University of Warsaw
Faculty of Economic Sciences

Working Papers
No. 42/2020 (348)

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Warsaw 2020

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Does Bitcoin Improve Investment Portfolio Efficiency?

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Abstract: The aim of the paper is to check if cryptocurrency Bitcoin – a new investable asset class representative – is able to improve the performance of an optimal portfolio. Using two Markowitz criteria of optimization – expected return maximization and expected shortfall (CVaR) minimization – we test the investment opportunities after adding Bitcoin to the portfolio of 10 traditional assets (among them equity, fixed income, money, commodities and money market indices). Using daily observations from 1.05.2013 till 24.05.2019, we examine the behavior of the portfolios without and with Bitcoin and check if the return-risk ratio improves for the latter. Discussing the results, we conduct the sensitivity analysis by changing the lookback window (LB) and rebalancing frequency (RB) parameters. Empirical analysis suggests that Bitcoin-inclusive portfolios provide an investor with wider diversification opportunities. Robustness check confirms the findings and also advocates for the cryptocurrency to be added to the portfolio.

Keywords: Portfolio optimization, portfolio theory, cryptocurrency, Bitcoin, Markowitz model, asset allocation, portfolio diversification, investment opportunities

JEL codes: C20, C22, C61, C80, G14, G17

1. Introduction

Recently, a new investable asset class appeared. Cryptocurrencies, with Bitcoin, their first and most popular instance, drew the attention of the whole world because of their decentralized nature, advanced technology behind, untypical features and fresh perspective on the financial system. Among others, investment environment was affected strongly since investors noticed various unique opportunities: low correlation with other assets, diversification and hedging opportunities, possibility of an outstanding gain. Market players quickly started perceiving Bitcoin as an attractive investment class since they all the time seek for the vehicles to make profit. Recent history shows that innovative instruments are often quiet promising. Not long ago, volatility was that "new" asset to be actively studied and employed. Options on S&P 500 index and VIX index together with its futures contracts and options, for example, were the subject of analysis for various researchers.[1] For the past decade, however, Bitcoin has been the phenomenon to watch and test.

Bitcoin (BTC) is the first ever cryptocurrency with the largest market capitalization. According to Satoshi Nakamoto (2008), it is a pioneer peer-to-peer version of electronic cash that let do transactions on the internet without the intermediation of the financial system. The motivation behind Bitcoin creation was pretty clear – to allow for decentralized, secure, lower-fee and transparent mean of payment with no rely on third party, as oppose to the classical banking infrastructure. Since its launch in 2009 there has been lots of discussions regarding its nature – if it is an alternative payment method or an investment vehicle. As any innovation, cryptocurrency faced major unacceptance, especially during the emerging stage. Also, not that long ago it has become finally being accepted and adopted in use by various huge financial institutions and corporations[2]. Despite many disputes, as of 2019 the number of digital currencies and tokens exceeded 2000 and no one can deny the presence of this technological know-how in the financial environment. The uniqueness of Bitcoin is that it combines the characteristics of several asset classes. It is also worth noting, a 10 years history proves that it can undoubtedly be perceived as an investment instrument and in this research we will examine it.

[1] See, for example, Alexander et al. (2011), Egloff et al. (2010), Chen et al. (2011)
[2] https://www.coindesk.com/microsoft-launches-decentralized-identity-tool-on-bitcoin-blockchain
https://cointelegraph.com/news/morgan-stanley-report-shows-strong-institutional-investment-for-bitcoin

Investors conduct time- and resource-consuming research to find the strategies which will provide them with the highest return and/or the lowest risk. Modern Portfolio Theory (MPT) approach, Mean-Variance framework, pioneered by Markowitz (1952), is one the most popular theories for portfolio optimization purposes. It claims that investor, given the particular level of risk, may find the-so-called efficient portfolio by maximizing the expected return. He proved that investment in a group of assets appeared to be more beneficial than in any single individual one within the portfolio. The idea is that positive performance of one could neutralize the very negative of another one, for example, providing the investor with lower risk. That is why the assets are supposed to be chosen based on their correlation – with negative one being an ideal case. Knowing from previous studies that Bitcoin does have a weak correlation with other asset classes (e.g. Briere et al. 2013, Eisl et al. 2015, Smales et al. 2018), potential investor is interested in adding it to the portfolio and checking the potential opportunities.

Coming back to the framework itself, Mean-Variance analysis is used to define the optimal portfolio out of a given number of risky assets. Rational investor is willing to make an efficient choice, that is why he/she will seek to maximize the expected return for given variance or lower the risk for an expected return. So, to check the potential difference in the approaches used by criteria, we conduct the research within both of them:

- expected return maximization,
- expected shortfall (CVaR) minimization.

To sum up, employing the Modern Portfolio Theory (MPT), we examine the efficient diversification properties of cryptocurrency Bitcoin from the perspective of US international investor. Namely, we want to verify if an inclusion of Bitcoin into the portfolio consisting of different markets' assets moves the efficient frontier to the north-west direction. We check if Bitcoin is able to gain some substantial weight and improve risk-return profile of the whole portfolio. So, the questions to be answered are:

1. Does Bitcoin improve return-risk ratio of the portfolio? (In other words, does Bitcoin improve investment portfolio efficiency?
2. Are the results of optimization robust depending on the criteria of optimization?
3. How does the change of parameters (look back window, rebalancing frequency) affect the results?

The remainder of the paper is organized as follows. Chapter 1 walks the reader through the findings of topic-related academic studies, forming the particular benchmark for results discussion. Chapter 2 describes the data together with all the transformations made along the empirical analysis. Chapter 3 describes the methodology used in the paper, chosen to either confirm or reject the main thesis hypothesis. Chapter 4 presents the results of the study and the sensitivity analysis conducted for robustness check. The last chapter concludes the study.

2. Literature review

Although it has only been a decade since Bitcoin exists as an asset class, there have been already made a huge research. Attention of both theorists and practitioners was drawn to Bitcoin since the inception of this innovation. It has been analyzed from the perspective of its nature, properties, price drivers, correlation with other markets, legitimacy, security and obviously investment possibilities. Bitcoin diversification properties were checked by various researchers and there have been already obtained quite promising findings on the issue.

The universe of investment instruments is pretty vast nowadays, some of them are more popular, some – less but all those are exploited by investors in various configurations to draw profits. Markowitz framework is applied for many asset classes to test the profitability of constructed portfolios, robustness of the results depending on the set of parameters and the behavior of the assets within it. Ivanova et al. (2017), for example, employed Markowitz Portfolio optimization on the Bulgarian stock market. Caldeira et al. (2012) optimized bond portfolios. Sakowski et al. (2016) examined the behavior of developed and emerging indices in the context of Markowitz framework. Rajani (2016), in turn, applied MPT to construct the optimum portfolio of funds of funds on Indian market.

Bitcoin is the recent asset class, which has also been tested within the context of Markowitz optimization. Kajtazi et al. (2017), adopting the Chinese market perspective (where 99% of trading was noted), empirically confirm the fact that optimal portfolio improves with Bitcoin inclusion, though not in all the scenarios considered in their analysis. They haven't managed to prove the robustness of the results in time and depending on the Markowitz optimization constraints or parameters. Extending their paper in 2019 by adding the U.S. and European perspectives and including most recent observations, Kajtazi et al. (2019) again found out the evident benefits of crypto added to the optimal portfolio. As well as in their prior research, the portfolio performance improvement was noticed only in case of long-only and naive (equally-weighted) scenarios.

Approximately, same results were obtained by Gangwal (2016) who checked, along with the reliability and popularity of Bitcoin, the ability of cryptocurrency to diversify. "Analyzing the Effects of Adding Bitcoin to Portfolio" concludes that Bitcoin insures a better risk adjusted return portfolio, also suggesting that Bitcoin can be seen by investors as a new beneficial asset class. Klabbers (2017) in his master thesis: "Bitcoin as an investment asset: the added value of bitcoin in a global market portfolio" employed the mean-variance framework to test the hedge, safe heaven and diversifier properties of cryptocurrency from the perspective of an international investor. And his findings happened to be consistent with the ones mentioned above. Again, Bitcoin is confirmed to bring diversification benefits for investors, providing them with higher returns. In this paper the robustness of this conclusion was proved with 4 tests. However, in terms of being safe heaven or hedge instrument, it failed to show unambiguous results, especially from the global perspective.

Eisl et al. (2015), using the CVaR optimisation criterion of Markowitz framework, several portfolio scenarios and international perspective, also show that Bitcoin is worth to be included. It contributes to the performance, increasing both return and risk. They claim, however, that according to the results the return obtained outweigh the additional risk imposed for investors. The results' robustness is also confirmed by the findings of the empirical research.

Carpenter (2016), analyzing the effects of adding Bitcoin to the optimal portfolio, concluded that those including BTC outperform those traditionally constructed. However, he also showed his concerns about the possibility of results skewing because of the speculative bubble in 2013-2014. Nonetheless, his findings support all previously described.

Brauneis et al. (2019) used the standard Markowitz framework to test how the portfolio behaves while including not only one but several cryptocurrencies. Findings are consistent with a trend – there was found a substantial potential for risk reduction. However, naive (equally-weighted) portfolio outperformed the mean-variance optimization.

Platanakis et al. (2018) focused on the bitcoin inclusion into traditional stock-bond portfolio, using various assets allocation strategies. The findings happened to be in favor of cryptocurrency. According to the conclusions, it positively affects the Sharpe, Omega and Sort ino ratios and substantially adds up to investor's benefits.

So far, we haven't found any papers which in the context of diversification with Bitcoin presented unambiguously negative results. In the works presented above Bitcoin seems to possess diversifier property, although not all researches are able to confirm the robustness of

the results. To sum up this section, the literature overview suggests the positive expectations about Bitcoin's ability to improve the portfolio performance.

3. Data

The goal of the paper is to check if Bitcoin, being a new asset class, could add up to the portfolio efficiency improvement within the Markowitz framework. The empirical analysis is based on the comparison of the portfolios with and without Bitcoin in the context of two Markowitz criteria of optimization. We track the behavior of the portfolio consisting of 10 traditional assets (representing equity, fixed income, money market, commodities, real estate markets) over a certain period of time. Then we add to it Bitcoin and look at the performance measures.

3.1 Portfolio composition

The whole research is conducted from the perspective of an international US-based investor, which has in the portfolio 10 assets, representing different traditional markets and a new one – cryptocurrency. Table 1 summarizes all the eleven instruments within the portfolio.

Table 1. List of the assets used in the empirical study.

#	**Name**	**Asset class**	**Mnemonic**
1	MASCI ACWI	Equity	ACWI
2	S&P500	Equity	^SPX
3	Nikkei 225	Equity	^N225
4	DAX	Equity	^GDAXI
5	Dow Jones Industrial Average	Equity	^DJI
6	iPath Bloomberg Commodity Index Total Return	Commodity	DJP
7	MSCI EAFE Currency Index	Money Market	EFA
8	FTSE EPRA/NA REIT Developed Real Estate Index	Real Estate	IFGL
9	SPDR Bloomberg Barcalys International Treasury	Fixed income	BWX
10	iShares Core US aggregate Bond ETF	Fixed income	AGG
11	Bitcoin	Cryptocurrency	BTC

Source: www.yahoo.finance.com for 1-10, www.coinmarketcap.com for 11.

Those are five liquid broad equity indices, presenting the international arena – both emerging and developed markets: Morgan Stanley Capital International All Country World Index, The Standard & Poor's 500, Nikkei 225, DAX, Dow Jones industrial average. Besides, the global assets bundle is diversified with the two representatives of fixed income: SPDR Bloomberg Barclays International Treasury Bond ETF and iShares Core US Aggregate Bond; a single one of commodity market: iPath Bloomberg Commodity Index; a single one of money: Morgan Stanley Capital International EAFE Currency index. The real estate market is

represented by FTSE EPRA/NAREIT Developed Real Estate Index. Daily data on the ten traditional asset is gathered from Yahoo Finance.

The eleventh, additional, asset is Bitcoin which is classified as a cryptocurrency. Daily data on Bitcoin is scrapped from the https://coinmarketcap.com website.

As for the time frames, since we examine the Bitcoin capabilities, the whole analysis period was adjusted to the cryptocurrency development. The observations of Bitcoin price are available since January 2009, however, during first 3-4 years we observed quite slight price changes. What is most important, there have also been insignificant capitalization volumes at that times. we decided to start tracking its behavior since April 2013 – right before first dynamics appear (Figure 1).

Consequently, the same applies to the rest of the assets in the portfolio. So, the historical observations cover the period from 1.05.2013 to 24.05.2019 providing us with 73 months of daily prices of 11 assets. All the assets' prices are denominated in US dollar. In case of Japanese (Nikkei 225) and German (DAX) indices, the data was obtained in the local currency and then transferred into US dollar-denominated one, using spot exchange rates.

As for the daily observations, one important adjustment was made. Bitcoin has an extraordinary nature. Not only is it characterized by noteworthy returns and high volatility[3], it has an untypical feature – it is traded also on weekends, when traditional exchanges are closed. In case of Bitcoin, there are 7 observations per week, meaning weekends are included in the sample, whereas other ten assets provide us with the prices only on 5 trading days without Saturday and Sunday ones. To make the optimization process fair we have to make some data transformations. Hence, we define the two datasets of returns while constructing the portfolios:

1. ***5days returns*****,** which applies to all the portfolios without Bitcoin.
2. ***7days returns,*** which applies to all the Bitcoin-inclusive portfolios.

