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# Business Cycles, Innovation and Growth: Welfare Analysis

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# Business cycles, innovation and growth: welfare analysis

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**Abstract**

Endogenous growth literature treats deliberate R&D effort as the main engine of long-run growth. It has been already recognized that R&D expenditures are procyclical. This paper builds a microfounded model that generates procyclical aggregate R&D investment as a result of optimizing behavior by heterogeneous monopolistically competitive firms.

I find that business cycle fluctuations affect the aggregate endogenous growth rate of the economy so that transitory shocks leave lasting level effects on the economy's Balanced Growth Path. This result stems from both procyclical R&D expenditures of the incumbents and procyclical firm entry rates.

This mechanism generates economically significant hysteresis effects, increasing the welfare cost of business cycles by two orders of magnitude relative to the exogenous growth model. Coupled with potential to affect endogenous growth rates, ample space for welfare improving policy interventions arises. The paper evaluates the effects of selected subsidy schemes and finds some of them welfare improving.

**Keywords:**

business cycles, firm dynamics, innovation, growth, welfare analysis

**JEL:**

E32, E37, L11, O31, O32, O38, O40

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# 1 Introduction

Three notions have lately come to the center of attention in macroeconomics. First, the behavior of research and development expenditures is highly procyclical, which may have significant consequences for growth in face of severe shocks. Second, the behavior of incumbent firms is of utmost importance to growth in the long run, while exhibiting very turbulent patterns over the business cycle. Finally, changes in entry and exit patterns of firms have significant consequences on other macroeconomic variables.

This paper builds a model of heterogeneous monopolistically competitive establishments (plants) who endogenously choose the intensity of research and development, while subject to endogenous entry and exit. The objective of this paper is to integrate several strands of literature on the long-lasting effects of temporary shocks and provide a new assessment of the welfare cost of business cycles, as well as novel guidance on industrial policy over the business cycle.

I find that business cycle fluctuations have a noticeable impact on the endogenous growth rates. Two main channels are responsible for this effect. First, incumbents behave procyclically, investing more in R&D in good times, and less in bad times. Second, net entry is also strongly procyclical. As the innovations performed by entrants tend to be more radical than those by incumbents, reduced entry rates put downward pressure on aggregate growth rates.

The results from the model indicate that almost 6% of a temporary shock is translated to the level shift in the Balanced Growth Path, and 4% of the shock is embedded within the first 5 years. This has significant welfare consequences, as the cost of business cycle fluctuations is of two orders of magnitude higher than in the exogenous growth variant of the model. The presence of large welfare effects and the ability to potentially affect the growth rates and volatility of the economy through appropriate industrial policy creates space for policy intervention via countercyclical subsidies. Of those the most positive welfare effect is achieved through countercyclical subsidies to incumbents' operating cost, as it prevents excessive exits and encourages more R&D spending.

The late 1980s and early 1990s have seen the emergence of the first generation of endogenous growth theory with the breaking-ground contributions by Romer (1987), Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). Jones (1995) pointed out, however, that the first generation models generate counterfactual scale effects. As a response to this critique, a second generation of endogenous growth models was proposed. From the perspective of this paper, the contributions by Dinopoulos and Thompson (1998), Peretto (1998), Young (1998) and Howitt (1999) all share the interesting feature in that the attention is redirected from the entire economy toward individual establishments. In short, while in the aggregate the population of R&D scientists may rise, the important statistic is the R&D labor per establishment, which remains constant under mild assumptions regarding the market structure. Indeed, Laincz and Peretto (2006) show that since 1964 the number of full-time equivalent R&D employees per establishment has been almost constant and does not trend over time.

The idea that business cycles and endogenous growth are intertwined is not new. In one of the early contributions, Ozlu (1996) shows that shocks to learning-by-doing and human capital investment processes can generate aggregate fluctuations similar to business cycles. Further work in this vein of endogenous business cycles includes papers by e.g. Maliar and Maliar (2004), Jones et al. (2005), Walde (2005), Phillips and Wrase (2006), where various factors influencing endogenous growth rate are found to generate persistent business cycles. In an interesting vein of research Gabaix (2011) and Acemoglu et al. (2012) utilize the granularity and networking features of real economies to argue that in such circumstances idiosyncratic shocks do not average out and give rise to aggregate fluctuations. Rozsypal (2015) models the consequences of market complementarities and imperfect information that lead to a "contagion"-type effects of idiosyncratic innovative activities, causing persistent business cycles even if idiosyncratic shocks are not persistent themselves.

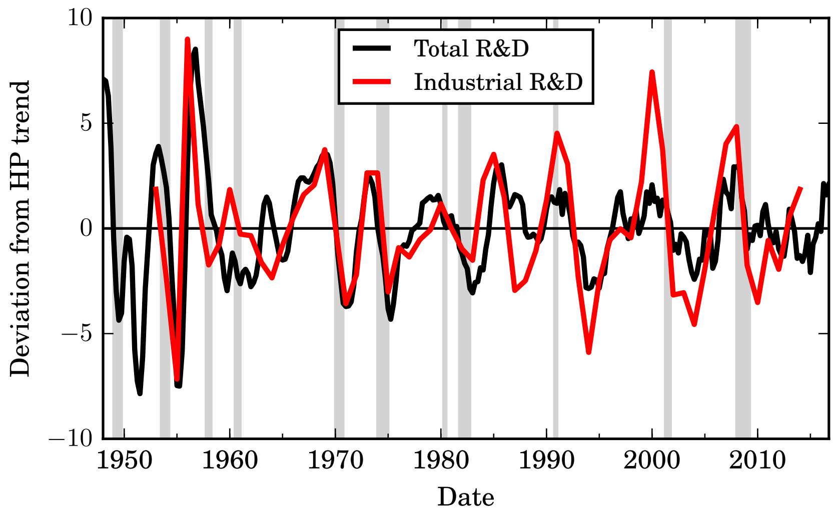
The wealth of works analyzing endogenous business cycles contrasts with the scarcity of literature that considers the causality going in other direction, i.e. the impact of business cycle fluctuations on the endogenous growth rates. Among the few papers that do so, Comin and Gertler (2006) employ the notion of medium-term business cycles. In their work, transitory TFP shocks procyclically influence invention of new technologies and adoption of existing ones, creating more persistent effects[1]. Anzoategui et al. (2016) successfully extend this framework to argue that large demand shocks at the onset of the Great Recession and subsequent drop in R&D activity may explain the weak recovery.

As argued by Barlevy (2004), if business cycles influence endogenous growth rate of the economy, the welfare cost of business cycles as estimated by Lucas (1987) may be biased downward by a few orders of magnitude. Barlevy (2007) provides ample evidence that R&D expenditures in the US are volatile and procyclical[2], as can be seen in Figure 1. However, under standard endogenous growth framework a rational firm manager would find it optimal to engage in R&D more intensively during recessions, as the opportunity cost of R&D relative to production drops, generating countercyclical R&D expenditure patterns. Therefore, Barlevy (2007) relies on dynamic externalities to R&D making entrepreneurs "short-sighted" to counteract the aforementioned effect.

[1]The key difference between Comin and Gertler (2006) and my work is that they use an ad-hoc, rather than microfounded aggregate innovation functions. This obviously precludes analysis of industrial policy.

[2]Walde and Woitek (2004) find similar patterns among other G-7 countries for the 1973-2000 period.

Figure 1: Research and Development expenditures in the US, 1948q1-2016q2

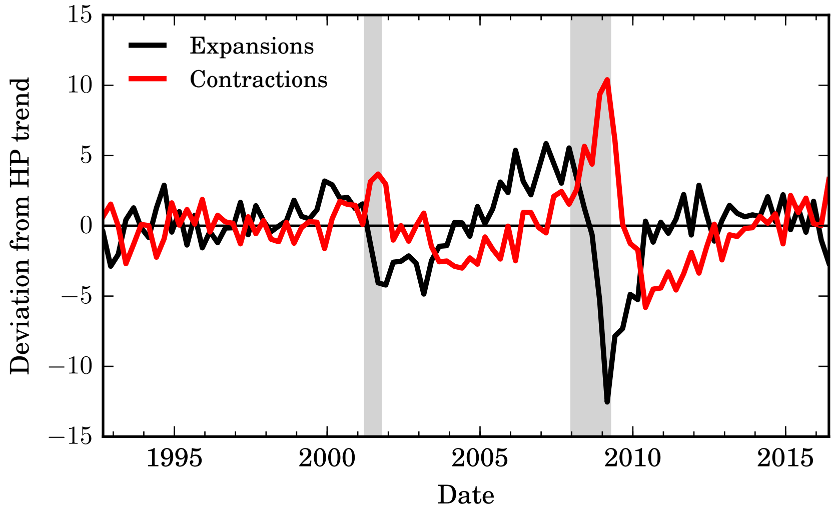



Source: US Bureau of Economic Analysis, Gross Domestic Product: Research and Development (Total R&D) and National Science Foundation, Industrial R&D performed in the United States, by source of funds, expenditures financed from all private and nonfederal sources (Industrial R&D). Deflated by GDP deflator and detrended with Hodrick-Prescott filter. Shaded bars indicate NBER recessions.

