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INTERGENERATIONAL REDISTRIBUTIVE EFFECTS  
OF MONETARY POLICY

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## Intergenerational redistributive effects of monetary policy

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**Abstract:** This paper investigates the distributional consequences of monetary policy across generations. We use a life-cycle model with a rich asset structure as well as nominal and real rigidities calibrated to the euro area using both macroeconomic aggregates and microeconomic evidence from the Household Finance and Consumption Survey. We show that the life-cycle profiles of income and asset accumulation decisions are important determinants of redistributive effects of monetary shocks and ignoring them can lead to highly misleading conclusions. The redistribution is mainly driven by nominal assets and labor income, less by real and housing assets. Overall, we find that a typical monetary policy easing redistributes welfare from older to younger generations.

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**Keywords:** monetary policy, life-cycle models, wealth redistribution

**JEL codes:** E31, E52, J11

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# 1 Introduction

It is fairly well known that central bank policy has distributional consequences. For instance, by raising interest rates, monetary policy can benefit rich households who earn more on interest bearing assets, but is usually harmful for poorer households, who rely more on labor income and credit. However, the existing literature looking at redistributive effects of monetary policy mainly relies either on a purely empirical approach or on structural models where heterogeneity arises from idiosyncratic shocks affecting ex ante identical agents as introduced in a seminal paper by Bewley (1977). Interestingly, life-cycle models, which introduce another very important dimension of heterogeneity, have very rarely been used in this debate so far.

Young and old households systematically differ in asset positions and labor market participation, making the age dimension of heterogeneity crucial for redistributive effects of monetary policy as central bank decisions affect asset prices and labor market developments. Life cycle motives also generate strong and predictable patterns in future asset positions of each cohort, which we demonstrate to be much more relevant than their current asset holdings. The importance of the age dimension of household heterogeneity is well documented in the empirical literature, see e.g. Kuhn and Rios-Rull (2016) for the US and HFCN (2016) for the euro area. De Nardi (2015) demonstrates how including the overlapping generations structure in a basic incomplete markets economy increases the wealth Gini coefficient from 0.38 to 0.67.

To analyze the distributional effects of monetary policy across generations, we construct a life-cycle model with a rich asset structure and new-Keynesian characteristics. The model features three types of assets – housing, as well as nominal and real financial assets. In the language of Kaplan et al. (2018), this allows us to analyze in detail the direct effects of monetary policy shocks, while the presence of nominal and real rigidities ensures a realistic representation of indirect effects. The life-cycle framework generates a natural environment where agents are heterogeneous with respect to many key economic characteristics, including asset positions, labor market participation, or propensity to consume. As a consequence, households are not only affected by monetary policy via different channels and to a different degree, but also adjust their consumption, housing and financial savings differently, depending on their age.

Focusing on the age dimension and abstracting from other sources of household heterogeneity allows us to include in the model a fairly rich set of frictions that have been found key in ensuring a good empirical fit according to the representative agent DSGE literature, especially in the context of dynamic responses to monetary shocks. The model is calibrated to the euro area using standard macroeconomic data and microeconomic evidence collected in the Household Finance and Consumption Survey (HFCS). Its aggregate implications are consistent with the empirical evidence on the effects of unanticipated monetary shocks identified in standard vector autoregressions.

Our main contribution to the literature is twofold. First, to the best of our knowl-

edge, we are the first to investigate the redistributive effects of monetary policy across generations in a fully-fledged structural life-cycle framework, where agents differ in net worth and composition of asset portfolios along the age dimension. Second, we put into a lifecycle context and demonstrate the quantitative relevance of the recent seminal insight of Auclert (2019), that what matters for redistribution is the distribution of *maturing* assets and liabilities. For instance, we show that taking into account the effective maturity of housing assets overturns the wealth effect of house price appreciation for most cohorts. This type of conclusions is missing in the literature where all heterogeneity arises due to idiosyncratic shocks.

Our model simulations show that the life-cycle dimension of income and asset accumulation decisions is an important determinant of redistributive effects of monetary policy shocks. We decompose these effects into a few sources, and evaluate their relative importance. We find that the distributional effect via changes in house prices is of relatively minor importance, since what matters is not the housing wealth per se, but rather the housing transaction flows. This means that appreciation of house prices following a monetary expansion hurts households that are in the process of accumulating housing, even if their current stock of this asset is high. However, what turns out to be much more impactful are changes in returns on nominal assets. By lowering them, a monetary easing benefits borrowers, and this effect dwarfs the effect of house price changes. Additionally, the working population benefits from the expansion on the labor market, but loses from changes in returns on real assets. The net impact of all these effects results in welfare redistribution from older to younger generations. This outcome is consistent with recent survey-based evidence on the relationship between age and the impact of a monetary easing on self-reported happiness and life satisfaction, see Bunn et al. (2020).

The rest of this paper is organized as follows. Section 2 reviews the related literature. Section 3 lays out the model used in our analysis. Section 4 discusses the construction of aggregate and age-specific input data underlying our calibration strategy, and presents the empirical evidence from the euro area on the aggregate effects of monetary policy. Section 5 covers our main simulation results and Section 6 concludes.

## 2 Related literature

The literature relating monetary policy and asset distribution has been growing dynamically over the recent years. Here we only refer to the selected channels and studies, and direct the reader interested in an extensive review of the existing literature to Colciago et al. (2019).

Inflation stands out as a prominent driver of redistribution, and obviously is related to monetary policy. When contracts are nominal, unexpected inflation benefits debtors at the expense of creditors. One immediate consequence is the impact of inflation on real money balances, which redistributes wealth from cash holders to the government (Imrohorglu,

1992). Holders of other nominally fixed assets (e.g. government bonds) are affected in a similar way. What ultimately matters for the impact of inflation on household wealth are net nominal asset positions (NNP). Doepke and Schneider (2006) offer a detailed analysis of household NNPs in the US and conclude that rich households are hurt by unexpected inflation because of their large holdings of nominal bonds, while (usually younger) holders of mortgage debt gain. Adam and Zhu (2016) reach similar conclusions using HFCS data for the euro area.

Changes of real interest rates matter as well. The problem is analyzed in detail by Auclert (2019), who accentuates the role of unhedged interest rate exposures (URE). In particular, he points out that what matters for the impact of real interest rates on wealth is the difference between maturing assets and liabilities. An unexpected tightening of monetary policy raises real interest rates and hurts those households whose maturing liabilities are higher than maturing assets, since they will have to acquire new debt at a higher cost. Again, relating this theoretical concept to real world asset distributions reveals that, for instance, households with fixed rate mortgages have higher UREs than those with adjustable rate mortgages. As a consequence, the former gain and the latter lose from tighter monetary policy.

Another important channel of potential redistribution works indirectly, via macroeconomic effects of monetary policy actions. A monetary tightening results in an economic slowdown. As a consequence, wages decline and unemployment increases. This hurts people disproportionately, with relatively poor and credit constrained agents being affected more severely. Furceri et al. (2018) show that the impact of monetary policy shocks on inequality is larger the higher are the labor shares. This finding seems in line with a more general result of Heathcote et al. (2010), who find that labor income of the poor fluctuates most over the business cycle.

All in all, monetary policy seems to have statistically and economically significant consequences for income distribution and inequality. Several studies investigated not only the selected channels described above, but also aggregate effects of central bank policy on economic inequality. Coibion et al. (2017) use data from the US Consumer Expenditure Survey and find that contractionary monetary policy shocks increase several measures of inequality. Similar results are found by Guerello (2018), who analyzes the impact of monetary policy on income dispersion in the euro area, by Mumtaz and Theophilopoulou (2017) who look at the case of UK, and by Furceri et al. (2018) who investigate a panel of 32 countries.

On the modeling side, we are closest to papers that use overlapping generations (OLG) models with New Keynesian features. These include for instance Galí (2021) on rational bubbles, Del Negro et al. (2012) about the forward guidance puzzle, Nisticò (2012) about stock prices and monetary policy, or studies about demographic forces and monetary policy (e.g. Carvalho et al., 2016; Bielecki et al., 2018). However, to our knowledge, this class of models has not been used to analyse the distributional consequences of monetary

policy. Moreover, most of the OLG studies use a simplified Blanchard-Yarri setup of perpetual youth, while our paper considers a fully-fledged life cycle structure, thus providing a more accurate description of intergenerational differences.

### 3 Model

To analyze the distributional effects of monetary policy, we construct a New Keynesian model with overlapping generations of finitely-lived households. The households face age-dependent mortality risk and have access to housing and two types of financial assets. As it is standard in the Keynesian literature, nominal rigidities encompass both price and wage stickiness. The model economy is also populated by two types of producers, investment funds, as well as fiscal and monetary authorities. The problems that the agents solve are described below. While denoting prices, we employ the convention of using upper case for nominal values and lower case for their ratio to the aggregate price index  $P_t$ . Variables without time indices denote their non-stochastic steady state levels. The model frequency is annual.

#### 3.1 Households

Each household consists of a single agent, who is assumed to enter the model at age 20 and is assigned age index  $j = 1$ . The maximum lifespan of a household is 99 years ( $J = 80$ ) and at each year the household faces age-dependent mortality risk  $\omega_j$ . Thus, at each time period, there are 80 cohorts of overlapping generations, with their size denoted by  $N_{j,t}$ . Within a cohort, households differ only by the amount of labor supplied due to staggered wage contracts. However, we assume that idiosyncratic wage risk can be perfectly insured so that all other allocations chosen by agents in the same cohort are identical.

##### 3.1.1 Optimization problem

A  $j$ -aged household  $\iota$  maximizes its expected remaining lifetime utility that depends on consumption  $c_{j,t}$ , end-of-period housing stock  $\chi_{j+1,t+1}$  and hours worked  $h_{j,t}$  according to

$$U_{j,t}(\iota) = \mathbb{E}_t \sum_{s=0}^{J-j} \beta^s \frac{N_{j+s,t+s}}{N_{j,t}} \left[ \begin{array}{l} \log(c_{j+s,t+s} - \varrho \bar{c}_{j+s,t+s-1}) + \\ + \psi_{j+s} \log \chi_{j+s+1,t+s+1} - \phi_{j+s} \frac{h_{j+s,t+s}(\iota)^{1+\varphi}}{1+\varphi} \end{array} \right] \quad (1)$$

where  $\beta$  is the subjective discount factor, the ratio  $N_{j+s,t+s}/N_{j,t}$  represents the probability of surviving for at least  $s$  more years,  $\psi_j$  and  $\phi_j$  are the age-dependent parameters regulating preference for housing and leisure,  $\varphi$  is the inverse of the Frisch elasticity of labor supply, and  $\varrho$  controls the strength of external habits, expressed relative to average consumption of the same age group in the previous period.

Households face the following budget constraint

$$\begin{aligned} c_{j,t} + p_{\chi,t}[\chi_{j+1,t+1} - (1 - \delta_{\chi})\chi_{j,t}] + a_{j+1,t+1} = \\ = (1 - \tau_t) w_t(\iota) z_j h_{j,t}(\iota) + \frac{R_{j,t}^a}{\pi_t} a_{j,t} + beq_{j,t} + beq_{j,t}^{\chi} + \Xi_{j,t}(\iota) \end{aligned} \quad (2)$$

where  $P_t$  denotes the aggregate price level,  $\pi_t \equiv P_t/P_{t-1}$  is (gross) inflation,  $p_{\chi,t}$  denotes the real price of housing,  $\delta_{\chi}$  is the housing depreciation rate,  $a_{j,t}$  stands for the beginning-of-period  $t$  real stock of financial assets that are managed by investment funds and that yield the gross nominal rate of return  $R_{j,t}^a$  (where the rate of return is age-specific, see below),  $\tau_t$  is the proportional tax on labor income,  $w_t$  is the real wage per effective hour,  $z_j$  represents age-specific labor productivity,  $beq_{j,t}$  and  $beq_{j,t}^{\chi}$  denote, respectively, received bequests of financial and housing wealth, and  $\Xi_{j,t}$  collects net real payments from the wage insurance scheme.