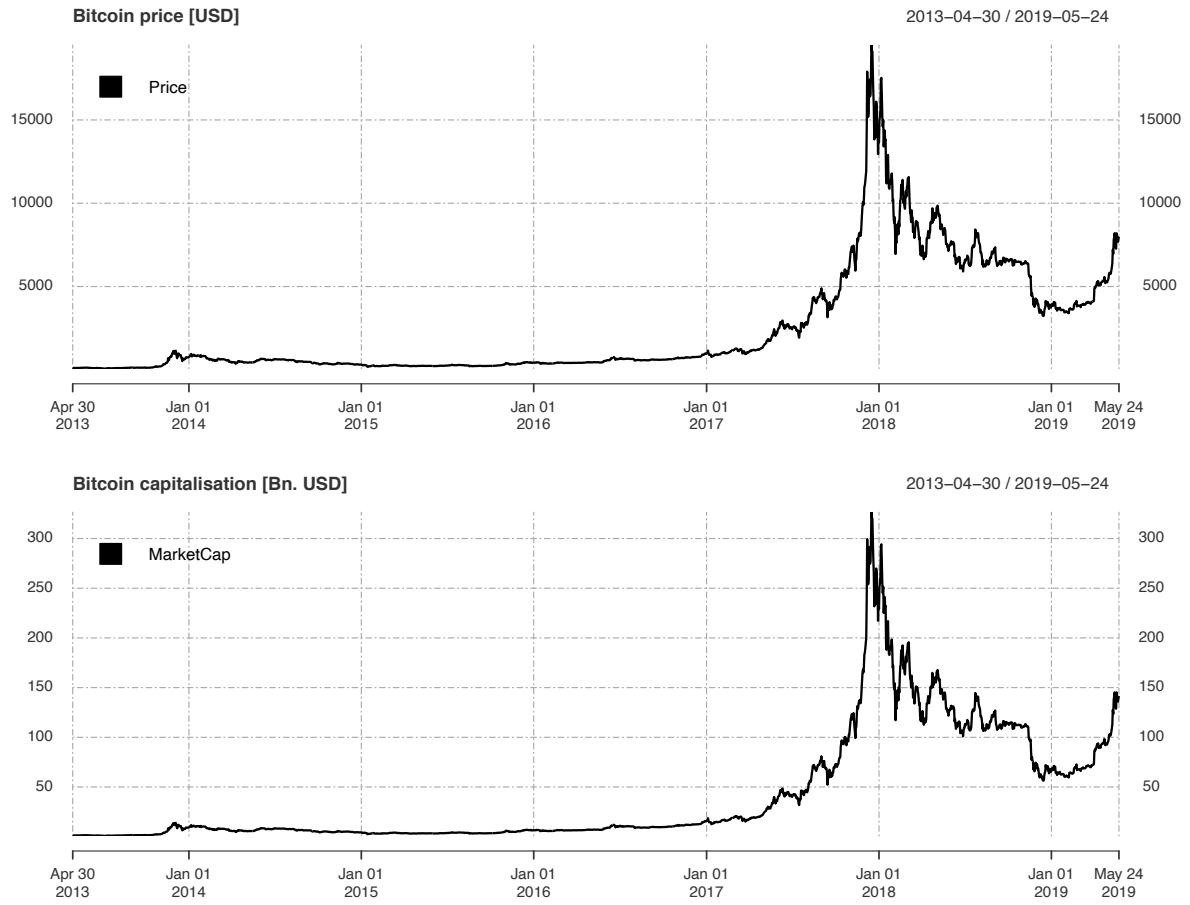
Since the traditional assets are present in those Bitcoin-inclusive portfolios, we have to adjust them to "7days returns" model, to make the dataset consistent. Namely, we fulfill those non-tradable days with the artificially created prices. To avoid the look-ahead bias[4], we substitute both missing values on Saturday and Sunday with Friday's one. This transformation

[3] For example, Eisl et al. (2015), Balcilar et al. (2017).
[4] We create the observations on weekends by fulfilling them with Friday price – not the average of Friday and Monday. That means that Saturday & Sunday returns are equal to zero.

results in the increasing number of zero-returns in the whole *7days returns* dataset, which we should mind when discussing the descriptive statistics.

Other missing prices (exchange holidays) in the both datasets are also filled with last non-missing observation.

Figure 1. Historical prices of Bitcoin [USD] and market capitalization of Bitcoin [Bn. USD] over sample period 30.04.2013 – 24.05.2019

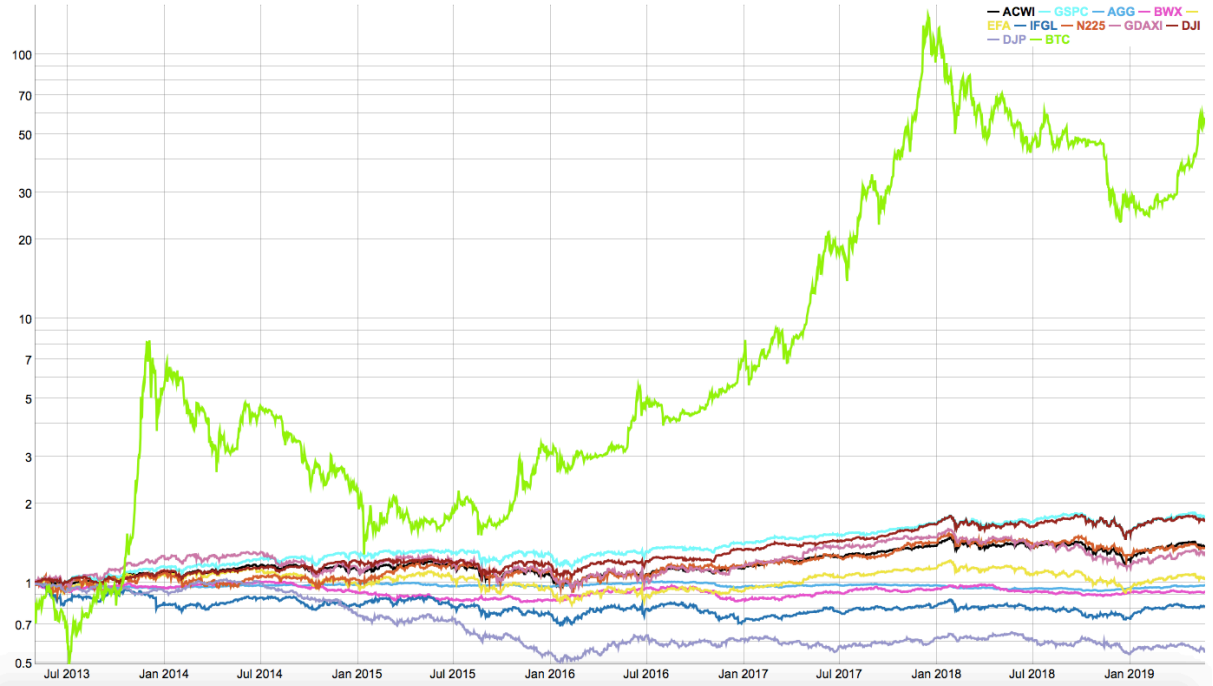


Source: www.coinmarketcap.com

3.2 Data description

To compare the behavior of assets within the portfolio, we visualize their normalized prices (first value = 1) on the log scale (Figure 2). We can see that Bitcoin significantly outperforms 10 assets in terms of returns obtained, what is promising in the context of portfolio optimization. It also shows much wider fluctuations in its returns, whereas all the traditional assets' behavior could be described as stable through the whole analysis period.

Figure 2. Portfolio assets' normalized prices (equity lines) with 1 being the first value over sample period 01.05.2013 – 24.05.2019



Source: own empiric study. Y axis: cumulative assets' returns (log scale)

To explore the particular characteristics of the assets in more detail, one needs to look at their descriptive statistics. Those are presented in Table 2 and Table 3. The classical statistical measures are calculated on the *5days returns* and *7days returns* of the assets' prices respectively.

While looking at the statistics of traditional assets, we see no outstanding anomalies. Variance of the returns of all 10 assets is not exceeding 0.00008. Almost all the returns distributions are left-skewed – apart from BWX's one – and quite symmetrical around mean. Only EFA & IFGL relatively high negative skewness: -0.89 and -1.32 respectively. Those two assets also draw our attention in terms of the possibility of extreme returns in the data. Kurtosis values are quite high: 6.97 and 11.39 respectively. This could be explained by some outliers in the returns observations.

Table 2. Descriptive statistics of the daily returns of the Portfolio 10 assets' prices for *5days returns* set for the period 01.05.2013 – 24.05.2019

	ACWI	SPX	AGG	BWX	EFA	IFGL	N225	GDAXI	DJI	DJP
nobs	1570	1570	1570	1570	1570	1570	1570	1570	1570	1570
Minimum	−0.05384	−0.04098	−0.01093	−0.01726	−0.08586	−0.08972	−0.06293	−0.09530	−0.04605	−0.03889
Maximum	0.03694	0.04959	0.00840	0.02097	0.03326	0.03628	0.07296	0.04974	0.04985	0.03634
Mean	0.00023	0.00040	−0.00001	−0.00004	0.00006	−0.00009	0.00027	0.00022	0.00038	−0.00035
Variance	0.00007	0.00006	0.00000	0.00002	0.00008	0.00007	0.00015	0.00012	0.00006	0.00007
Stdev	0.00816	0.00806	0.00202	0.00431	0.00887	0.00867	0.01220	0.01101	0.00805	0.00830
Skewness	−0.54076	−0.40624	−0.41238	0.02609	−0.88727	−1.32022	−0.23352	−0.52572	−0.39839	−0.04810
Kurtosis	3.27828	3.80701	1.56845	1.53279	6.96969	11.39493	3.84349	4.80817	3.91711	1.30673

Source: own empiric analysis; nobs - number of observations in the set; Stdev - standard deviation. Mean is presented in the decimal form (e.g. 0.00023 = 0.023%)

Table 3. Descriptive statistics of the daily returns of the Portfolio 11 assets' prices for *7days returns* set for the period 01.05.2013 – 24.05.2019

	ACWI	SPX	AGG	BWX	EFA	IFGL	N225	GDAXI	DJI	DJP	BTC
nobs	2215	2215	2215	2215	2215	2215	2215	2215	2215	2215	2215
Minimum	−0.05384	−0.04098	−0.01093	−0.01726	−0.08586	−0.08972	−0.06293	−0.09530	−0.04605	−0.03889	−0.23371
Maximum	0.03694	0.04959	0.00840	0.02097	0.03326	0.03628	0.07296	0.04974	0.04985	0.03634	0.42968
Mean	0.00016	0.00028	−0.00001	−0.00003	0.00004	−0.00007	0.00019	0.00016	0.00027	−0.00025	0.00276
Variance	0.00005	0.00005	0.00000	0.00001	0.00006	0.00005	0.00010	0.00009	0.00005	0.00005	0.00188
Stdev	0.00687	0.00678	0.00170	0.00363	0.00747	0.00730	0.01027	0.00927	0.00678	0.00698	0.04334
Skewness	−0.61301	−0.43137	−0.49532	0.02130	−1.04699	−1.57957	−0.25452	−0.60340	−0.42435	−0.10040	0.52385
Kurtosis	5.83226	6.56268	3.45110	3.39668	11.06030	17.33821	6.64771	8.00013	6.72094	3.07669	9.97896

Source: own empirical analysis; nobs - number of observations in the set; Stdev - standard deviation. Mean is presented in the decimal form (e.g. 0.00023 = 0.023%)

Considering the *7days return* set for 11 assets (including Bitcoin), we obviously notice some interesting results. The kurtosis of Bitcoin returns is relatively high with the value 9.98. We expected it to be several times higher than in the normal distribution because of the nature of cryptocurrency. The kurtosis for EFA and IFGL are however even higher in this set. Those values could be biased by the data transformations.[5] As for the skewness, oppositely to nine traditional portfolio assets, Bitcoin exhibits the positive skew. It is consistent with papers on the alike subject (e.g. Eisl et al. 2015, Briere et al. 2017, Liu et al. 2018). Out of the whole set Bitcoin has also the largest values for the mean 0.0028 and the variance 0.0019. Those findings are in line with the nature of cryptocurrency.

3.3 Assets' correlation

According to the studies within the area of crypto, the historical returns of Bitcoin show noticeably low correlation with traditional assets.[6] Hence, theorists and especially practitioners are extremely enthusiastic about investigating the diversification properties of cryptocurrency. Sample returns of the data used in this research also support the above mentioned. The correlation matrix of 11 assets constituting the global portfolio is presented below in Table 4.

[5] At least 2/7 of the returns are equal to 0 because of the weekends' transformation (Saturday and Sunday are fulfilled with Friday's closing price)

[6] For example, Briere et al. (2013), Eisl et al. (2015), Smales et al. (2018)

Bitcoin shows slightly different from zero correlation coefficient values, with the highest one (in the absolute value) being equal to 0.019. Such a characteristic is quite promising in further analysis. In turn, the correlation between the traditional assets could suggest that we have constructed quiet a diversified portfolio. Just 4 pairs out of 55 combinations (below the "ones" diagonal) are strongly positively correlated with coefficients between 0.9 and 1.