While procyclicality of R&D expenditure may be the main factor affecting long-run endogenous growth, impacting welfare via shifts in the BGP, another source of welfare consequences of business cycles are firm and establishment dynamics. The early seminal contributions in the field of industry equilibrium and dynamics are Jovanovic (1982) and Hopenhayn (1992). Both of those papers assume that a firm's productivity is drawn from a certain distribution at entry, and remains fixed afterwards. Klette and Kortum (2004) merges endogenous growth theory with industry dynamics, with firms innovating in order to enhance their product lines portfolio and growth emerging as the result of creative destruction. Lentz and Mortensen (2008) use a panel of Danish firms and are able to provide empirical support for the model. Acemoglu et al. (2013) develop a parsimonious model where firms have either high or low innovative capacity and show that they can generate steady-state behavior consistent with the stylized facts such as high growth rates and high exit probability among young firms.

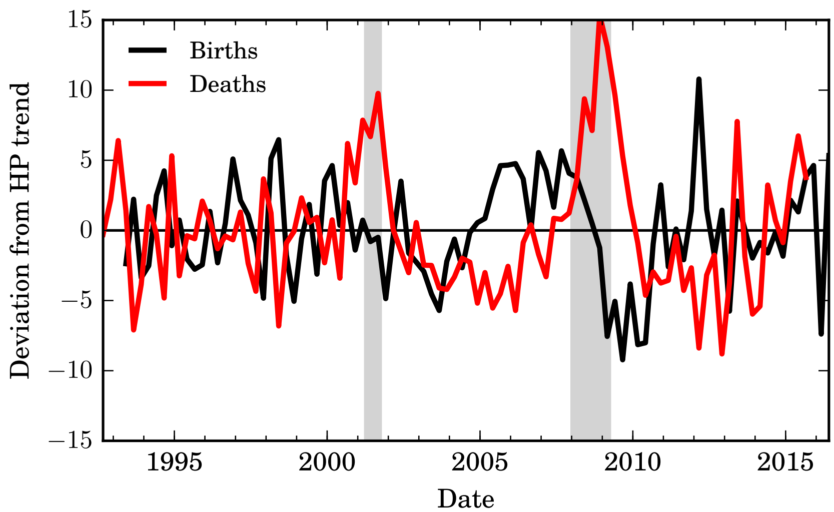
Recently, the experience of the Great Recession and the subsequent slow recovery motivated researchers to investigate possible links between cyclical changes in firm and establishment dynamics and other macroeconomic variables, a phenomenon dubbed missing generation of firms. Messer et al. (2016) show using regional US data that low entry rates in the 2007-2009 period contributed significantly to low employment and labor productivity growth. Siemer (2014) finds that tight financial constraint during the Great Recession were responsible for both low employment growth and firm entry rates. Indeed, a cursory look at establishment dynamics in Figures 2 and 3 reveals very a very turbulent picture, with significant deviations from the trend during the dot-com bubble recession of 2001 and very large deviations during the Great Recession.

Figure 2: Cyclical behavior of US establishment expansions and contractions, 1992q3-2016q2




Source: US Bureau of Labor Statistics, Business Employment Dynamics. Detrended with HP filter.

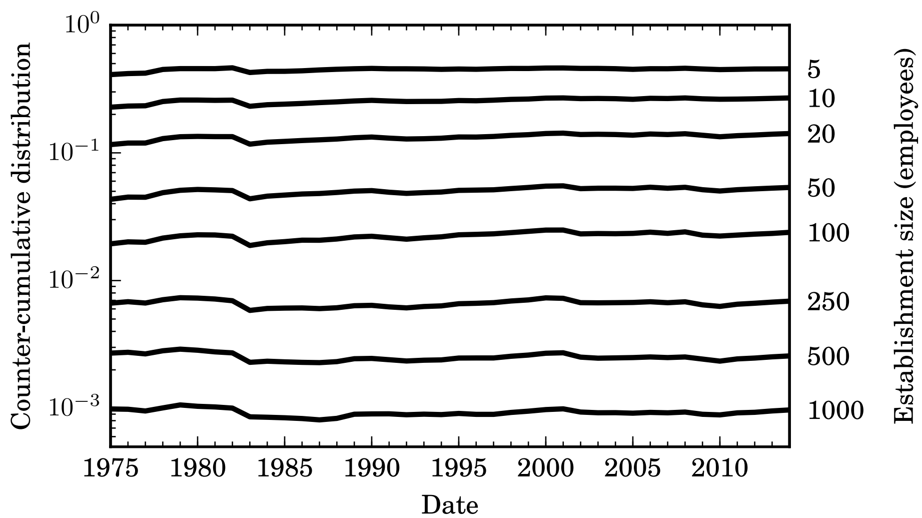
Figure 3: Cyclical behavior of US establishment births (entries) and deaths (exits), 1992q3-2016q2




Source: US Bureau of Labor Statistics, Business Employment Dynamics. Detrended with HP filter.

In the light of above facts, a surprising feature of the US economy is the remarkably stable distribution of establishment size (measured in number of employees), a fact emphasized by Luttmer (2010). Figure 4 documents a distribution that is stationary over time, paying no attention to booms and busts. The only visible shift in the distribution around year 1983 is a result of the break in the time series due to a change in the definition of establishment. Therefore any model that aims to explain the cyclical behavior of the US economy, has to generate a stationary distribution of firms that is invariant to aggregate fluctuations.

Figure 4: Establishment size distribution over time, 1975-2014




Source: County Business Patterns. The figure plots the counter-cumulative distribution of establishments' size in terms of employment. The numbers at the right hand size axis denote the size cutoff points, e.g. the curve corresponding to number 5 tracks the proportion of establishments that employ at least 5 employees.

In the literature closest in spirit to my work, Annicchiarico et al. (2011), Annicchiarico and Rossi (2015) and Annicchiarico and Pelloni (2016) employ a New Keynesian model with endogenous growth as a result of learning-by-doing to study optimal monetary policy. Their work however has no scope for deliberate R&D investment nor firm dynamics. Clementi and Palazzo (2016) build a model where procyclical entries and countercyclical exits amplify and propagate business cycle shocks and apply the model to study the behavior of US economy during the Great Recession. Cozzi et al. (2017) estimate a business cycle model with Schumpeterian components such as creative destruction to assess the relative importance of several shocks on the recent performance of the US economy. The latter two works abstract from welfare or policy analysis. Welfare considerations and optimal industrial policy are at the heart of paper by Atkeson and Burstein (2011), although their analysis employs a deterministic model. All of these papers abstract from endogenous R&D expenditures by incumbents over the business cycle, a key mechanism in my model.

The remainder of the paper is organized as follows. The next section describes the model, with particular emphasis on the problem of an incumbent establishment. The third section provides the solution for the Balanced Growth Path of the model economy to gain some intuition on the model mechanisms and discusses the stochastic solution procedure. The fourth section presents the data and calibration, as well as stochastic properties of the model economy in comparison to the data. This section offers also an application of the model to the aftermath of the Great Recession and points out to sources of seemingly permanent level shift in the US GDP. The fifth section is devoted to welfare analysis, providing an estimate of the welfare cost of business cycles for the US economy, an analysis of sensitivity of model outcomes to assumed parameter values, and a discussion on welfare improving subsidy schemes. The last section concludes.

# 2 Model

The model in this paper is mostly inspired by a closed economy version of the model sketched in Endogenous Firm Productivity section of Melitz and Redding (2014), especially with regards to the market structure and the innovation process. Some inspiration is also drawn from Acemoglu et al. (2013), especially the distinction between skilled and unskilled labor. This assumption effectively breaks the effect present in other endogenous growth models that since in recession the cost of labor is smaller, it is optimal to increase R&D intensity.

The paper focuses heavily on the dynamics among the intermediate goods producers, and so I consciously keep the complexity of setup along other dimensions at a reasonable minimum. In the following exposition I employ the following notational convention. All aggregate and nominal variables are written in uppercase. Lowercase is reserved for real prices and variables related to individual establishments or households.

