Events take place within the period \( t \) so there is no time preferences and the borrower is protected by limited liability (entrepreneur income cannot take on negative values). For now we drop the time subscript \( t \) for ease of exposition. Lenders are competitive and make zero profit. The loan contract between the lender and the borrower stipulates the following. First the contract specifies whether investment is financed and then if so, how the profits are shared between lenders and borrower. The borrower’s limited liability means the in case of success, the two parties share profits \( R_i \) (the verifiable amount); \( R^b \) goes to the borrower and \( R^l \) goes to the lenders, so \( R^b + R^l = R_i \). The incentive scheme for the entrepreneur is of the following form: \( R^b \) in the case of success and 0 in the case of failure. The zero profit condition for the lenders can therefore be written as:

\[
p_H R^l = i - A
\]

assuming the loan contract induces the borrower to chose the safe technology The rate of interest is given by

\[
R^l = (1 + \text{Interest}) (i - A) \text{ or } (1 + \text{Interest}) = \frac{1}{p_H}
\]
which reflects a default premium: Interest = \( \frac{1}{p_H} - 1 > 0 \) unless \( p_H = 1 \); i.e., the interest rate exceeds the expected zero rate of return demanded by investors.\(^2\) We assume that the project has positive NPV per unit of investment only if the entrepreneur chooses the safe technology: \( p_H R_i - i > 0 \) or \( p_H R > 1 \) and negative NPV even when the entrepreneur’s private benefit is included: \( p_L R_i - i + B_i < 0 \).\(^3\)

The entrepreneur faces the following trade-off once financing has been obtained: by choosing the risky technology, she obtains private benefit \( B_i \), but reduces the probability of success from \( p_H \) to \( p_L \). Because she has a stake \( R_b \) in the firm’s income, the entrepreneur will choose the safe technology if the following “incentive compatibility constraint” holds:

\[ p_H R^b \geq p_L R^b + B_i \quad \text{or} \quad R^b \geq \frac{B_i}{\Delta p} \]  \hfill \text{(IC}_b\text{)}

The highest income in the case of success that must be pledged to the lenders without jeopardizing the entrepreneur’s incentive is \( qR_i - \frac{B_i}{\Delta p} \) and the expected pledgeable income is the

\[ \mathcal{P} \equiv p_H \left(qR_i - \frac{B_i}{\Delta p}\right) = p_H(R_i - R^b) \]

and because the lender must break even in order to finance investment, a necessary condition for the borrower to receive a loan is that the expected pledgeable income exceed the borrowed amount:

\[ p_H \left(qR_i - R^b\right) \geq i - A \]  \hfill \text{(IR}_l\text{)}

which is the individual rationality or “breakeven” or “participation” constraint. The necessary condition for financing to occur is that the entrepreneur has initial income

\[ A \geq \bar{A} = i - p_H(qR_i - R^b) \]

\[ = \left[ 1 - p_H \left(qR - \frac{B}{\Delta p}\right) \right] i = (1 - \theta)i \]

Equation (2) is the credit constraint, i.e.: \( A \geq (1 - \theta)i \). The entrepreneur’s share of future returns is the fraction of investment financed from her net-worth: \( \frac{A}{i} = (1 - \theta) \) and the lender’s share is \( \frac{i - A}{i} = \theta \). If financing occurs, then \( \bar{A} \geq 0 \), which implies

\[ 1 > p_H \left(qR - \frac{B}{\Delta p}\right) \quad \text{or} \quad \theta < 1 \]  \hfill \text{(3)}

\(^2\) In calibrating the model in Section 3, \( p_H = .95 \) which implies a risk premium or “interest rate” of \( r_t \approx 5\% \).

\(^3\)Using \( p_H = .95, R = 1.75 \) gives \( p_H R = 1.66 > 1 \) and \( p_L = .51, B = .06 \) gives \( p_L R - 1 + B = -.0475 \).
i.e. the NPV is smaller than the minimum expected rent that must be left to the borrower to provide her with an incentive to choose the safe technology. Condition (3) means that the borrower can lever her wealth to invest \( i \leq mA \) for some multiplier

\[
m = \frac{1}{1 - \theta} > 1 \quad \text{and} \quad \theta = p_H \left( qR - \frac{B}{\Delta p} \right)
\]

(4)

The multiplier is smaller the higher the private benefit \( B \) and the lower the likelihood ratio \( \frac{p_H}{\Delta p} \).

### 2.2.1 Taxation

Assume that the government has access to a proportional tax \( \tau \) on final profit so that Profit after Tax = \((1 - \tau)(qR - 1)i = R^b + R^l\). We will introduce the possibility of misreporting profit, but for now we maintain the assumption that the entrepreneur always reports her income truthfully. The incentive compatibility constraint becomes:

\[
p_H R^b \geq p_L R^b + (1 - \tau)Bi \quad \text{or} \quad R^b \geq \frac{(1 - \tau)Bi}{\Delta p}
\]

\((\text{IC}^b)\)

The breakeven condition for the lender to participate now becomes:

\[
p_H \left[ (1 - \tau)(R - 1)i - R^b \right] \geq i - A
\]

\((\text{IR}^l)\)

Again, the credit constraint is \( A \geq (1 - \theta)i \) and the maximum investment satisfies \( i \leq \bar{m}A \), where

\[
\bar{m} = \frac{1}{1 - \theta} \quad \text{and} \quad \bar{\theta} = p_H (1 - \tau) \left[ (qR - 1) - \frac{B}{\Delta p} \right]
\]

(5)

so that the multiplier is smaller the higher the tax on profit \( \tau \). This formalizes the ideas found in both the empirical and theoretical literature that corporate tax lowers investment.\(^4\) Note that the incentive compatibility condition \((\text{IC}^b)\) for the borrower would be equivalent to \((\text{IC}^b)\) if we did not assume truthful reporting of income.

### 2.2.2 Tax Evasion

We now introduce the possibility of evading tax by the entrepreneur together with audits or monitoring. We analyse the audit or monitoring decision from the perspective of corporate governance

\(^4\)See for instance Jaimovich and Rebelo (2017) on the role of taxes in reducing private incentives to invest. Heider and Ljungqvist (2015) show that an increase in taxes increases leverage: firms tend to use more debt rather than equity to finance investment.
due to the relationship between tax evasion and corporate governance described by Desai, Dyck and Zingales (2007). These authors note that the government, due to its tax claim on cash flows, is the de-facto largest minority shareholder in almost all firms. Since the theory of corporate finance has been developed around providing incentives to managers (entrepreneurs) not to engage in risky behavior, and not extract rents from investors, it is the right framework from which to analyze the issue of evasion. The relation to tax revenue is described by Desai et al. (2007) as follows: when the corporate governance system is ineffective (i.e. when it is easy for the entrepreneur to divert income, high $B$), an increase in the tax rate can reduce tax revenues while a decrease can have the opposite effect. This induces a hump-shaped relationship between corporate taxes and tax revenues.