Table 4. Correlation matrix of the *7days returns* for all 11 assets for the period 01.05.2013 – 24.05.2019

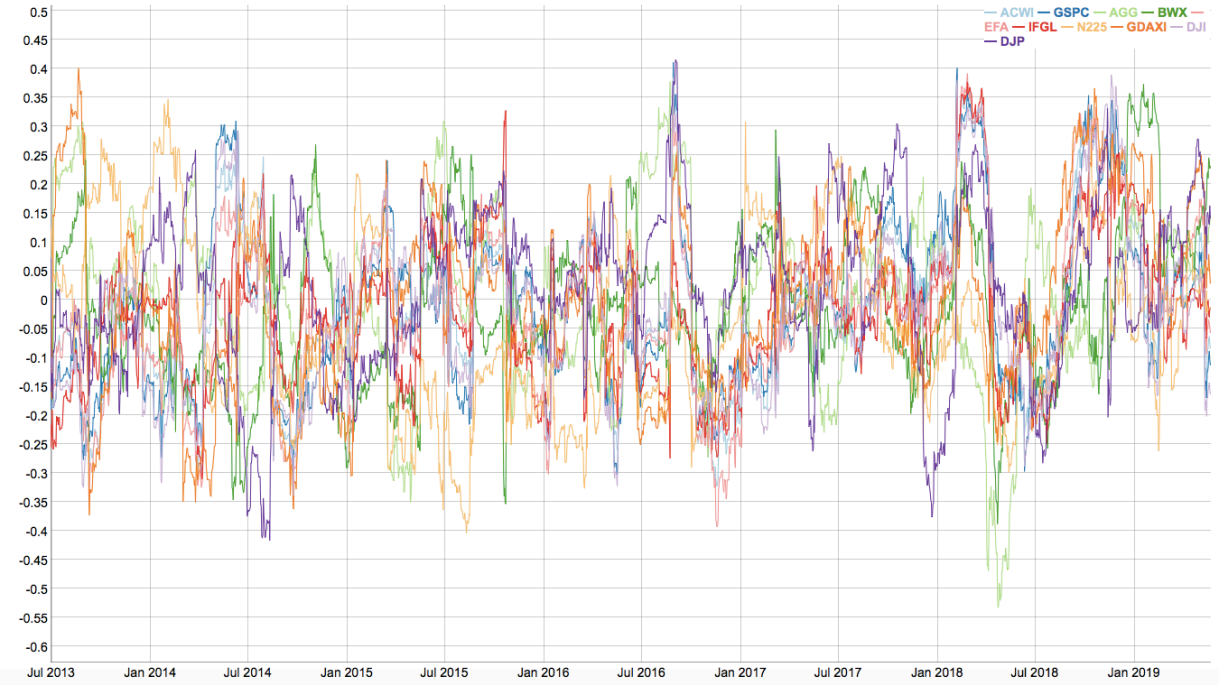
	ACWI	SPX	AGG	BWX	EFA	IFGL	N225	GDAXI	DJI	DJP	BTC
ACWI	1.000	0.950	−0.126	0.073	0.942	0.763	0.076	0.613	0.918	0.366	0.005
SPX	0.950	1.000	−0.162	−0.040	0.818	0.671	−0.003	0.489	0.967	0.286	0.004
AGG	−0.126	−0.162	1.000	0.546	−0.109	0.013	0.110	−0.057	−0.182	−0.004	0.015
BWX	0.073	−0.040	0.546	1.000	0.160	0.227	0.110	0.173	−0.050	0.251	−0.008
EFA	0.942	0.818	−0.109	0.160	1.000	0.788	0.147	0.712	0.796	0.378	−0.003
IFGL	0.763	0.671	0.013	0.227	0.788	1.000	0.074	0.498	0.638	0.325	−0.015
N225	0.076	−0.003	0.110	0.110	0.147	0.074	1.000	0.164	−0.002	0.078	−0.006
GDAXI	0.613	0.489	−0.057	0.173	0.712	0.498	0.164	1.000	0.491	0.266	−0.002
DJI	0.918	0.967	−0.182	−0.050	0.796	0.638	−0.002	0.491	1.000	0.278	0.008
DJP	0.366	0.286	−0.004	0.251	0.378	0.325	0.078	0.266	0.278	1.000	0.019
BTC	0.005	0.004	0.015	−0.008	−0.003	−0.015	−0.006	−0.002	0.008	0.019	1.000

very strong positive correlation
medium strong positive correlation
positive correlation
almost no correlation
slightly negative correlation
negative correlation

Source: own empiric analysis. Green color is most positive scenario in terms of diversification, meaning the negative correlation.

We also visualize the daily rolling correlation of Bitcoin against 10 traditional assets. We are interested in tracking it along the investment period and see if there are extreme values of those. We use 60 days lookback window. And results are quote promising (Figure 3). When it comes to a positive correlation, throughout the whole period BTC/DJP pair barely exceeds 0.4. If it comes to the negative one – BTC/AGG pair's coefficient correlation achieves the value of almost -0,55 (Figure 3).

Figure 3. Rolling correlation: Bitcoin against 10 traditional assets for the period 01.05.2013 – 24.05.2019



Source: own empirical analysis, lookback window: 60 days.

4. Methodology

4.1 Optimization criteria

The main objective of this study is by employing the Mean-Variance framework to check if Bitcoin indeed improves the performance of a global investment portfolio. Simultaneously the research is conducted within the two criteria:

1. expected return maximization [Max_Ret]
2. expected shortfall (CVaR) minimization [Min_CVaR]

The former criterion is intuitive – investor aims at returns maximization – however, when it comes to the risk minimization, we are using more sophisticated measure than just the simple standard deviation. The classical Markowitz portfolio framework defines *portfolio risk* as the variance of the portfolio return and seeks a portfolio weight vector which minimizes the portfolio risk subject to a target expected return Markowitz (1952). Mean-Variance approach, however, has some disadvantages, and one of them is that it oversimplifies investor's risk preferences (Kajtazi et al. 2017). Variance is a symmetric measure that incorporates both, the upside & downside volatility values, whereas in the real-world, for rational investor, only the

downside component is of the great importance. So, a measure better reflecting exactly the downside risk could be more favorable from the perspective of portfolio optimization. The possible one is Value at Risk (VaR). However, its main shortcoming is that it solely estimates the minimum potential loss, not quantifying the amount by which this threshold could be exceeded. Hence, it may potentially underestimate the tail risk.

So, one should think of a more favorable measure for the analysis. The so-called Conditional Value at risk (CVaR) appears to be such one. Its major advantage over VaR is that it delivers the average loss the threshold is exceeded by, supplying more information by this. Furthermore, Rockafellar et al. (2002) also showed that, in the context of portfolio optimization, while minimizing the CVaR one also minimizes VaR. It provides investors with a flexible and strong risk management tool Forghieri et al. (2014).

4.2 Parameters of the strategies

Within each of the two criteria we construct 5 strategies (Figure 4). A strategy refers to the set of portfolios constructed based on arbitrary chosen parameters. Those parameters are:

- lookback window [LB] – the period, on which the variance-covariance matrix is estimated,
- rebalancing frequency [RB] – the frequency of portfolio rebalancing (estimating the weights of the assets within the portfolio).

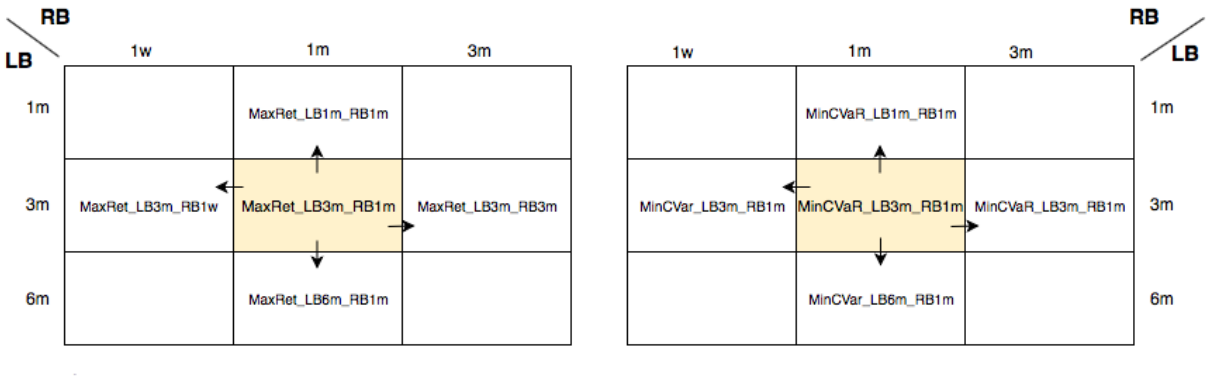
One of the 5 strategies is the Main Strategy for each of the two criteria. First, the results of that two will be discussed. We have picked 3 months as a lookback window and 1 month as a rebalancing frequency for it. Hence our two Main strategies are as follows:

- **MaxRet_LB3m_RB1m** for the returns' maximization,
- **MinCVaR_LB3m_RB1m** for expected shortfall (CVaR) minimization,

where LB corresponds to the lookback window, RB – to the rebalancing frequency.

The rest of the strategies we create for the sensitivity analysis. Those 8 are described in the same manner as the Main ones. Their names suggest the criterion and the values of LB and RB parameters.

Figure 4. Set of strategies in the analysis. 5 strategies for MaxRet criterion and 5 for MinCVaR.



Source: own empiric analysis. 1w – 1 week, 1m -1 months, 3m – 3 months, 6m – 6 months, LB – lookback window, RB – rebalancing frequency.

For the rebalancing we use 1 week, 1 months and 3 months frequencies. For the lookback window we picked 1 month, 3 months and 6 months. For the results to be comparable, we move the beginning of the analysed period from 1-05-2013 to 31-10-2013 – to the first date of rebalancing with the maximum lookback period (6 months).

4.3 Portfolios construction

We have already revealed that our analysis has two criteria, each of which has 5 strategies. But each of those 5 strategies also consists of 4 portfolios.

1. Markowitz optimisation without Bitcoin *(using 5days returns)*
2. Markowitz optimisation with Bitcoin *(using 7days returns)*
3. Equally weighted portfolio without Bitcoin *(using 5days returns)*
4. Equally weighted portfolio with Bitcoin *(using 7days returns)*

Certain constraints are imposed on each portfolio:

- full investment, namely sum of all the assets' weights is supposed to be equal to one or 100%,
- long-only portfolio, so no shorting is available, no negative weights for Markowitz optimization,
- transaction cost is equal to 1% for all the portfolios.

Following Eisl et al. (2015) we construct the optimization procedure in a following way. In the Markowitz approach we apply a back-testing technique, estimating the out-of-sample returns based on the initial optimal weights of each asset. Those initial ones are calculated based

on the look back window. In case of equally-weighted approach, where the weights are calculated as 1/N with N being the number of assets, on every rebalancing date the weights are the same. By multiplying obtained returns by the lagged values of corresponding weights we obtain the portfolio returns. To track the portfolio performance on the daily basis, we also adjust portfolio weights between rebalancing points (for every day).

4.4 Performance analysis

So, overall analysis includes 5 strategies, each having 4 portfolios, what results in 20 portfolios. However, we also set two benchmarks, to compare the results to. Those are Bitcoin buy&hold strategy and SP500 buy&hold[7], so the final summary table will have 22 portfolios for MaxRet and 22 portfolios for MinCVaR.

Comparison of the results is based on the reviewing the performance measures of each portfolio, equity lines performance and efficient frontiers of the portfolios. As for the measures we use the following:

- annualized rate of return of the strategy (aRC)

$$aRC = \left(\left(1 + \frac{P_{i,T}}{P_{i,0}}\right)^{\frac{365}{T}} - 1\right) * 100\%,$$

where $P_{i,T}$ is a price of i-th asset at the end of interval T,

- annualized standard deviation of the strategy daily returns (aSD)

$$aSD = \sqrt{\frac{365}{T}\sum_{t=1}^{T}(r_{i,t} - \bar{r})^2} * 100\%,$$

- Information Ratio (IR), which describes the relation of the portfolio annualized rate of return to the annualized volatility of the return (IR)

$$IR = \frac{aRC}{aSD},$$

- Maximum Drawdown (MD), which is the maximum loss from a peak to a minimum of a portfolio before a new peak is attained (MD)[8]

[7] Buy&hold strategy means just buying the asset and holding it throughout whole investment period.
[8] https://www.investopedia.com/terms/m/maximum-drawdown-mdd.asp

$$MD(T) = max_{\tau\in[0,T]}(max_{t\in[0,\tau]}R_{i,T}^{(p)} - R_{i,\tau}^{(p)})*100\%,$$

- the relation of Information Ratio to maximum drawdown (IRMD),

$$IRMD = \frac{IR}{MD}$$

- the relation of product of IR and annualized return to the maximum drawdown (IRaRCMD):

$$IRaRCMD = IR * \frac{aRC}{MD}$$

5. Results

Having constructed 22 portfolios (2 benchmark portfo lios and 5 strategies, each employed for 4 portfolios) for every of the two criteria – MaxRet and MinCVaR we obtained the performance measures for 44 portfolios. The findings are summarized in the Table 5 and Table 6 respectively.

Discussing the results, we look at the performance measures for portfolios with and without Bitcoin – mostly annualized rate of return (aRC) and Information Ratio (IR) values and compare the Bitcoin-inclusive portfolio with the benchmarks. Then we discuss efficient frontiers for the Markowitz-optimised portfolios with and without Bitcoin. After that we elaborate on Bitcoin-inclusive portfolio weights structures. Besides, we discuss the graphs of portfolios' equity lines together with the capital drawdowns. In the end we present the sensitivity analysis.

5.1 Performance measures

From the Table 5 and Table 6 we may conclude that the main thesis hypothesis seems to be supported. The performance of the Main Strategies for the two optimisation criteria show that including Bitcoin in the portfolio indeed improves its efficiency.

Information Ratio (IR) and annualized rate of return (aRC) for MaxRet_withoutBTC_LB3m_RB1m portfolio are considerably lower than those for MaxRet_withBTC_LB3m_RB1m portfolio (see Table 5, panel PortfolioStats__Main_Strategy__MaxRet_LB3m_RB1m). In case of the MaxRet portfolio without Bitcoin Information Ratio (IR) equals to 0.01, whereas in case of one with cryptocurrency the value is 1.22. The difference in the annualized rate of return (aRC) is a lot more substantial. MaxRet portfolio with Bitcoin shows 86% yearly against 0.16% for the one

without. When it comes to the annualized standard deviation (aSD), MaxRet_withBTC_LB3m_RB1m portfolio shows the value of 70% which is almost 6 times higher than MaxRet_withoutBTC_LB3m_RB1m.

If we compare the two equally weighted portfolios (no Markowitz framework employed), the one with Bitcoin outperforms that without new asset class in terms of all the performance measures analyzed. Information Ratio (IR) for Eq.W_withBTC_LB3m_RB1m portfolio equals to 1.21, whereas for Eq.W_withoutBTC_LB3m_RB1m only 0.21.

We also picked two benchmarks for the analysis – BTC buy&hold and S&P 500 buy&hold because we are curious about the relative performance of those two and the Markowitz-optimized Bitcoin-inclusive portfolios. BTC outperforms S&P 500 and MaxRet_withBTC_LB3m_RB1m in terms of annualized rate of return (aRC) with the value 93.24%. However, if we look at the Information Ratio (IR) measure, MaxRet portfolio with Bitcoin happens to be the most efficient out of the three compared with the value 1.22.

We can conclude that for the Main Strategy within the return maximization criterion the Bitcoin-inclusive portfolios perform better than those consisting solely of traditional assets.

The thesis hypothesis seems to be supported for the CVaR minimization criterion too. MinCVaR_withoutBTC_LB3m_RB1m portfolio also happens to show poorer results in terms of Information Ratio (IR) than MinCVaR_withBTC_LB3m_RB1m, 0.55 against 0.64 (see Table 6, panel PortfolioStats__Main_Strategy__MinCVaR_LB3m_RB1m). Although the annualized rate of return (aRC) does not show that noticeable difference as in the case of the return maximization Main Strategy. It is equal to 1.81% for MinCVaR portfolio without Bitcoin and 2.14% for the one with. In terms of annualized standard deviation measure, we can also see the slight increase from 3.29% in case of portfolio without Bitcoin and 3.33% - in case of Bitcoin-inclusive one.

