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Demographics and the natural interest rate in the euro area.

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Abstract: We investigate the impact of demographics on the natural rate of interest (NRI) in the euro area, with a particular focus on the role played by economic openness, migrations and pension system design. To this end, we construct a life-cycle model and calibrate it to match the life-cycle profiles from HFCS data. We show that population aging contributes significantly to the decline in the NRI, explaining about two-thirds of its secular decline between 1985 and 2030. Openness to international capital flows has not been important in driving the EA real interest rate so far, but will become a significant factor preventing its further decline in the coming decades, when aging in Europe accelerates relative to the rest of the world. Of two possible pension reforms, only an increase in the retirement age can revert the downward trend on the equilibrium interest rate while a fall in the replacement rate would make its fall even deeper. The demographic pressure on the Eurozone NRI can be alleviated by increased immigration, but only to a small extent and with a substantial lag.

Keywords: population aging, natural interest rate, life-cycle models, pension systems, migrations

JEL codes: E31, E52, J11

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1 Introduction

Many economies, developed and developing alike, are experiencing (or are soon expected to begin experiencing) a substantial demographic transition. Increasing longevity and sub-replacement fertility rates translate into aging of societies, with the speed of this process varying between different countries. Aging affects many aspects of economic activity, including aggregate output, pension system sustainability, structure and volume of fiscal expenditures or housing markets.

More recently, the demographics has also attracted attention as a possible force behind the secular decline in the natural rate of interest (NRI) observed in many advanced economies since about mid-1980s. A number of papers have documented this process using various empirical methods, see e.g. King and Low (2014), Bean et al. (2015) and Del Negro et al. (2019) for G7 countries, Holston et al. (2017) for the US, euro area, Canada and the UK, or Jordà et al. (2019) for 16 advanced economies. Following Summers (2014), who associated the fall in interest rates with what he dubbed "secular stagnation", many economists believe that it may have a permanent nature.[1] As argued by Rachel and Summers (2019), were it not for extraordinary fiscal policies (increase in public debt and social security spending), real interest rates would have fallen even more.

Two broad types of possible explanations for this trend have been proposed, namely productivity slowdown and shifts in saving and investment preferences (Rachel and Smith, 2015; Lunsford and West, 2019). This second category includes, among others, adjustments related to demographic processes. Economic theory predicts that a slowdown in population growth may translate into higher level of capital per worker, and hence into a decrease in the rate of return on capital. Increasing longevity lengthens the planning horizon of households, inducing them to save more, and thus exerting a further downward pressure on the NRI. All of this suggests that population aging has a potential to account for at least some of the recent medium-term trends in this important macroeconomic variable. However, a quantitative assessment of this effect is far from trivial and calls for a structural analysis.

A number of papers have looked at this issue using macroeconomic models where demographic processes play an important role. In their seminal work that long preceded the secular stagnation debate, Krueger and Ludwig (2007) used a multi-country overlapping generations model to predict a fall in the world interest rate due to demographic transition in OECD countries by about 1 percentage point between 2000 and 2060. More recently, Kara and von Thadden (2016) calibrated a Blanchard-Yaari model to the euro area and projected a decrease in the natural interest rate for this group of countries by 0.9 percentage points between 2008 and 2030. Carvalho et al. (2016) calibrated a similar model to the average of several developed countries and simulated a more significant

[1] It is important to note that this debate concerns risk-free interest rates, and not returns on risky assets. As documented by Marx et al. (2019), while the risk-free rates have been falling for several decades, the return on capital has not.

decline of the equilibrium interest rate (1.5 percentage points between 1990 and 2014). Based on a richer overlapping generations (OLG) model, Gagnon et al. (2016) argued that in the US demography is virtually the sole culprit of the recent permanent decline in real GDP growth, the rate of aggregate investment and safe asset yields, suggesting that this situation is the "new normal". Eggertsson et al. (2019) also used a quantitative life-cycle model to conclude that changes in the US mortality and fertility rates are the key forces behind the secular stagnation hypothesis.

Our paper is focused on the euro area. As evidenced by Figure 1, from 1970 to the mid-2000s, the European economy was experiencing a sharp decline in the fertility rate, which then stabilized at sub-replacement levels and is projected to remain persistently low. The outcome is a continuing drop in the number of people entering the working-age period of life. Moreover, the mortality rates in the EA are consistently falling and the currently born individuals are expected to face a life span that is longer by over 10 years compared to people born five decades ago. These two forces reinforce each other in leading to a rapid increase in the old-age dependency ratio (ratio of people aged 65 and more to population 20-64 years old), which is projected to exceed 60% by 2060, from well below 30% in the 1980s and 1990s. Similar processes can be observed in other developed economies, with two potentially important quantitative differences. First, in the aggregate of non-EA G7 economies, the fertility rate stopped declining already before 1980 and stabilized at higher levels than in Europe. Also, since about 1990 the life expectancy in this group of countries has been increasing at a slower pace. These processes affect the old-age dependency ratio with a delay, but eventually lead to significantly faster aging in the EA compared to other developing economies.

To analyze the impact of these massive demographic trends, we develop a detailed life-cycle model for the EA economy. Unlike the previous studies for this region that rely on the Blanchard-Yaari setup,[2] our model draws on the full-scale life-cycle framework pioneered by Auerbach and Kotlikoff (1987). This allows us to incorporate granular data on fertility and mortality rates as well as age profiles of labor productivity, estimated using the Household Finance and Consumption Survey (HFCS) for the euro area countries, and to closely match the resulting life-cycle patterns of consumption-savings decisions to this data. In contrast to the US-based studies by Gagnon et al. (2016) or Eggertsson et al. (2019), who use a similar level of detail in modeling demographics, we cast our model in an open economy setup. This allows us to study cross-border capital flows and the dependence of the NRI in the euro area on the real interest rate rate in the rest of the world. Another important feature of our paper is the presence of a pension system, which allows us to analyze how its possible reforms implemented in response to population aging can affect the NRI. Our final contribution is a discussion of the possible role for increased migration in mitigating the fall in the natural interest rate.

[2]In a related paper quantifying the impact of demographic changes on the euro area NRI in a closed economy model, Papetti (2019) applies the methodology proposed by Jones (2018) to approximate the solution of the optimization problem of OLG households via a representative agent's problem.

Our findings suggest that aging is the dominant force behind the medium-term trends in the EA real interest rate, and also significantly affects other macroeconomic variables. In particular, given the history and currently available projections for mortality, fertility, productivity, public debt, and the world interest rate, the NRI in the euro area is projected to decline by about 2 percentage points between 1985 and 2030, of which about two-thirds can be attributed to demographic forces alone. Being an open economy has not been significantly reducing the adjustment in the EA NRI, mainly because the rest of the world has been undergoing a similar demographic transition. However, this has already started to change and faster aging in Europe will eventually lead to a significant improvement in its net foreign assets position, which can be estimated at 10 percent of GDP between 2010 and 2030. A possible reform of the pension systems that reduces the effective replacement rate would put additional downward pressure on the NRI while an increase in the effective retirement age has the opposite consequences for this variable. Allowing for more migration could improve the situation, but the effect would be delayed by about two decades as migrants arrive with very low asset holdings.

The rest of this paper is organized as follows. Section 2 lays out the model used in our analysis. Section 3 documents our calibration strategy and construction of exogenous variables driving the model dynamics. In Section 4 we present our main simulation results. Section 5 concludes.

2 Model

We construct a structural open economy model with life-cycle features, where households face a hump-shaped productivity profile as well as age- and time-dependent mortality risk. The model economy is also populated by producers, investment funds, and a fiscal authority, and linked to the rest of the world via international trade in goods and assets. Below we describe the problems faced by each type of agents. While denoting real allocations, we employ the convention of using upper case letters for aggregates, and lower case letters for variables expressed in per capita terms.

2.1 Households

2.1.1 Optimization problem

Each household consists of a single agent, who appears in our model at the age of 20 and is assigned age index $j = 1$. Agents can live up to 99 years ($j = J = 80$), at each year t subject to age- and time-dependent mortality risk $\omega_{j,t}$. Hence, at each point in time, the model economy is populated by 80 cohorts of overlapping generations, with the size of cohort aged j denoted by $N_{j,t}$.

A representative j-aged household maximizes its expected remaining lifetime utility that depends on consumption streams $c_{j,t}$ according to

$$U_{j,t} = \sum_{i=0}^{J-j} \beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \ln c_{j+i,t+i} \tag{1}$$

where β is the subjective discount factor, and the ratio $N_{j+i,t+i}/N_{j,t}$ represents the probability of surviving for at least i more years.

Households work until reaching the age of 63 ($j = JR = 44$), after which they receive gross pension benefits pen_t that are independent of their age. Participation in the pension system is mandatory so that all workers have to pay a social security contribution that is levied together with other taxes on their labor income at total rate τ_t. Households can additionally smooth their consumption by trading assets, which consist of claims on domestic capital, domestic government debt and assets issued by the rest of the world. From an individual household perspective, all of these assets are perfect substitutes in a deterministic environment that we consider, and hence they pay the same real interest rate r_t. We do not restrict asset holdings $a_{j,t}$ to be non-negative at any age j so that borrowing across different cohorts is possible. Households also receive lump sum and age-specific dividends $\pi_{j,t}$ from intermediate goods producers. Since most agents die before reaching their maximum age, they leave unintentional bequests, which are redistributed equally across all living agents in the form of lump-sum transfers beq_t.

A j-aged household hence faces the following budget constraint

$$c_{j,t} + a_{j+1,t+1} = (1-\tau_t)\left[(1-\mathbf{1}_{j\geq JR})w_t z_j + \mathbf{1}_{j\geq JR}\, pen_t\right] + \pi_{j,t} + beq_t + (1+r_t)a_{j,t} \tag{2}$$

where $\mathbf{1}_{j\geq JR}$ is an indicator for being retired, w_t is the real gross wage rate and z_j is exogenous age-specific labor productivity.

2.1.2 Demography and aggregation

In our model, the demographic processes are governed by changes in the size of the youngest cohorts $n_{1,t+1} = \frac{N_{1,t+1}}{N_{1,t}} - 1$ and mortality risk $\omega_{j,t}$, both of which are assumed to be exogenous. The total number of living agents N_t and the population growth rate n_{t+1} are given by

$$N_t = \sum_{j=1}^{J} N_{j,t} \quad \text{and} \quad n_{t+1} = \frac{N_{t+1}}{N_t} - 1 \tag{3}$$

where the number of agents in each cohort evolves according to

$$N_{j+1,t+1} = (1-\omega_{j,t})N_{j,t} \tag{4}$$

The aggregate allocations over all living households can be expressed in per capita terms as follows

$$c_t = \sum_{j=1}^{J} \frac{N_{j,t} c_{j,t}}{N_t} \tag{5}$$

$$h_t = \sum_{j=1}^{JR-1} \frac{N_{j,t} z_j}{N_t} \tag{6}$$

$$a_{t+1} = \sum_{j=1}^{J} \frac{N_{j,t} a_{j+1,t+1}}{N_{t+1}} \tag{7}$$

$$beq_t = \sum_{j=1}^{J} \frac{(N_{j,t-1} - N_{j,t})(1 + r_t) a_{j,t}}{N_t} \tag{8}$$

2.2 Firms

There are two types of firms in our model economy – perfectly competitive final goods producers and monopolistically competitive intermediate goods producers, the latter indexed with ι. Consistently with demographic processes in the household sector, the mass of each type of firms is tied to the size of population.