For the sake of brevity and clarity this section presents the derivations in a compact manner. Detailed derivations are relegated to the Appendix.

## 2.1 Households

There is a unit mass of households. As in Acemoglu et al. (2013), there are two types of workers: those that supply skilled and unskilled labor. Each household is modeled as a large family with a fixed mass $s$ of skilled workers[3] and mass $1-s$ of unskilled workers who pool their incomes and consume identical amounts regardless of their labor market status.

A representative household maximizes the following social welfare function which takes into account the utilities of both types of households' laborers with proportional weights:

$$U_0 = (1-s)\,U_0^u + sU_0^s = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\theta}}{1-\theta} - (1-s)\,\psi_t^u \frac{(n_t^u)^{1+\kappa}}{1+\kappa} - s\psi_t^s \frac{(n_t^s)^{1+\kappa}}{1+\kappa} \right] \tag{1}$$

[3]See Laincz and Peretto (2006) for evidence that the share of employees engaged in R&D has been stationary in the time period between 1964-1999.

where $U_0^u$ and $U_0^s$ are the lifetime expected utilities of unskilled and skilled laborers, respectively, $c_t$ denotes per capita consumption while $n_t^u$ and $n_t^s$ denote labor supply of unskilled and skilled laborers, respectively. Parameter $\beta$ is the discount factor and the shape of the utility functions is regulated by the inverse of elasticity of intertemporal substitution parameter $\theta$ and the inverse of Frisch elasticity parameter $\kappa$. Both $\psi_t^u$ and $\psi_t^s$ are normalization factors that grow together with the aggregate quality index of the economy $Q_t$ and ensure that unskilled and skilled labor supply is equal to 1 along the Balanced Growth Path[4]. The shape of the utility function is regulated by the inverse of elasticity of intertemporal substitution parameter $\theta$ and the inverse of Frisch elasticity parameter $\kappa$.

The only asset in this economy are claims on the shares of establishments. Accordingly, the budget constraint of the household is constructed as follows:

$$c_t + p_t^{sh} sh_{t+1} = w_t^u n_t^u + w_t^s n_t^s + sh_t \left(p_t^{sh} + \Pi_t\right) \tag{2}$$

where $p_t^{sh}$ is the real price of a share of firm portfolio that pays real dividends $\Pi_t$, $sh_t$ is the mass of shares owned by the representative household, and $w_t^u$ and $w_t^s$ denote, respectively, real unskilled and skilled wages.

The first order conditions of the utility maximization problem reduce to the following two intratemporal equations and Euler equation:

$$(1-s)\,\psi_t^u \left(n_t^u\right)^{\kappa} = w_t^u c_t^{-\theta} \tag{3}$$

$$s\psi_t^s \left(n_t^s\right)^{\kappa} = w_t^s c_t^{-\theta} \tag{4}$$

$$1 = \mathrm{E}_t \left[\beta \left(\frac{c_{t+1}}{c_t}\right)^{-\theta} \frac{p_{t+1}^{sh} + \Pi_{t+1}}{p_t^{sh}}\right] \tag{5}$$

Since in the equilibrium the mass of establishments portfolio shares that households hold is constant and normalized to unity, budget constraint implies that consumption per capita is equal to production per capita. Furthermore, since there is a unit mass of households, production per capita is equal to aggregate production $Y_t$. The stochastic discounting kernel of the representative household is then defined as follows:

$$\Lambda_{t,t+i} = \mathrm{E}_t \left[\beta \left(\frac{c_{t+i}}{c_t}\right)^{-\theta}\right] = \mathrm{E}_t \left[\beta \left(\frac{Y_{t+i}}{Y_t}\right)^{-\theta}\right] \tag{6}$$

[4]While Boppart and Krusell (2016) show that balanced growth is possible when hours worked per worker are falling over time, a large body of research in macroeconomics employs utility functions of the convenient King et al. (1988) form which yield constant hours worked along the BGP. Trabandt and Uhlig (2011) prove that when the elasticity of intertemporal substitution differs from unity, only one functional form is characterized by both King-Plosser-Rebelo assumptions and constant Frisch elasticity of labor supply. Due to the restrictiveness of this specification, many authors, e.g. Mertens and Ravn (2011), employ a modeling shortcut in the form of time-dependent disutility of labor. Following this line of reasoning, I set the formula for labor disutility as $\psi_t^i = \psi^i Q_t^{1-\theta}$ (with $i = \{u, s\}$ and where $Q$ is a trending variable discussed further), which ensures constant labor supply along the BGP.

## 2.2 Final goods producer

The final goods producing sector is modeled as a single representative perfectly competitive firm that transforms a continuum of mass $M_t \in (0, 1)$ of intermediate good varieties[5] into final good using the CES aggregator:

$$Y_t = \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}} \,\mathrm{d}i\right]^{\frac{\sigma}{\sigma-1}} \tag{7}$$

where $y_t(i)$ denotes the output of $i$-th variety and $\sigma \in (1, \infty)$ is the elasticity of substitution between any two varieties. The standard solution of the cost minimization problem yields the price index of the final good as a function of the varieties' prices $P_t(i)$:

$$P_t = \left[\int_0^{M_t} P_t(i)^{1-\sigma} \,\mathrm{d}i\right]^{\frac{1}{1-\sigma}} \tag{8}$$

as well as the Hicksian demand function for the $i$-th variety:

$$y_t(i) = Y_t p_t(i)^{-\sigma} \tag{9}$$

where $p_t(i) = P_t(i)/P_t$ is the variety's price relative to the price index.

## 2.3 Intermediate goods producers

The intermediate goods producing sector is modeled as a single industry sector populated by monopolistically competitive continuum of mass $M_t$ of active establishments, each producing a distinct variety. Once an establishment hires $f$ units of skilled labor (this can be thought of as managers and other non-production employees), it gains access to the following production function:

$$y_t(i) = Z_t q_t(i) n_t(i) \tag{10}$$

where $Z_t$ is the stochastic aggregate productivity parameter, $q_t(i)$ is the quality level of $i$-th variety at time period $t$ and $n_t(i)$ denotes the employment of unskilled labor.

It is straightforward to show that the optimal pricing strategy given flexible prices and the demand for an individual variety given by Equation 9 follows the standard constant mark-up pricing formula:

$$p_t(i) = \frac{\sigma}{\sigma-1} \frac{w_t^u}{Z_t q_t(i)} \tag{11}$$

[5] The condition that the mass of intermediate goods varieties is bounded between 0 and 1 is supported by assuming that each individual possesses an idea for a product, but only a subset of those individuals are entrepreneurs and only a fraction of possible goods is actively produced.

## 2.4 Aggregation

The optimal pricing strategy of the establishments can be then used to produce certain useful results about the aggregate state of the economy.

It is very convenient to introduce a measure of an aggregate quality index of the economy. Similar to Melitz (2003), let us assume that the distribution of establishment-specic quality at time $t$ is described by some probability density function $\mu_t(q)$ with support on a subset of $(0, \infty)$. Then one can define an aggregate quality index $Q_t$ which is designed so that the aggregate state of the intermediate goods sector can be summarized as if it was populated by mass $M_t$ of establishments all with quality level $Q_t$. The index is given by the following formula:

$$Q_t = \left[ \int_0^\infty q^{\sigma-1} \mu_t(q) \, \mathrm{d}q \right]^{\frac{1}{\sigma-1}} \tag{12}$$

As a consequence, the aggregate output of the final good can be expressed as:

$$Y_t = Q_t Z_t M_t^{\frac{1}{\sigma-1}} N_t^u \tag{13}$$

where the dependence of output on $M_t$ reflects the love-for-variety phenomenon and $N_t^u$ is the aggregate unskilled labor supply.

The model features three aggregate state variables: aggregate quality level $Q_t$, stochastic productivity $Z_t$ and active establishment mass $M_t$. The aggregate quality level evolution will be discussed at length in Subsection 2.7.

Following the business cycles literature, I assume the exogenous common productivity shock to follow an AR(1) process in logs:

$$\ln Z_{t+1} = \rho_Z \ln Z_t + \varepsilon_{t+1} \tag{14}$$

where $\rho_Z$ is the autoregressivity of the process and $\varepsilon_{t+1}$ is a random, normally distributed innovation with mean 0 and standard deviation $\sigma_Z$.