Table 5. Performance measures table for the 22 portfolios within MaxRet criterium.

Portfolio statistics

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
PortfolioStats__Main_Strategy__MaxRet_LB3m_RB1m							
MaxRet_withoutBTC_LB3m_RB1m	0.155	12.683	28.333	0.012	0.043	0.000	1439
MaxRet_withBTC_LB3m_RB1m	86.004	70.467	74.305	1.220	1.643	1.413	2032
Eq.W_withoutBTC_LB3m_RB1m	1.769	8.389	18.392	0.211	1.147	0.020	1439
Eq.W_withBTC_LB3m_RB1m	14.387	11.941	22.173	1.205	5.434	0.782	2032
PortfolioStats_MaxRet_LB6m_RB1m							
MaxRet_withoutBTC_LB6m_RB1m	0.531	12.974	25.270	0.041	0.162	0.001	1439
MaxRet_withBTC_LB6m_RB1m	102.210	73.616	73.681	1.388	1.884	1.926	2032
Eq.W_withoutBTC_LB6m_RB1m	1.769	8.389	18.392	0.211	1.147	0.020	1439
Eq.W_withBTC_LB6m_RB1m	14.387	11.941	22.173	1.205	5.434	0.782	2032
PortfolioStats_MaxRet_LB1m_RB1m							
MaxRet_withoutBTC_LB1m_RB1m	-3.709	13.004	27.216	-0.285	-1.048	0.039	1439
MaxRet_withBTC_LB1m_RB1m	92.701	65.461	76.893	1.416	1.842	1.707	2032
Eq.W_withoutBTC_LB1m_RB1m	1.769	8.389	18.392	0.211	1.147	0.020	1439
Eq.W_withBTC_LB1m_RB1m	14.387	11.941	22.173	1.205	5.434	0.782	2032
Portfoliostats_MaxRet_LB3m_Rb3m							
MaxRet_withoutBTC_LB3m_RB3m	1.258	13.384	24.222	0.094	0.388	0.005	1439
MaxRet_withBTC_LB3m_RB3m	44.231	68.273	83.014	0.648	0.780	0.345	2032
Eq.W_withoutBTC_LB3m_RB3m	1.826	8.389	18.195	0.218	1.196	0.022	1439
Eq.W_withBTC_LB3m_RB3m	16.220	16.910	28.953	0.959	3.313	0.537	2032
Portfoliostats_MaxRet_LB3m_Rb1w							
MaxRet_withoutBTC_LB3m_RB1w	0.324	12.047	27.082	0.027	0.099	0.000	1439
MaxRet_withBTC_LB3m_RB1w	117.897	69.615	67.426	1.694	2.512	2.961	2032
Eq.W_withoutBTC_LB3m_RB1w	1.844	8.421	18.338	0.219	1.194	0.022	1439
Eq.W_withBTC_LB3m_RB1w	11.270	10.897	21.469	1.034	4.817	0.543	2032
PortfoliosBenchmarks							
BTCbenchmark	93.243	82.702	84.529	1.127	1.334	1.244	2032
SP500_benchmark	8.659	12.916	19.778	0.670	3.389	0.293	1439

Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. 5 strategies, 4 portfolios each + 2 benchmarks. aRC – annualized rate of return, aSD – annualized standard deviation, MD – maximum drawdown, IR – Information ratio, IRMD - the relation coefficient of IR to MD, IRaRCMD – the relation coefficient of IR multiplied by aRC to MD, nObs – number of observations in days. aRC, aSD, MD present the percentage change.

The equally weighted portfolios for the MinCVaR criterion show the same values as in the case of MaxRet one since they are not affected by Markowitz optimization criterion. Although if we compare Markowitz and equally-weighted frameworks - MinCVaR_withBTC_LB3m_RB1m and Eq.W_withBTC_LB3m_RB1m portfolios respectively – we may notice that the latter happens to be more efficient.

Information Ratio (IR) of 1.21 happens to be larger than 0.65 for Markowitz-optimized portfolio. We could have expected such an outcome at least in one of the scenarios, since those findings are described in the empirical studies of the similar subject (e.g. Brauneis et al. 2019).

If it comes to the benchmarks comparison, both S&P 500 buy&hold and BTC buy&hold outperform in terms of Information Ratio (IR). Value of the ratio for the first benchmark is 0.67 and for the second 1.13, whereas for MinCVaR portfolio it is 0.64. Annualized return ratio (aRC) and annualized standard deviation (aSD) for MinCVaR_withBTC_LB3m_RB1m are 2.14% and 3.33% respectively. Those are the lowest return and the lowest standard deviation of the three portfolios.

After discussing the results, we can conclude that our first two research questions got the positive answers. First of all, Bitcoin indeed improves the return-risk ratio of the portfolio, providing an investor with more efficient one. Second of all, the findings are robust in regard to the different optimization criteria, in our study – expected mean return maximization and expected shortfall (CVaR) minimization. Adding Bitcoin to Markowitz-optimized portfolios as well as to equally weighted one improves their performance.

Table 6. Performance measures table for the 22 portfolios within MinCVaR criterium.

Portfolios; statistics

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
PortfolioStats__Main_Strategy__MinCVaR_LB3m_RB1m							
MinCVaR_withoutBTC_LB3m_RB1m	1.806	3.291	6.262	0.549	8.763	0.158	1439
MinCVaR_withBTC_LB3m_RB1m	2.144	3.336	6.447	0.643	9.969	0.214	2032
Eq.W_withoutBTC_LB3m_RB1m	1.769	8.389	18.392	0.211	1.147	0.020	1439
Eq.W_withBTC_LB3m_RB1m	14.387	11.941	22.173	1.205	5.434	0.782	2032
PortfolioStats_MinCVaR_LB6m_RB1m							
MinCVaR_withoutBTC_LB6m_RB1m	1.453	3.047	5.869	0.477	8.128	0.118	1439
MinCVaR_withBTC_LB6m_RB1m	1.099	3.121	6.436	0.352	5.474	0.060	2032
Eq.W_withoutBTC_LB6m_RB1m	1.769	8.389	18.392	0.211	1.147	0.020	1439
Eq.W_withBTC_LB6m_RB1m	14.387	11.941	22.173	1.205	5.434	0.782	2032
PortfolioStats_MinCVaR_LB1m_RB1m							
MinCVaR_withoutBTC_LB1m_RB1m	0.499	4.428	8.511	0.113	1.324	0.007	1439
MinCVaR_withBTC_LB1m_RB1m	3.952	4.865	6.851	0.812	11.859	0.469	2032
Eq.W_withoutBTC_LB1m_RB1m	1.769	8.389	18.392	0.211	1.147	0.020	1439
Eq.W_withBTC_LB1m_RB1m	14.387	11.941	22.173	1.205	5.434	0.782	2032
PortfolioStats_MinCVaR_LB3m_RB3m							
MinCVaR_withoutBTC_LB3m_RB3m	1.101	3.495	7.689	0.315	4.099	0.045	1439
MinCVaR_withBTC_LB3m_RB3m	1.233	3.766	10.155	0.327	3.224	0.040	2032
Eq.W_withoutBTC_LB3m_RB3m	1.826	8.389	18.195	0.218	1.196	0.022	1439
Eq.W_withBTC_LB3m_RB3m	16.220	16.910	28.953	0.959	3.313	0.537	2032
PortfolioStats_MinCVaR_LB3m_RB1w							
MinCVaR_withoutBTC_LB3m_RB1w	1.261	3.136	6.933	0.402	5.799	0.073	1439
MinCVaR_withBTC_LB3m_RB1w	1.340	3.261	7.856	0.411	5.229	0.070	2032
Eq.W_withoutBTC_LB3m_RB1w	1.844	8.421	18.338	0.219	1.194	0.022	1439
Eq.W_withBTC_LB3m_RB1w	11.270	10.897	21.469	1.034	4.817	0.543	2032
PortfoliosBenchmarks							
BTCbenchmark	93.243	82.702	84.529	1.127	1.334	1.244	2032
SP500_benchmark	8.659	12.916	19.778	0.670	3.389	0.293	1439

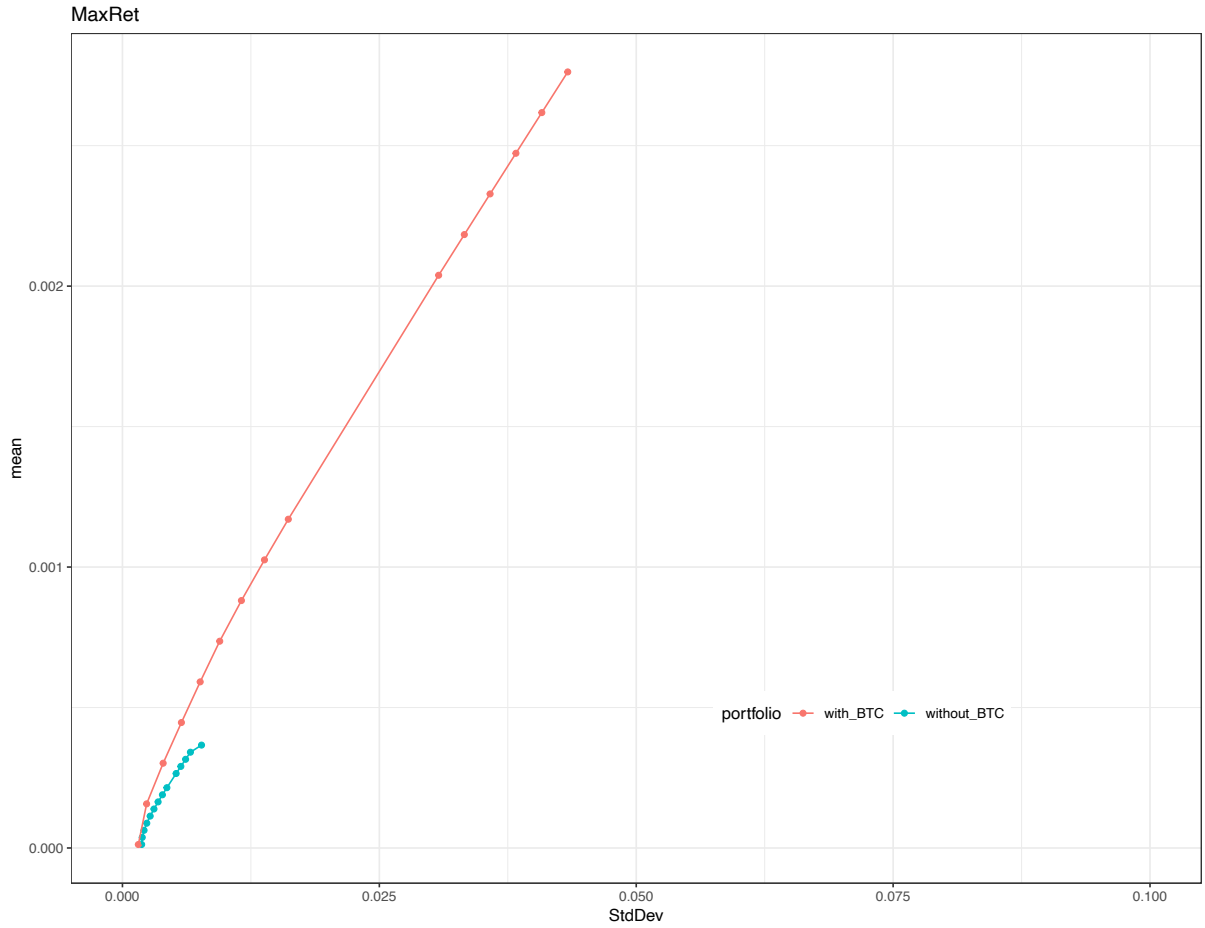
Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. 5 strategies, 4 portfolios each + 2 benchmarks. aRC – annualized rate of return, aSD – annualized standard deviation, MD – maximum drawdown, IR – Information ratio, IRMD -the relation coefficient of IR to MD, IRaRCMD – the relation coefficient of IR multiplied by aRC to MD, nObs – number of observations in days. aRC, aSD, MD present the percentage change.

5.2 Efficient frontiers

Apart from performance measures, we are also curious about the behavior of the efficient frontiers of the portfolios with and without Bitcoin since those are a great depiction of an investment portfolio efficiency in Markowitz model. We want to check if Bitcoin indeed moves the efficient frontier to the north-west direction, providing an investor with more optimal set of portfolios, with the higher expected return for a given level of risk. That is why we visualize the frontiers for two pairs within the two criteria - MaxRet and MinCVaR (Figure 5 and Figure 6 respectively).