Our model features exogenous retirement upon reaching the age of 64 ( $JR = 45$ ), and hence we set  $z_j = 0$  for all  $j \geq JR$ . Since most agents die before reaching the maximum age, they leave unintentional bequests in form of financial assets and housing. Both are redistributed equally across all agents that are at most ten years before retirement in form of lump-sum transfers so that  $beq_{j,t} = beq_{j,t}^{\chi} = 0$  for  $j \geq JR - 10$ .<sup>1</sup>

### 3.1.2 Wage stickiness

The differentiated labor services of households are bundled up by competitive aggregators who then transform them into standardized labor services

$$h_{j,t} = \left[ \int_0^1 h_{j,t}(\iota)^{1/\mu_w} d\iota \right]^{\mu_w} \quad (3)$$

where  $\mu_w$  captures the imperfect substitutability of differentiated labor services and is equal to the steady state markup over the competitive wage level.

We assume that, after adjusting for age-specific productivity  $z_j$ , labor services supplied by individual cohorts are perfect substitutes. For tractability, we additionally assume that wage setting is performed on behalf of each household by competitive labor unions, operating at the economy-wide level. Wage setting is subject to a nominal rigidity as in the Calvo scheme, i.e. each wage is reoptimized only after receiving a random signal, which arrives with probability  $1 - \theta_w$ . Wages of households who do not receive the signal are fully indexed to steady state inflation.

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<sup>1</sup>The upper age limit on receiving bequests is consistent with typical assumptions in the literature, see e.g. De Nardi and Yang (2014).

### 3.1.3 Age-specific rates of return

There are two financial assets traded in the model economy: claims on physical capital and bonds, the latter representing all nominal debt contracts between borrowers (including the government sector) and savers. These two assets will hence be also referred to as real and nominal financial assets, respectively. The age-specific shares of bonds in household portfolios are denoted with

$$s_{j,t} = \frac{b_{j,t}}{a_{j,t}} \quad (4)$$

where  $b_{j,t} \leq a_{j,t}$  stands for real bond holdings of a representative  $j$ -aged household. The gross nominal age-specific rates of return in household's budget constraint are thus given by

$$R_{j,t}^a = s_{j,t}R_{t-1} + (1 - s_{j,t})R_t^f \quad (5)$$

where  $R_t$  is the gross interest on bonds, which we assume to be risk-free in nominal terms (i.e. determined ex ante), and  $R_t^f$  is the gross return on real assets. The latter consists of the return on physical capital and dividends transferred from intermediate goods producers. We assume that these dividends are distributed across households proportionally to their claims on real financial assets  $a_{j,t} - b_{j,t}$ .

Rather than modeling age-specific portfolio choices by agents, we assume that real bond holdings by age  $b_{j,t}$  are constant over time, and set at values such that their steady state shares in household assets match the age profiles from the data.<sup>2</sup> It has to be stressed that, while this assumption makes the financial asset portfolio of households exogenous, it is innocuous for the types of model simulations that we perform in this paper, see section 4.4 for more discussion.

### 3.1.4 Demographics and aggregation

In our model, the relative size of cohorts is determined by mortality risk  $\omega_j$  and the growth rate of the number of youngest agents  $n$ , both of which are assumed to be exogenous. Then, the total number of living agents  $N_t$  is given by

$$N_t = \sum_{j=1}^J N_{j,t} \quad (6)$$

where the number of agents in each cohort evolves according to

$$N_{j+1,t+1} = (1 - \omega_j)N_{j,t} \quad (7)$$

Since we allow population growth in the steady state to differ from zero, the number of households within each cohort becomes nonstationary, and it is useful to define the size

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<sup>2</sup>Constant real bond holdings by cohort are consistent with our baseline assumption of their fixed total net supply, pinned down by the level of the public debt.

of cohorts relative to that of the youngest one (which are then time-invariant)

$$N_j^{rel} = \frac{N_{j,t}}{N_{1,t}} \quad (8)$$

This allows us to rewrite equations (6) and (7) in relative terms

$$N^{rel} = \sum_{j=1}^J N_j^{rel} \quad (9)$$

$$N_{j+1}^{rel} = \frac{(1 - \omega_j)N_j^{rel}}{1 + n} \quad (10)$$

where we omitted the time subscripts as both mortality risk and the growth rate of youngest agents are assumed to be constant.

The aggregate allocations over all living households can be then expressed in per capita terms as follows

$$c_t = \sum_{j=1}^J \frac{N_j^{rel} c_{j,t}}{N^{rel}} \quad (11)$$

$$h_t = \sum_{j=1}^J \frac{N_j^{rel} z_j h_{j,t}}{N^{rel}} \quad (12)$$

$$\chi_{t+1} = \sum_{j=1}^J \frac{N_j^{rel} \chi_{j+1,t+1}}{N^{rel}(1+n)} \quad (13)$$

$$a_{t+1} = \sum_{j=1}^J \frac{N_j^{rel} a_{j+1,t+1}}{N^{rel}(1+n)} \quad (14)$$

$$b_{t+1} = \sum_{j=1}^J \frac{N_j^{rel} b_{j+1,t+1}}{N^{rel}(1+n)} \quad (15)$$

$$beq_t = \sum_{j=1}^J \frac{[N_{j-1}^{rel} - N_j^{rel}(1+n)]}{N^{rel}(1+n)} \frac{R_{j,t}^a a_{j,t}}{\pi_t} \quad (16)$$

$$beq_t^x = \sum_{j=1}^J \frac{[N_{j-1}^{rel} - N_j^{rel}(1+n)]}{N^{rel}(1+n)} p_{\chi,t} \chi_{j,t} \quad (17)$$

### 3.2 Firms

There are four types of firms in our model economy – final good producers, intermediate goods producers, capital producers and investment funds. Except for intermediate goods producers, which are monopolistically competitive, all firms operate under perfect competition. Consistently with demographic processes in the household sector, the mass of each type of firms is tied to the size of population. Whenever firms generate period profits or

losses, they are treated as a component of gross return on real assets.<sup>3</sup>

### 3.2.1 Final goods producers

Final goods producers purchase intermediate inputs  $y_t(i)$ , where  $i$  indexes intermediate goods producers, and transform them into a homogeneous final good  $y_t$  according to the following CES aggregator

$$y_t = \left[ \frac{1}{N_t} \int_0^{N_t} y_t(i)^{\frac{1}{\mu}} di \right]^{\mu} \quad (18)$$

where  $\mu$  controls the degree of substitutability between intermediate inputs and can be interpreted as the steady state gross markup. The final good is then sold at price  $P_t$ .

### 3.2.2 Intermediate goods producers

Intermediate goods producers hire capital  $k_t$  and labor  $h_t$ , and produce differentiated output according to the following production function

$$y_t(i) = k_t(i)^{\alpha} h_t(i)^{1-\alpha} - \Phi \quad (19)$$

where  $\Phi$  is a fixed cost of production ensuring zero profits in the steady state. They face demand schedules implied by the solution to the final goods producers' maximization problem, and set their prices subject to the Calvo friction, with  $\theta$  representing the probability of not receiving the reoptimization signal, in which case prices are fully indexed to steady state inflation. Intermediate goods producers are risk neutral, i.e. they use the nominal risk-free rate  $R_t$  to discount expected future profit flows.

### 3.2.3 Capital producers

Capital producers purchase investment goods  $i_t$  at price  $P_t$  and combine them with existing undepreciated capital purchased at price  $Q_t$  to produce new capital, subject to flow investment adjustment costs. The resulting law of motion for aggregate capital per capita in the economy is

$$(1+n)k_{t+1} = (1-\delta)k_t + \left[ 1 - \frac{S_1}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t \quad (20)$$

where  $\delta$  is the capital depreciation rate and  $S_1 \geq 0$  captures the investment adjustment costs. The adjustment costs are zero in the steady state.

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<sup>3</sup>Final goods producers and investment funds earn zero profits every period while capital and intermediate goods producers generate zero profits on average. Cyclical fluctuations of profits are also small, i.e. they increase by only about 0.0045% of steady state output in response to a standard monetary easing of 20 bp. This feature is due to the presence of wage stickiness, in the absence of which fluctuations in profits would be an order of magnitude larger and countercyclical, as in the standard New Keynesian setup with flexible wages. Small movements of profits in our model also mean that their distribution across households does not matter quantitatively for the main results presented in the paper.

### 3.2.4 Investment funds

Investment funds intermediate nominal assets between borrowers and savers, and manage physical capital that they rent to intermediate goods producers at nominal rate  $R_t^k$ . Since investment funds are assumed to be risk-neutral, their portfolio choices imply equalization of ex ante rates of return on capital and bonds

$$\mathbb{E}_t \frac{R_{t+1}^k + (1 - \delta) Q_{t+1}}{Q_t} = R_t \quad (21)$$

Ex post returns earned by investment funds, including dividends from capital and intermediate goods producers, are transferred to households according to their age-specific portfolio composition described before.

## 3.3 Government

### 3.3.1 Fiscal authority

The fiscal authority collects taxes, finances government expenditures, and services nominal government debt  $b_t^g$ , subject to the budget constraint

$$(1 + n) b_{t+1}^g + \tau_t w_t h_t = \frac{R_{t-1}}{\pi_t} b_t^g + g_y y_t \quad (22)$$

where  $g_y$  is the constant share of GDP spent on government purchases. We assume that the fiscal rule stabilizes the government debt at a constant level  $\bar{b}$ , while the labor income tax adjusts to satisfy the constraint (22).<sup>4</sup>

### 3.3.2 Monetary authority

The monetary authority sets the nominal interest rate according to a Taylor-like rule

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left( \frac{y_t}{y_{t-1}} \right)^{\gamma_y} \right]^{1-\gamma_R} \exp(\varepsilon_t^R) \quad (23)$$

where the coefficients  $\gamma_R$ ,  $\gamma_\pi$  and  $\gamma_y$  control, respectively, the degree of interest rate smoothing, the response to deviations of inflation from the target, and the response to the output per-capita growth rate, while  $\varepsilon_t^R$  is a monetary policy innovation.

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<sup>4</sup>We have also experimented with a version of our model, in which the public debt is allowed to move in response to shocks, i.e. taxes only partially adjust to deviations of the government debt from the target. This setup is no longer compatible with our assumption of fixed bond holdings by cohorts, so we assumed that the additional public debt issuance (or repurchase) is absorbed by households proportionally to their steady state bond holdings. For reasonable parametrizations of such an alternative fiscal rule, this modification did not significantly affect any of our main results.

### 3.4 Market clearing conditions

The model is closed with a standard set of market clearing conditions. We assume that the per capita housing stock is fixed and the housing market clears

$$\chi_t = \chi \quad (24)$$

and the aggregate net supply of bonds is equal to the level of government debt

$$b_t = b_t^g \quad (25)$$

Equilibrium on the final goods market implies

$$y_t = c_t + i_t + \delta_\chi p_{\chi,t} \chi + g_y y_t \quad (26)$$

and the market clearing conditions for capital and labor can be written as

$$\frac{1}{N_t} \int_0^{N_t} k_t(i) di = k_t \quad (27)$$

$$\frac{1}{N_t} \int_0^{N_t} h_t(i) di = h_t \quad (28)$$

This allows us to write the aggregate production function as

$$y_t \Delta_t = k_t^\alpha h_t^{1-\alpha} - \Phi \quad (29)$$

where

$$\Delta_t \equiv \frac{1}{N_t} \int_0^{N_t} \left( \frac{P_t(i)}{P_t} \right)^{\mu/(1-\mu)} di \quad (30)$$

measures the price dispersion across intermediate goods.