2.2.1 Final goods producers

A representative final goods producer purchases intermediate goods $y_t(\iota)$ and produces a homogeneous final good y_t according to the following CES aggregator

$$y_t = \left[\frac{1}{N_t} \int_0^{N_t} y_t(\iota)^{\frac{1}{\mu}} \mathrm{d}\iota \right]^{\mu} \tag{9}$$

where $\mu \geq 1$ can be interpreted as gross markup resulting from imperfect substitution between intermediate varieties. The solution to the profit maximization problem implies the following demand function for intermediate goods

$$y_t(\iota) = p_t(\iota)^{\frac{\mu}{1-\mu}} y_t \tag{10}$$

where $p_t(\iota)$ is the real price (expressed relative to the aggregate price level) charged by intermediate goods producer ι.

2.2.2 Intermediate goods producers

Intermediate goods producers hire capital and labor, and produce differentiated output according to the following Cobb-Douglas production function

$$y_t(\iota) = x_t k_t(\iota)^{\alpha} h_t(\iota)^{1-\alpha} \tag{11}$$

where x_t is exogenous total factor productivity and k_t denotes physical capital accumulated according to the standard law of motion

$$k_{t+1}(\iota) = (1 - \delta)k_t(\iota) + i_t(\iota) \tag{12}$$

The period profit flows are then

$$\pi_t(\iota) = p_t(\iota)y_t(\iota) - w_t h_t(\iota) - i_t(\iota) \tag{13}$$

and each intermediate goods producing firm maximizes their present discounted value,[3] taking into account the demand schedules given by equation (10).

We assume that profits are transferred period by period to households, proportionally to their labor income.

2.3 Fiscal authority

The government purchases goods on the market in quantity G_t and manages the pension system, financing its expenditures with taxes levied proportionally on households' labor income and by issuing public debt B_t. The government's intertemporal budget constraint can then be written as

$$\tau_t w_t \sum_{j=1}^{JR-1} N_{j,t} z_j + B_{t+1} = G_t + (1 - \tau_t) pen_t \sum_{j=JR}^{J} N_{j,t} + (1 + r_t) B_t \tag{14}$$

Pensions per retired household are determined as a product of the replacement rate ϱ_t and the economy-wide average wage

$$pen_t = \varrho_t \tilde{w}_t \tag{15}$$

where $\tilde{w}_t = w_t (\sum_{j=1}^{JR-1} N_{j,t} z_j)/(\sum_{j=1}^{JR-1} N_{j,t})$. We assume that the paths of public debt, government purchases and the replacement rate are exogenously determined while the tax rate τ_t adjusts so that the government budget constraint (14) is satisfied.

2.4 Rest of the world

Domestic agents have access to international financial markets, and hence can accumulate foreign assets or borrow from abroad. To impose some realistic limits on financial integration, we assume that international transactions are intermediated by specialized agents, which additionally charge Γ_t per unit borrowed, passing the obtained revenue to savers. This cost shows up as a wedge in the uncovered interest parity condition that can

[3] For simplicity, the future profit flows are discounted with the real interest rate. A more sophisticated alternative using appropriately weighted stochastic discount factors of individual cohorts does not change our simulation results in any significant way.

be written as follows

$$1 + r_{t+1} = (1 + \Gamma_t)(1 + r^*_{t+1}) \tag{16}$$

where r^*_t is the world interest rate, which is exogenous from the home economy's perspective.[4] As it is common in the literature, we assume that the intermediation wedge depends on the country's external debt so that

$$\Gamma_t = \xi \left(\exp\left(-B^*_t / Y_t \right) - 1 \right) \tag{17}$$

where $\xi > 0$ and B^*_t denotes the aggregate net foreign assets position.

2.5 Market clearing conditions

The model is closed with a standard set of market clearing conditions. Since all intermediate goods producers are identical, they charge the same price in equilibrium so that $p_t(\iota) = 1$ for all ι. This makes aggregation of the production side in our economy trivial and in particular leads to the following aggregate production function

$$Y_t = x_t K_t^{\alpha} H_t^{1-\alpha} \tag{18}$$

Asset market clearing implies

$$A_t = K_t + B_t + B^*_t \tag{19}$$

which, after consolidating the constraints of all agents in the economy, results in the following standard law of motion for net foreign assets

$$B^*_{t+1} = (1 + r_t)B^*_t + Y_t - C_t - I_t - G_t \tag{20}$$

2.6 Exogenous processes

The model economy is driven by several exogenous forces. Demographic processes are characterized by the growth rate of initial young $n_{1,t}$ and age-specific mortality risk $\omega_{j,t}$ $(j = 1, \ldots, J)$. The fiscal authority exogenously determines its purchases of goods G_t, the replacement rate in the pensions system ϱ_t and the path of public debt B_t. Finally, two exogenous processes come from abroad, and these are the total factor productivity at the world technology frontier x_t and foreign real interest rate r^*_t.

[4] While strict exogeneity of the world interest rate from the EA perspective might look controversial, it is not uncommon in the literature. For example, the main structural macroeconomic model at the ECB assumes that the euro area is a small open economy, see Coenen et al. (2018). It also needs to be noted that we do allow for a fairly strong link between the domestic and foreign real interest rate in our model by assuming a common productivity productivity process in both regions, which we interpret as reflecting the world technology frontier.

3 Calibrated parameters and exogenous variables

The model is calibrated for the euro area, with the rest of the world represented by the remaining four G7 economies (Canada, Japan, the UK and the US), henceforth referred to as non-EA G7. We choose this group for two reasons. First, while obviously the rest of the world is much bigger than G7 alone, we believe that to take into account international capital flows the group of rich countries is much more relevant than possibly more populous, but poor and financially underdeveloped countries. Second, the availability of relevant data is limited for countries outside the EA and G7.

The model is first solved for the non-EA G7 as a closed economy, which generates the world interest rate. Then we solve the model again with EA as a small open economy which takes the world interest rate as given. Below we report the construction of the exogenous inputs to the model. We explain in detail how we parameterize the life-cycle characteristics, assumptions regarding demographic trends and other exogenous forces, as well as key structural parameters.

3.1 Life-cycle characteristics

We calibrate the EA age profiles associated with z_j ($j = 1, ..., JR - 1$) by relying on calculations presented in Jabłonowski (2018), who used the second wave of the Household Finance and Consumption Survey (HFCS), conducted in 18 euro area countries between 2012 and 2014. Our empirical measure of productivity z_j is hourly labor income, which includes wage employment and self-employment. The life-cycle characteristics are extracted at the household level, and the age of the household is determined by the age of the household head. Thus obtained profile is next smoothed with the Hodrick-Prescott filter[5] and the profile for the age groups 20-24 (for which the data is missing) is extrapolated linearly, basing on the productivity trend for the age groups 25-30.

The first panel in Figure 2 presents the matched age profile for labor productivity. It follows the well-documented pattern, increasing up to the late middle age of a household head, and then declining. In the remaining two panels, we show how our model matches two other important life-cycle profiles that we do not target directly, namely consumption and net assets. Again, as an empirical benchmark, we use the HFCS data described above, with assets defined as net wealth excluding public and occupational pensions, and consumption approximated by spending on food (at home and outside) and utilities. Since our model does not explicitly account for changes in household composition or family size, the extracted age profiles are next divided by the square root of the number of household members, which is one of the equivalence scales used while working with household level data, see Fernandez-Villaverde and Krueger (2007) and OECD (2008). Given the simple structure of our model, the profiles are matched remarkably well, even

[5] Whenever we use the Hodrick-Prescott filter, the smoothing parameter is consistently kept at 100.

though we somewhat underestimate the pace of asset accumulation at early stages of life, which is not surprising as our model does not feature housing.

Although we do not possess comparable, consistent micro-level household data for the non-EA G7 countries, we are able to construct the age profile for labor productivity by using the hourly wage profiles by age for the US workers from Keane and Wasi (2016) and the administrative aggregate data for earnings and hours worked by age groups in Japan.[6] The resulting profile is almost identical to the profile of EA workers, differing significantly only for active workers above the average effective retirement age.

3.2 Demographic forces

Our demographic scenarios use the past data and projections on mortality rates and the sizes of the cohorts of 20-year olds. We rely on the United Nations World Population Prospects 2019, covering years 1950-2020 (actual data) and 2020-2100 (projections). Since the data are available for the individual EA and G7 member states in 5-year intervals and for broad five-year age groups, we first interpolate them to annual frequency using cubic splines, and then use the population-weighted averages to construct the data for both country groups.[7] The resulting mortality rates and rates of growth of the population of 20-year olds are smoothed using the Hodrick-Prescott filter to avoid spurious jumps in the demographic input data produced by data revisions or by splicing data from not fully compatible sources.

At the end of our projection horizon, we assume that mortality rates stabilize while the rate of change of the 20-year old cohort size stays at the level projected for 2080. To ensure that the model predictions for the years that we focus on (i.e. 1980 and after) are not contaminated by frontloading effects and initial conditions, we start our deterministic simulations in year 1900. To that end, we construct artificial population data for years 1900-1950 and backcast the sizes of historical 20-year old cohorts while holding the historical mortality rates at the earliest available date to accurately match the existing population structure. Since due to the presence of migration flows we cannot exactly match the population structure in every year, we have chosen year 2010 as our base year,[8] while the modeled population structures for other years are constructed using the data on mortality rates and rates of growth of initially young. Note that, by following this procedure, we effectively account for migration of people below the age of 20.[9] The resulting

[6]Data provided by the Ministry of Health, Labour and Welfare, Year Book of Labour Statistics 2017, Table III.69.

[7]To ensure the validity of this approach, we cross-check the constructed data for the EA with comparable data from Eurostat which are available at the annual frequency, but cover a shorter timespan. We find no significant differences between the actual and constructed population structures.

[8]This choice is dictated by the fact that significant migration flows from the EU New Member States to EU-15 countries have occurred after the 2004-2007 enlargements. The stock of migrants has stabilized shortly after the Great Financial Crisis.

[9]As future migration flows are highly uncertain, we do not explicitly incorporate them in our baseline scenario. A more detailed treatment of post-2010 migration and the discussion of its consequences are

age pyramids for the EA population in years 1980, 2010 and 2040 are depicted in Figure 3, which confirms that we are able to capture accurately the underlying demographic trends despite the applied smoothing and incomplete treatment of migration flows.

3.3 Other exogenous forces

We feed into our model also a couple of other driving forces that have been discussed in the literature as potentially important drivers of the real interest rate. The obvious one is the total factor productivity (TFP) that we interpret as representing the world technology frontier, and hence assume that the growth rate of "potential" TFP is the same for the EA and other G7 countries. We construct a GDP-weighted TFP index for the entire G7 group[10] for years 1960-2020, apply the Hodrick-Prescott filter to isolate its trend growth rate, and splice it with the projections for the TFP growth rate in the euro area provided by the Aging Working Group (AWG) for years 2016-2070.

Another set of driving forces is related to the fiscal sector and pension system. The paths of general government gross and net debt are taken from the IMF WEO database, covering years 1980-2024. Since a non-trivial part of the gross debt is held by the public sector (mostly central banks) and is unavailable for households to trade, we opt to feed the model with time series for net debt. While this statistic is readily available for the EA group, it has to be constructed for the non-EA G7 group by first calculating its GDP-weighted gross debt to GDP, and then correcting this time series by the average observed ratio between net and gross debt in the entire group of advanced economies. The paths for public debt in years preceding 1980 and following 2024 are fixed at the 1980 and 2024 levels, respectively,[11] and smoothed using the Hodrick-Prescott filter to abstract from swings in public debt due to business cycles. Given little time variation in the share of government purchases in EA output, we fix it at 20%, which corresponds to the long-term average observed in the data. We also fix the replacement rate at 44%, which corresponds to the average replacement rate[12] observed in the last decade in the EA countries. However, we will also consider alternative paths for this indicator while discussing the impact of potential pension system reforms.