**Random inspections or audits**  We assume that when an audit is undertaken, the monitor or government with probability $y$ discovers the undeclared private benefit $B$ (evaded amount) and with probability $(1 - y)$ discovers nothing. The probability of discovery depends on the amount invested in monitoring, a cost $: y = y(\cdot)$, with $y(0) = 0, y(\infty) = 1$ and $y'(\cdot) > 0$. If evasion is discovered, the entrepreneur faces a surcharge $s$ in addition to her tax liabilities. The assumption that audits only discover evasion with probability $y$ is without loss of generality equivalent to a model where firms are audited with the same probability and evasion is always discovered. Introducing audits in this manner makes the exposition simpler and more aligned with the corporate governance literature.

**Income declaration under auditing**  In the presence of audits, the entrepreneur can either declare her true income or report untruthfully. To formalize the choice, we write the entrepreneur decision as a game where she chooses the best response to the audit ($A$) or no-audit ($NA$) strategies of the tax authority (auditor/monitor). The entrepreneur’s strategies are to play truthful ($T$) and not-truthful ($NT$). We assume that when a firm is audited, it faces audit response cost proportional to its level of investment $i$ for some small number $\tau$. The game matrix is as follows:

<table>
<thead>
<tr>
<th>Firm</th>
<th>Tax Authority</th>
<th>Not Audit (NA)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Truth (T)</strong></td>
<td>$p_L R^b + (1 - \tau)Bi - \eta i$</td>
<td>$p_L R^b + (1 - \tau)Bi$</td>
</tr>
<tr>
<td><strong>No Truth (NT)</strong></td>
<td>$(\tau + s)Bi - \eta i$</td>
<td>$-(\tau + s)Bi$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Audit (A)</th>
<th>Not Audit (NA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau Bi$</td>
<td>$\tau Bi$</td>
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</tbody>
</table>
Entries inside the matrix are payoffs or gains to each player when they play a given strategy, where we follow the standard convention of left entries in each cell being the row player’s payoff. In the first-row first-column entry, the entrepreneur is audited even though she is reporting true income. The audit costs her $i$ while the auditor incurs the cost without any gains in tax revenue. Similar arguments apply to all other entries. The second row entries include the surcharge $s$ for tax evasion when detected.

The game has no Nash equilibrium in pure strategies (no saddle point i.e. the minimax and maximin values are not equal). To solve the game, we consider mixed strategies. Letting the entrepreneur declare her true income with probability $x$ while auditing takes place with probability $y$, the expected payoff of this randomising strategy is:

$$E(x) = x \left[ y(p_L R^b + (1 - \tau) Bi - i) + (1 - y)(p_L R^b + (1 - \tau) Bi) \right] + (1 - x) \left[ y(p_L R^b + (1 - (\tau + s)) Bi - i) + (1 - y) Bi \right]$$

(6)

The entrepreneur then chooses $x$ such that any changes in $y$ have no effect on $E(x)$ which occurs when $\frac{\partial E(x)}{\partial y} = 0$.\textsuperscript{5} This implies that the probability of declaring true income is given by:

$$x = 1 - \frac{i}{p_L R^b - (\tau + s) Bi}$$

(7)

which indicates that the probability of declaring truthful income is decreasing in the tax rate $\tau$ and the private benefit $B$ so that jurisdictions characterized by high tax and poor corporate governance are more likely to experience tax evasion. An increase in audit costs reduces the probability of truthful reporting: $\frac{\partial x}{\partial y} < 0$, in line with the theoretical results of Lipatov (2012).

**Pledgeable Income with Evasion and Random Audits** We now turn our attention to pledgeable income and the investment rate under tax evasion and auditing. Let $U^b_S$ and $U^b_R$ be the borrowers return when choosing the safer and riskier technologies respectively. When the safe technology is chosen, the borrower always reports truthfully so $U^b_S = p_H R^b$. Since when the risky technology is chosen, the borrower can either report truthfully with probability $x$ or untruthfully with probability $(1 - x)$ in which case an inspection discovers her evasion with probability $y$. Her expected return when the risky technology is chosen is:

$$U^b_R = x(p_L R^b + (1 - \tau) Bi) + (1 - x) \left[ y\left(p_L R^b + [1 - (\tau + s)] Bi \right) + (1 - y)\left(p_L R^b + Bi \right) \right]$$

\textsuperscript{5}See Chapter 7 of Binmore (2007).
The incentive compatibility for the borrower to choose the safe technology requires that $U^b_S \geq U^b_R$ which implies that:

$$R^b \geq \left( 1 - y(\tau + s) - x(\tau - y(\tau + s)) \right) \times \frac{Bi}{\Delta p} \equiv \omega \times \frac{Bi}{\Delta p} \quad (IC''_b)$$

Similar to the results in the corporate governance literature, the existence of audits reduces the private benefit enjoyed by the entrepreneur whenever $\omega \leq 1$ (see e.g. Admati, Pfleiderer and Zechner, 1994). While investors do not get a higher share of the profits because the private benefit is lower, the reduction in this benefit reduces the incentive to choose the riskier technology and consequently the amount of initial assets required by the lender in order to provide financing. An alternative interpretation, $\frac{\partial R^b}{\partial y} = - (\tau + s)(1 - x) \frac{Bi}{\Delta p} < 0$ means that increasing the audit probability by a small amount $dy$, allows the lender (principal) to reduce the transfer to the entrepreneur (agent) $R^b$ by an amount proportional to $(\tau + s)$ (see e.g. Laffont and Martimort, 2002, page 125). The breakeven condition for the lender to participate is:

$$p_H \left[ (1 - \tau)(qR - 1) - R^b \right] \geq i - A \quad (IR''_t)$$

Again, the **credit constraint** is $A \geq (1 - \tilde{\theta})i$ and the maximum investment satisfies $i \leq \tilde{m}A$, where

$$\tilde{m} = \frac{1}{1 - \tilde{\theta}} \quad \text{and} \quad \tilde{\theta} = p_H \left( (1 - \tau)(qR - 1) - \omega \times \frac{B}{\Delta p} \right) \quad (8)$$

Without auditing, $y = 0$ and nobody reports truthfully, $x = 0$ which gives $\omega = 1$ and $\tilde{m} > \tilde{m}$. Without tax evasion, $x = 1$ (truthful reporting) and $\omega = (1 - \tau)$ implies $\tilde{m} = \tilde{m}$. Since an increase in $\omega$ lowers the multiplier and hence investment; an increase in evasion which also lowers $\omega$ would decrease investment. An increase in the probability of being caught has a positive effect on the multiplier $\tilde{m}$:

$$\frac{\partial \tilde{m}}{\partial y} = \frac{p_H}{\tilde{m}^2} \left[ (\tau + s)(1 - x) \frac{B}{\Delta p} \right] > 0$$

so a higher probability of an audit revealing evasion increases the level of financing an entrepreneur can obtain. This result is similar to Ellul, Jappelli, Pagano and Panunzi (2016) who find that firms

---

6With $\omega = 1$, we have $\tilde{\theta} = p_H \left( (1 - \tau)(qR - 1) - \frac{B}{\Delta p} \right) < p_H \left( (1 - \tau)(qR - 1) - (1 - \tau) \frac{B}{\Delta p} \right) = \tilde{\theta}$. This happens because we assumed truthful reporting when introducing taxation. Naturally, if entrepreneurs can artificially inflate costs to reduce the tax burden without consequence, returns are higher and so is investment.