## 4 Calibration

The model is calibrated for the euro area, using annual time frequency. The asset structure, both at the aggregate level and its variation over the life cycle, can be expected to play a key role in our analysis. Detailed information about the age profiles of various types of assets in the euro area can be derived from the Household Finance and Consumption Survey (HFCS). Since aggregate quantities from this survey do not line up with national account statistics (see e.g. Hammer, 2015), we proceed as follows. We calibrate the standard macroeconomic parameters and match the key aggregate steady state proportions, including those describing the aggregate asset structure, on the basis of national accounts data. The age profiles, on the other hand, are taken from the HFCS.

#### 4.1 Aggregate structural parameters and macroeconomic proportions

We calibrate the economy-wide structural parameters using standard values from the literature to match the key macroproportions, or using econometric estimates performed outside of the model. The chosen values are reported in Table 1.

The age-invariant discount factor of households is calibrated at 0.992 to get the average real interest rate of 0.8%, observed in the euro area over the years 1997-2012. We cut the sample at 2012 as this was the last year during which the ECB monetary policy was not constrained by the effective lower bound. The Frisch elasticity of labor supply is set to 0.5, which is a conventional value in the business cycle literature. We calibrate both the price and wage markups at a standard value of 1.2, see e.g. Coenen et al. (2008). The Calvo parameters governing the price and wage reoptimization probabilities are set such that they imply the average contract duration of 3 and 5 quarters under quarterly frequency, respectively, which translates into values of 0.19 and 0.41 after annualization. The degree of habit persistence in consumption is set to 0.32 and the slope of investment adjustment cost is calibrated at 4. Both values are well within the range of estimates reported in the DSGE literature.

We use the balance sheets of financial and nonfinancial assets<sup>5</sup> for the euro area countries to calibrate the steady state stock of housing, non-residential fixed capital and nominal assets. Our empirical counterpart of the first category is dwellings owned by households and its ratio to GDP is 1.4. The empirical measure of non-residential capital is total fixed assets in the economy less dwellings owned by households, and its ratio to GDP is 1.9. We use these two ratios to pin down the capital share in output (set to 0.3) and residential capital depreciation rate (set to 1.3%). By calibrating the physical capital depreciation rate at 15%, we are able to match the ratio of investment to GDP observed in the euro area. Since in our model non-sovereign nominal assets are in zero net supply, the stock of nominal assets equals the stock of government bonds. Two different approaches can be used to calibrate the steady state stock of sovereign bonds. Holdings of government securities in the hands of households based on EA financial national accounts amount to 50% of GDP. Debt securities issued by the general government (same source) net of government debt holdings of the rest of the world (based on the balance of payments data) are about 58% of GDP. While calibrating the model, we use the average of these numbers.

The parametrization of the monetary policy feedback rule is based on the econometric estimation of a log-linearized version of the monetary policy feedback rule (23), using euro area data over the period 1980-2012 from the AWM database, converted to annual frequency. Prior to estimation, all series are detrended using the Hodrick-Prescott filter with the smoothing parameter set to 100, which is a conventional value used for annual data. Subtracting trends from the data is aimed to abstract from the time variation in the natural interest rate, potential output, and the implicit inflation target.

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<sup>5</sup>Eurostat data codes: `nasa_10_f_bs` and `nama_10_nfa_bs`.

## 4.2 Demographic variables

We use Eurostat data for the euro area to construct the age-dependent mortality risk and the rate of growth of 20-year olds.<sup>6</sup> In our calibration we use averages for the time period 1999-2018. For years where mortality rates were not documented for the oldest cohorts, we employ exponential extrapolation. Since the population structure in the model's steady state is stationary by construction, which also means that the 20-year olds and total population grow at the same rate, we have set the rate of growth of the youngest cohorts to 0.1% annually, which corresponds to the average growth rate in total EA population. Figure 1 depicts the profiles of mortality rates and the implied stationary population structure.<sup>7</sup>

## 4.3 Life-cycle profiles

Finally, we use the HFCS data to extract the steady state age profiles for labor income, hours worked, housing and financial assets at the household level. We use the second wave of this survey, conducted in 18 euro area countries between 2012 and 2014. See Appendix A.1 for more detailed definitions and HFCS codes, and Jablonowski (2018) for the method used to extract the age profiles.

Labor income is the sum of wage employment and self-employment, while hours worked are defined as time spent working at the main job. Housing is the sum of the household's main residence and other real estate property not for business activities. Real financial assets consist of household's business wealth, value of non-self-employment private business, publicly traded shares, as well as 50% of mutual fund holdings and 50% of voluntary pension and life insurance contracts. Nominal financial assets equal the value of deposits net of mortgage and non-mortgage debt, bonds and the remaining 50% of both mutual and voluntary pension funds.<sup>8</sup>

The age of the household is determined by the age of the household head. Since our model does not explicitly account for changes in the household composition or the family size, the extracted average age profiles are next divided by the square root of the number of household members, which is one of the equivalence scales used while working with household level data, see Fernandez-Villaverde and Krueger (2007) and OECD (2008). The thus obtained profiles are next smoothed using fourth-order polynomials and extrapolated for age groups not represented in the HFCS, subject to the model-consistent

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<sup>6</sup>Population data and age-specific death rates come from the `demo_pjan` and `demo_mlifetable` series, respectively.

<sup>7</sup>We have also performed simulations where the population was not stationary, but rather reflected actual population structure in a given year. The impulse response functions to a monetary shock were virtually identical.

<sup>8</sup>The 50-50 split of mutual and voluntary pension funds between nominal and real assets is supported by Table 1 in Euro area investment fund statistics for January 2014, where the amount invested by bond funds equals 2498 billions EUR and the sum of amounts invested by equity, real estate and hedge funds was 2516 billions EUR.

assumption that assets of the youngest and oldest cohort are zero.

While calibrating the model, we choose to match the steady state age profiles for labor income and hours worked exactly to their empirical counterparts. We achieve this by setting the age-specific productivity parameters  $z_j$  to the ratio of labor income to hours worked from the empirical profiles. Similarly, the age-specific weights on labor disutility  $\phi_j$  are chosen such that, in the steady state, the model-implied hours worked match exactly the empirical age profile for this variable. Rather than matching the age profile for housing exactly using housing weights in utility  $\psi_j$ , we assume that they increase linearly until retirement, and then stay flat. As we show below, choosing appropriately the intercept and slope over the pre-retirement period for this set of parameters is sufficient to obtain a very good fit of the life cycle housing profile.

Given other structural parameters, the age profile of total (real and nominal) financial assets is generated endogenously in the model, but its composition between nominal and real assets varies exogenously with age. We match it using the HFCS profiles for nominal assets. This is obtained by adjusting loans and deposits so that their net value is zero and the remaining nominal assets categories, representing government bonds in our model, to equal 54% of GDP. While making these adjustments we additionally impose that all debt is nominal and, as we do not have observations for households older than 70 from the HFCS, that after the age of 80 all financial assets are held in form of nominal assets. This seems to be a valid assumption as real asset holdings in the HFCS data decline sharply after the age of 60, by approximately 50% over the next 10 years.

Figure 2 presents the profiles which we match exactly. The profile of labor productivity follows the well-documented pattern, increasing up to the late middle age of the household head, and then declining. The profile of hours worked is almost flat, with some increase at a young age and a drop close to retirement. Nominal assets are negative for young cohorts, who finance their consumption and accumulation of housing by borrowing from older agents (who own positive nominal assets).

Figure 3 shows two key asset profiles that our model generates endogenously. The model reproduces very well the increasing pattern of housing and then its gradual decumulation. It is also quite successful at matching total net financial assets. In the HFCS data, households are net debtors until the age of 34, after which they accumulate assets until about retirement, and then start running them down. The model captures the timing of these episodes in asset accumulation very well, although the peak net financial asset position is somewhat higher than in the (smoothed) data.

#### 4.4 Solution method

We simulate the model in its non-linear form. More precisely, we use the extended path method developed by Fair and Taylor (1983), replacing the conditional expectations showing up in the model equilibrium conditions with their conditional means obtained from a deterministic simulation. The method hence ignores the effect of uncertainty on agents'

decisions in a way similar to standard first-order perturbation techniques that rely on the certainty equivalence assumption.<sup>9</sup> As in the case of linearized DSGE models, this solution feature does not allow for solving a portfolio problem within the model, hence justifying our strategy to fix the composition of financial assets within a cohort exogenously, as explained in section 3.1.3.

#### 4.5 Aggregate evidence from high-frequency identification

Before discussing the model simulations, we present some empirical evidence on how monetary policy affects key macroeconomic variables in the euro area. While it is well known that standard DSGE models can match these features of the data, it is far from obvious that an overlapping generations model like ours is capable of doing the same. To this end, we construct a monetary VAR model and apply a structural decomposition in the spirit of Gürkaynak et al. (2005), Mertens and Ravn (2012) and Gertler and Karadi (2015), based on high frequency identification of monetary policy shocks to simulate relevant impulse responses. This approach uses surprises to financial asset prices evaluated in small time windows around announcements of monetary policy decisions as exogenous instruments that allow to identify monetary policy shocks. To that purpose, we use two factors calculated by Altavilla et al. (2019) that are related to conventional policy surprises (called respectively Target and Timing factors in their paper). Unfortunately, the factors are available only since 2002, which is too short a period to estimate a quarterly VAR model. For this reason, we proceed as Hafemann and Tillmann (2017) and estimate the model on monthly data. In line with most standard information criteria, we allow for four lags. The obtained monthly impulse responses are transformed to annual figures to facilitate comparison with our model that operates at the latter frequency. The details of the data preparation and structural identification are presented in Appendix A.2.

Figure 4 presents the impulse responses of inflation and GDP growth to a typical expansionary monetary policy shock identified in the VAR model. In line with most of the existing literature on monetary transmission, such a shock generates an increase in output and inflation. Since a standard monetary surprise can be mapped into our structural model in a fairly straightforward way, we also use this shock to validate it. More specifically, the model-based responses are generated using the unexpected component in the Taylor rule (23), and setting the shock size such that it generates the same interest rate response on impact as the VAR. The reactions implied by our model are very well aligned with the VAR evidence, giving us confidence that it is consistent with aggregate empirical evidence on conventional monetary transmission.

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<sup>9</sup>We have also experimented with a stochastic version of the extended path method, using Gaussian quadrature to take into account the effects of future uncertainty one period ahead. The results turned out to be very similar to those presented below.

## 5 Model simulations

We are now ready to use our model to address several important questions about the distributional consequences of monetary policy across generations. We start by showing the impulse responses of key macroeconomic aggregates, including asset prices and labor market variables, to an unexpected monetary shock. We next present how the net worth of households of different age is affected by the shock on impact. Then we discuss its effect for welfare redistribution between the living cohorts. Finally, we show how the allocations of consumption and housing of various cohorts are affected by the shock.

### 5.1 Aggregate effects

To understand the effects of monetary policy across different age cohorts, we need two key ingredients, i.e. the life-cycle profiles for income and asset positions, which we have already discussed in Section 4, and the reaction of key macroaggregates to policy shocks, which we present now in Figure 5.

Consistently with standard DSGE models, a monetary expansion leads to a boom in aggregate output, which supports an increase in labor income, and hence allows for higher aggregate consumption spending. This eventually leads to an increase in inflation, while increased demand for housing generates an increase in real house prices.

For the redistributive outcomes that we will discuss later, it is important to comment on the differences between the reactions of returns on the two types of financial assets. Note that these differences can be observed only in the first period since, absent further surprises, the returns on all financial assets are the same as certainty equivalence holds in our model. The response of the return on non-housing capital is strongly positive on impact, reflecting an appreciation of stock prices, while the real return on nominal assets falls due to surprise inflation. Since the monetary policy easing persistently lowers the real interest rate, the real rate of return on financial assets stays significantly below its steady state level also in the next three years.

### 5.2 Income and wealth redistribution on impact

We now turn to the main topic of this paper, which is the redistributive effects of monetary policy across the age cohorts. To understand the effects associated with changes in asset prices described above, it is useful to recall their distribution over the life cycle. We summarize it in Figure 6, which combines the age profiles for the three types of assets that we include in our model, and which we plotted in Figures 2 and 3.