3.4 Structural parameters

The remaining structural parameters are calibrated to match some key macroeconomic proportions of the EA economy. The chosen values are reported in Table 1. The discount factor is calibrated at 1.0034 to match the average real interest rate of 1.2% observed

described in a separate section.

[10]The data are taken from AMECO and cover years 1960-2020.

[11]This seems to be a reasonable assumption for the EA, but not necessarily for other G7 countries, especially US and Japan. Nevertheless, the impact of foreign public debt on the EA variables in our simulations is small.

[12]Strictly speaking, we use the benefit ratio, which is the ratio between the average gross pension and average gross monthly earnings in the entire economy.

in the euro area over years 1999-2008.[13] This is the longest time span during which the Eurozone interest rates can be considered close to their equilibrium values. In this period inflation was relatively stable, and after 2008 the euro area faced a prolonged crisis that pushed the interest rates down for cyclical rather than structural reasons.

Physical capital is assumed to depreciate at a standard annual rate of 10%. The capital elasticity of output is calibrated at 0.25, which ensures that the investment rate is close to the average observed in the Eurozone. This parametrization, together with a standard value of product markup of 1.25, implies that the labor income share in output is 60%, which is close to numbers reported in the national accounts of EA economies. For non-EA G7 countries we assume the same parameter values as in the EA, except that the discount factor is set to a somewhat higher value to match the average real interest rate of 0.8% observed in the non-EA G7 block (using GDP as country weights) over the years 1999-2008. Finally, the risk premium parameter associated with international borrowing is set to 0.038, targeting the average international investment position of the euro area over the years 1999-2008 of about -10% of GDP.

4 Macroeconomic impact of the demographic transition

As it is well known from the literature, population aging can potentially have a sizable impact on the economy, and in this section we quantify this effect for the euro area. To this end, we feed our structural model of this region with the paths of exogenous variables described in Section 3, first producing the world rate of interest by simulating a closed economy version of our model calibrated to non-EA G7 countries. All of our simulations are obtained by running the model variants in a deterministic mode, i.e. we assume that all driving forces are known to agents in advance. As indicated before, we start the simulations in 1900. However, our discussion of the main results focuses on the period since mid-1980s, which roughly corresponds to the beginning of a secular fall in the NRI observed in developed countries.

4.1 Baseline scenario

We begin with presenting the scenario for the foreign interest rate. Since the focus of our paper is on the euro area, we do not show detailed simulations or decompositions for this variable, which is exogenous from the EA perspective. Figure 4 presents the world real interest rate generated from the foreign block of our model, together with the estimates

[13] Despite a low real interest rate, our model economy is not dynamically inefficient, at least using the criterion by Abel et al. (1989). At any time period in our simulations, the marginal product of capital is larger than the investment to capital ratio. More generally, our model can reconcile a relatively large marginal product of capital with a low real interest rate because of the presence of monopoly rents, which drive a wedge between the two. See also Eggertsson et al. (2019) for more discussion of this issue.

of the GDP-weighted average of the natural rate of interest estimates for the US, Canada and UK calculated by Holston et al. (2017) and for Japan by Han (2019) who uses a similar method. A comparison over the time period for which the empirical estimates are available shows that the foreign block of our model captures very well the downward trend in the natural interest rate observed during the last three decades, including its flattening towards the end of the sample. This gives us confidence that our key external driving force for developments in euro area is appropriately captured. As regards the future, the world real interest rate is projected to start rebounding around mid-2020s, but this increase will be very moderate. The main factor behind these developments is the expected recovery in TFP growth, mitigated by the continued demographic pressure associated with population aging.

Figure 5 shows the past realizations and projected evolution of other forces that, besides the foreign interest rate discussed above, drive the EA scenario. These are the old-age dependency ratio, which summarizes the impact of demographic processes, as well as productivity growth and the net public debt-to-GDP-ratio. To facilitate a later discussion on the role of financial openness, in the same figure we also show the drivers for the external block of our model, which was used to generate the foreign interest rate. Since late 1980s, population of the EA was aging at an increasing pace, reflecting low fertility and decreasing mortality rates. Given the available demographic projections, this trend is going to continue over the next decades. During the same period, trend TFP growth at the world technology frontier was decelerating, reaching a trough around 2010, after which it started a sluggish recovery. This recovery is expected to continue, but, according to the forecasts described in Section 3.2, the growth rate of productivity will not reach levels observed in the 1990s within the next two decades. The public debt in the euro area has been on a continuous rise at least since 1980, and its ratio to GDP started to decline only very recently. Due to the IMF projections ending in 2024, we assume that the path of public debt stabilizes around the year 2030 at 60%.

Figure 6 presents the decomposition of the model-generated natural interest rate in the euro area into the contributions of the driving forces. Since in our simulations TFP is defined as the world technology frontier (and hence its path is the same in the EA and the rest of the world), we combine its contribution with that of the world interest rate, postponing a detailed discussion of the role of openness to the next subsection. Clearly, the real interest rate in the EA follows a downward trend in the simulation period, and the decline is substantial, amounting to around 2 percentage points between 1985[14] and 2030. As the decomposition shows, demography is the main driving force behind these developments, accounting for approximately two-thirds of the decline. Of the two demographic factors, mortality is the more important one. Fertility also plays a significant role, but only since about 2000, and its contribution is even slightly positive in the 1990s

[14] We anchor our decomposition in year 1985, as then our simulated NRI was quite stable for a few years and the level closely approximates the mean level for the entire 1980s decade.

because of the echo effect of the post-war baby boom. Global factors contribute positively for about ten initial years, reflecting elevated TFP growth, but then pull the EA natural rate strongly down, and their role in the 2010s is similar to that of demography. It is also interesting to note that the projected recovery in the EA NRI is postponed by about a decade relative to the rest of the world due to worse demographic conditions. Finally, the evolution of public debt has a positive impact on the EA interest rate throughout the whole simulation sample, but its contribution is dwarfed by the demographic and global forces.

How do these two main drivers affect the interest rate? Given a fixed retirement age, lower mortality means a longer expected time without labor income, hence workers increase their savings. Low birth rates result in a declining population, and thus lead to higher per capita asset holdings. Sluggish growth in productivity means that households expect a slower increase in their labor income, which dampens their desire to borrow in order to smooth their lifetime consumption. These effects can be observed more directly on Figure 7, which also shows the evolution of other important macroeconomic variables under the baseline scenario. As can be expected, population aging means that labor becomes scarce, leading to an increase in real wages and, as a consequence, a higher capital-labor ratio chosen by firms. This shift towards more intensive use of capital in the production process initially generates an investment boom, but after 2010 shrinking working-age population relative to the number of retirees starts to prevail and the investment-to-output ratio begins to fall. These demographic processes together with a slowdown in productivity reduce the growth rate of GDP. As people expect a longer lifespan and future income growth prospects become weaker, they accumulate more assets. Since aging in the EA proceeds faster than in the rest of the world, this increase in savings leads to both higher domestic physical capital and accumulation of foreign assets. As a result, the net foreign asset position of the euro area, initially negative, starts increasing after 2010 and improves by about 10 percent of GDP before 2030. Since in our baseline simulation we keep the replacement rate and retirement age constant, a higher dependency ratio requires a steady increase in the total tax rate, which can be only temporarily postponed by (exogenously assumed) public debt accumulation. We discuss the alternative assumptions on the pension system later in a separate subsection.

It might be interesting to compare those of our simulated paths that are observable in the data to actual developments in the euro area. Figure 8 plots the outcomes of our simulation together with the data for hours worked per capita, GDP growth rate and the international investment position. Additionally, we plot the smoothed estimates of the EA natural real rate of interest from Holston et al. (2017), and compare it to our model-based trajectory. While (not surprisingly) the exact numbers differ, it is striking that our projected paths are much in line with the trends observed in the data. In particular, our model captures very well the magnitude of reduction in per capita working time, and the decline in per capita GDP growth rate over the last three decades. The model is less successful with respect to net foreign assets, but even here the general tendency seems to coincide (EA data is available only since 1999). In this context, it has to be born in mind that this variable has probably been, to a large extent, driven by cyclical developments related to the boom-bust cycles in the euro area periphery and hence cannot be reflected by our life-cycle framework. As regards the NRI, the model-based predictions are very well aligned with the data. Again, our model is not able to capture cyclical fluctuations exhibited by the data, but the comparison clearly suggests that, over the analyzed period, technological and demographic processes have been the main drivers of the downward trend in the Eurozone real rate of interest.

4.2 The role of openness

It is well known that, in a world with liberalized capital flows, domestic interest rates will, at least to some extent, depend on external developments, and in particular on the world interest rate. While the textbook offers a vision of a simple binary world, where economies can be either completely closed or open, the reality is more nuanced. Even under complete de jure openness of the capital account, capital does not flow freely across borders to perfectly equalize expected returns. One of the reasons are investors' preferences to invest on the domestic market, where they are more familiar with the rules of the game (e.g. contract enforcement). Moreover, capital flows, especially if they occur in one direction, may be perceived as risky as they trigger default concerns. Such risk premia create additional incentives to invest on the domestic market and form a barrier to capital flows.

In our model, similarly to a standard open economy framework, the degree of financial openness is determined by the debt elasticity parameter ξ in equation (17). As described earlier, its value was calibrated to match the average international investment position of the euro area in our calibration sample. While formally the parameter determines the debt elasticity of the risk premium, a more appropriate interpretation would be that it is responsible for all impediments to capital flows, including formal restrictions and behavioral features mentioned above. We now discuss the role of this parameter in driving our findings for the euro area discussed above.

Figure 9 presents the outcomes for the real interest rate and the net foreign asset position. It is useful first to consider one extreme, which is when ξ is infinitely large so that the euro area becomes financial autarky. It is interesting to note that in this scenario the EA natural real interest rate would have been somewhat higher than under our baseline, but the trends are otherwise very similar. This is because the demographic processes in the euro area have not been very different from those observed in the rest of the G7 group treated as one region. After 2020, however, the closed economy variant implies a bit steeper decline in the NRI so that it eventually falls below the baseline path. The reason behind this divergence is that the euro area is expected to age at a faster pace than the rest of the world. Recalling Figure 1, this can be traced back to both a much steeper fall in the fertility rate during the last three decades of the previous century, which is only going to kick in at full force in the coming years, and a steeper increase in longevity since the mid-1980s. As a result, the downward pressure on the real interest rate in the EA can be somewhat alleviated if its households are allowed to save abroad, but this effect is not quantitatively large.

The other extreme that could be considered is full financial openness. In this case the real interest rate in the euro area would be simply equal to the world rate, leaving no role for EA-specific processes in shaping this variable. However, as it is well known from standard open economy models, perfect equalization of the interest rates across the borders implies unrealistically high international imbalances.[15] Therefore, rather than showing this extreme, we consider a much lower, but still positive value of ξ, in line with the estimates of Brzoza-Brzezina and Kotłowski (2020). As this number reflects only the impact of international debt on the country risk premium, it can be considered very close to a completely open capital account. As can be seen from Figure 9, the EA and world real interest rates are now very closely aligned over the whole presented simulation period. However, the magnitude of international capital flows needed to achieve this outcome are about four times larger than under the baseline – the EA international investment position improves by about 40 percentage points over just two decades from 2010 to 2030, compared to only 10 percentage points under our baseline scenario.