The mass of active establishments evolves according to the following, endogenously determined law of motion:

$$M_{t+1} = M_t - M_t^x - \delta_t \left( M_t - M_t^x \right) + M_t^e \tag{15}$$

where $M_t^x$ denotes mass of establishments exiting "voluntarily" due to product obsolence, $\delta_t$ is the state-dependent, endogenous probability of receiving an exit shock and $M_t^e$ is the mass of successful entrants who attempted entry at time period $t$. Due to the presence of fixed costs, the mass of active establishments stabilizes around the value characteristic for the balanced growth path, although it fluctuates over the business cycle. The only source of sustained long-run economic growth is the continuing improvement in the aggregate quality level.

## 2.5 Incumbents

### 2.5.1 Research and development

Each incumbent establishment can engage in R&D activities in order to attempt to raise the quality of its variety in the next period[6]. Innovations performed by incumbents should be interpreted as incremental, rather than radical. The success probability function of an establishment is modeled as in Pakes and McGuire (1994) and Ericson and Pakes (1995):

$$\alpha_t(i) = \frac{a x_t(i)}{1 + a x_t(i)} \tag{16}$$

where $\alpha_t(i)$ denotes the probability of making a quality improvement, $a$ is a parameter that describes the efficacy of R&D input $x_t(i)$ in generating improvements. The R&D input has the following formula:

$$x_t(i) = \frac{n_t^x(i)}{\left(q_t(i)/Q_t\right)^{\sigma-1}} \tag{17}$$

where $n_t^x(i)$ denotes the skilled R&D labor and $\left(q_t(i)/Q_t\right)^{\sigma-1}$ is a relative quality adjustment factor. As a consequence, establishments with higher quality products need to employ more R&D labor to have the same success probability as their lower quality competitors.

The logic behind introducing this adjustment is based on two reasons. First, since the incumbents' innovations are incremental, a lot of those improvements stem from imitation rather than pure innovation. It is then reasonable that establishments producing low quality varieties have a bonus due to their distance from the "average" quality frontier, since they can easily imitate what the ones producing better quality do. Conversely, establishments far ahead with respect to their products' quality have little opportunities to imitate, and rather have to innovate themselves, raising the input requirement.

The second reason is empirical, and relates to the Gibrat's law (Gibrat (1931)), which postulates that there is no correlation between firm size (which in the model is directly related to quality) and firm growth rates (which in the model result from the innovation success probability $\alpha$). Without the adjustment, establishments producing higher quality varieties would have comparative advantage of performing R&D relative to ones producing varieties of lower quality. However, empirical evidence on the evolution of firms shows that either the Gibrat's law cannot be rejected for large enough firms (see e.g. Hall (1987)) or that the larger firms have slower rates of growth (see e.g. Evans (1987), Dunne et al. (1989) or Rossi-Hansberg and Wright (2007)).

It is convenient to introduce a new variable $\phi_t(i)$ for this relative quality adjustment factor, so that:

$$\phi_t(i) \equiv \left(q_t(i)/Q_t\right)^{\sigma-1} \tag{18}$$

Given the above functions, one can derive the demand for R&D labor as a function of

[6] A plausible alternative assumption would be that an innovation lowers the marginal cost of production. The model setup and results would remain largely the same.

target success probability $\alpha_t(i)$:

$$n_t^x(i) = \frac{1}{a}\frac{\alpha_t(i)}{1-\alpha_t(i)}\phi_t(i) \tag{19}$$

and the real cost of innovation equals the R&D employment times the real skilled wage.

Making further use of the relative quality variable $\phi_t(i)$, it can be shown that the real operating profit of an establishment at time $t$ can be rewritten as:

$$\pi_t^o(i) = \frac{Y_t}{\sigma M_t}\phi_t(i) - w_t^s f \tag{20}$$

and the real profit function, after taking into account the R&D expenditure, is given by:

$$\pi_t(i) = Y_t\left[\left(\frac{1}{\sigma M_t} - \frac{\omega_t}{a}\frac{\alpha_t(i)}{1-\alpha_t(i)}\right)\phi_t(i) - \omega_t f\right] \tag{21}$$

where $\omega_t \equiv w_t^s/Y_t$ denotes the ratio between the skilled wage and aggregate output.

### 2.5.2 Recursive formulation

I will now recast the incumbents' problem into the dynamic programming form. Since all of the intermediate goods producers that share the same value of $\phi$ will make the same decisions, I drop the subscript $i$. Moreover, if the expected continuation value is negative, an establishment will exit at the end of the current period. An incumbent establishment maximizes its real discounted stream of profits, conditional on the expected paths of state variables and the control variable $\alpha$:

$$V_t(\phi_t) = \max_{\alpha_t\in[0,1)}\left\{\pi_t(\phi_t,\alpha_t) + \max\left\{0, \mathrm{E}_t\left[\Lambda_{t,t+1}(1-\delta_t)V_{t+1}(\phi_{t+1}|\phi_t,\alpha_t)\right]\right\}\right\} \tag{22}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor consistent with the households' valuation of current and future marginal utility from consumption (Equation 6) and the relative quality of a variety in $t+1$ period is subject to the following lottery[7]:

$$\phi_{t+1} = \begin{cases} \iota\phi_t/\eta_t & \text{with probability } \alpha_t \\ \phi_t/\eta_t & \text{with probability } 1-\alpha_t \end{cases} \tag{23}$$

where $\iota$ denotes the size of the innovative step and $\eta$ is the rate of growth of the aggregate quality index (raised to the $\sigma-1$ power):

$$\eta_t \equiv \left(\frac{Q_{t+1}}{Q_t}\right)^{\sigma-1} \tag{24}$$

[7]The underlying absolute quality levels evolve according to the lottery:

$$q_{t+1} = \begin{cases} \iota^{1/(\sigma-1)}q_t & \text{with probability } \alpha_t \\ q_t & \text{with probability } 1-\alpha_t \end{cases}$$

Here I assume that the value of $\eta$ is taken as given by the individual establishments. This assumption is justified in the case of a continuum of atomistic agents[8].

As is common in the dynamic programming literature, I shall proceed with the following notational convention. Variables without the time subscript shall denote time period $t$ variables, while variables with a prime $'$ shall denote the $t+1$ variables. Let the vector $\boldsymbol{S} \equiv \{Q, Z, M\}$ denote the aggregate state of the economy. I shall also normalize the value function by expressing it as a product of aggregate output, dependent only on the economy's state, and time-independent component $v$, which depends both on the aggregate state and idiosyncratic relative quality:

$$Y(\boldsymbol{S})\, v(\phi, \boldsymbol{S}) = \max_{\alpha \in [0,1)} \left\{ \begin{array}{c} Y(\boldsymbol{S}) \left[ \left( \frac{1}{\sigma M} - \frac{\omega(\boldsymbol{S})}{a} \frac{\alpha}{1-\alpha} \right) \phi - \omega(\boldsymbol{S})\, f \right] \\ + \max \left\{ 0, \mathrm{E} \left[ \Lambda(\boldsymbol{S}) (1 - \delta(\boldsymbol{S}))\, Y(\boldsymbol{S}')\, v(\phi', \boldsymbol{S}') \right] \right\} \end{array} \right\} \tag{25}$$

Both sides of the above expression can be divided by the aggregate output, reducing the problem to the form that can be handled via standard contraction mapping procedures. Note that the "relative" skilled wage $\omega$ is established on a period by period basis via the skilled labor market equilibrium, upon which I expand later. Moreover, the state space is effectively reduced to contain only two aggregate variables: stochastic productivity $Z$ and active establishment mass $M$. The normalized value function of an establishment thus equals:

$$v(\phi, \boldsymbol{S}) = \max_{\alpha \in [0,1)} \left\{ \begin{array}{c} \left[ \left( \frac{1}{\sigma M} - \frac{\omega(\boldsymbol{S})}{a} \frac{\alpha}{1-\alpha} \right) \phi - \omega(\boldsymbol{S})\, f \right] \\ + \max \left\{ 0, \mathrm{E} \left[ \Lambda(\boldsymbol{S})\, \gamma(\boldsymbol{S}) (1 - \delta(\boldsymbol{S}))\, v(\phi', \boldsymbol{S}') \right] \right\} \end{array} \right\} \tag{26}$$

where $\gamma(\boldsymbol{S}) \equiv Y(\boldsymbol{S}')/Y(\boldsymbol{S})$ denotes the growth rate of aggregate output between two subsequent periods.