It is instructive to start with how monetary policy affects households' wealth and income on impact, and we present these effects in Figure 7. For comparability with welfare calculations presented in the next subsection, all gains or losses are expressed in units of marginal utility of consumption. Given the profile of housing over the lifecycle and reaction

of house prices discussed before, the outcomes for housing wealth are straightforward. The gains follow a piecewise linear shape that increases until retirement and then stabilizes, and are positive for all cohorts as each of them holds some housing. The gain related to financial wealth is the sum of two effects. First of all, agents face a redistribution associated with nominal asset holdings that has been well documented in the literature, and which arises from unexpected inflation. Given our profiles for nominal assets, it benefits households aged 48 or less, with most gains accruing to households in their 30s as they are most heavily indebted, while the remaining cohorts lose. Second, we observe gains associated with higher returns on real assets, the holdings of which rise sharply until around the age of retirement, and then decline.

Asset price revaluation is only a part of the story as monetary policy has also indirect effects, operating mainly through the labor market. An expansion in economic activity increases employment and also (with a lag due to nominal wage rigidity) real wages, which benefits the working population proportionally to their working time and productivity. As Figure 7 reveals, this effect is sizable, especially for younger individuals, even when we consider only the first period reaction to monetary shocks. Overall, putting all these effects together shows that a monetary policy easing boosts resources available for spending for all age groups that we consider in our analysis, with most gains accruing to households aged around 60 as their housing and real assets positions, as well as labor income, peak around this age.

### 5.3 Welfare over the remaining lifetime

Naturally, the analysis presented above does not give us the full answer on who really gains and loses from a monetary policy easing. As a matter of fact, this simple reasoning can be even misleading. One obvious reason is that monetary shocks have persistent effects on the economy, especially on labor income and (to a lesser extent) on real returns on assets. Another very important, though often neglected argument is related to asset prices in the presence of life cycle decisions. Whether and to what extent an individual benefits from an increase in asset prices depends not only on the current net exposition, but also on whether he/she is in the process of accumulating or decumulating this particular type of assets. For example, a household with a lot of housing might not necessarily gain from house price appreciation if it is still accumulating this asset due to life cycle motives, because the planned increase in housing becomes more costly. A similar argument applies to accumulation of financial assets before retirement and their decumulation during the remaining lifetime.<sup>10</sup> Last but not least, labor market expansion has also some negative effects on households as it increases labor disutility.

In a microfounded model like ours, we have a natural measure that captures all aspects

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<sup>10</sup>This kind of considerations are related to a debate on whether and when housing is net wealth, see e.g. Buiters (2010). A recent formalization of these arguments, though not directly referring to life cycle choices, can be found in Auclert (2019).

relevant to an individual household, namely expected utility defined by equation (1). In what follows we present a decomposition of this measure that allows us to calculate the welfare effect of a typical monetary policy expansion via its impact on aggregate quantities and prices. Using total differentiation of the indirect utility function implied by household's optimization problem and applying the envelope theorem (see Appendix A.3), we can decompose the first-order accurate impact of a monetary policy shock hitting the economy (that at time  $t = 0$  is in the steady state) on expected welfare over the remaining lifetime of cohort  $j$  into the following components<sup>11</sup>

$$\Delta U_j = \Gamma_j^x + \Gamma_j^b + \Gamma_j^f + \Gamma_j^l + \Gamma_j^t + \Gamma_j^h + \mathcal{O}(2) \quad (31)$$

where  $\mathcal{O}(2)$  collects terms of order two or higher and the other symbols capture, respectively, the effects of changes in house prices, returns on nominal assets, returns on real assets, labor market conditions, bequests, and external habit formation. The expressions describing the impact via these channels are

$$\Gamma_j^x = \mathbb{E}_0 u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} [(1-\delta_\chi)\chi_{j+s} - \chi_{j+s+1}](p_{\chi,s} - p_\chi) \quad (32)$$

$$\Gamma_j^b = \mathbb{E}_0 u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} s_{j+s} a_{j+s} \left( \frac{R_{s-1}}{\pi_s} - (1+r) \right) \quad (33)$$

$$\Gamma_j^f = \mathbb{E}_0 u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} (1-s_{j+s}) a_{j+s} \left( \frac{R_s^f}{\pi_s} - (1+r) \right) \quad (34)$$

$$\Gamma_j^l = \mathbb{E}_0 u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} z_{j+s} \left[ \begin{array}{l} (1-\tau)h_{j+s}(w_s - w) - wh_{j+s}(\tau_s - \tau) \\ + (\mu_w - 1)/\mu_w \cdot (1-\tau)w(h_{j+s,s} - h_{j+s}) \end{array} \right] \quad (35)$$

$$\Gamma_j^t = \mathbb{E}_0 u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} (beq_{j+s,s} - beq_{j+s} + beq_{j+s,s}^x - beq_{j+s}^x) \quad (36)$$

$$\Gamma_j^h = -\varrho \mathbb{E}_0 \sum_{s=1}^{J-j} \beta^s \frac{N_{j+s,t+s}}{N_{j,t}} u_{j+s}^c (c_{j+s,s-1} - c_{j+s}) \quad (37)$$

where  $u_j^c \equiv [(1-\varrho)c_j]^{-1}$  denotes the steady state marginal utility of consumption for cohort  $j$  and  $r \equiv \frac{R}{\pi} - 1$  is the steady state real interest rate.

The first five components are associated with the household budget constraint. They essentially describe how its individual components are affected by changes in prices or aggregate quantities that households take as given while solving their problem. Since aggregate effects of monetary shocks are persistent, each component is a discounted sum over the remaining lifetime of the household. Multiplying the effects by marginal utility converts them into utility units. Hence, their interpretation is the extra utility that an

<sup>11</sup>Note that this approach actually allows us to decompose the effects of any aggregate shock that hits the economy.

agent at given age derives from the (current and expected future) changes in prices or aggregates associated with a given income or non-consumption expenditure component. It is also easy to see that the welfare effect goes much beyond the pure income effect that occurs on impact and which we described in the previous section – it is the whole remaining lifetime path of relevant prices and aggregate allocations that matters. The final first-order component in (31) stems from the presence of external habits and its structure is different from that of the first five terms associated with the budget constraint as habits are a part of the utility function.

The resulting decomposition is summarized in Figure 8. The black line denotes the total impact of a standard expansionary monetary policy shock on welfare of each cohort. The bars denote the contribution of each component derived above. Below we discuss each one separately, ignoring only the role of bequests, which play a largely technical role in the model, and whose impact on welfare is not particularly large.

Let us begin with the house price component as it stands in a particularly sharp contrast with the on-impact effect, and hence has the biggest instructive potential. Unlike in Figure 7, the impact of changes in house prices is negative for households below 65 and only positive thereafter. This is because what matters for wealth is not the level of assets, but their accumulation pattern, which is in our model dictated by life cycle motives. Using the words of Auclert (2019), one should look not at the total, but only at maturing assets. Increasing house prices benefit agents only if they are in the process of running down their housing stock. Since agents aged 65 or less accumulate housing, their welfare suffers from real house price increases as they have to pay more for additional purchases of residential property.

The second component is associated with changes in returns on nominal assets, and is the most important one. Its contribution is positive for young households (until their mid-forties) and negative thereafter. Clearly, this pattern is related to the fall in the ex post real rate of return on nominal assets, especially on impact due to surprise inflation. The effect is thus positive for young (debtor) and negative for old (creditor) households. However, as is obvious from comparing with Figure 7, there must be something else at play. Youngest households (below 35) are in the process of lowering their nominal asset holdings as they increase their debt, middle-aged households accumulate them, while oldest households (over 70) decumulate again. Since these adjustments happen at a higher price (as the interest rate is persistently lower), the youngest and oldest benefit additionally at the expense of middle-aged agents. Note that this component of welfare effects is much more important than the one associated with house prices, because the life cycle profile of nominal assets is much steeper.

The next component is related to the return on real assets. Again, there are two competing forces. First, owners of real assets benefit from the high ex post rate of return on impact, which mainly reflects the increase in the price of capital, and this is the effect we have seen in Figure 7. However, simultaneously, households that want to follow their

life cycle pattern of real asset accumulation must do it by paying more for additional claims on capital. Consequently, being on the accumulation path of real assets lowers welfare. The net effect is hence negative for all households aged 54 and below. Only after this age do the initial capital gains outweigh the additional accumulation cost, and the gains are highest for capital-rich households in their mid-sixties, who start decumulating real assets.

Let us now move to the labor income component. Here the story is relatively simple as working age households benefit from the persistent macroeconomic expansion that follows a monetary easing, which drives gross wages up and allows the fiscal authority to decrease the labor income tax rate. The difference between the shape of this component and its on-impact counterpart from Figure 7 is mainly due to future expected gains. In particular, these decline for agents close to retirement, and hence they benefit less despite being relatively productive.<sup>12</sup>

Last but not least, let us turn our attention to the effect of external habits. Their character is somewhat different than that of the first five components, which were associated with the household budget constraint. Why do habits redistribute welfare after a monetary policy shock? The explanation is as follows. Habits in the utility function (1) capture the “keeping up with the Jones” motive – agents value consumption growth relative to other agents. In our model, the reference level is previous period consumption of the preceding cohort, i.e. of households who were at the same age a year ago. For example, this means that, if a monetary policy easing brings financial benefits (summarized by the first five components in equation 31) to a given cohort, but at the same time raises consumption of the reference group, the presence of habits makes the total welfare gain smaller. As we will show in the next section, the reaction of consumption to a monetary easing is positive for young and middle-aged households, and (except on impact) negative for older cohorts, and so is the sign of the contribution of habits to the change in the remaining lifetime welfare.

The net outcome of all these factors is denoted with the black line. Clearly, younger households (aged 50 or below) gain from a monetary easing, while older ones lose. Especially when contrasted with Figure 7, our estimates show the perils of relying on simplistic conclusions based on the initial impact on wealth or income. This is particularly true for the effect via housing wealth, but also for the aggregate picture, where everybody seems to be a winner if only the initial price adjustments are taken into account.

It is worth noting that our picture squares very nicely with the evidence based on survey data in the UK and presented in Bunn et al. (2020), that old households were less satisfied with the monetary expansion which followed the financial crisis than the young ones.

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<sup>12</sup>Apart from net wages, the labor income effect  $\Gamma_j^l$  also includes an aggregate labor externality. Its presence is related to monopolistic competition on the labor market, which means that employment in the steady state is inefficiently low, and hence an increase in this variable is welfare improving.

## 5.4 Impact on consumption and housing

In reaction to the direct effects of monetary shocks associated with asset prices, and indirect ones coming from general equilibrium reactions of other prices, and wages in particular, households modify their allocations.

Figure 9 shows how selected cohorts adjust their consumption and housing following a negative 20 bp interest rate shock. Regarding consumption, it increases on impact for all cohorts, but very soon declines for oldest agents. As to housing, the impact is negative from the very beginning for older cohorts, while young agents increase their housing stock.

The reactions are generally consistent with the estimated welfare effects described in the previous section. In particular, following a monetary policy easing, younger and middle-aged households feel wealthier, and so can increase consumption and accumulate more housing. Older generations feel poorer and so, in general cut down on their consumption plans and decumulate housing. The only exception from this rule is the initial increase of their consumption. This follows the short-term income boost from selling houses at elevated prices. Since old households have relatively low effective discount rates (due to higher mortality), part of the proceeds are consumed immediately..

## 6 Conclusions

In this paper we investigate the redistributive effects of monetary policy in an overlapping generations model with real and nominal frictions. We calibrate the model economy to match the salient features of the euro area and pay considerable attention to replicate the asset structure of European households.

Our contribution to the literature is twofold. First, to our knowledge, this is the first paper to investigate the redistributive effects of monetary policy across generations in a structural, fully-fledged life-cycle model where, depending on their current age, agents differ significantly in net worth and composition of asset portfolios. This gives rise to a natural quantitative environment where monetary policy affects households through multiple channels.