7$\frac{\partial \omega}{\partial y} = - (\tau - y(\tau + s)) < 0$ for $y < \frac{\tau}{\tau + s}$ which is likely to hold for most jurisdictions. For example if $\tau = 25\%$ and the surcharge for evasion when caught is twice the evaded amount, then we would need $y < 30\%$ which is highly likely for most countries.

8In a fixed investment model, a lower $\tilde{m}$ would increase the initial wealth required by an entrepreneur to make an investment.
choose higher transparency in countries with higher audit quality and consequently enjoy better access to finance.

The more interesting effect is that of a change in taxes. This depends on the relationship between $\omega$ and $\tau$. The effect of a change of taxes on the multiplier $\bar{m}$ is

$$\frac{\partial \bar{m}}{\partial \tau} = -\frac{p_H}{\bar{m}^2} \left[ R + \frac{\partial \omega}{\partial \tau} \right]$$

which depends on the sign of $\frac{\partial \omega}{\partial \tau}$. From equation (7), we have $\frac{\partial x}{\partial \tau} = -\frac{(1-x)^2}{B}$ which together with the definition of $\omega$ from condition IC$_b^\prime$ gives:

$$\frac{\partial \omega}{\partial \tau} = \left[ \tau(1-y)(1-x)^2 B - (y+x(1-y)) \right]$$

If the expression in square brackets is positive, then an increase in taxes reduces $\bar{m}$ and investment. If negative, then an increase in taxes may either increase or reduce investment, depending on the other parameters of the economy. For instance, if $y = 0.5$, then $\frac{\partial \omega}{\partial \tau} < 0$ only requires that $x+ > 1$ which is unlikely to hold in most economies. In other words, when the rate of auditing is high, then an increase in taxes lowers investments. We now look at some examples of the values of these parameters and how they would affect the level of tax evasion and financing. To do this, we need to first solve for the equilibrium value of $x$ and $\omega$. Using $R^b = \omega \frac{B_i}{\Delta p}$ from (IC$_b''$) in equation (7) gives a quadratic equation in $x$:

$$[\tau - y(\tau+s)] x^2 + [\Delta p(\tau+s) - p_L (1 + \tau - 2y(\tau+s))] x + [p_L (1 - y(\tau+s)) - \Delta p \left( (\tau+s) + \eta B \right)] = 0$$

which we solve for the root $< 1$: The values in Table 1 are based on various estimates found in the literature. The value of $B = 0.05$ comes from estimates by Dyck and Zingales (2004, Table XI: Private Benefits of Control and Legal Origin) for English origin legal jurisdictions. The audit
response cost $\eta = 1\%$ of project value is not based on any estimates. The probability of success for an investment project is calibrated to match a risk premium of $5\%$ and $p_L = 0.51$ is chosen such that the risky technology has a slightly better than fair chance of success. $\tau = 35\%$ is based on estimates of the effective corporate tax rate on investment from the Tax Foundation Special Report 2014.\footnote{See footnote 2 and calibration of macro model in Section 3} The surcharge $s$ on misreporting costs, is based on IRS Code Section 6662 which includes the Accuracy Related Penalty of 20\% of total understatement of tax or 40\% for Gross Valuation Misstatements. We choose the latter value and calibrate $s = 1.4$. The probability of an audit $y$ is calibrated based on Table 9a of the United States Internal Revenue Service Databook.\footnote{https://files.taxfoundation.org/legacy/docs/SR214.pdf} For the period October 1, 2015 to September 30, 2016, the audit rate is $y = 9.5\%$.\footnote{See e.g. https://www.irs.gov/pub/irs-soi/16databk.pdf} For earlier periods, such as the fiscal year 2014, $y = 12.2\%$ which we use here. For these parameter values, $x = 0.85$ and $\omega = 0.69$.

Figure 1 shows how changes in the parameters related to auditing, tax rates and governance affect the level of evasion $x$. The top and bottom panels show a smooth increase and decrease as the audit probability ($y$) and the private benefits ($B$), respectively increase. The intuition for these results is straightforward: a higher audit probability simply reduces the expected gain from misreporting profit/costs and consequently the probability of a true report rises. The converse is true for private benefits: a deterioration of the quality of corporate governance mechanisms as measured by the value of the private benefit parameter $B$ increases the value of misreporting income and consequently $x$ falls.

The effects of tax changes tends to vary with the quality of corporate governance. To see this, we solve for $x$ under different estimates of the private benefits of control from Dyck and Zingales (2004). In their Table XI, they give estimates of $B$ based on the legal origin of different jurisdictions. These are Scandinavian, English, German, French and Soviet, for which the private benefits are respectively, $B = 0.048, 0.055, 0.109, 0.212$ and $B = 0.356$. We plot the values of $x$ for these values of $B$ while increasing tax rates but holding all other parameters as in Table 1. The results are shown in Figure 2, while Figure A1 in Appendix A.3 shows the variation of $x$ for a more refined grid of $\tau$ and $B$ values. One immediate result is that for high levels of $B$ (poor corporate governance), the probability of truthful reporting is lower at all tax rates. In the higher quality governance environments (Scandinavian and English legal origins), increasing taxes result into modest rises in evasion (falling $x$). The other legal environments are characterised by an initial dip in truthful reporting at low levels of taxation. The solid line in Figure 2 traces $x$ as $\tau$ rises in Soviet origin jurisdictions. For a modest 5 percentage points increase in taxation, there is an
approximately 20 percentage points drop (0.55 to 0.35) in the probability of truthful reporting. The values of $x$ bottom out as $\tau$ increases given we maintain the assumption $s = 1.4$, so as higher tax rates, the effect of penalties reduce the expected gain from misreporting income.
2.3 Macroeconomic Equilibrium

Since our model is very close to that of Kiyotaki and Moore (2012), the exposition henceforth will closely follow their work. We omit some details and refer the interested reader to the original paper. The timing of events in the economy is as follows:

1. Aggregate productivity $\chi_t$ is realized and production takes place.
2. $\pi$ is revealed. Investing agents choose consumption, sell a fraction $\phi_t$ of their depreciated asset holdings. Non-investing agents choose consumption and purchase assets from investing agents.
3. Within period capital production occurs subject to moral hazard.
4. $[x p_H + (1 - x) p_L] R_t$ units of new capital are added to the economy at the end of the period.