### 2.5.3 Tractability and obsolence

For large enough $\phi$ the probability that the establishment will in the future exit due to obsolence is negligible. The problem then simplifies to:

$$v(\phi, \boldsymbol{S}) = \max_{\alpha \in [0,1)} \left\{ \begin{array}{c} \left[ \left( \frac{1}{\sigma M} - \frac{\omega(\boldsymbol{S})}{a} \frac{\alpha}{1-\alpha} \right) \phi - \omega(\boldsymbol{S})\, f \right] \\ + \mathrm{E} \left[ \Lambda(\boldsymbol{S})\, \gamma(\boldsymbol{S}) (1 - \delta(\boldsymbol{S}))\, v(\phi', \boldsymbol{S}') \right] \end{array} \right\} \tag{27}$$

and in the optimum the R&D success probability does not depend on establishment variety's quality. The above problem is affine in $\phi$, as both the immediate payoff (profit) function and the expected continuation value function are affine in $\phi$[9]. On the low end of the $\phi$ distribution, if an establishment chooses to exit, it does not invest in R&D at all ($\alpha = 0$) and its current value is also affine in $\phi$:

$$v(\phi, \boldsymbol{S}) = \frac{1}{\sigma M} \phi - \omega(\boldsymbol{S})\, f \tag{28}$$

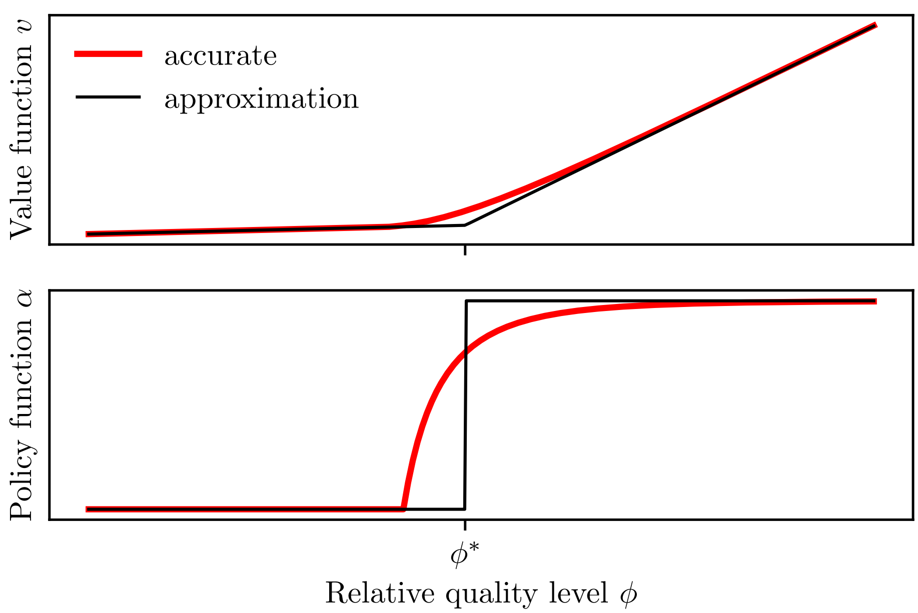
[8] For the case of a finite number of firms, see the "oblivious equilibrium" concept developed in Weintraub et al. (2008).

[9] It is straightforward to prove that the sum of two affine functions is also an affine function. Consider $f(x) = a + bx$ and $g(x) = c + dx$. Then their sum $h(x) = (a+c) + (b+d)\, x$, clearly an affine function. This procedure generalizes to the case of an infinite sum.

The true value function is nonlinear in the area where establishments are not yet willing to exit (the expected continuation value is still positive), but the probability that they choose to do so in the future is not negligible. This leads them to invest in R&D less than their higher quality competitors do. Furthermore, some establishments may choose not to invest in R&D at all if their expected continuation value is positive but small. Keeping this nonlinearity would make the entire distribution of establishments' qualities (or at least a set of its moments) a state variable, and would require much more advanced numerical techniques to solve[10].

In order to improve the problem's tractability and sidestep the issue of distribution tracking, I consider an approximation of the true problem, as depicted in Figure 5. I extend the linear parts of the true value function and construct an approximating piecewise linear value function. This simplification implies that the approximate policy function becomes a piecewise constant function of $\phi$, with all continuators choosing exactly the same R&D success probability $\alpha$. The establishments that are active but will exit in the current period choose not to innovate at all. The Appendix contains a proof[11] that if the innovation probabilities are establishment size independent but there is a lower bound on establishment quality, the distribution of establishment qualities converges in the upper tail to an ergodic Pareto distribution regardless of the initial distribution of entrants' qualities.

Figure 5: Accurate and approximated value and policy functions




[10]Krusell and Smith (1998) is the pioneering seminal paper in the heterogeneous agents with aggregate shocks literature.

[11]The proof is based on the Web Appendix for the Melitz and Redding (2014).

Let $\phi^*$ denote a level of relative quality such that an establishment is indifferent between exiting at the end of the current period and continuing into the next period, conditional on setting the common R&D success probability $\alpha$. This cutoff value is given implicitly by the following condition:

$$\frac{\omega(\boldsymbol{S})}{a}\frac{\alpha}{1-\alpha}\phi^* = \mathrm{E}\left[\Lambda(\boldsymbol{S})\gamma(\boldsymbol{S})(1-\delta(\boldsymbol{S}))v\left((\phi^*)',\boldsymbol{S}'\right)\right] \tag{29}$$

The cyclical movements in $\phi^*$ are responsible for the endogenous "voluntary" exit margin. To provide a closed form equation of this process, I will assume that the distribution of establishment qualities follows exactly the Pareto distribution with power parameter equal to 1 for the entirety of its support[12]. The mass of establishments exiting due to obsolence is equal to:

$$M_t^x = M_t(1-\alpha_t)\left(1-\frac{\phi_{t-1}^*}{\phi_t^*\eta_{t-1}}\right) \tag{30}$$

## 2.6 Entrants

There is an unconstrained mass of prospective entrants. Within each period, they can exert innovative effort to attempt to successfully enter the market. To do that, they hire skilled labor in the similar manner the incumbents do. I assume also a possibility of a positive additional skilled labor fixed cost, $f^e$. The real cost of attempting entry conditional on success probability $\alpha_t^e$ is equal to:

$$tc_t^e = Y_t\omega_t\left(f^e+\frac{1}{a^e}\frac{\alpha_t^e}{1-\alpha_t^e}\right) \tag{31}$$

If the entry is successful, the entrant draws the initial quality level from the distribution of incumbent qualities, scaled up by $\left(\frac{\sigma}{\sigma-1}\right)^{1/(\sigma-1)}$[13]. This assumption reflects the fact that innovations developed by entrants tend to be more radical, as stressed by i.a. Acemoglu and Cao (2015) and Garcia-Macia et al. (2016). A successful entrant begins operation at the beginning of the next period, and discounts this fact accordingly. The expected value of entry is expressed as:

$$V_t^e = \max_{\alpha_t^e\in[0,1)}\left\{-Y_t\omega_t\left(f^e+\frac{1}{a^e}\frac{\alpha_t^e}{1-\alpha_t^e}\right)+\alpha_t^e\mathrm{E}_t\left[\Lambda_{t,t+1}V_{t+1}\left(\phi_{t+1}^e\right)\right]\right\} \tag{32}$$

where $\phi_{t+1}^e$ is the relative quality draw upon entry. Again, using the dynamic programming notation and after normalization with aggregate output, one gets:

$$v^e(\boldsymbol{S}) = \max_{\alpha^e\in[0,1)}\left\{-\omega(\boldsymbol{S})\left(f^e+\frac{1}{a^e}\frac{\alpha^e}{1-\alpha^e}\right)+\alpha^e\mathrm{E}\left[\Lambda(\boldsymbol{S})\gamma(\boldsymbol{S})v\left((\phi^e)',\boldsymbol{S}'\right)\right]\right\} \tag{33}$$

[12] This assumption is common in the firm size distribution literature. For empirical support see e.g. Axtell (2001). Note that this is the only place in the entire paper where I need to assume a specific functional form for the distribution of quality levels. All other results hold for generic distributions, although only the distributions whose upper tail converge to Pareto are consistent with the other assumptions of the model.

[13] This assumption ensures that each variety is produced by a single establishment and that no establishment needs to resort to limit pricing as opposed to the regular markup pricing.

Since the pool of entrants is unbounded, in each state of the economy the following free entry condition applies:

$$v^e(\boldsymbol{S}) \leq 0 \tag{34}$$

where in case of a negative value of entry the mass of entrants is 0. Finally, if the mass of successful entrants is $M^e$ and the success probability is $\alpha^e$ then in equilibrium the mass of prospective entrants is determined by $M^e/\alpha^e$ in case of positive $\alpha^e$ and 0 otherwise.