Second, our findings document the key practical importance of the results obtained by Auclert (2019), who stresses the role of *maturing* assets and liabilities in household portfolios. We find that the life-cycle paths of consumption and asset accumulation decisions are important determinants of the relevant asset maturity structure. We assess the redistributive effects of monetary policy over the remaining lifetime of agents, and decompose them into selected sources. As a consequence, we can for instance show that the welfare effects of house price appreciation that follows a monetary expansion are in fact negative for the majority of population. This stands in stark contrast to the analysis that ignores life-cycle motives, which rather suggests that every age cohort gains from a monetary stimulus. We also document the key role of changes in nominal financial asset returns and labor market flows for welfare redistribution. The net impact of all these

effects is a wealth transfer from older to younger generations after a monetary easing.

Admittedly, our account of the effects of conventional monetary policy shocks has its limits. Following the 2007-2009 financial crisis, central banks worldwide have employed unconventional policy tools, including forward guidance and various asset purchase programs. Considering such measures would require introducing nominal assets with different maturities and adding further frictions limiting the arbitrage along the yield curve, like e.g. in Chen et al. (2012). We leave such an extension for future research.

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## Tables and figures

Table 1: Calibrated structural parameters

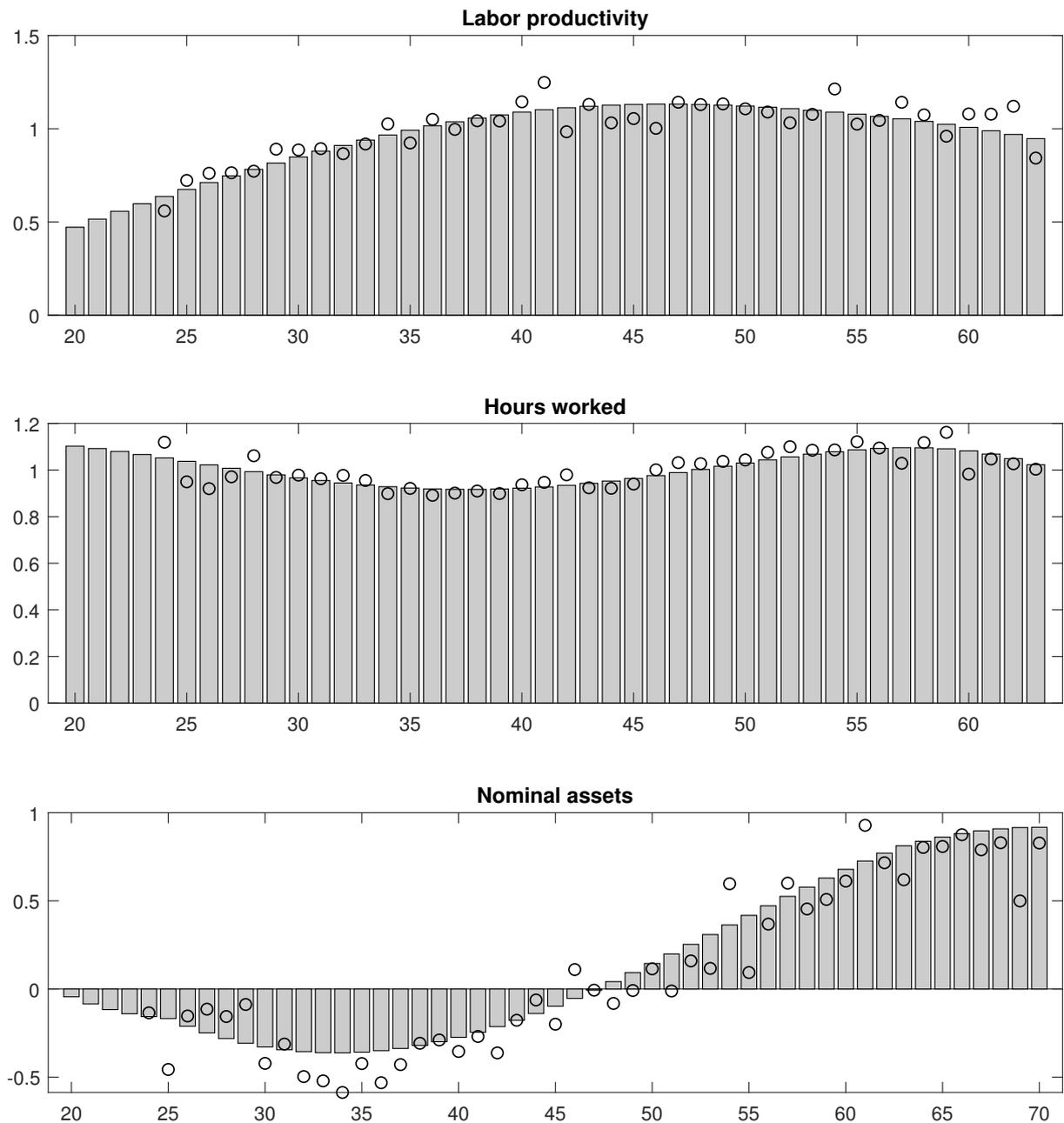
Parameter	Value	Description
$\beta$	0.992	Discount factor
$\varphi^{-1}$	0.5	Frisch elasticity of labor supply
$\rho$	0.32	Habit persistence
$\delta_x$	0.013	Housing depreciation rate
$\delta$	0.15	Capital depreciation rate
$\alpha$	0.3	Capital share in output
$S_1$	4	Investment adjustment cost curvature
$\mu$	1.2	Steady state product markup
$\theta$	0.19	Calvo probability (prices)
$\Phi$	0.04	Intermediate goods producers fixed cost
$\mu_w$	1.2	Steady state wage markup
$\theta_w$	0.41	Calvo probability (wages)
$g_y$	0.2	Share of government purchases in GDP
$b^g/y$	0.54	Steady state government bonds to GDP ratio
$\pi$	1.02	Inflation target
$\gamma_R$	0.41	Interest rate smoothing
$\gamma_\pi$	1.97	Reaction to inflation
$\gamma_y$	0.42	Reaction to GDP growth

Figure 1: Population structure and mortality risk by age



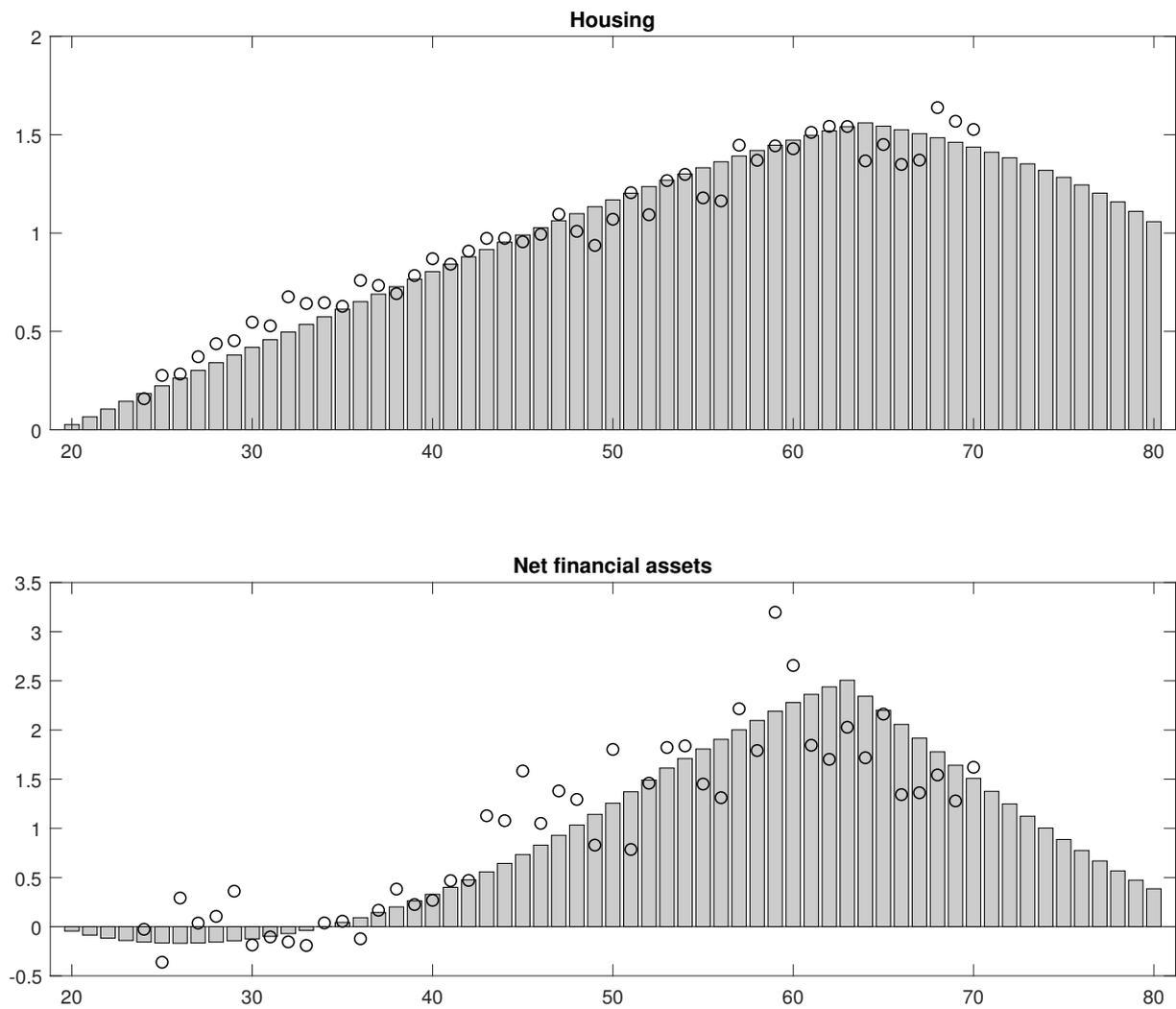
Note: Mortality risk is calculated as the death probability, taken from the Eurostat and averaged over the period 1999-2018. The population structure is the stationary distribution given constant mortality rates and annual growth of total population equal to 0.1%.