From section 2.1, recalling that $c_t$ is consumption, $a_t$ is the equity holding, priced at $q_t$, and that the return to equity equals that of capital $r_t$, the investing entrepreneur’s flow of funds constraint is:

$$c_t^b + i_t = [(1 - \tau) r_t + \tau \delta q_t + (1 - \delta) \phi_t q_t] a_t^b + (1 - \tau) r_{z,t} z_t^b + \theta_t i_t$$  \hspace{1cm} (9)

where the superscript $b$ on variables stands for borrower. This equation says that in order to finance consumption $c_t$ and investment $i_t$, the entrepreneur issues equity $\theta_t i_t$ priced at unity together with the maximum after tax liquidity obtained from dividends $[(1 - \tau) r_t + \tau \delta q_t] a_{t-1}^b$, liquid assets $(1 - \tau) r_{z,t} z_{t-1}$ and the resalable fraction of depreciated equity $(1 - \delta) \phi_t q_t a_t^b$. In equation (9), $\tau \delta q_t$ is depreciation allowance and $z_{t-1} = (1 - (z_t / z_0)) \bar{z}$, $z_0, \bar{z} > 0$. The investing entrepreneur’s end of period asset holding is:

$$a_{t+1}^b = (1 - \theta_t) [x p_H + (1 - x) p_L] R_t + (1 - \phi_t) (1 - \delta) a_t^b$$  \hspace{1cm} (10)

which is her retained fraction $(1 - \theta_t)$ of the newly produced capital $[x p_H + (1 - x) p_L] R_t$ plus the unsold fraction of her depreciated initial asset holding $(1 - \phi_t) (1 - \delta) a_t^b$. Solving this equation forward for investment:

$$i_t = \frac{a_{t+1}^b - (1 - \phi_t) (1 - \delta) a_t^b}{(1 - \theta_t) [x p_H + (1 - x) p_L] R}$$

and substituting into (9) gives:

$$c_t^b + q_t^R a_{t+1}^b = [(1 - \tau) r_t + \tau \delta q_t + (1 - \delta) (\phi_t q_t + (1 - \phi_t) q_t^R)] a_t^b + (1 - \tau) r_{z,t} z_t^b$$  \hspace{1cm} (11)
where \( q_t^R = \frac{1}{xp_H + (1-x)p_L + R} \) is the effective replacement cost of equity for the investing entrepreneur: she has to make a downpayment \((1 - \theta_t)\) for each unit of investment for which she retains a fraction \((1 - \theta_t)(xp_H + (1-x)p_L + R)\), so she needs \( q_t^R \) to retain a unit claim to the capital she has produced. The RHS of the equation is her net worth which is gross-dividend from equity and storage plus the value of her depreciated equity \((1 - \delta)a_t^b\) of which the resalable fraction \(\phi_t\) is valued at the market price \(q_t\) and the non-resaleable fraction \((1 - \phi_t)\) is valued at the effective replacement cost \(q_t^R\). Using (9) we obtain investment as:

\[
i_t = \frac{\left[(1 - \tau)r_t + \tau \delta q_t + (1 - \delta)\phi_t q_t\right]a_t^b + (1 - \tau)rz_t^b - c_t^b}{(1 - \theta_t)}
\]

which simply says that \(i_t\) equals the ratio of liquidity available after consumption to the required downpayment for investment. Next consider the entrepreneur who does not have an investment opportunity (lender) and in line with previous notation, let the superscript \(l\) tag her variables. Her flow of funds constraint is

\[
c_l^t + q_t a_{t+1}^l = [(1 - \tau)r_t + \tau \delta q_t + q_t(1 - \delta)]a_t^l + (1 - \tau)rz_t^l - c_l^t
\]

The LHS is her purchase of consumption and new equity holdings and the RHS is her income from dividends and storage plus the market value of her depreciated equity holdings, assuming the resaleability constraint does not hold. We now determine the optimality conditions. Let the superscript \(i, j = b, l\) tag variables for an agent who is of type \(i\) in period \(t - 1\) and type \(j\) in period \(t\). For instance, date \(t\) consumption of an agent who was a borrower in the previous period and is currently a lender is denoted by: \(c_{t}^{bl}\). The optimality conditions with respect to \(a_{t+1}^{l}, a_{t+1}^{b}\) for agents of type \(ij\) together with the trade-off between holding equity and storage, are given by: (see
A.1 in the Appendix):

\[
\frac{q_t}{c_t} = \beta \pi E_t \left\{ \frac{(1 - \tau)q_{t+1} + \tau \delta q_{t+1} + (1 - \delta)(\phi_{t+1}q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R)}{c_{t+1}^{ll}} \right\} \\
+ \beta (1 - \pi) E_t \left\{ \frac{(1 - \tau)q_{t+1} + \tau \delta q_{t+1} + (1 - \delta)(\phi_{t+1}q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R)}{c_{t+1}^{ll}} \right\} \\
\frac{q_t^R}{c_t^{ll}} = \beta \pi E_t \left\{ \frac{(1 - \tau)q_{t+1} + \tau \delta q_{t+1} + (1 - \delta)(\phi_{t+1}q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R)}{c_{t+1}^{ll}} \right\} \\
+ \beta (1 - \pi) E_t \left\{ \frac{(1 - \tau)q_{t+1} + \tau \delta q_{t+1} + (1 - \delta)(\phi_{t+1}q_{t+1} + (1 - \phi_{t+1})q_{t+1}^R)}{c_{t+1}^{ll}} \right\} \\
\frac{(z_t / z_0)^\bar{z}}{c_t^{ll}} = \beta E_t \left\{ \frac{\pi - 1}{c_{t+1}^{ll}} + (1 - \pi) \frac{1}{c_{t+1}^{ll}} \right\}
\]

(13a) \quad (13b) \quad (13c) \quad (13d) \quad (13e)

We now consider the aggregate economy. The linearity of consumption, investment and savings choices means aggregation can be done without the need to keep track of distributions. Aggregate holdings of equity equals the aggregate capital stock \( K_{t-1} \). At the start of date \( t \), a fraction \( \pi \) of \( K_{t-1} \) is held by entrepreneurs who have an investment opportunity. Letting \( Z_t \) denote aggregate storage, then from (12), total investment \( I_t \) in new capital satisfies:

\[
I_t = i_{t}^{bb} + i_{t}^{ll} = \frac{[(1 - \tau)q_t + \tau \delta q_t + (1 - \delta)\phi_t q_t] (a_{i_{t-1}^{bb}} + a_{i_{t-1}^{ll}}) + (1 - \tau)r_t Z_{t-1} (z_{t-1}^{bb} + z_{t-1}^{ll}) - c_{t}^{bb} - c_{t}^{ll}}{(1 - \theta_t)}
\]

(14)

where the last equality has used (i) \( a_{i_{t-1}^{bb}} = \pi a_{i_{t-1}^{bb}} = \pi^2 K_{t-1}, a_{i_{t-1}^{ll}} = \pi a_{i_{t-1}^{ll}} = \pi(1 - \pi)K_{t-1} \) and \( a_t = K_t \) and (ii) \( z_{t-1}^{bb} = 0, z_{t-1}^{ll} = (1 - \pi)\pi Z_t \), since agents who had an investment opportunity in the previous period do not accumulate storage. Goods market clearing requires total output \( Y_t = \chi_t^s K_{t-1}^{\alpha} \) plus storage brought forward \( Z_{t-1} \) to be equal to investment \( I_t \) plus consumption \( C_t = c_{t}^{bb} + c_{t}^{ll} + c_{t}^{ll} \).