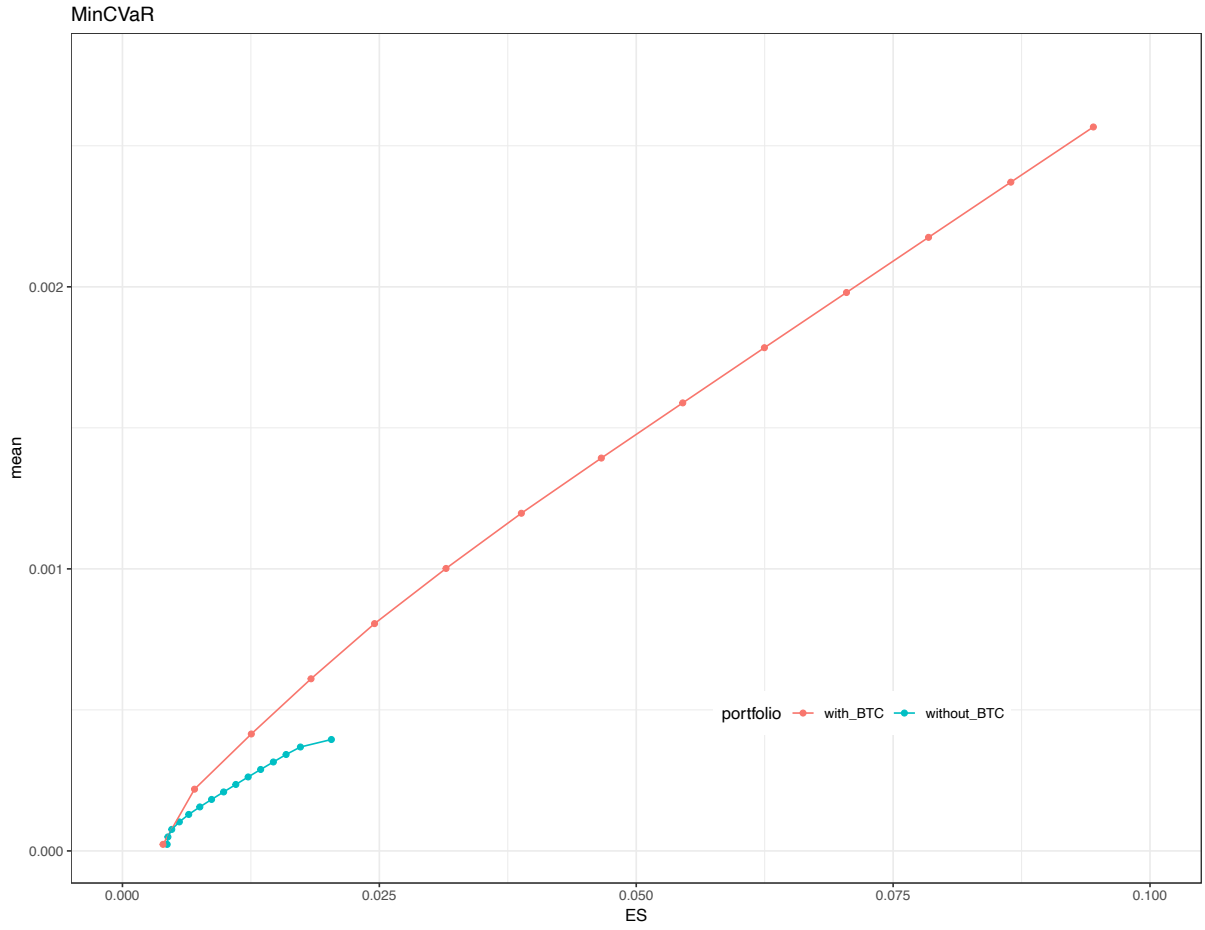
The findings described earlier in this paper seem to be confirmed. The frontiers indeed move up and left, demonstrating the more efficient set of investment portfolios, widening the opportunities for a potential investor.

Figure 5. Efficient frontiers for the portfolios with and without Bitcoin within the criteria MaxRet.



Source: own empirical analysis. Efficient frontiers are constructed for 15 optimal portfolios. Red line is the portfolio with BTC.

Figure 6. Efficient frontiers for the portfolios with and without Bitcoin within the criteria MinCVaR.



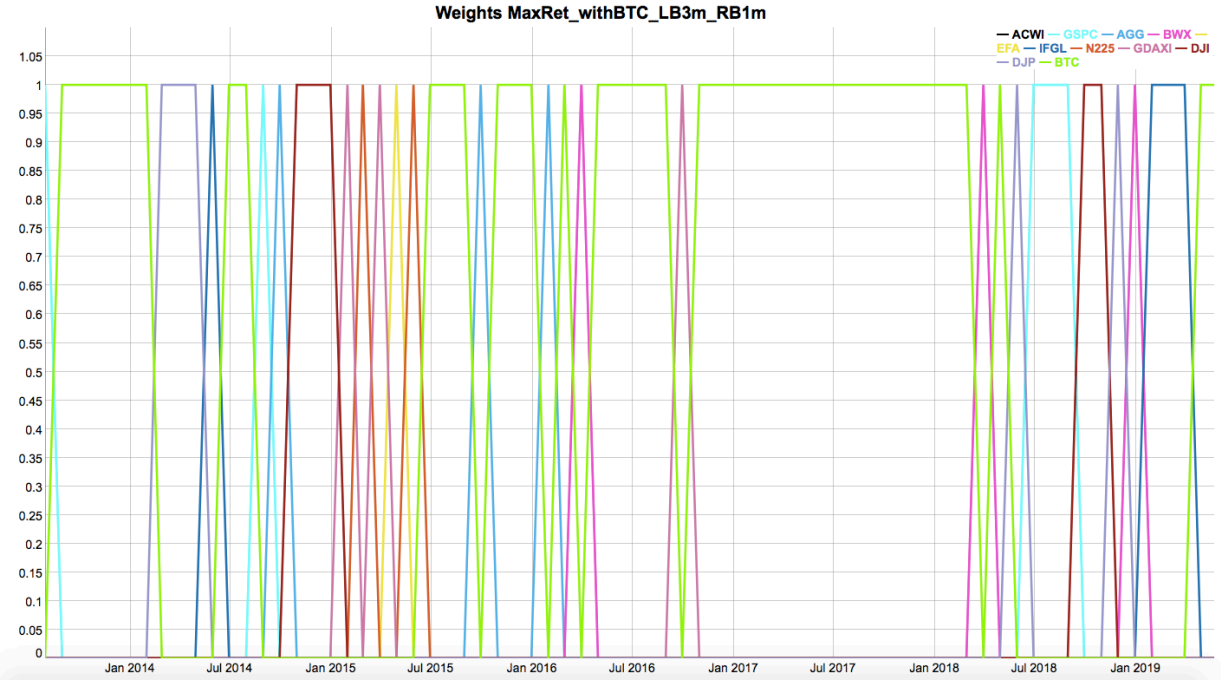
Source: own empirical analysis. Efficient frontiers are constructed for 15 optimal portfolios. Red line is the portfolio with BTC.

5.3 Portfolio weights

Besides, in the context of Markowitz framework we are also wondering about the important optimization element - the weight structure within the portfolio. The analysis aimed to check if Bitcoin is able to gain reasonable weight, along other traditional assets. Figure 7 and Figure 8 show the weights of assets in time for the MaxRet_withBTC_LB3m_RB1m and MinCVaR_withBTC_LB3m_RB1m portfolios.

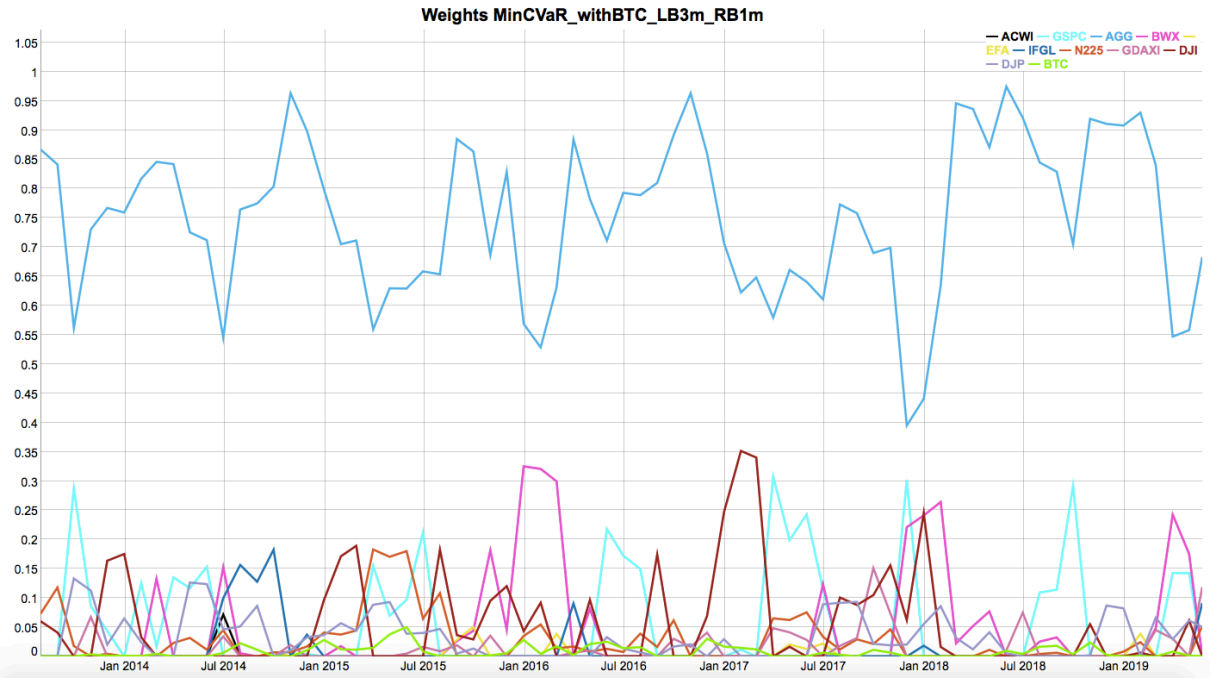
The visualization suggests that Bitcoin indeed gains substantial weights, adding up to portfolio optimization. The two criteria show, however, quite different results for the Main Strategies. MaxRet shows impressive Bitcoin weights (Figure 8). For the most of investment

Figure 7. Weights of the 11 assets in time for the Markowitz-optimized portfolio with Bitcoin MaxRet_withBTC_LB3m_RB1m.



Source: own empirical calculation. Period 31.10.2013 – 24.05.2019

Figure 8. Weights of the 11 assets in time for the Markowitz-optimised portfolio with Bitcoin MinCVaR_withBTC_LB3m_RB1m.



Source: own empirical calculation. Period 31.10.2013 – 24.05.2019

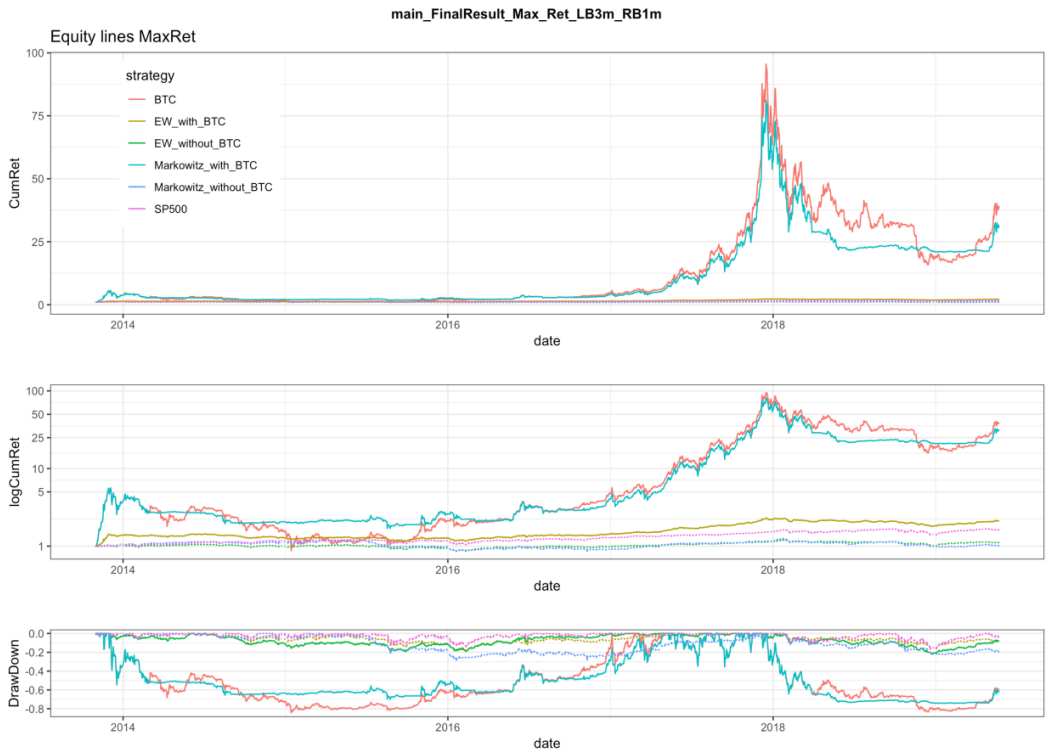
period it prevails over other assets, often gaining the maximum weight of 100% of the portfolio.

Nonetheless, if we consider MinCVaR criterion, we can noticeably see that the Bitcoin weights throughout the whole investment period are smaller than the other assets' ones. It is also worth noting that there are no longer breaks in the BTC curve, meaning that Bitcoin gains some weight all the time. Even with the small portion it is able to improve the portfolio, what is highlighted by the performance measures obtained in this analysis.

5.4 Equity lines & drawdowns

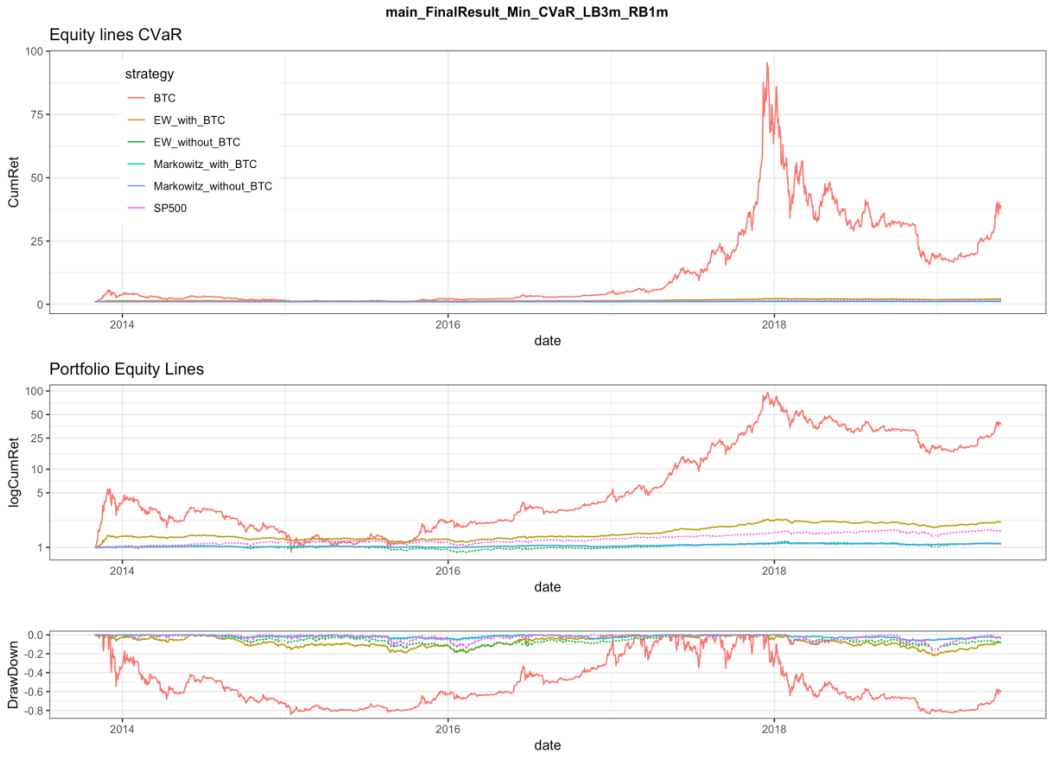
We also were interested in tracking the relative performance of the Main Strategies and the benchmarks: Bitcoin buy&hold and S&P500 buy&hold along whole investment period. Figure 10 and Figure 11 depict the equity lines (for 2 Markowitz portfolios, 2 equally-weighted portfolios and 2 benchmarks), for the normal and log scales, and capital drawdowns for the MaxRet and MinCVaR criteria respectively.