All in all, the conclusion from these exercises is that financial openness has not played a pronounced role in accounting for the observed secular fall in the EA natural interest rate. However, this situation has recently started to change quite dramatically. As a result, the ability to save abroad is becoming an important factor limiting the fall in the EA natural interest rate, and its importance is likely to increase significantly in the coming decades.

4.3 Impact of pension system reforms

In our baseline scenario we assume that the pension system in a typical EA country will not undergo any major changes in the foreseeable future, i.e. both the retirement age and replacement rate will be kept at their currently observed levels. We have already seen that, given the limited room for further increase in the public debt, this implied a steady increase in the total tax rate. In this section we discuss some alternative assumptions about the future shape of the pension system and their possible impact on the euro area NRI.

To better see the set of options faced by the government, it is instructive to substitute the definition of pensions (15) into the government budget constraint (14), and rewrite it

[15] This assumption also creates technical difficulties with our model solution as the steady state level of the net foreign assets position becomes indeterminate, and hence its initial value (and, in consequence, average value over our calibration period) would have to be simply imposed.

as follows

$$(1+\gamma_{t+1})b^y_{t+1} - (1+r_t)b^y_t = g^y_t + \frac{1-\alpha}{\mu}\left[(1-\tau_t)\varrho_t d_t - \tau_t\right] \tag{21}$$

where $d_t = (\sum_{j=JR}^{J} N_{j,t})/(\sum_{j=1}^{JR-1} N_{j,t})$ is the old-age dependency ratio (i.e. ratio of retirees to workers), $\gamma_t = Y_t/Y_{t-1} - 1$ is the growth rate of aggregate real GDP, $b^y_t = B_t/Y_t$ and $g^y_t = G_t/Y_t$ are the shares of, respectively, public debt and government purchases in output, and we used the result that in equilibrium the labor share in output is equal to $\mu^{-1}(1-\alpha)$. Therefore, an increase in the dependency ratio associated with population aging does not need to be fully accommodated by changes in the contribution rate to the public pension system, which is part of the total tax rate τ_t. Alternatively, the demographic pressure on the pension system can be reduced by increasing the retirement age JR, which automatically reduces the dependency ratio, or by adjusting the replacement rate ϱ_t.

In what follows we entertain the effects of possible pension reforms that modify these two variables. The scenarios are based on the projections regarding the effective retirement age and replacement rate prepared by the European Commission's Ageing Working Group (AWG) in their 2018 Ageing Report.[16] To feed changes in the retirement age in a smooth way, we allow for "fractional" retirement age, where within year t a j-aged individual spends fraction $\rho_{j,t}$ of time in retirement, still being active in the labor market for the remaining time. This boils down to modifying the budget constraint (2) as follows

$$c_{j,t} + a_{j+1,t+1} = (1-\tau_t)\left[(1-\rho_{j,t})w_t z_j + \rho_{j,t}\, pen_t\right] + \pi_{j,t} + beq_t + (1+r_t)a_{j,t} \tag{22}$$

The two alternative ways to reform the pension system result in the opposite incentives towards the accumulation of private savings, given that they are known to the agents well in advance, allowing them to adjust their lifetime decisions. These differing adjustments can be visible in Figure 10, where we show the results of three scenarios: an increase in the retirement age from 63 to 66, a decrease in the replacement rate from 44% to below 34%, and a combination of the two that replicates the AWG assumptions. The reforms are implemented gradually, starting shortly after 2010 and being finalized by 2060.

While all three scenarios reduce the strain on public finances by decreasing the expenditure on pensions and allowing for a reduction in the tax rate, they have markedly differing effects on the NRI and the euro area's international investment position. A decrease in the replacement rate reduces income received during the period of retirement and induces households to accumulate more assets. In comparison with the baseline outcomes, this results in deepening of the fall in the EA NRI by around 14 basis points in 2030 and in a stronger improvement in the net foreign assets position by around 5 percentage points. In contrast, a hike in the retirement age increases the pre-retirement income and

[16] We note that the ratios of gross old pensions to GDP generated from our model by feeding these projections closely match the levels observed in years 2010-2019 and are broadly in line with the comparable statistic projected by the AWG for the upcoming years.

shortens the period during which households deplete their assets to support consumption, reducing the need to accumulate private savings prior to retirement. This limits the fall in the real interest rate, which is higher by around 26 basis points in 2030, and leads to lower accumulation of foreign assets. Importantly, these two alternative scenarios comfortably house in the middle of their range both the baseline and the AWG scenario that combines both reforms.

Summing up, possible pension system reforms can have a significant impact on the future evolution of the NRI in the euro area, but its direction depends on the implementation details. A somewhat perverse implication of our simulations is that the more the fiscal authority will do to preserve the long-run sustainability of the pension system by moving towards a defined contribution design (which effectively decreases the replacement rate), the more downward pressure on the NRI it will create, thus constraining the monetary authority in its ability to prevent deep recessions.

4.4 The role of migration

Another potentially important and relevant question is to what extent our findings can be affected by alternative assumptions on future migration flows. There exists a relatively large pool of potential migrants who, provided there is political will, could to some degree make up for the declining labor force in the euro area. Our baseline scenario was based on the population projections published by the UN, which do assume positive imigration into advanced economies. As we mentioned before, we are effectively able to capture migrants arriving before they are 20 years old. This obviously ignores a fraction of actual flows.

We do not attempt to make our own predictions of possible migration, and instead construct two extreme variants that demonstrate the potential impact of migratory processes, basing on the EUROPOP 2018 projections. In the "zero migration" scenario we make use of the *no migration* variant to construct the rate of growth of initially young individuals. Compared to the baseline, this means that we are ignoring the migration of minors. In the "high migration" case, we use the Europop's *baseline* scenario and introduce corrections to the rate of growth of initial young such that the model generates the same number of 50-year olds as it is projected in this variant. Technically, we assume that all migrants (counterfactually) arrive at the age of 20 and share the features of domestic population, i.e. initially own no assets and have the same productivity profile as workers born in the euro area. This is clearly a shortcut, but it allows us to model immigration in a model-consistent way without needing to track migrants by their arrival age.

As a new baseline for these alternative scenarios we assume the evolution of the pension system as in the AWG scenario described in the previous section. Since migration flows consist of relatively young individuals, they affect the old-age dependency ratio in the recipient country, and hence the financial situation of the pension system. Consequently, as we have seen before, the effects on the economy will depend on whether this is accommodated by adjustments in the replacement rate or in social security contributions (part of taxes). For this reason, Figure 11 shows the simulation outcomes by interacting the two alternative migration assumptions with these two types of pension system adjustments.

If the replacement rate follows the same path as in the AWG baseline, increased migration can ultimately have some positive effect on the NRI and reduce accumulation of foreign assets. However, this impact is highly delayed. The explanation is related to the feature of migrants described above -- they arrive with no assets and it takes years before they accumulate them in quantity sufficient to have a visible impact on the real interest rate. The effects are about twice as large and materialize earlier if we instead assume that increased revenue to the public pension system is used to limit or delay the fall in the replacement rate. This is because adjustments in this pension system parameter directly affect the savings decisions of incumbent population. Turning to the zero migration case, it shows that without inflow of young people from abroad the natural real interest rate in the EA would not have been significantly different from what is currently observed or projected over the medium term, but the strain on the pension system would be significantly bigger.

All in all, immigration to the EA can to some extent mitigate the downward pressure on its natural interest rate exerted by demographic forces. However, its role over the nearest decades is not large, at least under the currently observed inflows or even their more optimistic paths in the future. In particular, they are unable to significantly offset the impact of population aging and reduced productivity growth.

5 Conclusions

To what extent can population aging, resulting from lower fertility rates and higher life expectancy, account for the secular downward trend in the natural rate of interest observed in many developed economies at least since the mid-1980s? While this impact has been recently investigated by several studies, also using structural models, its magnitude is not entirely clear, especially in the European context. We believe that our modeling approach, based on a fully-fledged open economy OLG framework for the euro area, featuring a realistic pension system and carefully calibrated to closely match the age profiles of key decisions made at the household level, is able to deliver arguably more precise simulations.

We show that the effects are substantial. In particular, out of the estimated 2 percentage point decline in the EA equilibrium interest rate between 1985 and 2030, about two-thirds can be attributed to demographic forces. Since the euro area has been aging at a similar pace as in the group of other advanced economies, openness to international capital flows has not been able to alleviate the demographic pressure on its NRI. More recently, aging in Europe started to outpace that in the rest of the world, and the the outcome is reshuffling in international borrowing. According to our estimates, demographics can account for approximately 10 percentage point increase in the euro area international investment position relative to GDP between 2010 and 2030. However, despite this capital outflow and projected rebound in productivity growth, the natural rate of interest in Europe is projected to continue falling until the early 2030s.

To some extent, these developments may be modified by possible pension system reforms that are likely to be undertaken in Europe in response to growing old-age dependency ratio. One of them is to increase the retirement age, which could reduce the downward pressure on the real interest rate. In contrast, improving pension system sustainability by decreasing the replacement rate would make the fall in the NRI deeper. Increased migration has some potential to alleviate the downward pressure on the natural rate somewhat, but its effect is highly delayed unless increased revenue from social security contributions is used to mitigate or postpone the planned fall in the replacement rate.

All in all, our conclusion is that the euro area should get accustomed to live in a world of relatively low real interest rates in the foreseeable future. This bears important consequences for monetary policy (higher risk of hitting the effective lower bound), macroprudential policy (more likely financial bubbles) or fiscal policy (implications for fiscal space). We leave the details of these interesting consequences for further research.