13From equation (1), \( Y_t = \chi_t^s K_{t-1}^{\alpha} \). Since the all \( k_{j,t-1} \) are perfect substitutes, the elasticity of substitution \( \sigma = \frac{1}{1 - \phi} \to \infty \), which implies \( \phi \to 1 \), so aggregate output equals \( Y_t = \chi_t^s K_{t-1}^{\alpha} \).
new storage \((z_t/z_0)\bar{z}_t\) and tax revenue \(T_t\):

\[
Y_t + Z_{t-1} = I_t + C_t + (z_t/z_0)\bar{z}_t + T_t
\]

where the tax revenue comprises of the tax from returns to capital and storage plus the tax on profit from investment of truthful and untruthful reporters. The untruthful reporters consist of two groups: audited and unaudited.

\[
T_t = \tau(r_t - \delta q_t)K_{t-1} + \tau r_{\bar{z}_t}Z_{t-1} + x \left[ p_H \tau(q_t R - 1) \right] I_t + (1 - x) \left[ (1 - y) \left\{ p_L \tau(q_t R - 1) + (\tau + s)B \right\} \right] I_t
\]

Finally, investing entrepreneurs sell a fraction \(\theta_t\) of claims to the outcome of their investment \([xp_H + (1 - x)p_L]RI_t\) together with a fraction \(\phi_t\) of their depreciated equity holdings \(\pi(1 - \delta)K_{t-1}\). The stock of equity held by the non-investing entrepreneurs at the beginning of period \(t + 1\) is therefore:

\[
\theta_t[xp_H + (1 - x)p_L]RI_t + \phi_t \pi(1 - \delta)K_{t-1} + (1 - \delta)(1 - \pi)K_{t-1} \equiv A^1_{t+1}
\]

The stock held by investing entrepreneurs is their retained equity from investment outcome \((1 - \theta_t)[xp_H + (1 - x)p_L]RI_t\) plus their unsold depreciated equity holdings \((1 - \phi_t)\pi(1 - \delta)K_{t-1}\):

\[
(1 - \theta_t)[xp_H + (1 - x)p_L]RI_t + (1 - \phi_t)(1 - \delta)\pi K_{t-1} \equiv A^b_{t+1}
\]

The aggregate capital stock therefore evolves according to:

\[
K_t = A^1_{t+1} + A^b_{t+1} = [xp_H + (1 - x)p_L]RI_t + (1 - \delta)K_{t-1}
\]

Define the prices \(q^R_t\), \(r_t\) and the fraction of pledgeable future returns \(\theta_t\) as:

\[
q^R_t = \frac{1}{[xp_H + (1 - x)p_L]R}
\]

\[
r_t = \alpha \chi_t K_{t-1}^{\alpha - 1}
\]

\[
\theta_t = p_H \left[ (1 - \tau)\left(q_t R - 1\right) - \omega \times \frac{B}{A_p} \right]
\]

and the evolution of aggregate productivity \(\chi_t\) and liquidity \(\phi_t\) as

\[
\chi_t = (1 - \rho_z)\chi_t + \rho_z \chi_{t-1} + e_{\chi,t}
\]

\[
\phi_t = (1 - \rho_\phi)\phi_t + \rho_\phi \phi_{t-1} + e_{\phi,t}
\]
We can now define the equilibrium. A recursive competitive equilibrium is a function \((q_t^R, q_t, r_t, \theta_t, C_t, I_t, T_t, K_t, Z_t)\) of the aggregate state \((K_{t-1}, Z_{t-1}, \chi_t, \phi_t)\) that satisfies equations (13)–(22).

3 Quantitative Analysis

In this section, we explore the model’s quantitative predictions by calibrating and solving it numerically using perturbation methods. We employ the Dynare suite of programs.

3.1 Calibration

We calibrate the model following current standards the macroeconomic literature. We divide parameters into two categories and fix the time period to a quarter. Table 1 lists parameters related to the agency problem and taxation system. Table 2 lists parameters related to the macroeconomic model and their values. Most of our parameters are set to follow the quantitative analysis of the KM model by Bigio and Schneider (2017) and Del Negro, Eggertsson, Ferrero and Kiyotaki (2017).

We set the discount factor \(\beta = 0.99\), the capital share to \(\alpha = 0.36\) and \(\delta = 0.025\) to match the steady state investment to capital ratio \(\frac{I}{K} \approx \delta\). These numbers are well established in the RBC literature. We set our value of \(R\), the gross-return to investment to 1.75, so that our value of effective equity replacement cost matches that of Bigio and Schneider (2017). Following the same authors, we set the arrival of investment opportunities to \(\pi = 0.012\) and the persistence of shocks to productivity and resaleability \(\rho_\chi = \rho_\phi = 0.95\). The steady state resaleability value is set at the maximum value of \(\bar{\phi} = 0.375\) which is slightly higher than the number used in the literature so far which is about 0.3. We require the slightly higher value in order to obtain reasonable values of \(\theta\) and \(q\). We set the aggregate productivity level at \(\bar{\chi} = 0.35 \approx \alpha\), because our model has capital as the only factor of production, so that aggregate output equals the return to holding capital. This choice also ensures that our steady state value of the capital stock is close to the theoretical moments found in the literature. The parameter values in Table 1 imply a probability of truthful income reporting by an investing entrepreneur is \(x = 0.79\) which reduces the private benefit by a factor \(\omega = 0.70\). These are the last two entries in Table 2.

3.2 Steady State

In the literature, the mortgageable fraction of new investment \(\theta_t\) is usually calibrated as a model parameter. In our case, the value of \(\theta\) is determined in equilibrium as it depends on the market value
Table 2: Baseline Calibration: Macroeconomic Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital Share</td>
<td>0.36</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation Rate</td>
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</tr>
<tr>
<td>$\bar{z}, z_0$</td>
<td>Storage Parameters</td>
<td>0.1, 0.5</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Arrival of investment opportunity</td>
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</tr>
<tr>
<td>$R$</td>
<td>Gross Return to Capital Investment</td>
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</tr>
<tr>
<td>$\bar{Z}$</td>
<td>Productivity</td>
<td>0.35</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Resaleable fraction of equity</td>
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</tr>
<tr>
<td>$\rho, \chi$</td>
<td>Persistence of shocks</td>
<td>0.95</td>
</tr>
<tr>
<td>$x$</td>
<td>Truthful Report Probability</td>
<td>0.79</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Private Benefit Multiplier</td>
<td>0.70</td>
</tr>
</tbody>
</table>

of capital $q_t$. As noted by Kiyotaki and Moore (2012) and Bigio and Schneider (2017), the value of $\theta$ sets the upper bound for the spot price of equity: $q_t \leq 1/\theta_t$ which our equilibrium outcome should satisfy. Furthermore, the equilibrium outcome should also satisfy $0 \leq \theta_t \leq 1 − \pi = 0.998$. Our basic calibration and the ensuing model solution satisfies these conditions as shown in Table 3.