### 2.6.1 Creative destruction

Although entry is undirected, there is a possibility of an entrant leapfrogging an incumbent. To model that possibility, I assume that the space of all possible varieties occupies a unit interval, and active establishments occupy its subset $M_t$. This can be justified by assuming that each individual person has a potential business idea, but only a subset of them is realized at any given time. It is then natural that the mass of potential ideas is proportional to (here equal to) the unit household mass. A successful entrant draws its location from the entire interval, and a fraction $M_t$ of all entrants replaces previously active establishments[14].

To account for the fact that the entrants who replace incumbents have leapfrogged them, I assume that the quality advantage of entrants is high enough to ensure no limit pricing. Thus, an establishment in which an incumbent successfully innovated but was replaced by an entrant will be characterized by $\left(\iota\frac{\sigma}{\sigma-1}\right)^{1/(\sigma-1)}$ times higher quality compared to the previous period and the establishment where an incumbent was uncuccessful in innovating and got replaced will be characterized by $\left(\frac{\sigma}{\sigma-1}\right)^{1/(\sigma-1)}$ times higher quality.

It is now possible to characterize the aggregate establishment mass dynamics. Incumbents exit for three reasons, which I assume to happen in the following order. First, their relative quality becomes low enough that they decide to exit due to the obsolence of their product. Second, they may receive an exogenous exit shock, $\delta^{exo}$. Finally, their establishment may be creatively destroyed by a successful entrant.

A successful entrants' variety may or may not overlap with already produced varieties. Creative destruction is a churning process, where ouflows and inflows are by definition equal. Thus, I only need to account for exogenous exit and entry into previously inactive varieties:

$$M_{t+1} = (1-\delta^{exo})(M_t - M_t^x) + \left[1 - (1-\delta^{exo})(M_t - M_t^x)\right] M_t^e \tag{35}$$

where I use the fact that the mass of active establishments just before entry is equal to $(1-\delta^{exo})(M_t - M_t^x)$.

A comparison between Equations 15 and 35 reveals that the following relationship holds:

$$\delta_t = 1 - (1-\delta^{exo})(1-M_t^e) \tag{36}$$

[14] One could make an alternative assumption that the mass of potential varieties is unbounded. Although it is a frequent assumption in case of expanding varieties models, it would be hard to reconcile with creative destruction.

## 2.7 Aggregate quality evolution

Following Melitz (2003), I consider the current period distribution of quality levels $\mu_t(q)$ to be a truncated part of an underlying distribution $g_t(q)$, so that:

$$\mu_t(q) = \begin{cases} 1/\left[1 - G_t\left(q^*_{t-1}\right)\right] g_t(q) & \text{if } q \geq q^*_{t-1} \\ 0 & \text{otherwise} \end{cases} \tag{37}$$

where the lowest quality level among all active establishments at period $t$ is equal to previous period's cutoff quality level $q^*_{t-1}$[15]. The aggregate quality level defined by Equation 12 can be rewritten as:

$$Q_t = \left[\frac{1}{1 - G_t\left(q^*_{t-1}\right)} \int_{q^*_{t-1}}^{\infty} q^{\sigma-1} g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}} \tag{38}$$

There are two mechanisms that change the distribution of incumbents' quality levels. First, the lowest quality establishments exit, raising the next period's lowest quality level to $q^*_t$. Second, while $1 - \alpha_t$ share of incumbents do not change their quality levels, share $\alpha_t$ of incumbents innovate successfully and raise their quality levels by $\iota^{1/(\sigma-1)}$. The aggregate quality level after exits[16] and innovation resolution but before entry can be expressed as follows:

$$Q^*_t = \left[(1 - \alpha_t + \alpha_t \iota) \frac{1}{1 - G_t\left(q^*_t\right)} \int_{q^*_t}^{\infty} q^{\sigma-1} g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}} \tag{39}$$

Entrants draw their quality levels from the above distribution, upscaled by factor $\sigma/(\sigma-1)$. The share of new establishments in the $t+1$ period equals $M^e_t/M_{t+1}$ and the aggregate quality level in $t+1$ is described by:

$$Q_{t+1} = \left[(1 - \alpha_t + \alpha_t \iota)\left(1 - \frac{M^e_t}{M_{t+1}} + \frac{M^e_t}{M_{t+1}} \frac{\sigma}{\sigma-1}\right) \frac{1}{1 - G_t\left(q^*_t\right)} \int_{q^*_t}^{\infty} q^{\sigma-1} g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}} \tag{40}$$

Finally one can derive the transformed aggregate growth rate $\eta_t$[17]:

$$\eta_t = \left(\frac{Q_{t+1}}{Q_t}\right)^{\sigma-1} = (1 - \alpha_t + \alpha_t \iota)\left(1 - \frac{M^e_t}{M_{t+1}} + \frac{M^e_t}{M_{t+1}} \frac{\sigma}{\sigma-1}\right) \tag{41}$$

Ceteris paribus, aggregate quality grows faster when incumbents engage more intensively in R&D and when entry rates are elevated. In general equilibrium those two forces moderate each other, as both incumbents and potential entrants compete for scarce skilled labor and incumbents face a direct threat of being creatively destroyed by entrants, reducing their willingness to innovate.

[15] The absolute level of cutoff quality can be recovered from the relative cutoff quality via the formula $q^*_t = (\phi^*)^{1/(\sigma-1)} Q_t$.

[16] Note also that due to the assumption that probabilities of both exogenous exit shock and creative destruction do not depend on establishment quality, they do not affect the quality distribution.

[17] If the density $g$ is assumed to be Pareto, which is immutable with respect to the truncation point, the argument holds with equality. Otherwise, this result is an approximation relying on assuming that the shifts in cutoff quality levels are small enough to not affect significantly the entire distribution.

## 2.8 Market clearing

There are two labor markets that clear each period. Since the aggregate mass of unskilled workers is equal to $1-s$, the following relationship between aggregate and individual labor supply holds:

$$N_t^u = (1-s)\, n_t^u \tag{42}$$

As the production function depends linearly on unskilled labor, the unskilled wage is independent of the supply side. Unskilled labor is determined via the unskilled intratemporal condition (Equation 3) and is given by:

$$n_t^u = \left[\frac{\sigma-1}{\sigma} Z_t^{1-\theta} M_t^{\frac{1-\theta}{\sigma-1}} (1-s)^{-(1+\theta)} / \psi^u \right]^{\frac{1}{\kappa+\theta}} \tag{43}$$

where I use the assumption about the labor disutility normalization factors that $\psi_t^u = \psi^u Q_t^{1-\theta}$ and with $\psi^u$ chosen so that along the BGP individual unskilled labor supply is equal to unity.

As opposed to the unskilled labor market, which behaves rather mechanically, finding equilibrium in the skilled labor market is more involved. First, using the skilled intratemporal condition (Equation 4) I obtain the relationship between skilled labor supply and the relative skilled wage $\omega_t$:

$$n_t^s = \left[\omega_t \left(Z_t M_t^{\frac{1}{\sigma-1}} N_t^u\right)^{1-\theta} / (s\psi^s)\right]^{\frac{1}{\kappa}} \tag{44}$$

where again I use the assumption about the labor disutility normalization factors that $\psi_t^s = \psi^s Q_t^{1-\theta}$ and with $\psi^s$ chosen so that along the BGP individual skilled labor supply is equal to unity.

Second, the relative skilled wage influences the R&D intensity choices made by incumbents and prospective entrants. The skilled labor demand, which implicitly depends on $\omega_t$, is expressed as:

$$N_t^s = M_t f + (M_t - M_t^x)\left(\frac{1}{a}\frac{\alpha_t}{1-\alpha_t}\right) + \frac{M_t^e}{\alpha_t^e}\left(\frac{1}{a^e}\frac{\alpha_t^e}{1-\alpha_t^e}\right) \tag{45}$$

and emerges from three sources: "managerial" demand from all active establishments, R&D demand from active and non-obsolete establishments, and "managerial" and R&D demand from prospective entrants. In equilibrium, relative skilled wage $\omega_t$ adjusts to equate skilled labor demand and supply, so that:

$$N_t^s = s n_t^s \tag{46}$$

# 3 Solution procedure

## 3.1 Balanced growth path

To provide more intuition on the working of the model and discuss its stability properties, I first present the salient parts of the solution for the BGP which can be to a large extent derived analytically. The labor market is assumed to equilibrate when both individual skilled and unskilled labor equals to 1 and the households' stochastic discount factor (Equation 6) can be expressed as:

$$\Lambda = \beta \gamma^{-\theta} \tag{47}$$

where $\gamma$ is the gross rate of growth of output along the BGP.