Figure 2: Age profiles imposed exogenously



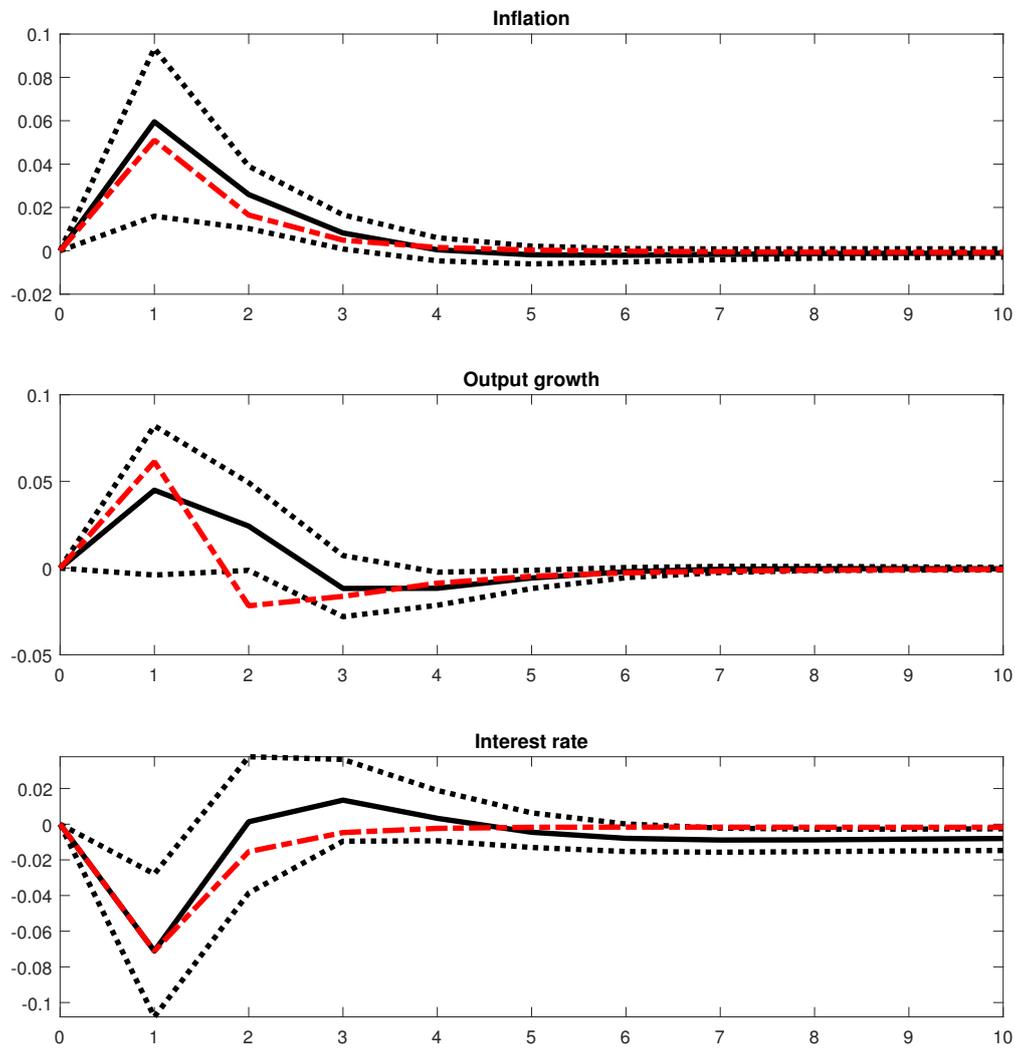
Note: Dots denote raw data adjusted by household size and aggregated by cohorts, bars present the smoothed profiles which are used in the structural model. The age profiles for labor productivity and hours worked are normalized by their average values over the working age. The nominal assets are expressed relative to the mean of the total net financial assets position.

Figure 3: Age profiles for housing and net financial assets



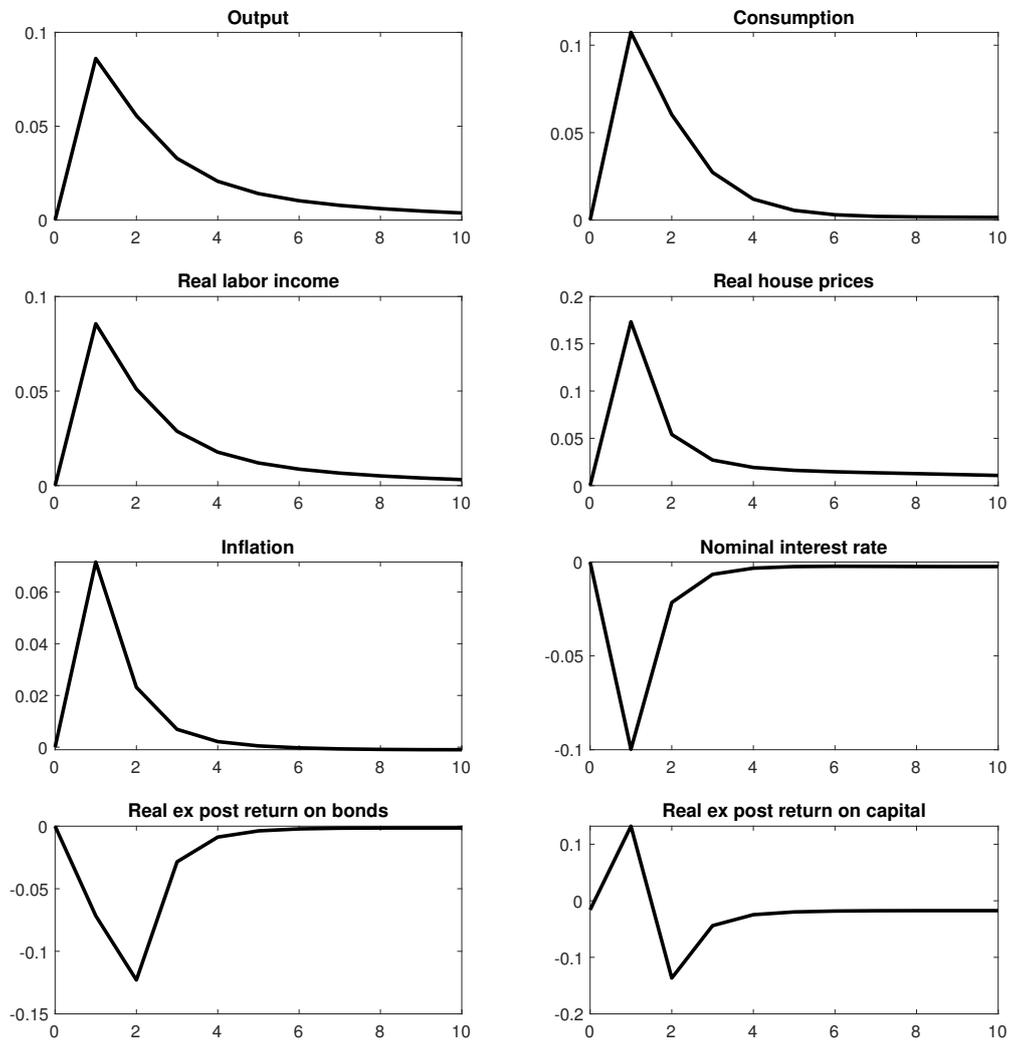
Note: The figure compares the age profiles from the HFCS data (dots) to those implied by the baseline model (bars). All age profiles are expressed relative to their mean values over the life cycle. Raw data were adjusted for household size and aggregated by cohorts.

Figure 4: VAR evidence on monetary transmission



Note: The mean annual VAR impulse responses are presented as black solid lines, dotted lines denote  $\pm$  one standard deviation based on 25000 bootstrap repetitions. Red dashed lines plot the responses from the life-cycle model. All variables are in percent.

Figure 5: Aggregate responses to a monetary expansion



Note: The figure shows the reactions to an expansionary monetary policy shock of 20 basis points, obtained with the structural model. All variables are in percent deviations from the steady state.