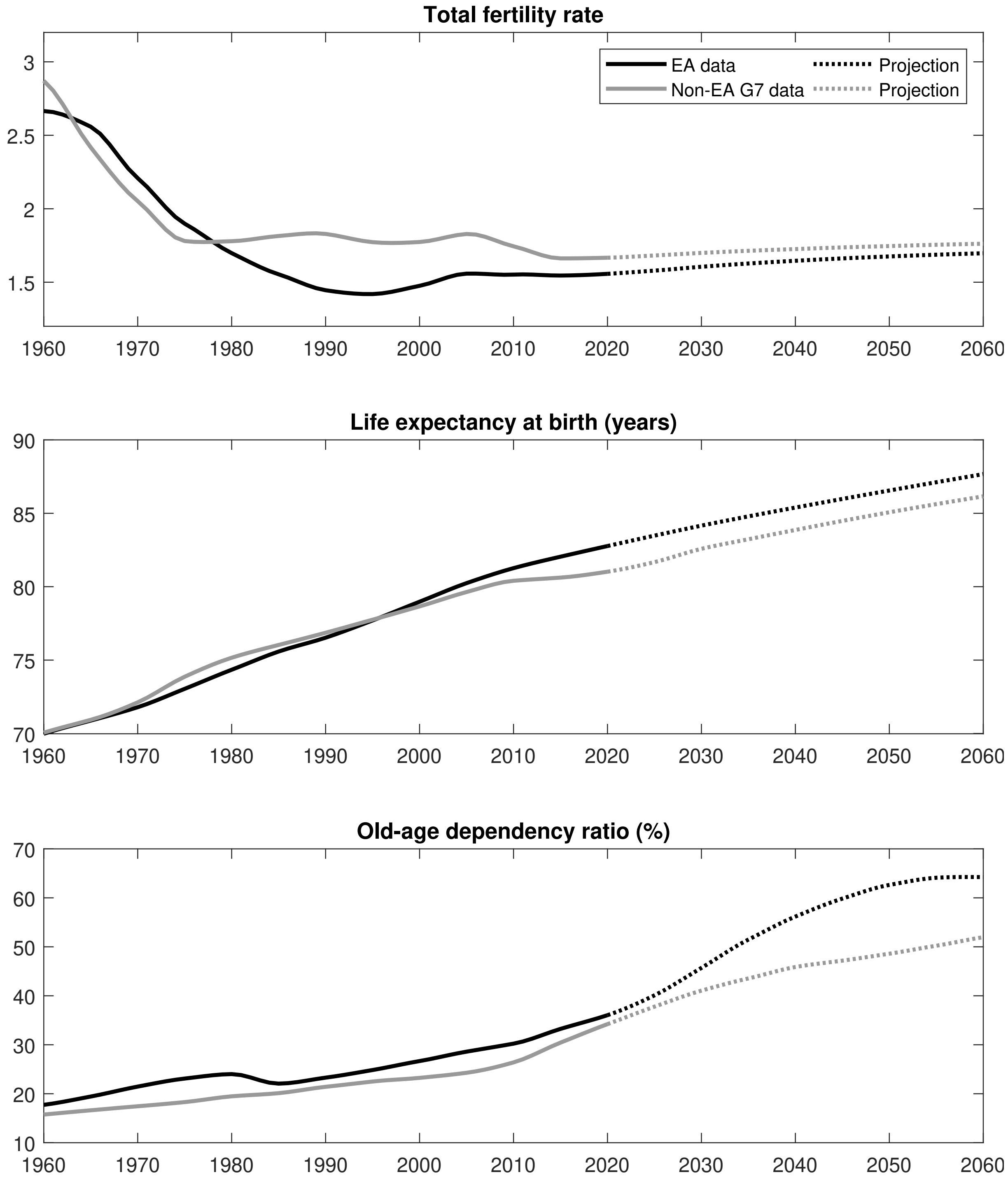
Tables and figures

Table 1: Calibrated structural parameters

Parameter	Value	Description
β	1.0047	Discount factor in the EA
β^*	1.0093	Discount factor in the non-EA G7
δ; δ^*	0.1	Capital depreciation rate
α; α^*	0.25	Capital share in output
μ; μ^*	1.25	Product markup
ξ	0.038	International borrowing friction

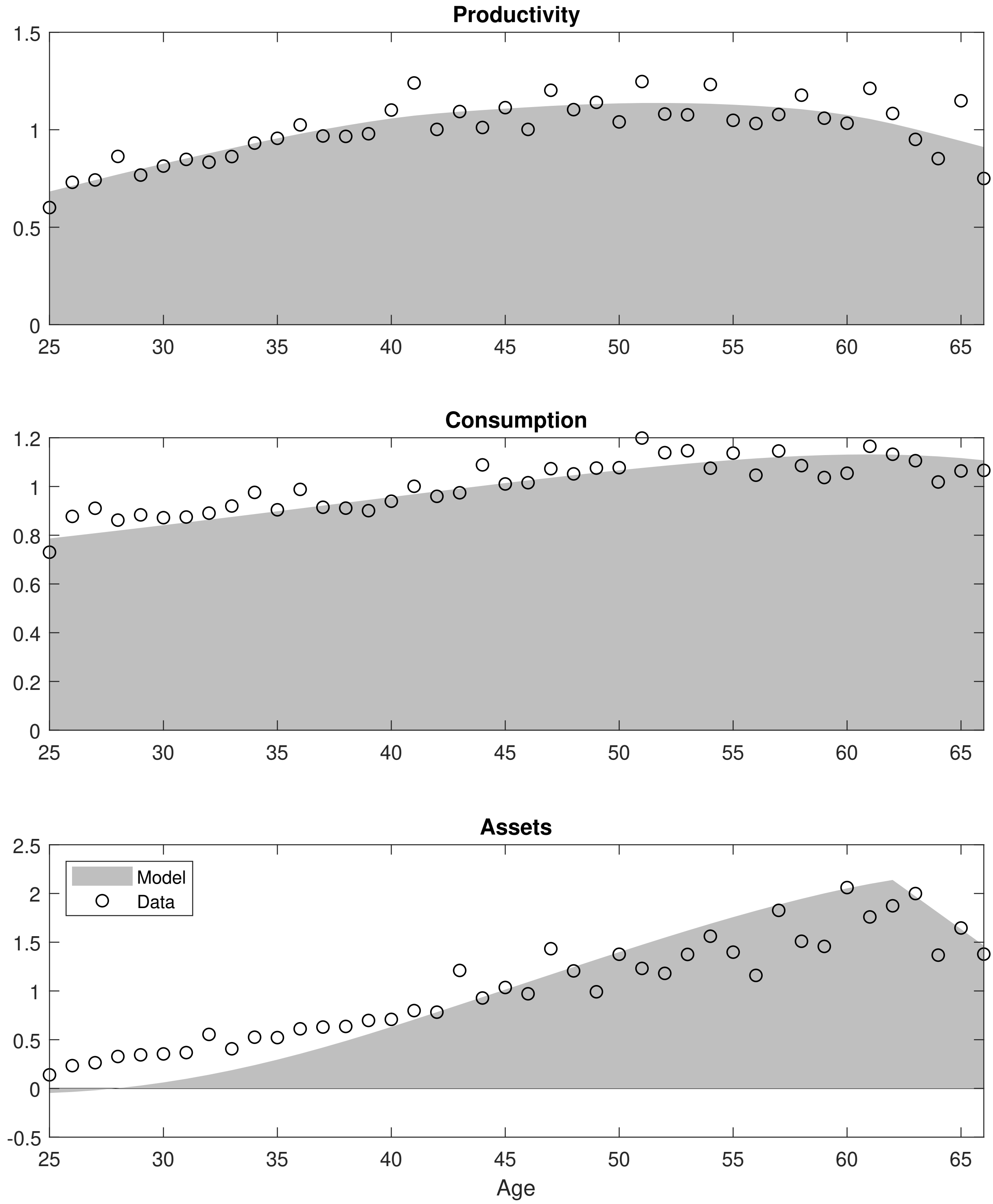
Notes: Stars indicate the parameters used to calibrate the rest of the world, modeled as a closed economy version of the baseline model.

Figure 1: Demographics



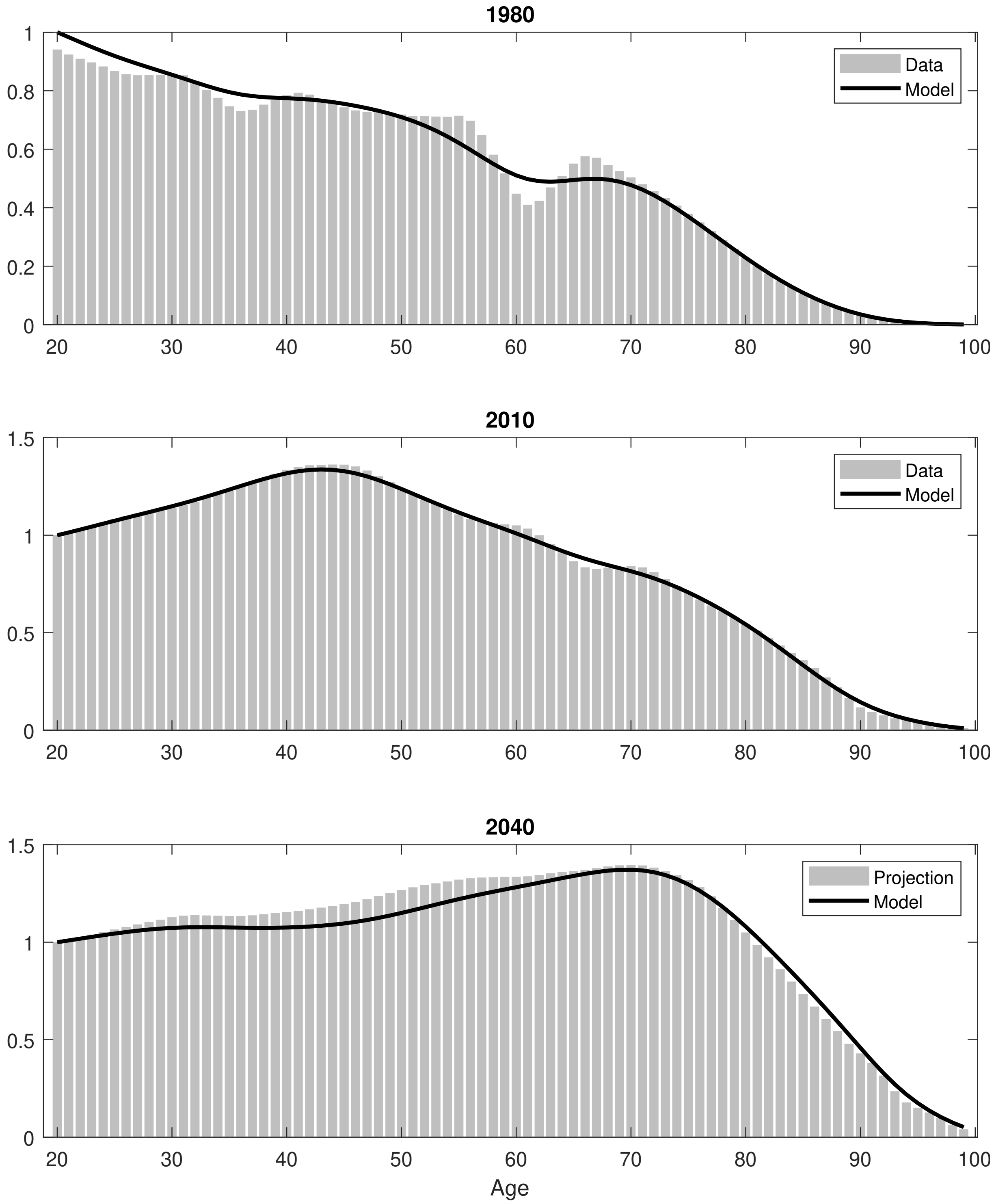
Source: United Nations World Population Prospects 2019. Within group averages are weighted by total population. The old-age dependency ratio is defined as the number of people 65 years old and above relative to the number of people aged 20-64.