### 3.1.1 Incumbents

I exploit the property that the value function of the incumbent with high enough $\phi$, given by Equation 27 is affine. As along the BGP the reduced aggregate state is time-invariant, all variables present in the incumbents' problem are constant, and the value function can be stated as:

$$A + B\phi = \max_{\alpha \in [0,1)} \left\{ \begin{array}{c} \left[ \left( \frac{1}{\sigma M} - \frac{\omega}{a} \frac{\alpha}{1-\alpha} \right) \phi - \omega f \right] \\ + \xi \left[ \alpha \left( A + B \frac{\iota \phi}{\eta} \right) + (1-\alpha) \left( A + B \frac{\phi}{\eta} \right) \right] \end{array} \right\} \tag{48}$$

where $\xi \equiv \beta \gamma^{1-\theta} (1-\delta)$ is the effective incumbents' discount factor.

The resulting first order and envelope conditions are:

$$0 = -\frac{\omega}{a} \frac{1}{(1-\alpha)^2} \phi + \xi B \frac{\phi (\iota - 1)}{\eta} \tag{49}$$

$$B = \left( \frac{1}{\sigma M} - \frac{\omega}{a} \frac{\alpha}{1-\alpha} \right) + \xi B \frac{\alpha (\iota - 1) + 1}{\eta} \tag{50}$$

There are two approaches that can be used to solve this problem -- partial equilibrium and explicit solutions[18]. Both solutions are derived in the Appendix. Here I present only the result of the partial equilibrium solution where an establishment treats "aggregate" and individual R&D success probabilities asseparate objects. This allows to focus on the intuition behind the factors driving incumbents' decisions. The optimal R&D success probability equals:

$$\alpha = \frac{\frac{1}{\sigma M} \frac{a}{\omega} - \frac{1 - \xi \zeta}{\xi} \frac{\eta}{\iota - \zeta \eta}}{1 + \frac{1}{\sigma M} \frac{a}{\omega}} \tag{51}$$

where $\zeta \equiv [\alpha (\iota - 1) + 1] / \eta$ can be interpreted loosely as the contribution of incumbents to raising the aggregate quality level and is treated as given by an individual establishment.

Note that while $\alpha$ is smaller than one for any economically sensible parametrization, one needs to ensure that it is positive. For that to be true, the sign of the numerator has to be positive, which implies that the real revenue for the average establishment needs to be high enough to justify investment in R&D.

The following set of partial derivatives conforms with intuition and previous research in endogenous growth theory literature. Incumbents invest less in R&D when their monopolistic

[18] Obviously the problem can be also solved numerically via value function iteration which can then be subsequently extended to the stochastic environment case.

power is low, market is fragmented among many firms and when the R&D labor cost is high. Incumbents invest more in R&D when R&D labor is more productive, discount factor is closer to unity and the innovative step size is high.

$$\begin{aligned} &\frac{\partial \alpha}{\partial \sigma} < 0 \qquad \frac{\partial \alpha}{\partial M} < 0 \qquad \frac{\partial \alpha}{\partial \omega} < 0 \\ &\frac{\partial \alpha}{\partial a} > 0 \qquad \frac{\partial \alpha}{\partial \xi} > 0 \qquad \frac{\partial \alpha}{\partial \iota} > 0 \end{aligned} \tag{52}$$

A second set of partial derivatives relates to the stability properties of the problem. Partial derivatives with respect to transformed aggregate quality growth rate and incumbents' contribution to growth are negative[19] which ensures that the BGP is a stable equilibrium of the system and can be reached by a trial-and-error process (i.e. the equilibrium is learnable[20]):

$$\frac{\partial \alpha}{\partial \eta} < 0 \qquad \frac{\partial \alpha}{\partial \zeta} < 0 \tag{53}$$

Once the optimal R&D success probability level is found, the parameters of the closed form of the value function can be easily recovered:

$$B = \frac{\frac{1}{\sigma M} - \frac{\omega}{a}\frac{\alpha}{1-\alpha}}{1-\xi\zeta} \qquad A = \frac{\omega f}{1-\xi} \tag{54}$$

### 3.1.2 Entrants

The problem of entrants from Equation 33 remains unchanged, except for the fact that growth rates and entry value are known with certainty:

$$v^e = \max_{\alpha^e \in [0,1)} \left\{ -\omega \left( f^e + \frac{1}{a^e}\frac{\alpha^e}{1-\alpha^e} \right) + \alpha^e \xi^e \left( A + B\phi^e \right) \right\} \tag{55}$$

where $\xi^e \equiv \beta\gamma^{1-\theta}$ is the effective entrants' discount factor.

The first order condition for potential entrants results in:

$$\alpha^e = 1 - \sqrt{\frac{\omega}{a^e \xi^e v\left(\phi^e\right)}} \tag{56}$$

that is, entry success probability is higher when R&D labor is more productive, discount factor is closer to unity and the expected entry value is higher. Entry success probability is lower when R&D labor is more costly.

[19]The partial derivative w.r.t. incumbents' contribution to growth is negative for reasonable parametrizations. The formal condition for the negativity is $\xi\iota > \eta$, which is easily satisfied when the effective discount factor is close to unity.

[20]For a game-theoretic discussion on learning in non-cooperative games see e.g. Milgrom and Roberts (1990).

The free entry condition, which in BGP has to hold with equality, pins down the entry success probability in equilibrium:

$$\alpha^e = \frac{a^e f^e \pm \sqrt{a^e f^e}}{(a^e f^e - 1)} \tag{57}$$

In the above expression I need to assume that $a^e f^e > 1$ and then only one root lies within the unit interval. Note that entry success probability along the BGP depends neither on the expected entry value nor on the R&D labor cost. This result does not carry over outside the BGP and in general changes in entry rates are driven both by the extensive (mass of potential entrants) and intensive (success probability) margins.

### 3.1.3 General equilibrium

To close the model, I need to pin down the variables responsible for establishment dynamics, which are determined by the following system of equations:

$$\begin{aligned} M^x &= (1-\alpha)(1-1/\eta) M \\ M^e &= \delta M + (1-\delta) M^x \\ \delta &= 1 - (1-\delta^{exo})(1-M^e) \\ \eta &= (1+\alpha(\iota-1))(1+\delta/(\sigma-1)) \end{aligned} \tag{58}$$

Finally, the skilled labor market clears. Since by assumption the individual skilled labor supply along the BGP is unity, the aggregate skilled labor supply equals the mass of skilled households $s$ and the equilibrium in the skilled labor markets occurs whenever:

$$s = Mf + (M - M^x)\left(\frac{1}{a}\frac{\alpha}{1-\alpha}\right) + \frac{M^e}{a^e}\left(f^e + \frac{1}{a^e}\frac{\alpha^e}{1-\alpha^e}\right) \tag{59}$$

Thanks to the favorable stability properties of the model, the solution can be easily found. Using an initial guess for "relative" skilled wage rate $\omega$, endogenous exit probability $\delta$ and the mass of active establishments $M$, one can iterate the system forward until convergence is reached. I keep iterating as long as the $L^\infty$ between subsequent iterations is higher than $10^{-12}$.

## 3.2 Global solution

Under stochastic environment obtaining analytical results is not possible. Therefore, I solve the model using global methods, and employ stochastic value function iteration. The two state variables, exogenous aggregate productivity level $Z$ and endogenous mass of active establishments $M$ are discretized on a 15 by 15 grid. The grid for establishments spans the range of $\pm 10\%$ deviation from the BGP value.

The stochastic process for the changes in aggregate productivity is recast in the form of a Markov chain. Following the analysis in Kopecky and Suen (2010), I employ the method proposed by Rouwenhorst (1995), instead of more popular methods by Tauchen (1986) or

Tauchen and Hussey (1991), as the former generates a better approximation to the underlying continuous process for highly persistent ($\rho_Z > 0.9$) processes. For the chosen parameters of the productivity process ($\rho_Z = 0.95$, $\sigma_Z = 0.0055$) and the desired grid density the Rouwenhorst (1995) method generates a grid for aggregate productivity level spanning the range of $(0.94, 1.06)$.

The initial values for the endogenous variables are set to be equal to their BGP values. The stochastic value functions given by Equations 27 and 33 are iterated over each point on the grid as long as the $L^\infty$ between subsequent iterations is higher than $10^{-9}$. Also, for each point on the grid the general equilibrium consistency is ensured.