Table 3: Steady State

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_t$</td>
<td>Mortgageable fraction of new investment</td>
<td>0.79</td>
</tr>
<tr>
<td>$q_t$</td>
<td>Market value of equity</td>
<td>1.13</td>
</tr>
<tr>
<td>$q^R_t$</td>
<td>Effective equity replacement cost</td>
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<tr>
<td>$C_t$</td>
<td>Consumption</td>
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<tr>
<td>$I_t$</td>
<td>Investment</td>
<td>0.63</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Capital</td>
<td>37.48</td>
</tr>
<tr>
<td>$Y_t$</td>
<td>Output</td>
<td>2.53</td>
</tr>
<tr>
<td>$Z_t$</td>
<td>Storage</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Our basic calibration gives a value of $\theta_t = 0.79 \leq 1 − \pi = 0.998$ which implies an upper bound of $q_t \leq 1.27$ which our equilibrium equity price of $q_t = 1.13$ satisfies. Our equilibrium value of $\theta$ matches the calibrations of Bigio and Schneider (2017) and Del Negro et al. (2017) who use the values $\theta = 0.77, 0.79$ respectively. Our market value of equity is very close to that obtained by Bigio and Schneider (2017, Table 4, Theoretical Moments) which is $q = 1.09$ and our choice of $R = 1.75$ implies that our effective equity replacement cost, $q^R_t = 0.67$ exactly matches the value implied by their equilibrium where $q^R_t = \frac{1−\theta_t}{1−\theta} = 0.67$. Our equilibrium also fits the values of the investment-output ratios targets for macroeconomic models: $\frac{I_t}{Y_t} = 0.25$ which is equivalent to the equilibrium target matched by Del Negro et al. (2017). In our case, the steady state investment to
capital ratio is given by $I_K = \frac{\delta}{\rho_H + (1-x)p_L} = 0.0326$ but our equilibrium investment-capital ratio = 1.68% which is slightly higher than the approximately 1.23% quarterly estimates of capital-growth matched by Perez-Orive (2016).

In our economy changes in asset prices and macroeconomic variables are dependent on parameters that are subject to policy changes such as the tax rate $\tau$, the audit rate $y$ and/or regulations that affect the quality of corporate governance $B$. These have direct and indirect effects on the equilibrium asset prices. We now evaluate the effect of changes in some of these policy parameters on equilibrium outcomes.

### 3.2.1 Tax Changes

To evaluate the effects of tax changes, we perform a deterministic simulation where the tax rate $\tau$ enters our model as an exogenous variable. We let all parameters in our model economy at the values in Table 2 while changing the tax rate $\tau$ in two directions, each by 7.5 percentage points. We use this value as it is close to the tax rate at which $x$ bottoms out for $B = 0.06$ in Figure 2 for a tax decrease. In the first case, the tax rate increases from $\bar{\tau} = 35\%$ to 42.5% and in the second case, it decreases to 27.5%. These changes have effects on both the tax evasion parameter $x$ and the multiplier $\omega$. When the tax rate increases to $\tau = 42.5\%$, then $x = 0.81$ and $\omega = 0.63$. The equilibrium equity price is $q_t = 1.12$ which is slightly lower than the steady state value. The lower equity price means that the equilibrium mortgageable fraction of new capital falls to $\theta_t = 0.78$. In the opposite direction, when $\tau = 27.5\%$, there is a steep fall in truthful reporting $x = 0.71$ while the multiplier slightly rises to $\omega = 0.78$. This raises the equilibrium equity price to $q_t = 1.17$ which also raises the mortgageable fraction of new capital to $\theta_t = 0.83$. While the mortgageable fraction of equity is higher, the overall output, consumption and investment is higher with high taxes.

We summarize these effects in Figure 3 where we trace the path of transition from the initial steady state defined by the baseline calibrations in Tables 1 & 2. The simulation assumes a perfect foresight economy where there is a permanent change in $\tau$ occurring after 16 periods (quarters). A decrease in taxes lowers output, consumption, capital while raising asset prices. The higher asset prices consequently increase the mortgageable fraction of new capital $\theta_t$. There is a simple explanation for this result. Lower taxes have two effects. First, a low tax reduces the required threshold of wealth for borrowing, so $\theta_t$ and the investment multiplier $\bar{\bar{m}}$ increases. Second, a lower tax reduces the penalty of evasion from $s = 1.4 \times \tau = 49\%$ to $s = 35\%$ so the incomes of entrepreneurs is higher. However, the low evasion penalty implies that more entrepreneurs are using the risky technology, which has the overall effect of reducing investment, output and consumption.
Figure 3: Effects of Tax Changes in Deterministic Equilibrium

Note: The figure compares the response of output ($Y_t$), consumption ($C_t$), investment ($I_t$), capital ($K_t$), mortgageable fraction of new capital ($\theta_t$) and market value of equity ($q_t$) to a decrease (solid line) and an increase (dotted line) in the tax rate. The simulation starts at the baseline calibration (short horizontal line) with tax changes occurring after 16 periods (vertical line). Final steady states are marked by +s (decrease) and ×s (increase).

3.2.2 Corporate Governance

One of the policies that can have an impact on our equilibrium is a change in the quality of corporate governance. Such changes can occur for instance when there are changes in financial regulations that affect the value of $B$. As an example of how regulations are related to the parameter $B$, suppose capital in our economy represents housing and equity was consequently a claim to the stream of payments a buyer (mortgage holder) promised to make upon purchase. In this scenario, the producer of new capital goods is a bank that bundles together housing loans that promise to pay dividends every time period in the form of mortgage repayments, similar to the “securitization” of loans that preceded the US sub-prime crisis.

In this case, we can think of the safe technology as a bank that does its due diligence by incurring the costs of screening potential borrowers. This cost lowers $B$ but increases the chance of success $p = p_H$, i.e. a borrower that will make repayments. However, the bank can also spend less in screening and originate loans with a high likelihood of default, which is cheaper so $B$ is high and $p = p_L$. A loosening of financial regulations would allow banks to divert more resources (high
B) without consequence and seek to benefit from such behaviour by mimicking the returns of a bank with high B, that is, engage in tax evasion. In our model, the effect of this policy/regulatory change is simply captured by changes in B. We perform the change by raising and lowering B by 0.02. The lower value is similar to moving to a scenario with Scandinavian origin legal jurisdiction described in Figure 2. The effects of these changes on evasion are $x = 0.96$, $\omega = 0.7$ when $B = 0.04$ and $x = 0.66$, $\omega = 0.73$ when $B = 0.08$, i.e. tighter regulations reduce evasion. The results on equilibrium are as summarized in Figure 4. A higher B raises the mortgageable fraction of new capital $\theta_t$ as the price of equity is increasing. Since new capital production now entails more risk, the market value of equity rises to a higher steady state value.

### 3.3 Productivity and Liquidity Shocks

We evaluate the effects of a negative productivity and liquidity shocks in our economy. Figures 5 and 6 show the impulse response functions to a 1\% and a 10\% decrease in $\chi_t$ and $\phi_t$ respectively, under three scenarios: baseline calibration, high and low taxes.
Because capital is predetermined and $\bar{X} = 0.35$, the 1% shock in $\chi_t$ decreases output by $0.35^\alpha - (0.35 - 0.035)^\alpha \approx 3.7\%$. Then from the goods market clearing condition, asset prices have to fall in line with productivity in order to reduce consumption and investment in line with the lower output. Investment is more sensitive to asset prices and falls more than output, given that the mortgageable fraction of equity is also falling as a result of lower asset prices.