Figure 2: Life-cycle profiles



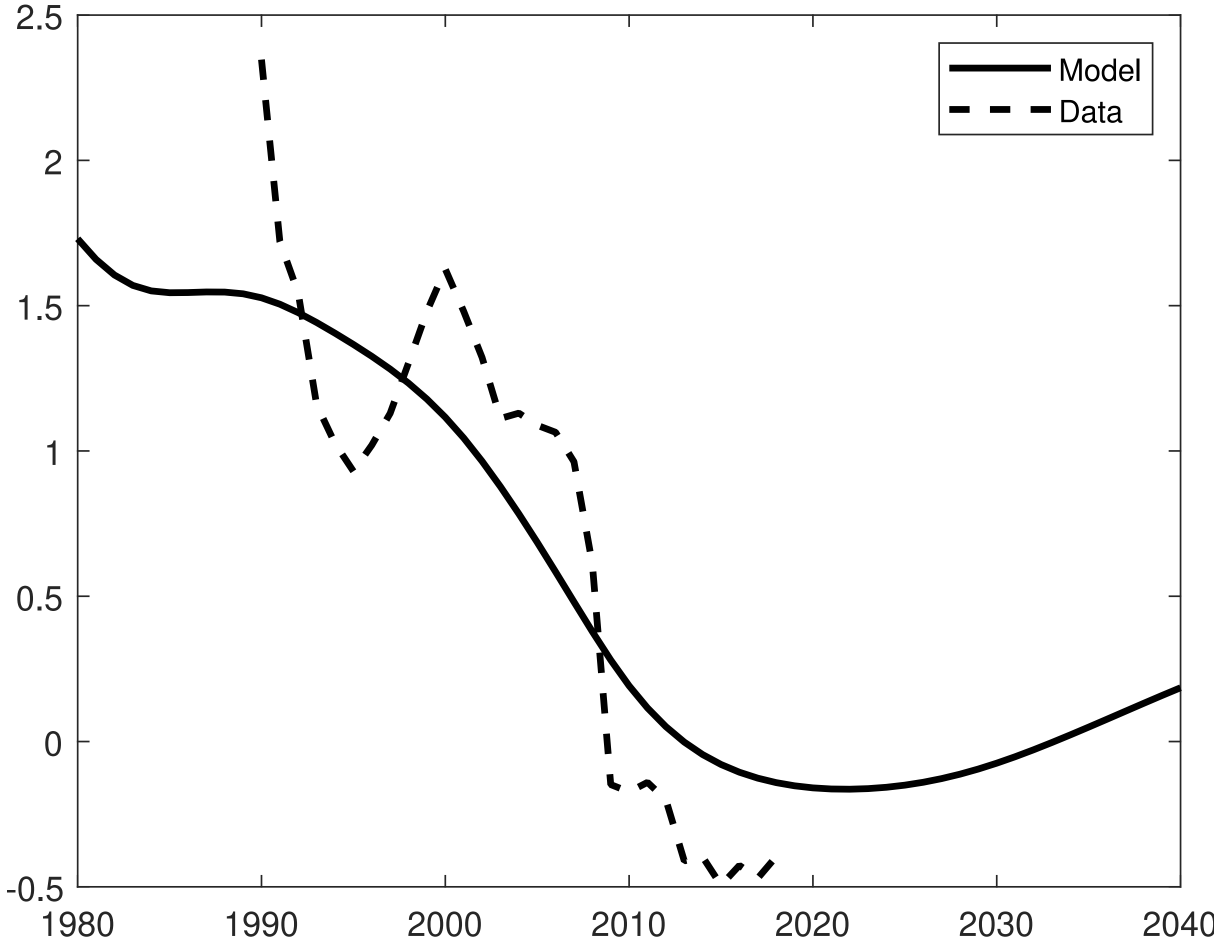
Source: HFCS data for the euro area and model simulations.

Figure 3: Population pyramids in the euro area



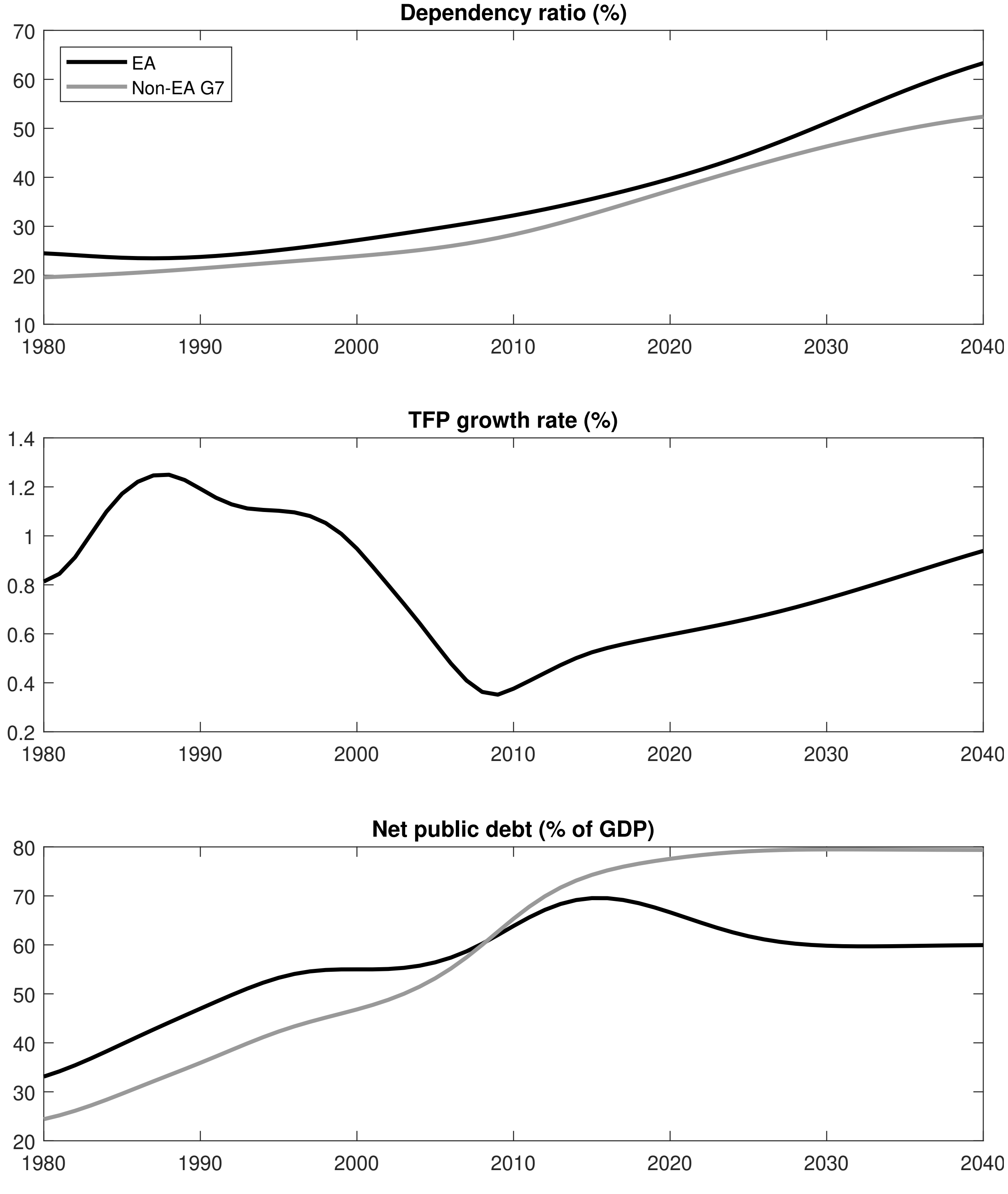
Source: United Nations World Population Prospects 2019 and model simulations. Cohort sizes have been normalized by the size of the 20-year old cohort.

Figure 4: Natural rate of interest in non-EA G7 countries



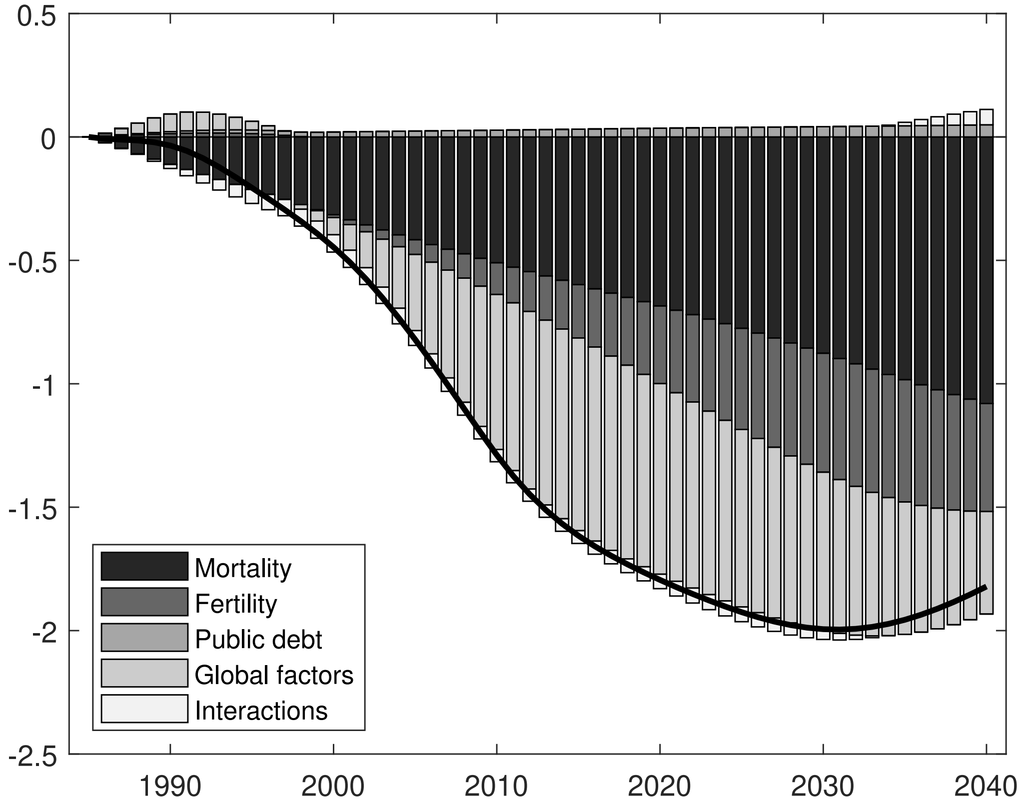
Notes: The data series is the GDP-weighted average of NRI estimates for Canada, Japan, the UK and US. The numbers come from Holston et al. (2017), except for Japan, in which case they are taken from Han (2019). Thus obtained aggregated are corrected such that their means over the presented sample are the same as in the series generated with the model. This correction is motivated by the fact that the Holston et al. (2017) estimates of the US NRI at every point in the presented sample are higher than the plain real interest rate data (defined as the short-term nominal interest rate less inflation), to which we calibrate the mean of the world interest rate in our model. For this reason, in this part of our discussion we stress the fit of trends rather than levels of the NRI between our model and the data.

Figure 5: Exogenous driving forces from EA and non-EA G7



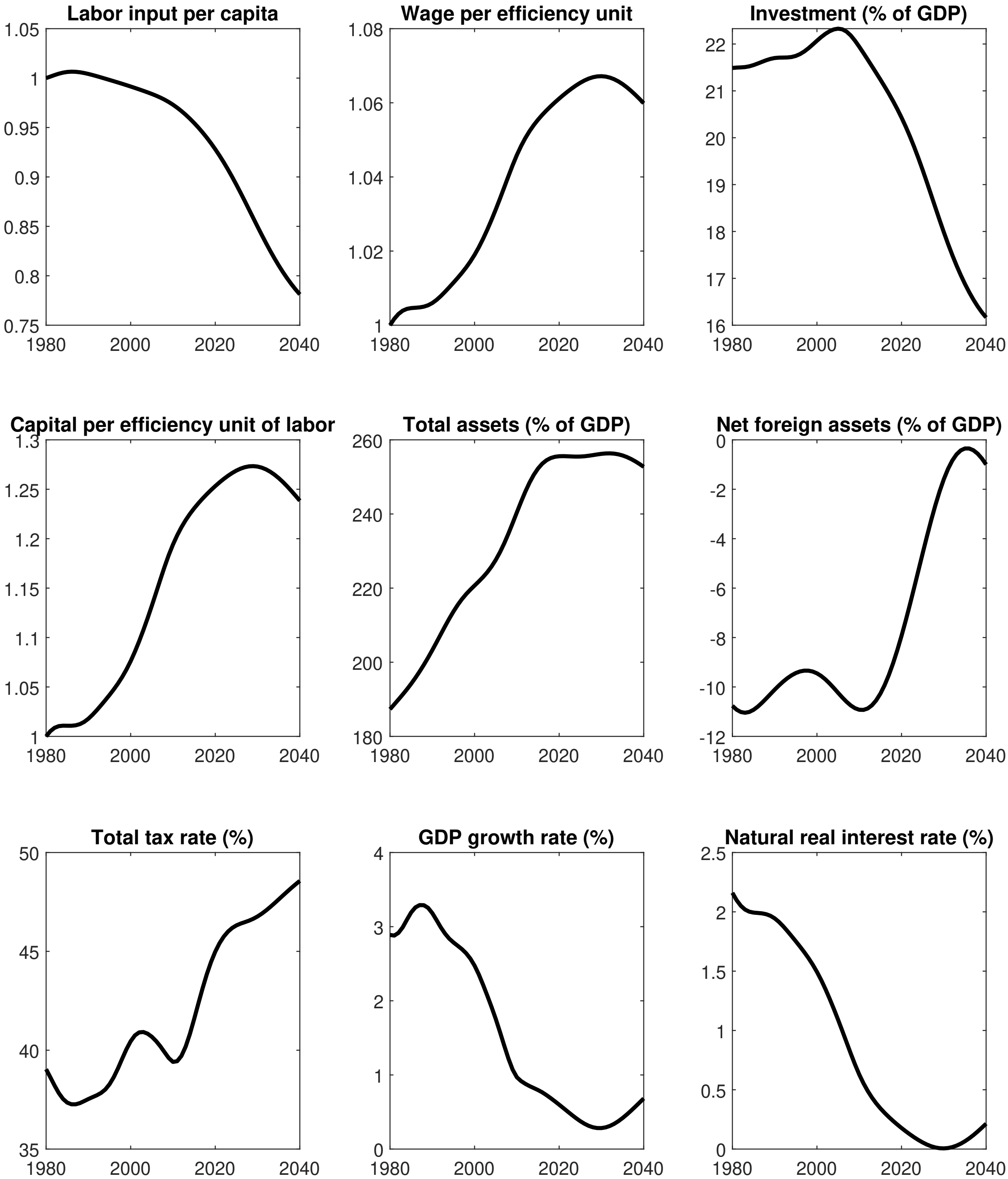
Notes: The dependency ratio is here defined as the number of retirees per working population.