Figure 9. MaxRet_LB3m_RB1m Main Strategy performance illustration.



Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. Panel 1 presents the equity lines of 4 portfolios within the Main strategy + 2 benchmarks BTC buy&hold and S&P500 buy&hold. Panel 2 is the log scale of the panel 1. Panel 3 visualizes the drawdowns of the 6 portfolios.

Figure 10. MinCVaR_LB3m_RB1m Main Strategy performance illustration.



Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. Panel 1 presents the equity lines of 4 portfolios within the Main strategy + 2 benchmarks BTC buy&hold and S&P500 buy&hold. Panel 2 is the log scale of the panel 1. Panel 3 visualizes the drawdowns of the 6 portfolios.

In the case of Max_Ret criterion, Markowitz-optimized Bitcoin-inclusive portfolio (MaxRet_withBTC_LB3m_RB1m) outperformed equally-weighted ones. We can see that its pattern is very alike the Bitcoin buy&hold strategy. As we already showed, Bitcoin occupies significant parts of the portfolio along whole investment period. The Main MaxRet strategy demonstrates the two clear portfolio leaders: BTC buy&hold and Markowitz_with_BTC which outperform other portfolios. Those results obviously suggest that within such a framework and with given parameters (LB = 3m, RB = 1m) Bitcoin could become quiet an attractive asset for an investment portfolio diversification. However, the pair also shows most noticeable drawdowns – 74.31% for MaxRet and 84.53% for BTC benchmark. Those findings confirm the very risky and volatile nature of cryptocurrency.

In the case of Min_CVaR criterion, Markowitz-optimized Bitcoin-inclusive portfolio (MinCVaR_withBTC_LB3m_RB1m) happened to be less efficient than equally-weighted Bitcoin-inclusive portfolio and both benchmarks. That is visible on the Figure 10, panel 2 – its curve is at the bottom of the graph. Passive Bitcoin investment, in turn, happened to gain the largest accumulated return within the MinCVaR criterion. If we consider the performance of

the main MinCVaR strategy, we see the outstanding Bitcoin returns but no other portfolios even close to its level (especially after 2017). The visualization is confirmed by the performance measures. As for the drawdowns for this strategy, apart from Bitcoin all the portfolios show no surprising volatility and no significant drops. The maximum drawdown equals to 22.17% for MinCVaR_withBTC_LB3m_RB1m.

5.5. Sensitivity analysis

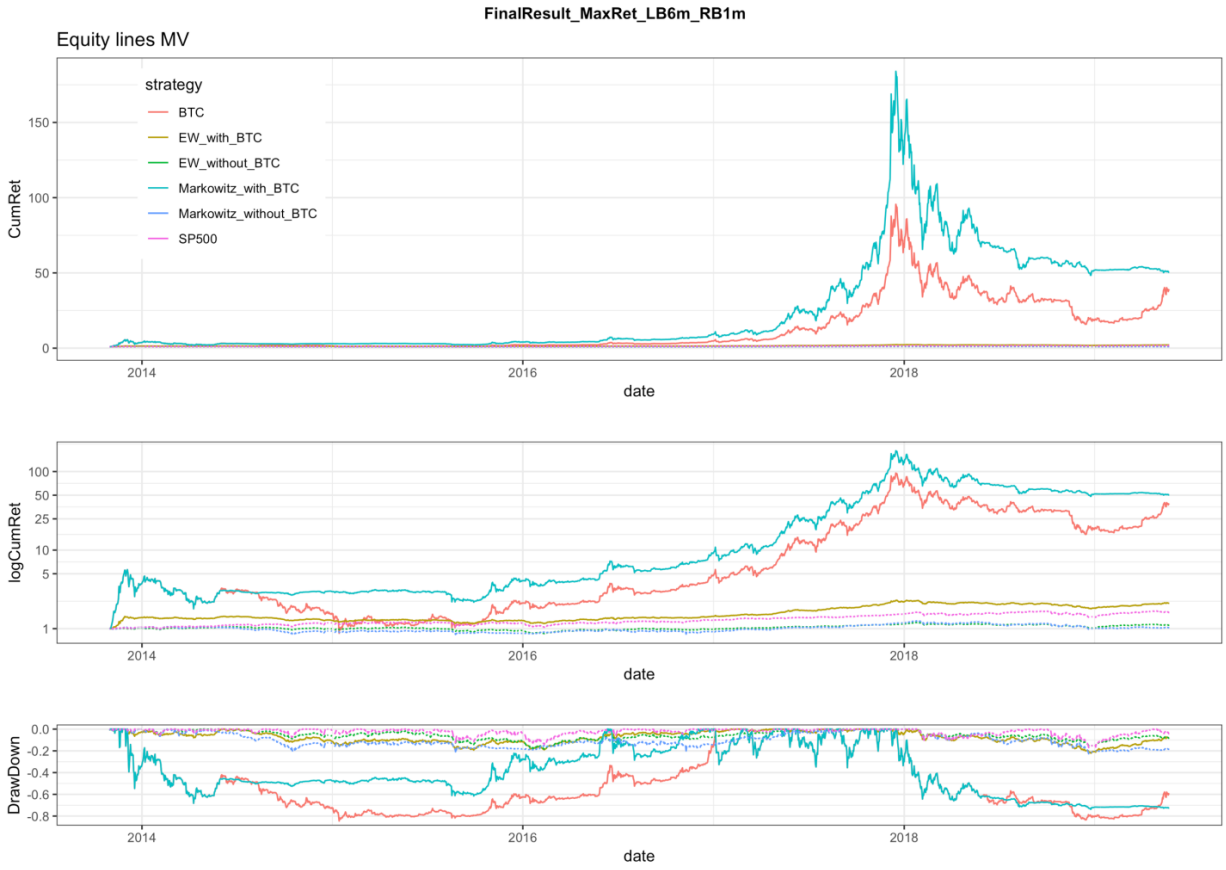
Apart from the Main Strategies we conducted the sensitivity analysis by creating 4 more strategies (4 portfolios each) with the changed lookback window (LB) and rebalancing frequency (RB). Having obtained the promising results for the two Main Strategies, we are now stressing the robustness of those findings.

By comparing the annualized rate of return (aRC), Information Ratio (IR) and annualized standard deviation (aSD) for Markowitz-optimized portfolio without Bitcoin and the one with (Table 5 and Table 6), we may observe that only one pair out of eight[9] fails to support the thesis hypothesis. In the strategy PortfolioStats_MinCVaR_LB6m_RB1m (Table 6) the portfolio without Bitcoin MinCVaR_withoutBTC_LB6m_RB1m happens to outperform the one including it with Information Ratio (IR) being equal to 8.13 over 5.47. What is interesting, the Bitcoin-inclusive equally-weighted portfolio performs best out of the whole strategy (6 portfolios) in terms of both annualized rate of return (aRC) and Information Ratio (IR). That again confirms the importance of the weight Bitcoin takes.

Rest of the sensitivity analysis strategies behave similarly to the Main Strategies, namely they show the improvement in the performance after adding Bitcoin to it. That let us consider the findings of our empirical research to be sufficiently robust in the context of Markowitz portfolio optimization.

[9] 8 panels out of 10 from the Table 5 and Table 6. since we are already considered the 2 Main Strategies, now we omit those in the discussion.

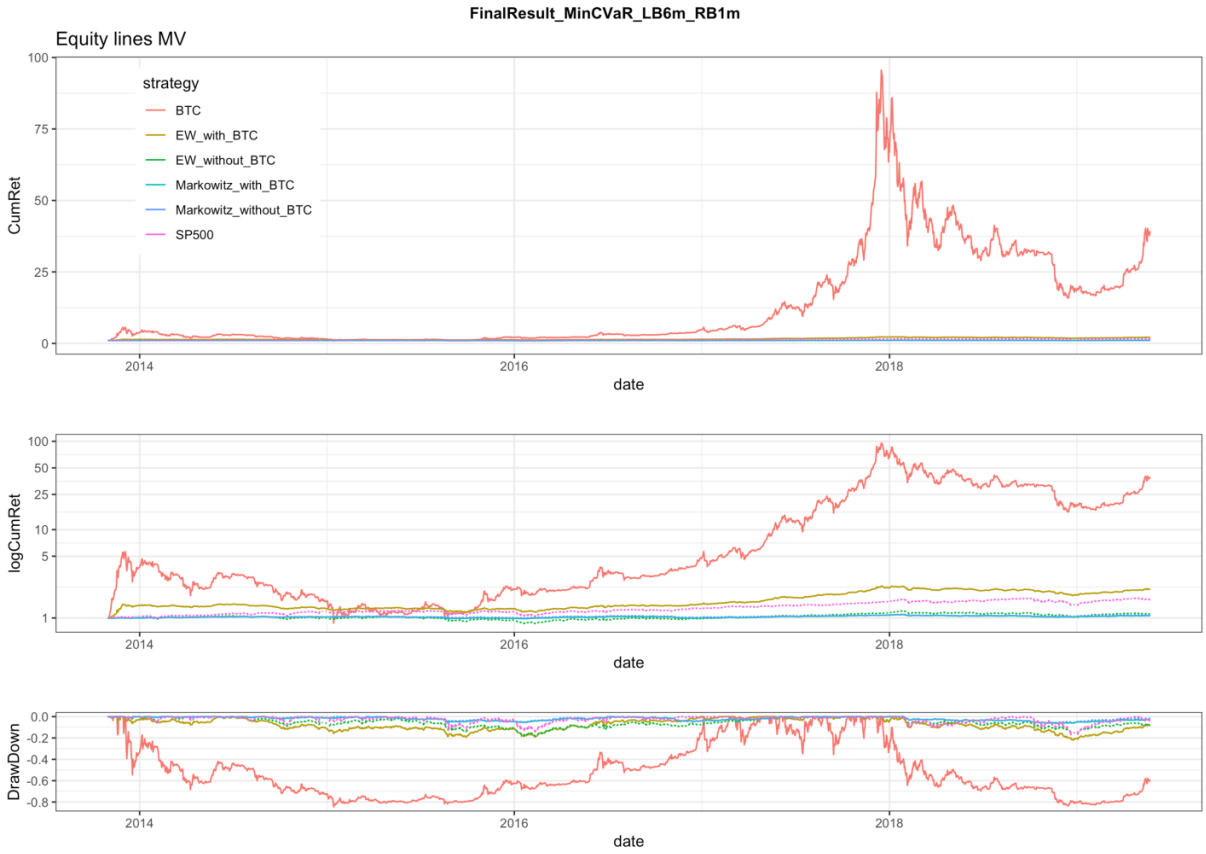
Figure 11. MaxRet_LB6m_RB1m Strategy performance illustration.



Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. Panel 1 presents the equity lines of 4 portfolios within the Main strategy + 2 benchmarks BTC buy&hold and S&P500 buy&hold. Panel 2 is the log scale of the panel 1. Panel 3 visualizes the drawdowns of the 6 portfolios.

MaxRet criterion strategy with the longest lookback window – 6 months and monthly rebalancing (Table 5) shows more or less the same results as for the Main Strategy. Markowitz-optimized Bitcoin-inclusive portfolio and BTC buy&hold benchmark outperform the rest of the strategy portfolios in terms of return generated, what is visible on Figure 11. Drawdowns, however, for those two are also significant – 73.69% and 84.53% respectively. Equally-weighted portfolios with such LB and RB look quite stable. However, it is worth noting that the Information ratio (IR) for the equally weighted portfolio with Bitcoin Eq.W_withBTC_LB6m_RB1m is higher than the one for Bitcoin benchmark portfolio. It is equal to 1.21, whereas BTC buy&hold shows the value of 1.13.