When a liquidity shock occurs, the investing entrepreneurs are less able to finance down payment from selling their equity holdings, so investment falls substantially. Given constant depreciation, capital and output gradually fall with persistently lower investment. Lower investment means consumption must rise to maintain the goods market equilibrium. This occurs through a wealth effect in rising equity prices, which also increase the value of $\theta_t$. Similar mechanisms are observed in the case with different values of $B$ displayed in Figures 7 and 8. The fall in investment following a liquidity shock is larger with low taxes and the gradual falls in output and capital also larger.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Impulse response a 1% negative shock in productivity $\chi_t$ under different tax regimes}
\end{figure}

\textit{Note:} The figure compares the impulse response of output ($Y_t$), consumption ($C_t$), investment ($I_t$), mortgageable fraction of new capital ($\theta_t$), capital ($K_t$) and the market price of equity ($q_t$) to a 1% shock to the productivity process $\chi_t$ in baseline calibration and with high and low taxes. The simulations start at the respective steady states.
Figure 6: Impulse response a 10% negative shock in resaleability of equity $\phi_t$ under different tax regimes

Note: The figure compares the impulse response of output ($Y_t$), consumption ($C_t$), investment ($I_t$), mortgageable fraction of new capital ($\theta_t$), capital ($K_t$) and the market price of equity ($q_t$) to a 10% shock to the process $\phi_t$ in baseline calibration and with different tax rates. The simulations start at the respective steady states.

4 Conclusion

This paper is part of a recent literature on macroeconomics with financial frictions, that includes the seminal papers of Holmstrom and Tirole (1997), Gertler and Kiyotaki (2010) and recently Del Negro, Eggertsson, Ferrero and Kiyotaki (2017). The common theme of this literature has been some kind of credit constraint, which has always been rationalized on the basis of asymmetric information between lenders and borrowers. In most of this literature, the borrowing constraint is frequently specified in a reduced form manner.

Building on the seminal work of Holmstrom and Tirole (1997), and the prominent role of taxation and auditing in business decisions, we have developed a microfounded economic environment in which the size of the borrowing constraint arises as an equilibrium outcome rather than a parameter calibrated from the data. While we have used the tax system and invoked the existence of evasion to motivate our moral hazard problem, our model still replicates many standard features of macroeconomic models with financial frictions. An important contribution of our work is in providing a link between aspects of corporate governance and macroeconomic outcomes.

Our goal was to show that the tax system in interaction with the quality of corporate governance, has an effect on the impact of productivity and liquidity shocks. Our results suggest that
Figure 7: Impulse response a 1% negative shock in productivity $\chi_t$ under different Corporate Governance regimes

Note: The figure compares the impulse response of output ($Y_t$), consumption ($C_t$), investment ($I_t$), mortgageable fraction of new capital ($\theta_t$), capital ($K_t$) and the market price of equity ($q_t$) to a 1% S.D. shock to the productivity process $\chi_t$ with different values of $B$. Simulations start at the respective steady states.

Poor corporate governance accompanied by low taxes result into both lower levels of output and consumption but also deeper recessions and slower recoveries.
Figure 8: Impulse response a 10% negative shock in resaleability of equity φ_t under different Corporate Governance regimes

Note: The figure compares the impulse response of output (Y_t), consumption (C_t), investment (I_t), mortgageable fraction of new capital (θ_t), capital (K_t) and the market price of equity (q_t) to a 10% shock to the process φ_t under different B values.
References


A Appendix

A.1 First Order Conditions

The first order conditions used in Sub-Section 2.3 are similar to those in the Kiyotaki and Moore (2012) model, but also with differences arising from the absence of labour income and money in our setup. It is therefore necessary to flesh out how we obtain our optimality conditions. Recall that, excluding storage, the budget constraint, for an investing (borrower) and non-investing (lender) entrepreneur, are respectively:

\[ c^b_i + i_t = [(1 - \tau) r_t + \tau \delta q_t + (1 - \delta) \phi_t q_t] a^b_i + \theta_t i_t \]  
\[ c^l_i + q_t a^l_{i+1} = [(1 - \tau) r_t + \tau \delta q_t + q_t (1 - \delta)] a^l_i \]  

(A.1)  

(A.2)

Solving (A.1) for investment \( i_t \) and substituting into the asset holding equation \( a^b_{i+1} = (1 - \theta_t)[x p_H + (1 - x) p_L] R_i + (1 - \phi_t)(1 - \delta) a^b_t \) gives the investing entrepreneur’s consolidated budget constraint:

\[ c^b_i + q^R_t a^b_i = [(1 - \tau) r_t + \tau \delta q_t + (1 - \delta) \phi_t q_t + (1 - \phi_t) q^R_t] a^b_i \]  

(A.3)

where \( q^R_t = \frac{1}{[x p_H + (1 - x) p_L] R} \) is the effective replacement cost of equity for an investing entrepreneur as described before.

Letting the superscript \( i, j = b, l \) tag variables for an agent who is of type \( i \) in period \( t - 1 \) and type \( j \) in period \( t \). For instance, date \( t \) consumption of an agent who was a borrower in the previous period and is currently a lender is denoted by: \( c^{bl}_t \). The Lagrangians for each of the four agent types are given by:

\[
\begin{align*}
\mathcal{L}^{ll} &= u(c^{ll}_t) - \lambda^{ll}_t \left[ c^{ll}_t + q_t a^l_{t+1} - [(1 - \tau) r_t + \tau \delta q_t + (1 - \delta) q_t] a^l_t \right] + \pi \beta E_t \left\{ u(c^{lb}_{t+1}) - \lambda^{lb}_{t+1} \left[ c^{lb}_{t+1} + q^{R}_{t+1} a^{lb}_{t+1} - [(1 - \tau) r_{t+1} + \tau \delta q_{t+1} + (1 - \delta) (\phi_{t+1} q_{t+1} + (1 - \phi_{t+1}) q^{R}_{t+1})] a^{lb}_{t+1} \right] \right\} \\
&\quad + (1 - \pi) \beta E_t \left\{ u(c^{ll}_{t+1}) - \lambda^{ll}_{t+1} \left[ c^{ll}_{t+1} + q_{t+1} a^{l}_{t+2} - [(1 - \tau) r_{t+1} + \tau \delta q_{t+1} + (1 - \delta) q_{t+1}] a^{l}_{t+1} \right] \right\} + \ldots
\end{align*}
\]

(A.4)
\[ \mathcal{L}_{lb}^{bl} = \left. \mathcal{L}_{lb}^{bb} \right|_{r_t} - \lambda_{r_{t+1}}^{bb} \left[ c_{l_t}^{bb} + q_t^{R} a_t^{b_{t+1}} - [(1 - \tau) r_t + \tau q_t + (1 - \delta) \left( \phi_t q_t + \phi_{t+1} R_{t+1} \right)] a_t^{b_t} \right] + \pi \beta E_t \left\{ u(c_{l_{t+1}}^{bb}) \right\} \\
- \lambda_{r_{t+1}}^{bb} \left[ c_{l_{t+1}}^{bb} + q_t^{R} a_t^{b_{t+2}} - [(1 - \tau) r_{t+1} + \tau q_t + (1 - \delta) \left( \phi_t q_t + \phi_{t+1} R_{t+1} \right)] a_t^{b_{t+1}} \right] \right\} + \left( 1 - \pi \right) \beta E_t \left\{ u(c_{l_{t+1}}^{bb}) \right\} \\
+ \left( 1 - \pi \right) \beta E_t \left\{ u(c_{l_{t+1}}^{bl}) \right\} - \lambda_{r_{t+1}}^{bl} \left[ c_{l_{t+1}}^{bl} + q_{t+1} a_{t+1}^{l_{t+1}} - [(1 - \tau) r_{t+1} + \tau q_{t+1} + (1 - \delta) q_{t+1}] a_{t+1}^{l_{t+1}} \right] \right\} + \ldots \\
(A.5) \\