Figure 6: Decomposition of changes in EA NRI



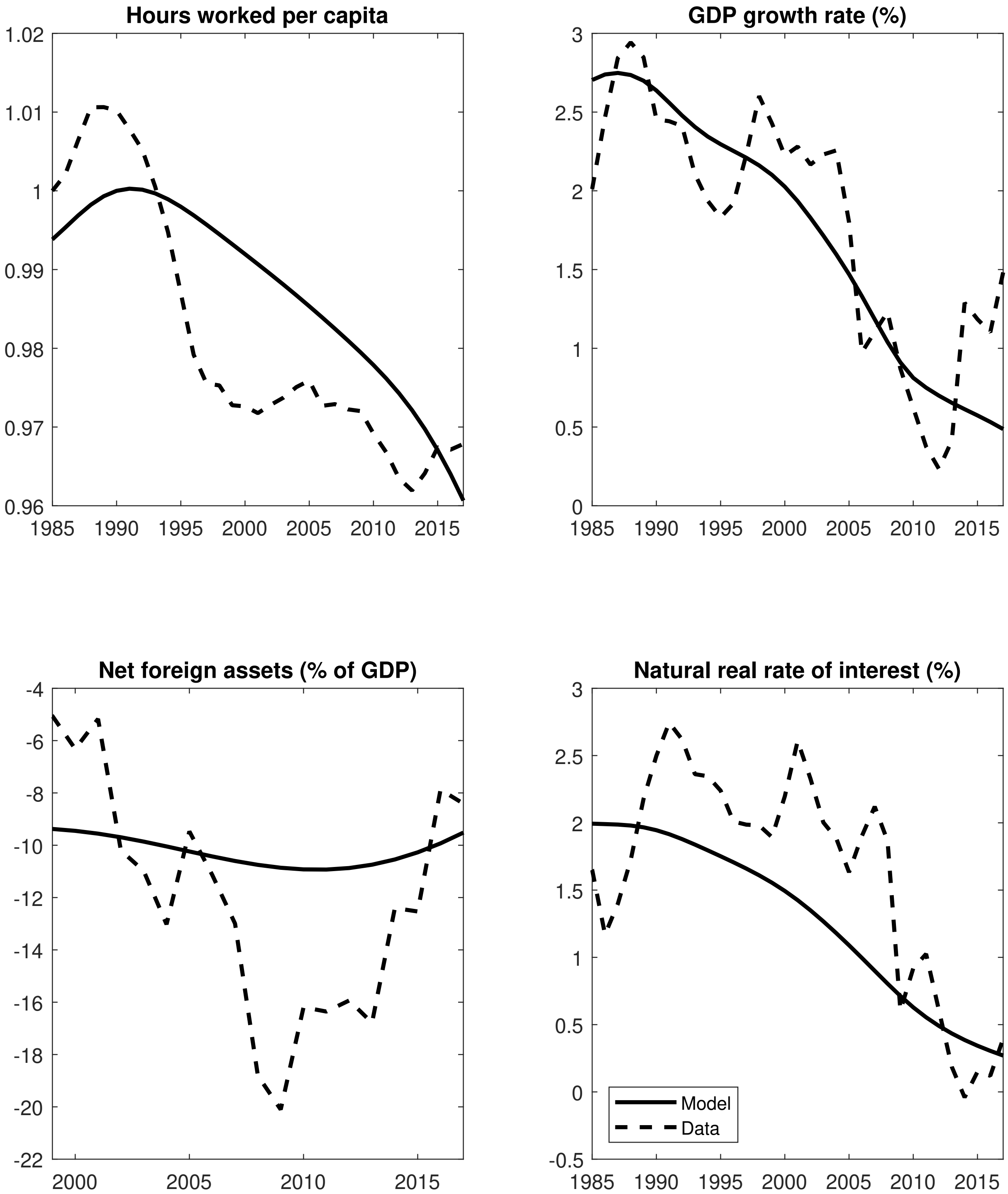
Notes: All numbers are presented as a difference from the 1985 levels. Global factors combine the impact of TFP growth and foreign interest rate.

Figure 7: Baseline model simulation



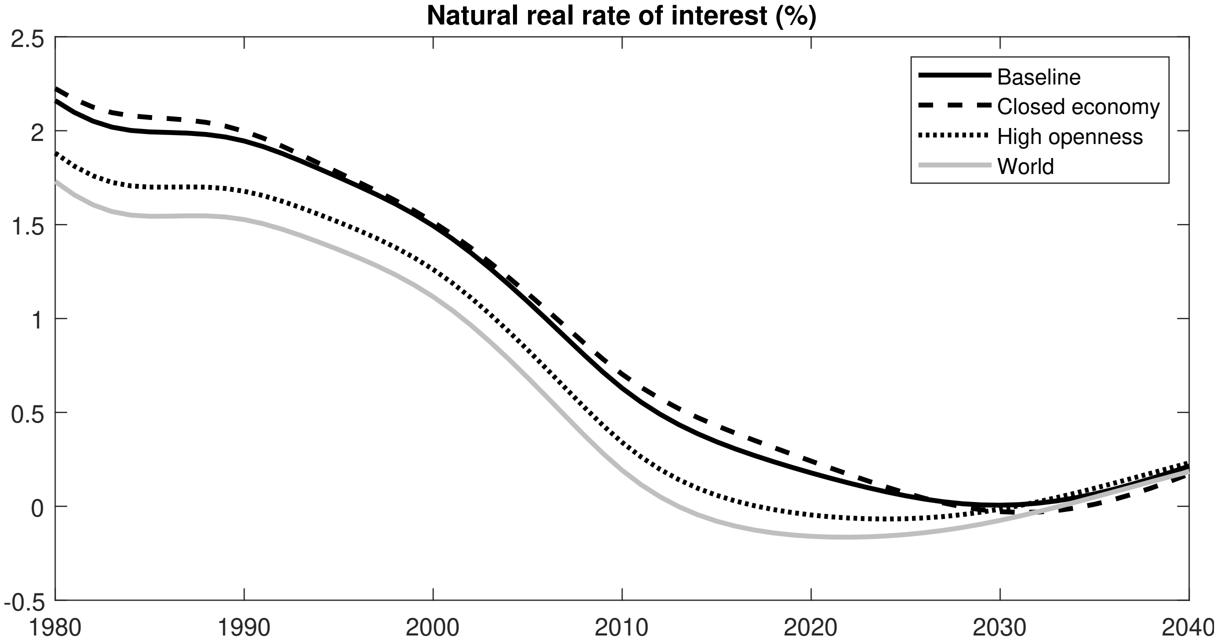
Notes: Labor input per capita, wage per efficiency unit and capital per efficiency unit of labor are normalized to unity in 1980.

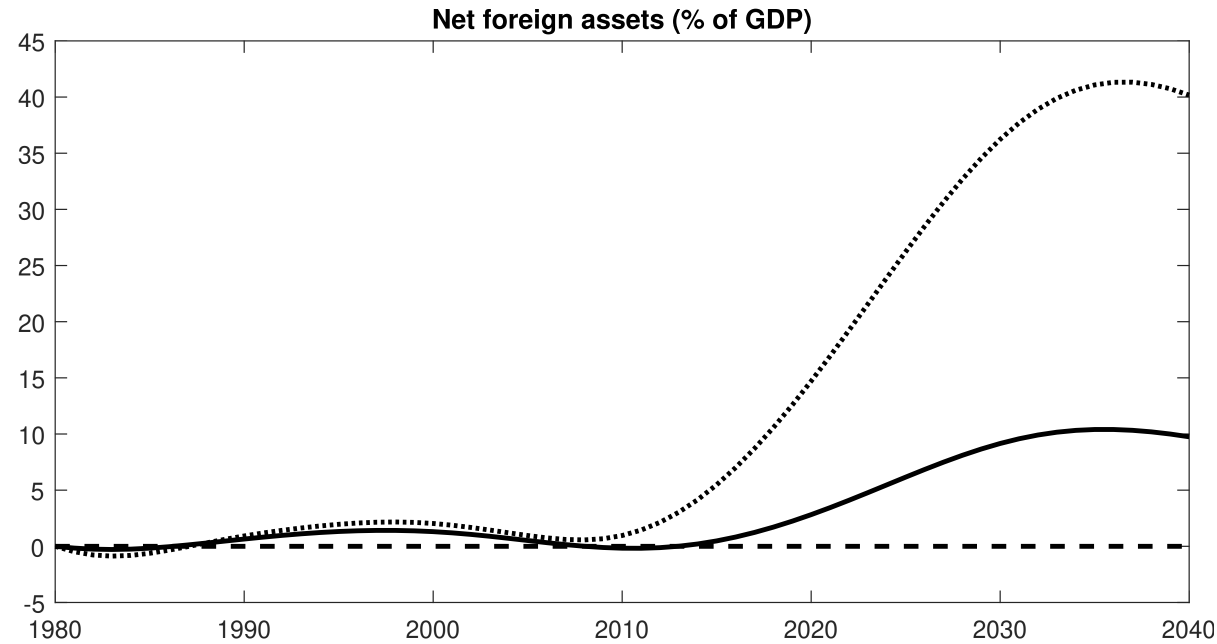
Figure 8: Model predictions against EA data



Notes: Hours worked per capita are for three largest euro area economies (Germany, France and Italy), and normalized to unity in 1985. The natural real rate of interest is taken from Holston et al. (2017). To smooth out fluctuations at business cycle frequencies, the series for hours and GDP growth are shown as 8-year moving averages.