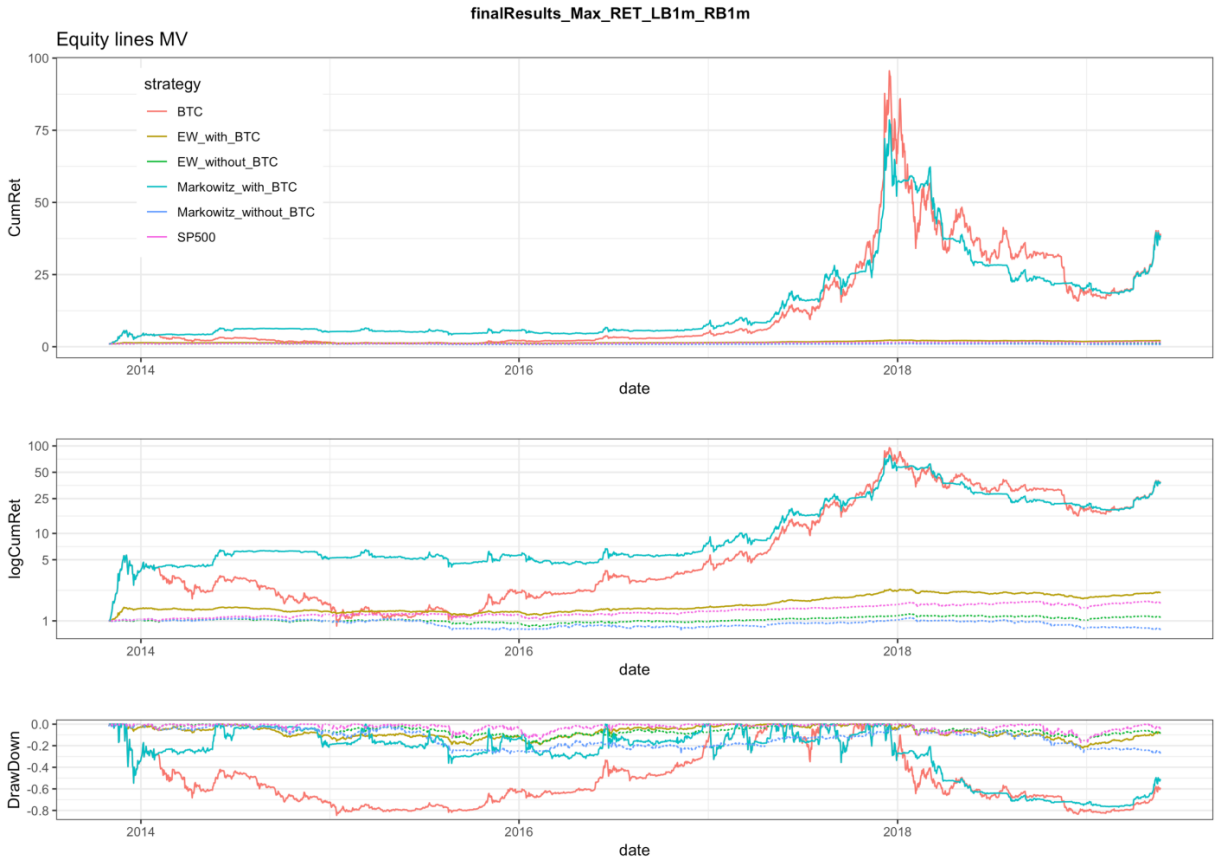
Figure 12. MinCVaR_LB6m_RB1m Strategy performance illustration.



Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. Panel 1 presents the equity lines of 4 portfolios within the Main strategy + 2 benchmarks BTC buy&hold and S&P500 buy&hold. Panel 2 is the log scale of the panel 1. Panel 3 visualizes the drawdowns of the 6 portfolios.

MinCvaR_LB6m_RB1m is the unique in the whole analysis strategy, which does not support the main hypothesis of this thesis about efficiency improvement properties of Bitcoin. Adding it to the portfolio of 10 traditional assets does not improve the risk-return ratio and does not provide an investor with additional opportunities. Best strategy portfolio for the parameters lookback - 6 months and rebalancing frequency – 1 month in terms of Information Ratio (IR) is Markowitz-optimized without Bitcoin with the value 0.48 (Table 6). It is, however, outperformed by both of the benchmarks and equally-weighted portfolio with Bitcoin while we considering the annualized rate of return (aRC) obtained (Figure 12). The maximum drawdown for the strategy is Bitcoin benchmark. Although, if we consider the five other portfolios, we can say that there have not appeared any outstanding drawdown among them. The highest value is 22.17%.

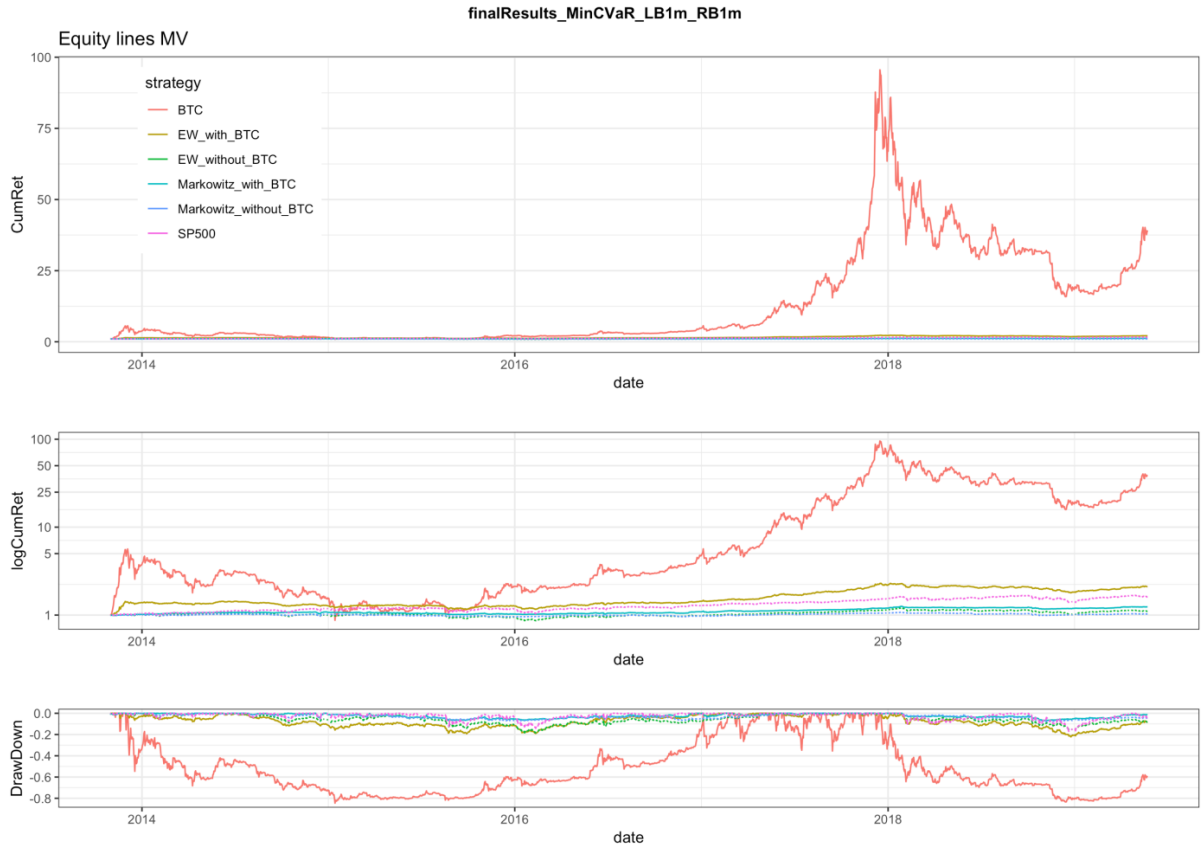
Figure 13. MaxRet_LB1m_RB1m Strategy performance illustration.



Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. Panel 1 presents the equity lines of 4 portfolios within the Main strategy + 2 benchmarks BTC buy&hold and S&P500 buy&hold. Panel 2 is the log scale of the panel 1. Panel 3 visualizes the drawdowns of the 6 portfolios.

The third strategy out of the four employed for the sensitivity analysis is based on one month lookback window and one month rebalancing frequency. Markowitz-optimized Bitcoin-inclusive portfolio shows considerable 92.7% of annualized rate of return (aRC), which is very close to 93.24% for Bitcoin benchmark. That portfolio also demonstrates the highest value for the Information Ratio (IR) measure among the 6 portfolios in a strategy. It happened to be equal to 1.42. That value is even higher than the one for BTC buy&hold strategy. That performance is well visible on the Figure 13. As for the drawdowns, still the largest one refers to the passive Bitcoin investment. However, MaxRet_withBTC_LB1m_RB1m notifies 76.9% of capital loss in the framework we employ.

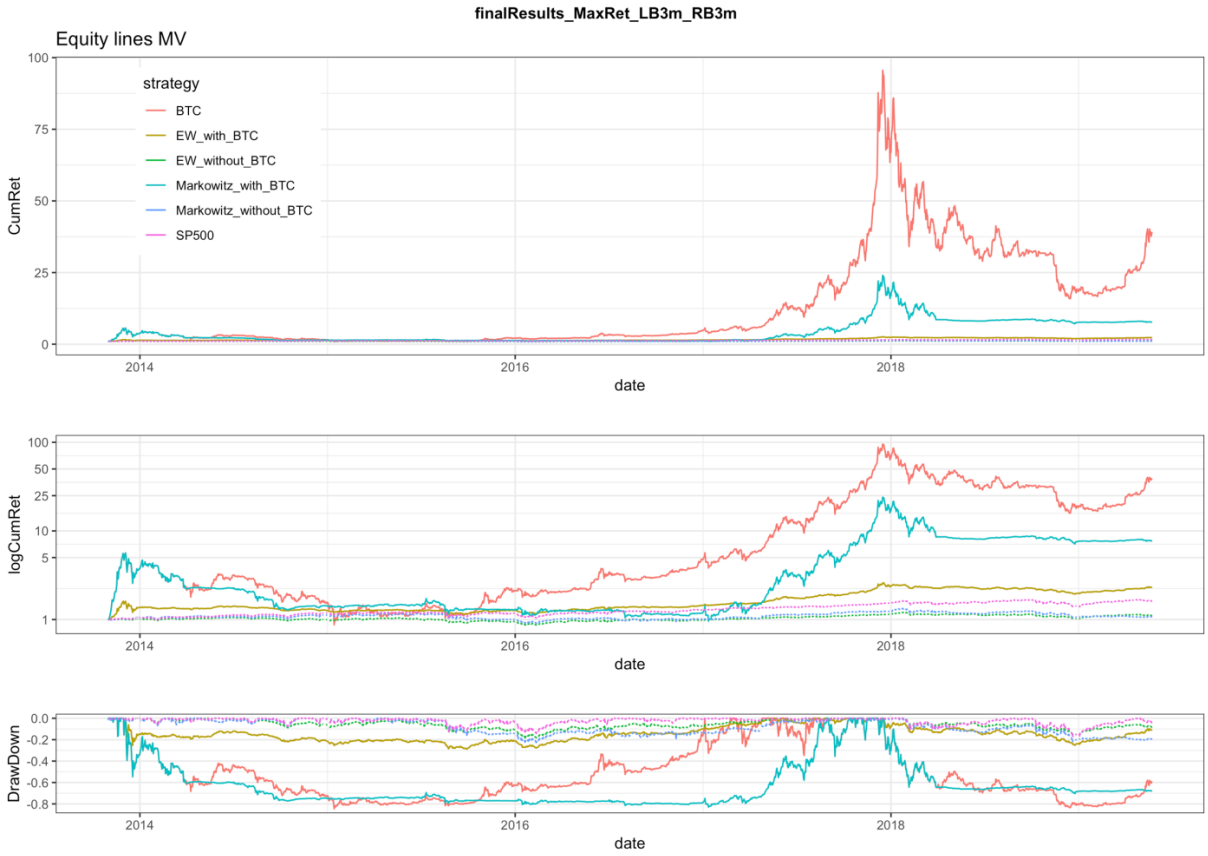
Figure 14. MinCVaR_LB1m_RB1m Strategy performance illustration.



Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. Panel 1 presents the equity lines of 4 portfolios within the Main strategy + 2 benchmarks BTC buy&hold and S&P500 buy&hold. Panel 2 is the log scale of the panel 1. Panel 3 visualizes the drawdowns of the 6 portfolios.

Within the MinCVaR criterion the arbitrary values of parameters RB and LB equal to 1 month make the equally-weighted portfolio with Bitcoin outperform the other 5 in the strategy. Exactly this portfolio follows the Bitcoin buy&hold one, when it comes to the annualized rate of return (aRC). However, if we look at the Information Ratio (IR), we see that exactly equally weighted portfolio shows the best value of it, 1.205 (Table 6). The maximum drawdown (not taking BTC benchmark into consideration) is same as in the previous case for MinCVaR and equals to 22.17% (Figure 14).

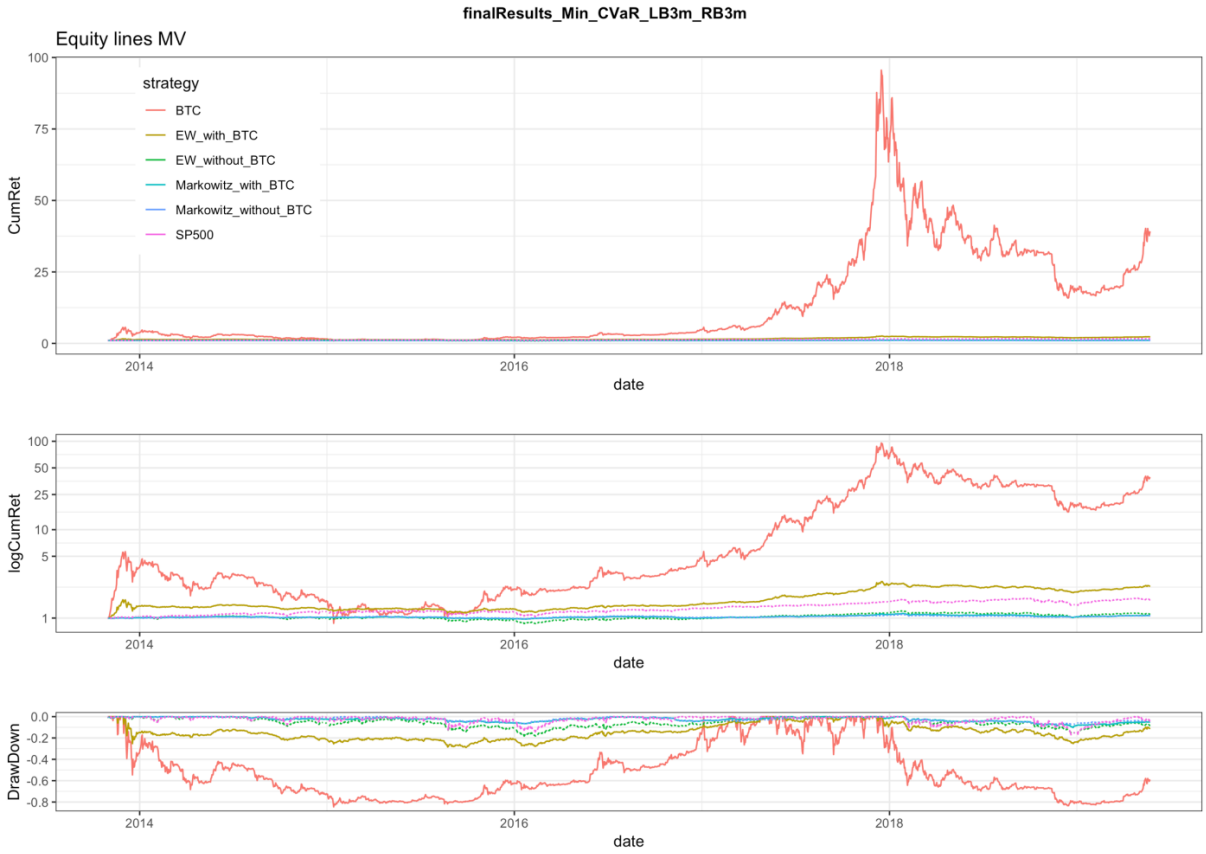
Figure 15. MaxRet_LB3m_RB3m Strategy performance illustration.



Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. Panel 1 presents the equity lines of 4 portfolios within the Main strategy + 2 benchmarks BTC buy&hold and S&P500 buy&hold. Panel 2 is the log scale of the panel 1. Panel 3 visualizes the drawdowns of the 6 portfolios.

When it comes to the MaxRet strategy with 3 months rebalancing frequency and lookback window, we observe the equally weighted portfolio with Bitcoin to have the highest Information ratio (IR) out of the 4 considered portfolios in the panel (Table 5, panel 4). Its value is 0.96%. However, the highest return belongs to the Markowitz optimized portfolio with Bitcoin MaxRet_withBTC_LB3m_RB3m what is also illustrated on the Figure 15. Although the return is substantial, risk is too. Over 83% of capital loss is probable according to the empirical results.

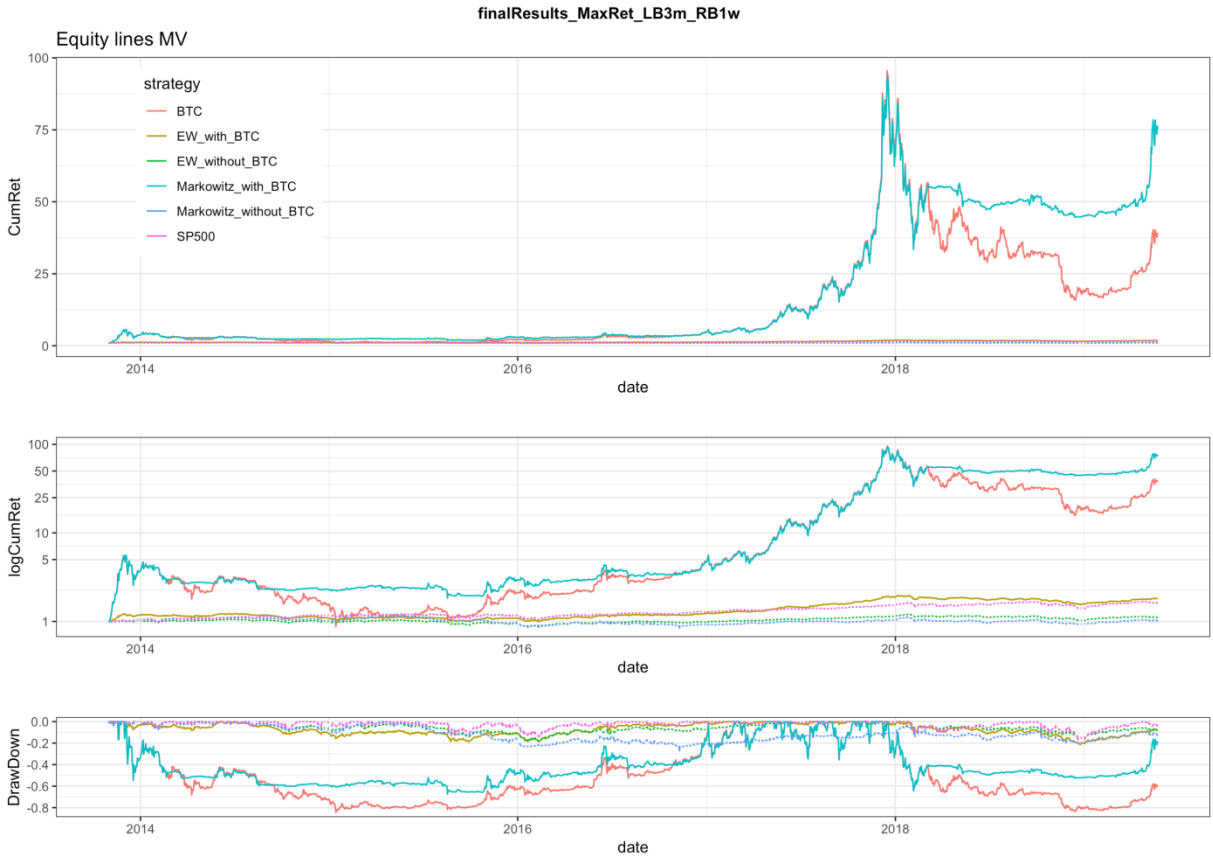
Figure 16. MinCVaR_LB3m_RB3m Strategy performance illustration.



Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. Panel 1 presents the equity lines of 4 portfolios within the Main strategy + 2 benchmarks BTC buy&hold and S&P500 buy&hold. Panel 2 is the log scale of the panel 1. Panel 3 visualizes the drawdowns of the 6 portfolios.

For the strategy with parameters of lookback window and rebalancing frequency equal to 3 months no anomalies are detected (Figure 16). The strategy behaves in line with the others within the MinCVaR criterion, namely the equally-weighted portfolio appears to be the most efficient in terms of risk-return ratio. The largest drawdown, apart from 84.53% of Bitcoin capital, shows the value of 28.95%.

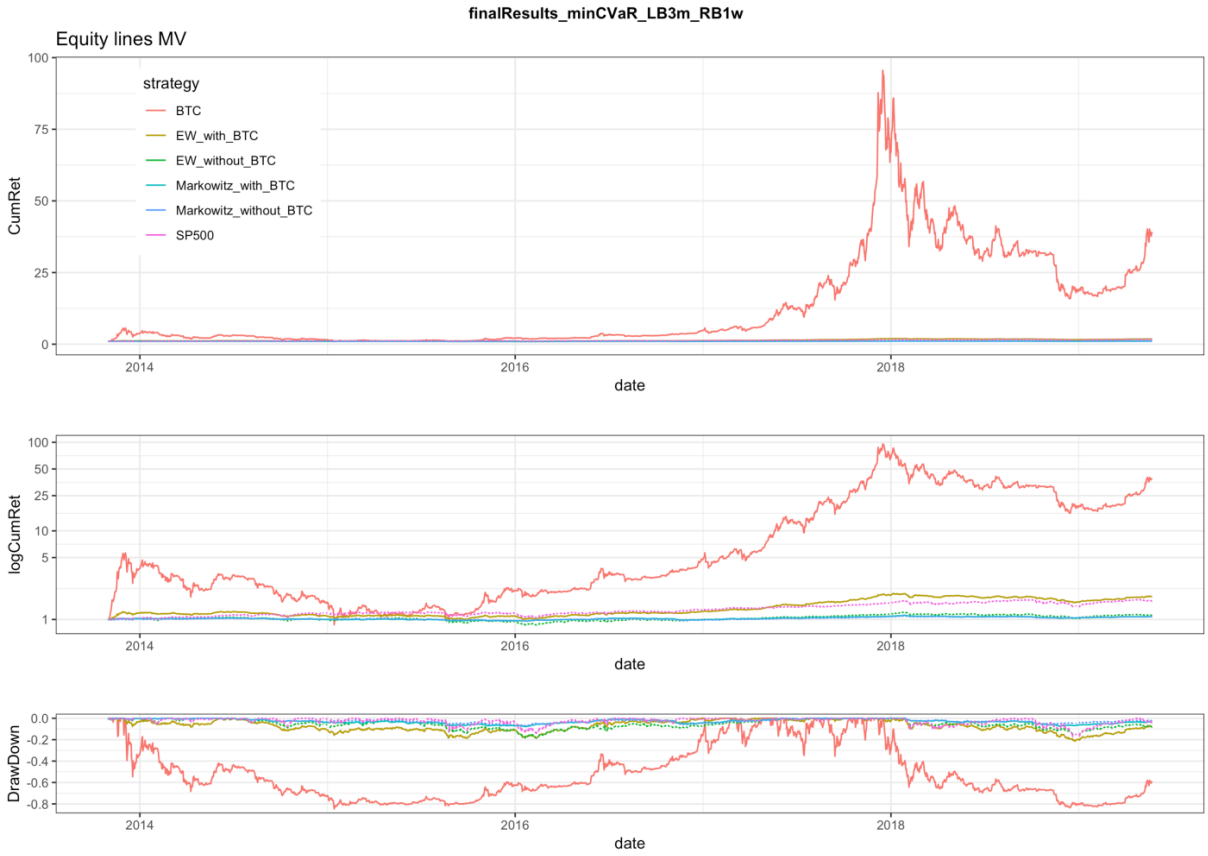
Figure 17. MaxRet_LB3m_RB1w Strategy performance illustration.



Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. Panel 1 presents the equity lines of 4 portfolios within the Main strategy + 2 benchmarks BTC buy&hold and S&P500 buy&hold. Panel 2 is the log scale of the panel 1. Panel 3 visualizes the drawdowns of the 6 portfolios.

The last strategy within the MaxRet criterion with 3 months lookback window and 1 month rebalancing frequency appears to be an outstanding one. The annualized rate of return of Markowitz Bitcoin-inclusive portfolio MaxRet_withBTC_LB3m_RB1w happens to be higher than for the one for the Bitcoin itself. The value of 117,9% is the largest annualized rate of return (aRC) in the empirical analysis. Bitcoin benchmark, in turn, shows 93.24% of aRC. The information Ratio (IR), being equal to 1.7%, is also the biggest among the strategies. We can track that untypical behavior on the Figure 17. The drawdown, however, is also quite noticeable – 67.43%.

Figure 18. MinCVaR_LB3m_RB1w Strategy performance illustration.



Source: own empirical analysis. Period 31.10.2013 – 24.05.2019. Panel 1 presents the equity lines of 4 portfolios within the Main strategy + 2 benchmarks BTC buy&hold and S&P500 buy&hold. Panel 2 is the log scale of the panel 1. Panel 3 visualizes the drawdowns of the 6 portfolios.

The last parameters set is 3 months lookback window and 1 week rebalancing frequency. Consistent with all the previous strategies from the MinCVaR criterion equally weighted portfolio with Bitcoin appears to be the most efficient of the whole set. If we look at the Information ratio (IR), it appears to be better than both Markowitz portfolios (with and without Bitcoin), equally-weighted one without BTC and S&P 500 benchmark. Only BTC buy&hold performs better (Figure 18). The maximum drawdown for the strategy is 18.34%

6. Conclusions

As a recent attractive new asset class, Bitcoin drew the attention of an international investment ecosystem. Thanks to its untypical characteristics it was examined from the different perspectives. In this research we decided to study the properties of Bitcoin in the context of Markowitz model. We wondered if it improves the investment portfolio efficiency, if the findings are robust and persistent to the optimization criteria (expected mean return

maximization and expected shortfall (CVaR) minimization) and optimization parameters' variations (lookback window LB and rebalancing frequency RB). Using the daily observations of 10 traditional asset classes (among which equity, fixed income, money, commodities, real estate) and Bitcoin from 01.05.2013 till 24.05.2019, we checked how capable in portfolio optimization Bitcoin is.

We designed 44 portfolios, 22 for each of the optimization criteria. Each criterion included 5 strategies, which were applied to the four portfolios each + 2 benchmark portfolios – BTC buy&hold and S&P 500 buy&hold. The first out of the five strategies was the Main Strategy, with the following parameters: LB 3 months, RB 1 month. The other 4 were used to check the robustness of the results by changing the set of the above parameters in the Markowitz optimization. We compared the portfolios with and without Bitcoin, criteria between themselves, and all the sensitivity analysis strategies in term of their performance measures, efficient frontiers, weights structure, equity lines and drawdowns.

The results suggest that Bitcoin should be definitely considered to be included into an investment portfolio since it adds up to the portfolio risk-return ratio improvement [research question #1]. In 90% of analysis (together with the robustness check) Bitcoin-inclusive strategies clearly outperformed those consisting solely of traditional assets. The results are not confronted by the different Markowitz optimization criteria [research question #2]. Arbitrary choice of the lookback window and rebalancing frequency parameters does not show any confronting findings [research question #3]. Sensitivity analysis (lookback window and rebalancing frequency parameters change) show that almost in all the checked cases the results are confirmed and the better performance of the portfolios are tracked. The results are consistent with the academic literature on the alike subjects (e.g. Klabbers 2017, Gangwal 2016, Kajtazi et al. 2017).

The past decade shows that Bitcoin became a very attractive investment asset. Bitcoin derivatives were employed by traditional exchanges. CME's Bitcoin futures are quite popular now. More than 2,960 accounts have traded CME Group's bitcoin (BTC) futures across all client types and time zones since launch.[10] Average daily volume (ADV) equals to $515 million notional value or 68,000 equivalent Bitcoin.[11] There appear specialized platforms for Bitcoin derivatives trading, for example Deribit.[12] Cryptocurrency market has a tremendous

[10] https://www.nasdaq.com/article/june-sets-records-for-cme-bitcoin-futures-as-sign-ups-surge-30-cm1171924
[11] https://www.coindesk.com/may-was-best-month-for-cme-bitcoin-futures-volume-since-2017
[12] https://www.deribit.com

capitalization, about 300 bn. USD[13]. There is a huge space for institutional capital placement. Phenomenon is undoubtedly very curious to observe. However, it is worth bearing in mind that Bitcoin is an extremely volatile asset with a sophisticated technology behind it. Along with the higher returns, the larger risk is coming.

To conclude, for the time being, with parameters and empiric model chosen we obtained quiet convincing and promising results, which go a line with what we observe both in the academic and business areas. Nevertheless, further investigation could be considered to make an unambiguous conclusion in favor of Bitcoin.

Potential extensions proposed for the further analysis are regarded the investment period, portfolio assets chosen, arbitrary chosen values of the LB and RB parameters, the origin of an investor – we could consider solely the Europe or Americas as the markets of investment, other Markowitz criteria, other limitations for the portfolio construction – allow short selling or change the transactional cost, for example, change the degrees of financial leverage or the empirical model itself. Even though this study contributes to the literature on investment portfolio diversification with Bitcoin in Markowitz model, still there is a huge variety of directions to proceed with.

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