\[ \mathcal{L}_{lb}^{bb} = \left. \mathcal{L}_{lb}^{bl} \right|_{r_t} - \lambda_{r_{t+1}}^{bl} \left[ c_{l_t}^{bl} + q_t^{l} a_t^{l_{t+1}} - [(1 - \tau) r_t + \tau q_t + (1 - \delta) \left( \phi_t q_t + \phi_{t+1} R_{t+1} \right)] a_t^{l_t} \right] + \pi \beta E_t \left\{ u(c_{l_{t+1}}^{bl}) \right\} \\
- \lambda_{r_{t+1}}^{bl} \left[ c_{l_{t+1}}^{bl} + q_t^{l} a_t^{l_{t+2}} - [(1 - \tau) r_{t+1} + \tau q_{t+1} + (1 - \delta) q_{t+1}] a_{t+1}^{l_{t+1}} \right] \right\} + \left( 1 - \pi \right) \beta E_t \left\{ u(c_{l_{t+1}}^{bl}) \right\} \\
+ \left( 1 - \pi \right) \beta E_t \left\{ u(c_{l_{t+1}}^{bl}) \right\} - \lambda_{r_{t+1}}^{ll} \left[ c_{l_{t+1}}^{ll} + q_{t+1} a_{t+1}^{l_{t+1}} - [(1 - \tau) r_{t+1} + \tau q_{t+1} + (1 - \delta) q_{t+1}] a_{t+1}^{l_{t+1}} \right] \right\} + \ldots \\
(A.6) \\

\[ \mathcal{L}_{lb}^{bl} = \left. \mathcal{L}_{lb}^{bb} \right|_{r_t} - \lambda_{r_{t+1}}^{ll} \left[ c_{l_t}^{ll} + q_t^{l} a_t^{l_{t+1}} - [(1 - \tau) r_t + \tau q_t + (1 - \delta) q_t] a_t^{l_t} \right] + \pi \beta E_t \left\{ u(c_{l_{t+1}}^{ll}) \right\} \\
- \lambda_{r_{t+1}}^{ll} \left[ c_{l_{t+1}}^{ll} + q_t^{l} a_t^{l_{t+2}} - [(1 - \tau) r_{t+1} + \tau q_{t+1} + (1 - \delta) q_{t+1}] a_{t+1}^{l_{t+1}} \right] \right\} + \left( 1 - \pi \right) \beta E_t \left\{ u(c_{l_{t+1}}^{ll}) \right\} \\
+ \left( 1 - \pi \right) \beta E_t \left\{ u(c_{l_{t+1}}^{ll}) \right\} - \lambda_{r_{t+1}}^{ll} \left[ c_{l_{t+1}}^{ll} + q_{t+1} a_{t+1}^{l_{t+1}} - [(1 - \tau) r_{t+1} + \tau q_{t+1} + (1 - \delta) q_{t+1}] a_{t+1}^{l_{t+1}} \right] \right\} + \ldots \\
(A.7) \]
The first order conditions with respect to $c_{i}^{j}, a_{t+1}^{j}$ for $i, j = \{l, b\}$ are given by:

\[
\frac{\partial L_{i}^{II}}{\partial c_{i}^{j}} = \frac{1}{c_{i}^{j}} - \lambda_{i}^{II} = 0
\]

(A.8)

\[
\frac{\partial L_{i}^{II}}{\partial a_{t+1}^{j}} = -\lambda_{i}^{II} q_{t} + \pi \beta E_{t} \left\{ \lambda_{t+1}^{lb} \left[ (1 - \tau) r_{t+1} + \tau \delta q_{t+1} + (1 - \delta) \left( \phi_{t+1} q_{t+1} + (1 - \phi_{t+1}) q_{t+1}^{R} \right) \right] \right\} + (1 - \pi) \beta E_{t} \left\{ \lambda_{t+1}^{II} \left[ (1 - \tau) r_{t+1} + \tau \delta q_{t+1} + (1 - \delta) q_{t+1} \right] \right\} = 0
\]

\[
\frac{\partial L_{i}^{II}}{\partial c_{I}^{bb}} = \frac{1}{c_{I}^{bb}} - \lambda_{I}^{bb} = 0
\]

(A.9)

\[
\frac{\partial L_{i}^{II}}{\partial c_{I}^{bl}} = \frac{1}{c_{I}^{bl}} - \lambda_{I}^{bl} = 0
\]

(A.10)

\[
\frac{\partial L_{i}^{II}}{\partial a_{t+1}^{b}} = -\lambda_{i}^{bb} q_{t}^{R} + \pi \beta E_{t} \left\{ \lambda_{t+1}^{bb} \left[ (1 - \tau) r_{t+1} + \tau \delta q_{t+1} + (1 - \delta) \left( \phi_{t+1} q_{t+1} + (1 - \phi_{t+1}) q_{t+1}^{R} \right) \right] \right\} + (1 - \pi) \beta E_{t} \left\{ \lambda_{t+1}^{bl} \left[ (1 - \tau) r_{t+1} + \tau \delta q_{t+1} + (1 - \delta) q_{t+1} \right] \right\} = 0
\]

\[
\frac{\partial L_{i}^{II}}{\partial c_{I}^{bl}} = \frac{1}{c_{I}^{bl}} - \lambda_{I}^{bl} = 0
\]

(A.11)

\[
\frac{\partial L_{i}^{II}}{\partial a_{t+1}^{l}} = -\lambda_{i}^{bl} q_{t} + \pi \beta E_{t} \left\{ \lambda_{t+1}^{bl} \left[ (1 - \tau) r_{t+1} + \tau \delta q_{t+1} + (1 - \delta) \left( \phi_{t+1} q_{t+1} + (1 - \phi_{t+1}) q_{t+1}^{R} \right) \right] \right\} + (1 - \pi) \beta E_{t} \left\{ \lambda_{t+1}^{ll} \left[ (1 - \tau) r_{t+1} + \tau \delta q_{t+1} + (1 - \delta) q_{t+1} \right] \right\} = 0
\]

(A.12)

Replacing for the $\lambda_{i}^{ij}$s and $\lambda_{i+1}^{ij}$s with the corresponding $\frac{1}{c_{i}^{ij}}$s and $\frac{1}{c_{i+1}^{ij}}$s gives the optimality conditions (13).

A.2 Steady States

A.3 Truthful Report Probabilities
Table A1: Steady States

<table>
<thead>
<tr>
<th>Variable</th>
<th>Tax Rates</th>
<th>B values</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$\tau = 35%$</td>
<td>$\tau = 27.5%$</td>
</tr>
<tr>
<td>$Y$</td>
<td>2.528</td>
<td>2.461</td>
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<td>$C$</td>
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</tr>
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<td>$w$</td>
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<td>0.781</td>
</tr>
</tbody>
</table>

Figure A1: How changes in taxation and governance affect evasion