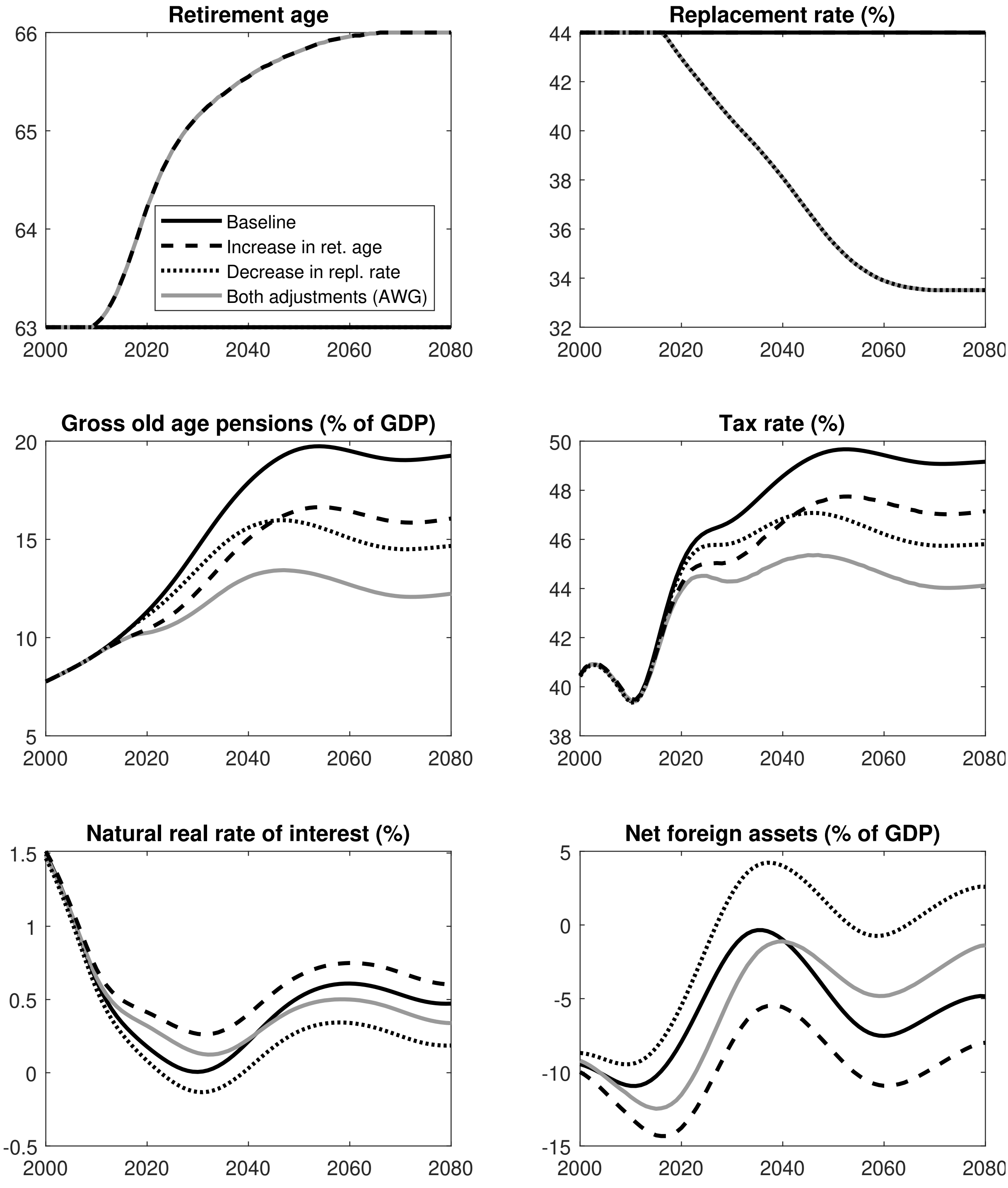
Figure 9: The role of financial openness





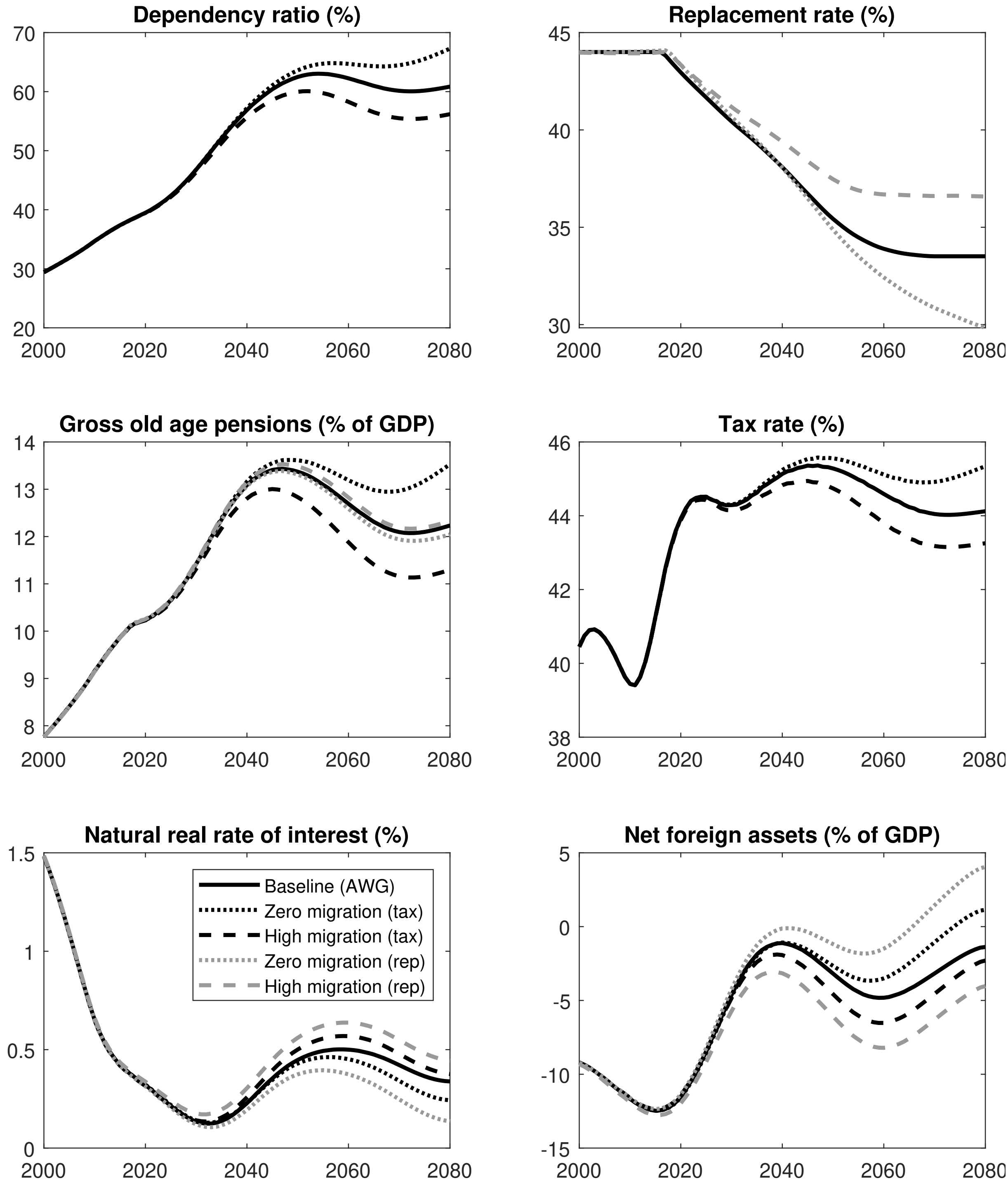
Notes: The net foreign assets to GDP ratio is normalized to zero in 1980.

Figure 10: The role of pension system reforms



Notes: Increases in retirement age, decreases in replacement rate and both adjustments scenarios are based on the AWG's 2018 Ageing Report projections.

Figure 11: The role of migration



Notes: Zero migration and high migration variant are based on the Eurostat's EUROPOP 2018 projections. In the (tax) scenarios replacement rate is kept at the AWG path, while tax rate adjusts. In the (rep) scenarios tax rate is kept at the AWG-implied path, while replacement rate adjusts.

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A Appendix

A.1 List of model equations

This appendix presents the full set of model equilibrium conditions that jointly determine the evolution of real per capita allocations and real prices, for given initial conditions and for given paths of exogenous deterministic variables: $n_{1,t}$, $\omega_{j,t}$ $(j = 1, ..., J-1)$, x_t, ϱ_t, b_t, g_t and r_t^*.

Households

$$c_{j,t} + a_{j+1,t+1} = (1 - \mathbf{1}_{j\geq JR}) \left(1 - \tau_t + \frac{\pi_t}{w_t h_t}\right) w_t z_j + (1-\tau_t)\mathbf{1}_{j\geq JR} pen_t + beq_t + (1+r_t)a_{j,t} \tag{A.1}$$

$$a_{0,t} = 0 \tag{A.2}$$

$$a_{J,t} = 0 \tag{A.3}$$

$$c_{j+1,t+1} = \beta(1-\omega_{j,t})(1+r_{t+1})c_{j,t} \tag{A.4}$$

Demographics

$$n_{1,t+1} = \frac{N_{1,t+1}}{N_{1,t}} - 1 \tag{A.5}$$

$$N_{j+1,t+1} = (1-\omega_{j,t})N_{j,t} \tag{A.6}$$

$$N_t = \sum_{j=1}^{J} N_{j,t} \tag{A.7}$$

$$n_{t+1} = \frac{N_{t+1}}{N_t} - 1 \tag{A.8}$$

Aggregation over households

$$c_t = \sum_{j=1}^{J} \frac{N_{j,t} c_{j,t}}{N_t} \tag{A.9}$$

$$h_t = \sum_{j=1}^{JR-1} \frac{N_{j,t} z_j}{N_t} \tag{A.10}$$

$$a_{t+1} = \sum_{j=1}^{J} \frac{N_{j,t} a_{j+1,t+1}}{N_{t+1}} \tag{A.11}$$

$$beq_t = \sum_{j=1}^{J} \frac{(N_{j,t-1} - N_{j,t})(1+r_t)a_{j,t}}{N_t} \tag{A.12}$$

Firms

$$(1+n_{t+1})k_{t+1} = (1-\delta)\, k_t + i_t \tag{A.13}$$

$$r_t^k = r_t + \delta \tag{A.14}$$

$$w_t = \frac{1-\alpha}{\mu} x_t k_t^{\alpha} h_t^{-\alpha} \tag{A.15}$$

$$y_t = x_t k_t^{\alpha} h_t^{1-\alpha} \tag{A.16}$$

$$\pi_t = y_t - w_t h_t - i_t \tag{A.17}$$

Government

$$pen_t = \varrho_t w_t \frac{\sum_{j=1}^{JR-1} N_{j,t} z_j}{\sum_{j=1}^{JR-1} N_{j,t}} \tag{A.18}$$

$$\tau_t w_t \sum_{j=1}^{JR-1} \frac{N_{j,t}}{N_t} z_j + (1+n_{t+1})b_{t+1} = g_t + (1-\tau_t) pen_t \sum_{j=JR}^{J} \frac{N_{j,t}}{N_t} + (1+r_t)b_t \tag{A.19}$$

External sector

$$1 + r_{t+1} = [1 + \xi\,(\exp{(-b_t^*/y_t)} - 1)]\,(1 + r_{t+1}^*) \tag{A.20}$$

Market clearing

$$a_t = k_t + b_t + b_t^* \tag{A.21}$$

$$(1+n_{t+1})b_{t+1}^* = (1+r_t)b_t^* + y_t - c_t - i_t - g_t \tag{A.22}$$

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