Figure 6: Model-implied age profiles of assets

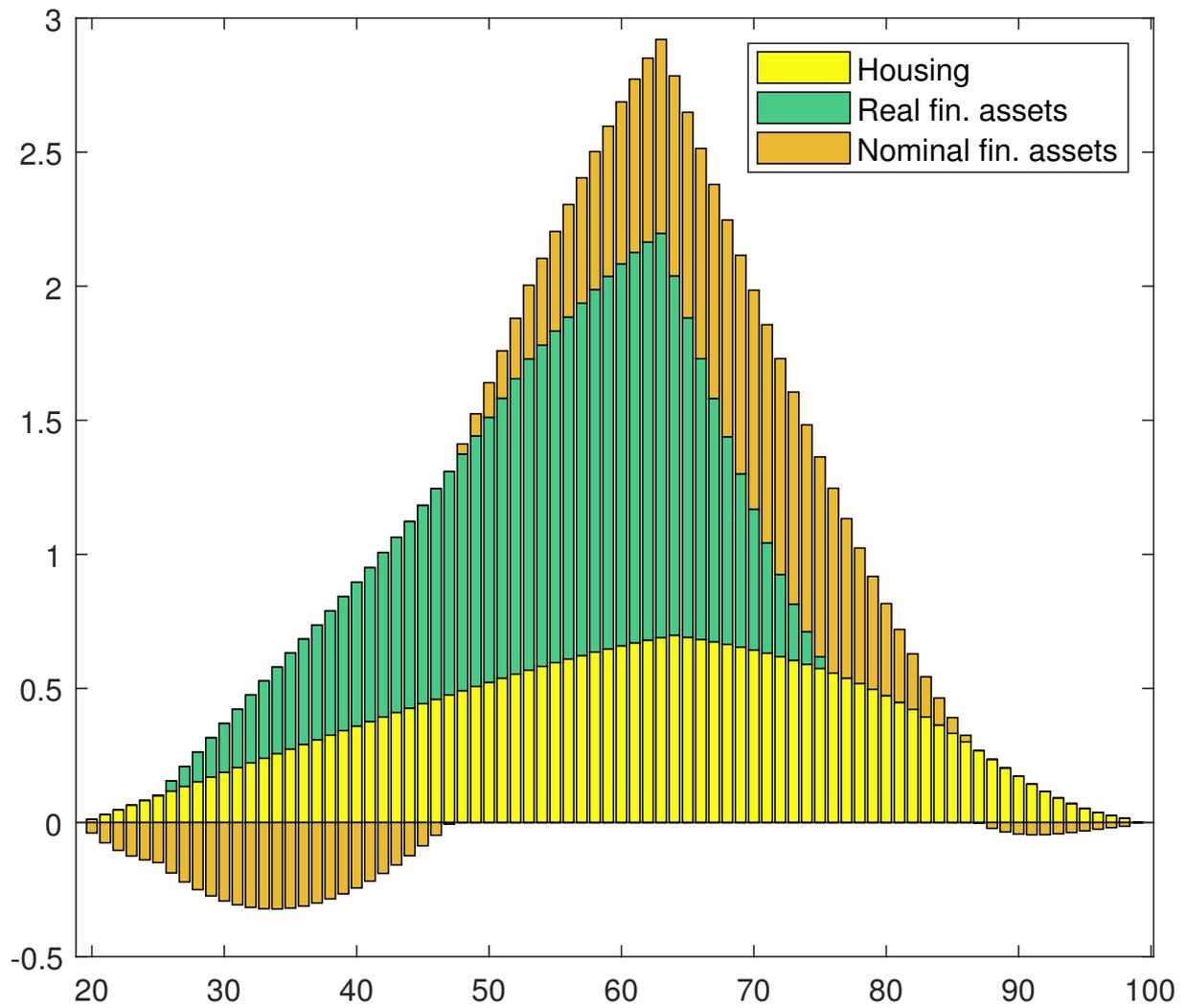


Figure 7: Redistributive effects of monetary policy on impact

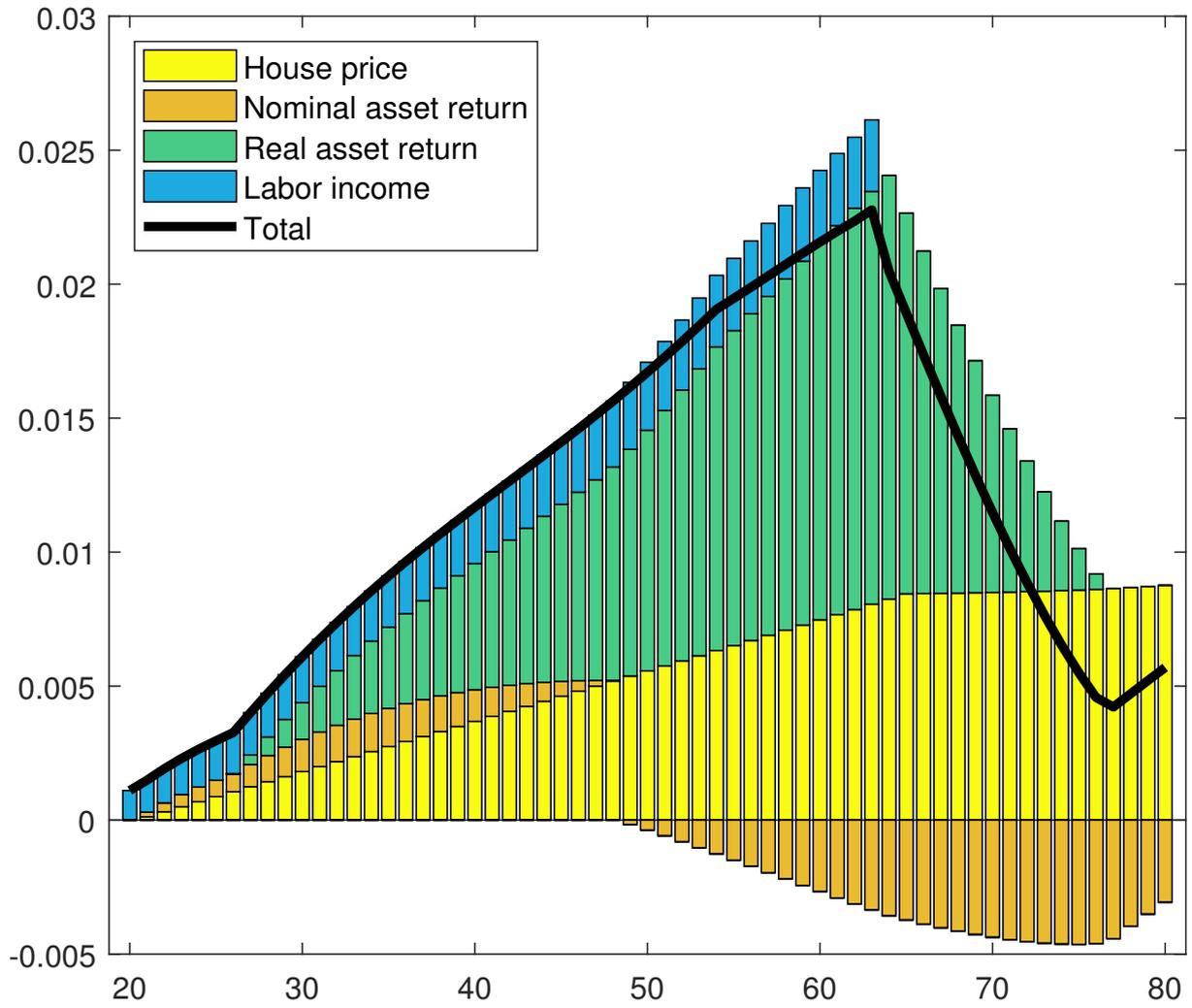


Figure 8: Welfare effects of monetary policy

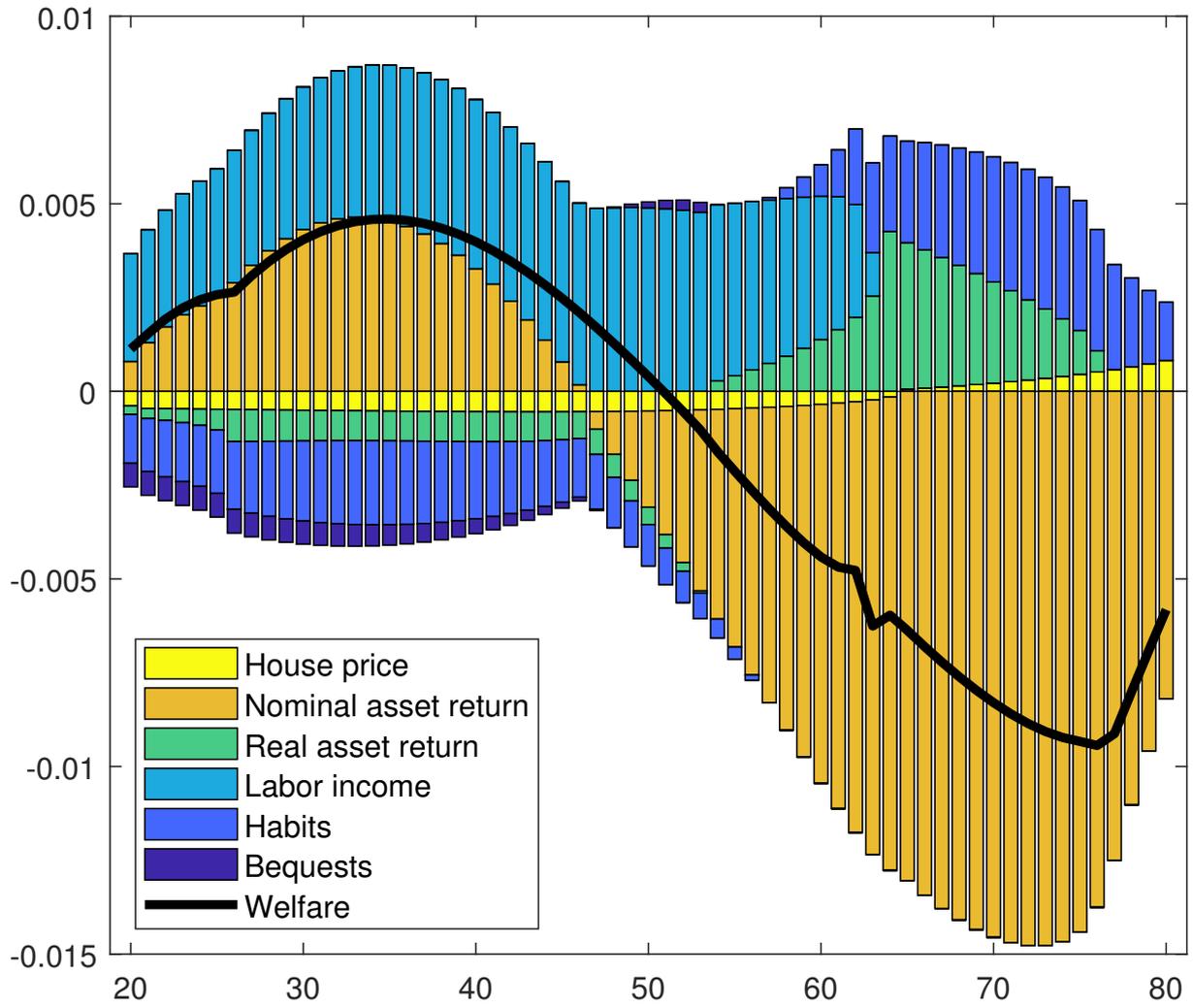
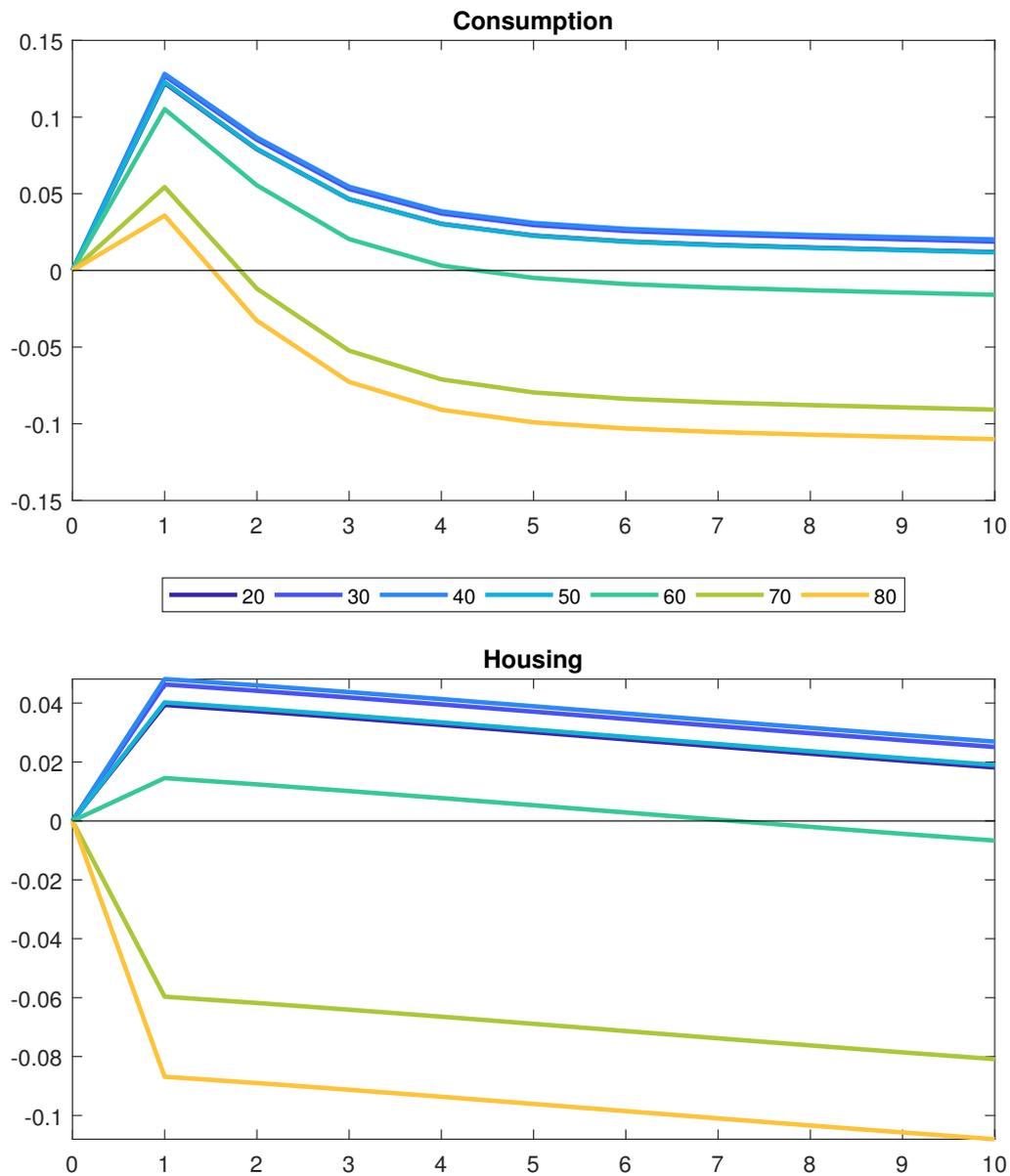


Figure 9: Impulse responses of consumption and housing to monetary shocks by cohort



Note: The responses are presented in percent deviations from the steady state and are drawn for selected cohorts (20, 30, 40, 50, 60, 70 and 80 years old).

## Appendix

### A.1 HFCS definitions

The table below maps our definitions of the life-cycle variables showing up in our structural model to the categories and their codes from the second wave of the Household Finance and Consumption Survey.

Category in the paper	HFCS name	HFCS code
Labor income	= Employee income	DI1100
	+ Self-employment income	DI1200
Hours worked	= Hours working a week - main job	PE0600
Housing stock	= Value of household's main residence	DA1110
	+ Value of other real estate property not for business activities	DA1122
Real financial assets	= Business wealth	DA1200
	+ Value of non self-employment private business	DA2104
	+ Shares, publicly traded	DA2105
	+ 50% Mutual funds	DA2102
	+ 50% Voluntary pension/whole life insurance	DA2109
Nominal financial assets	= Deposits	DA2101
	- Outstanding balance of mortgage debt	DL1100
	- Outstanding balance of other, non-mortgage debt	DL1200
	+ Bonds	DA2103
	+ 50% Mutual funds	DA2102
+ 50% Voluntary pension/whole life insurance	DA2109	

### A.2 VAR model and the structural decomposition

#### A.2.1 Data

The VAR model that we use in section 4.5 is estimated with monthly euro area data for the period Jan 2002 - Sep 2018. The model consists of three endogenous variables (source Eurostat): the log of the harmonized index of consumer prices (HICP), log of GDP, and the 3-month money market interest rate.<sup>13</sup> To identify monetary shocks, we also include

<sup>13</sup>Given potential estimation issues related to the presence of the lower bound on interest rates, we have also tried various government bond rates instead of the money market rate, but the results remained similar.

two exogenous instruments, i.e. target and timing factors from Altavilla et al. (2019). Finally, the model also contains the log of oil prices as an exogenous variable to alleviate the price puzzle.

Regarding the output variable, it is common to use industrial production in monetary VAR models estimated on monthly frequency. However, our structural model does not have such a variable. Therefore, to allow for a comparison, we generate GDP data of monthly frequency by interpolating quarterly GDP series using industrial production as an indicator variable in the Chow and Lin (1971) procedure.

### A.2.2 Identification of shocks

Our implementation of the high-frequency identification approach follows Ouliaris et al. (2018). Let  $\mathbf{y}_t = [p_t, gdp_t, i_t, target_t, timing_t]'$  be the vector of endogenous variables and the surprise factors. The variables denote: HICP, GDP, the interest rate, and the target and the timing factors. Our SVAR model can be written in matrix notation as follows

$$\mathbf{A}\mathbf{y}_t = \mathbf{B}(L)\mathbf{L}\mathbf{y}_t + \mathbf{C}\mathbf{x}_t + \mathbf{D}\boldsymbol{\varepsilon}_t \quad (\text{A.1})$$

where  $\boldsymbol{\varepsilon}_t = [\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t}, \varepsilon_{4,t}, \varepsilon_{5,t}]'$  is a vector of shocks,  $\mathbf{x}_t = [1, oil_t]'$  a vector of exogenous variables (constant, oil price) and  $L$  denotes the lag operator. Matrix  $\mathbf{A}$  describes the contemporaneous interactions between variables, matrices  $\mathbf{B}(L)$  the impact of lagged variables,  $\mathbf{C}$  the impact of exogenous variables, and  $\mathbf{D}$  the impact of disturbances. Identification via external instruments relies on instruments correlated with the identified shock and uncorrelated with all other shocks. In our case this implies  $E(target_t \varepsilon_{3,t}) \neq 0$ ,  $E(timing_t \varepsilon_{3,t}) \neq 0$ ,  $E(target_t \varepsilon_{i,t}) = 0$ ,  $E(timing_t \varepsilon_{i,t}) = 0$  for  $i = 1, 2$ . In contrast to the Cholesky identification scheme we allow for the policy variable to affect contemporaneously prices and GDP. Hence we have

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & a_{1,3} & 0 & 0 \\ a_{2,1} & 1 & a_{2,3} & 0 & 0 \\ a_{3,1} & a_{3,2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{A.2})$$

and

$$\mathbf{D} = \begin{bmatrix} \sigma_{1,1} & 0 & 0 & 0 & 0 \\ 0 & \sigma_{2,2} & 0 & 0 & 0 \\ 0 & 0 & \sigma_{3,3} & 0 & 0 \\ 0 & 0 & \sigma_{4,3} & \sigma_{4,4} & 0 \\ 0 & 0 & \sigma_{5,3} & 0 & \sigma_{5,5} \end{bmatrix} \quad (\text{A.3})$$

The oil price is assumed not to affect the monetary policy factors (the last two rows

of matrix  $\mathbf{C}$  contain zeros) and the factors are assumed to be correlated neither over time nor with other variables (the last rows and columns of matrices  $\mathbf{B}(\mathbf{L})$  contain zeros). The system is estimated via maximum likelihood.

### A.3 Decomposition of welfare effects over remaining lifetime

In this appendix we derive the decomposition of welfare effects presented in section 5.3.

Let us start by formally defining an indirect utility function of a  $j$ -aged household  $\iota$ , using the description of the household problem from section 3.1, as follows

$$\mathcal{W}_{j,t}(\iota) = \max \mathbb{E}_t \sum_{s=0}^{J-j} \beta^s \frac{N_{j+s,t+s}}{N_{j,t}} \left[ \begin{array}{l} \log(c_{j+s,t+s} - \varrho \bar{c}_{j+s,t+s-1}) + \\ + \psi_{j+s} \log \chi_{j+s+1,t+s+1} - \phi_{j+s} \frac{h_{j+s,t+s}(\iota)^{1+\varphi}}{1+\varphi} \end{array} \right] \quad (\text{A.4})$$

subject to

$$\begin{aligned} c_{j,t} + p_{\chi,t} [\chi_{j+1,t+1} - (1 - \delta_\chi) \chi_{j,t}] + a_{j+1,t+1} = \\ = (1 - \tau_t) w_t z_j h_{j,t}(\iota)^{\frac{1}{\mu_w}} h_{j,t}^{\frac{\mu_w-1}{\mu_w}} + \left( s_{j,t} \frac{R_{t-1}}{\pi_t} + (1 - s_{j,t}) \frac{R_t^f}{\pi_t} \right) a_{j,t} + beq_{j,t} + beq_{j,t}^\chi + \Xi_{j,t}(\iota) \end{aligned} \quad (\text{A.5})$$

where we used the equilibrium demand condition for individual labor variety implied by aggregation (3) to eliminate  $w_t(\iota)$  in the budget constraint (2). Indirect utility (A.4) is a function of (the sequences of) house prices  $p_{\chi,t}$ , return on nominal assets  $R_{t-1}/\pi_t$ , return on real assets  $R_t^f/\pi_t$ , aggregate wage rate  $w_t$ , labor income tax rate  $\tau_t$ , cohort-specific bequests  $beq_{j,t}$  and  $beq_{j,t}^\chi$ , as well as cohort-specific labor  $h_{j,t}$  and consumption  $\bar{c}_{j,t-1}$ . Note that the presence of  $h_{j,t}$  is due to imperfect competition on the labor market and it disappears when  $\mu_w \rightarrow 1$ , while the presence of  $\bar{c}_{j,t-1}$  reflects the external nature of consumption habits.

Suppose that at time  $t = 0$  the economy is hit by a shock. Then, up to first order of approximation, the effects of this shock on household welfare can be decomposed into the contributions of the above-listed nine arguments of indirect utility using its total derivative

$$d\mathcal{W}_{j,0}(\iota) = \mathbb{E}_0 \sum_{s=0}^{J-j} \frac{\partial \mathcal{W}_{j,0}(\iota)}{\partial p_{\chi,s}} dp_{\chi,s} + \dots \quad (\text{A.6})$$

Let us denote the Lagrange multiplier on the budget constraint of cohort  $j$  as  $\lambda_{j,t}$ , total lump sum transfers to households as  $tr_{j,t}(\iota) = beq_{j,t} + beq_{j,t}^\chi + \Xi_{j,t}(\iota)$ , the ex post real rates of return on nominal and real assets  $r_t \equiv R_{t-1}/\pi_t - 1$  and  $r_t^f \equiv R_t^f/\pi_t - 1$ , and the  $s$ -years ahead survival rate  $\omega_{j,t+1}^s \equiv \frac{N_{j+s,t+s}}{N_{j,t}}$ . Then we can work out all the individual components in (A.6), using the envelope theorem and taking the non-stochastic steady

state as the approximation point, as follows

$$\begin{aligned} \sum_{s=0}^{J-j} \frac{\partial \mathcal{W}_{j,0}(\iota)}{\partial p_{\chi,s}} dp_{\chi,s} &= - \sum_{s=0}^{J-j} \lambda_{j+s} [\chi_{j+s+1} - (1 - \delta_{\chi}) \chi_{j+s}] dp_{\chi,s} \\ &= - \sum_{s=0}^{J-j} \beta^s \omega_j^s u_{j+s}^c [\chi_{j+s+1} - (1 - \delta_{\chi}) \chi_{j+s}] dp_{\chi,s} \\ &= u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} [(1 - \delta_{\chi}) \chi_{j+s} - \chi_{j+s+1}] dp_{\chi,s} \end{aligned} \quad (\text{A.7})$$

$$\sum_{s=0}^{J-j} \frac{\partial \mathcal{W}_{j,0}(\iota)}{\partial r_s} dr_s = u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} s_{j+s} a_{j+s} dr_s \quad (\text{A.8})$$

$$\sum_{s=0}^{J-j} \frac{\partial \mathcal{W}_{j,0}(\iota)}{\partial r_s^f} dr_s^f = u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} (1 - s_{j+s}) a_{j+s} dr_s^f \quad (\text{A.9})$$

$$\sum_{s=0}^{J-j} \frac{\partial \mathcal{W}_{j,0}(\iota)}{\partial w_s} dw_s = u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} (1 - \tau) z_{j+s} h_{j+s}(\iota)^{\frac{1}{\mu_w}} h_{j+s}^{\frac{\mu_w - 1}{\mu_w}} dw_s \quad (\text{A.10})$$

$$\sum_{s=0}^{J-j} \frac{\partial \mathcal{W}_{j,0}(\iota)}{\partial h_{j+s,s}} dh_{j+s,s} = \frac{\mu_w - 1}{\mu_w} u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} (1 - \tau) w z_{j+s} h_{j+s}(\iota)^{\frac{1}{\mu_w}} h_{j+s}^{\frac{\mu_w - 1}{\mu_w}} dh_{j+s,s} \quad (\text{A.11})$$

$$\sum_{s=0}^{J-j} \frac{\partial \mathcal{W}_{j,0}(\iota)}{\partial \tau_s} d\tau_s = -u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} w z_{j+s} h_{j+s}(\iota)^{\frac{1}{\mu_w}} h_{j+s}^{\frac{\mu_w - 1}{\mu_w}} d\tau_s \quad (\text{A.12})$$

$$\sum_{s=0}^{J-j} \frac{\partial \mathcal{W}_{j,0}(\iota)}{\partial tr_{j,s}} dtr_{j,s}(\iota) = u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} dtr_{j,s}(\iota) \quad (\text{A.13})$$

$$\sum_{s=1}^{J-j} \frac{\partial \mathcal{W}_{j,0}(\iota)}{\partial \bar{c}_{j+s,s-1}} d\bar{c}_{j+s,s-1} = -\varrho \sum_{s=1}^{J-j} \beta^s \frac{N_{j+s,t+s}}{N_{j,t}} u_{j+s}^c d\bar{c}_{j+s,s-1} \quad (\text{A.14})$$

where we used the steady state relationship between the marginal utilities of various cohorts implied by the Euler equations, i.e.  $u_j^c = \beta^s \omega_j^s (1+r)^s u_{j+s}^c$  for  $0 \leq s \leq J-j$ .

Let us now define the welfare effect on an average (“representative”)  $j$ -aged household

as

$$d\mathcal{W}_{j,0} = \int_0^1 d\mathcal{W}_{j,0}(\iota) d\iota \quad (\text{A.15})$$

Then, using the definition of total labor input (3), the equilibrium result  $\bar{c}_{j,t} = c_{j,t}$ , as well as  $\int_0^1 \Xi_t(\iota) d\iota = 0$  so that  $\int_0^1 tr_{j,t}(\iota) d\iota = beq_{j,t} + beq_{j,t}^x \equiv tr_{j,t}$ , we can write (up to first order approximation)

$$d\mathcal{W}_{j,0} = \Gamma_j^x + \Gamma_j^b + \Gamma_j^f + \Gamma_j^l + \Gamma_j^t + \Gamma_j^h \quad (\text{A.16})$$

where

$$\Gamma_j^x = -\mathbb{E}_0 u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} [\chi_{j+s+1} - (1-\delta_\chi)\chi_{j+s}] dp_{\chi,s} \quad (\text{A.17})$$

$$\Gamma_j^b = \mathbb{E}_0 u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} s_{j+s} a_{j+s} dr_s \quad (\text{A.18})$$

$$\Gamma_j^f = \mathbb{E}_0 u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} (1-s_{j+s}) a_{j+s} dr_s^f \quad (\text{A.19})$$

$$\Gamma_j^l = \mathbb{E}_0 u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} z_{j+s} \left[ (1-\tau)h_{j+s} dw_s + \frac{\mu_w - 1}{\mu_w} (1-\tau)w dh_{j+s,s} - wh_{j+s} d\tau_s \right] \quad (\text{A.20})$$

$$\Gamma_j^t = \mathbb{E}_0 u_j^c \sum_{s=0}^{J-j} (1+r)^{-s} dtr_{j+s,s} \quad (\text{A.21})$$

$$\Gamma_j^h = -\varrho \mathbb{E}_0 \sum_{s=1}^{J-j} \beta^s \frac{N_{j+s,t+s}}{N_{j,t}} u_{j+s}^c dc_{j+s,s-1} \quad (\text{A.22})$$

which is the decomposition that we use in the main text.



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