The resulting policy functions, conditional on assumed parameter values dsicussed at length in Subsection 4.1, are displayed in subsequent figures. Figure 6 demonstrates the dependence of the incumbents' R&D success probability $\alpha$ on the state variables, $Z$ and $M$. The success probability increases with the value of the temporary productivity shock and decreases with the mass of active establishments. The intution for the former is straightforward – when current stochastic productivity is higher than average, it is expected to be higher than average also in the future. The latter result seems counterintuitive at first. However, whenever the mass of active establishments is lower than average, entry is more attractive. Therefore, incumbents face higher risk of being creatively destroyed, which lowers their effective discount factor, decreasing incentives to innovate.

The policy function of potential entrants, expressed in terms of desired entry rate, is depicted in Figure 7. As could be expected, entry is more desirable when current productivity is higher than average and the mass of active establishments is lower than average. Note that the combination of low productivity and high mass of active establishments may make entry unattractive enough so that no potential entrant is willing to invest in R&D (upper left portion of the graph). This creates a nonlinearlity in the policy functions of both entrants and incumbents, and emphasizes the usefulness of the global solution of the model.

Figure 6: Incumbents' policy function $\alpha$ (%)

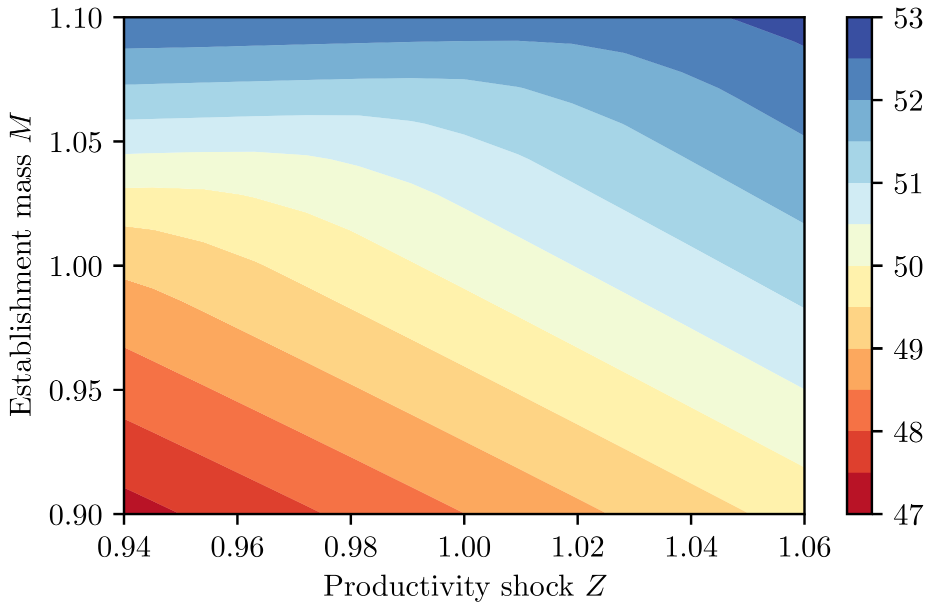
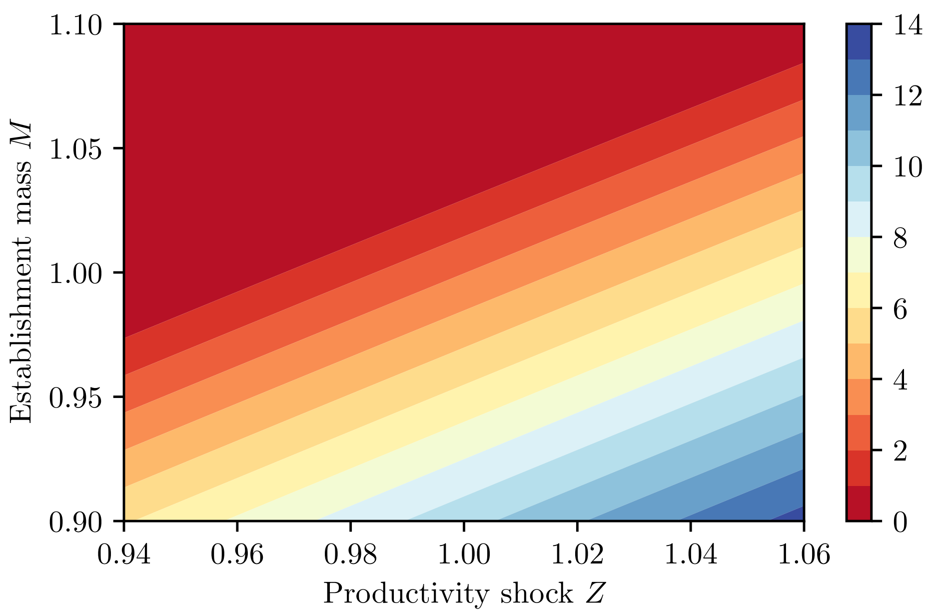



Figure 7: Entrants' policy function (expressed as desired entry rate $M^e/M$, %)



# 4 Data and results

## 4.1 Data and calibration

The data used in this paper come from four major sources. The primary source of data on establishment dynamics comes from the US Bureau of Labor Statistics (BLS) Business Employment Dynamics (BDM) database. The BDM, based on the Quarterly Census on Employment and Wages (QCEW) records changes in the employment level of more than 98% of economic entities in the US. Unfortunately, the data series is relatively short, starting as late as of 1992q3. Supplementary sources include the data from US Bureau of Economic Analyses (BEA), National Science Foundation (NSF) and County Business Patterns (CBP).

In the BDM, an establishment is defined as an economic unit that produces goods or services, usually at a single physical location, and engages in one, or predominantly one, activity for which a single industrial classification may be applied[21]. Thus an establishment, as measured by the BLS, corresponds quite closely to the theoretical concept of establishment considered in the model.

Expansions (contractions) are defined as units with positive employment in the third month in both the previous and current quarters, with a net increase (decrease) in employment over this period. Viewed through the lens of the model, expansions are the result of a successful innovation, while contractions are a consequence of being unable to innovate an thus declining relative quality level. Openings are defined as establishments with either positive third month employment for the first time in the current quarter, with no links to the prior quarter, or with positive third month employment in the current quarter following zero em- ployment in the previous quarter. Closings are defined as establishments either with positive third month employment in the previous quarter, with no employment or zero employment reported in the current quarter.

The problem with using these statistics directly is that both openings and closings are an upward biased measure of "true" entry and exit patters, as they are very sensitive to seasonal employment patterns. To correct for this issue, BLS produces data on establishment births and deaths, which are a subset of openings and closings, controlled for re-openings and temporary shutdowns via "waiting" for three quarters for status confirmation. While this correction introduces some discrepancies in the aggregate data, the gains from using data closer to the model objects should significantly outweigh the associated cost.

The model is calibrated to replicate key features of the US economy. The model BGP outcomes are compared to the long-run averages of the corresponding objects in the US data. Several parameters, such as the discount factor or the intertemporal elasticity of substitution (IES) are taken from literature.

The parameter value for the inverse of Frisch utility is non-standard, and the particular choice is dictated by the need to obtain sufficiently strong, positive reaction of labor supply in face of the productivity shock. Nevertheless, the resulting volatility of hours is still much smaller than that of output, a ubiquitous issue in the business cycle literature. The standard

[21] This and the following definitions are quoted from the Business Employment Dynamics Technical Note, available at http://www.bls.gov/news.release/cewbd.tn.htm.

deviation of the productivity shock is smaller than usually adopted in the literature, and this is due to the model having stronger amplification properties compared to the baseline real business cycle models.

The parameter pinning down the share of skilled workers in the economy is chosen to be well inside the plausible range of values discussed by Acemoglu et al. (2013) [22] and close to their estimated value of 7.8%. Since in their model a part of innovations arise spontaneously (due to e.g. learning by doing) and in this model all innovations result from deliberate R&D investment, it is natural that the share of skilled workers needs to be increased. Moreover, sensitivity analysis in Subsection 5.2 shows that this parameter does not have a major role in influencing the most salient outcomes.

Parameters specific to the model are chosen to match the targeted moments. As I have 6 free parameters to match 5 moments, I impose an additional restriction on the efficiency of R&D labor in the potential entrants sector to be equal to the efficiency of the R&D labor in the incumbents sector, as there is no specific a priori reason why they should differ. Table 1 documents chosen parameter values and the justification for the choices. Note that in the Targeted section of the table the justification should be viewed as a joint system of conditions rather than a 1:1 correspondence between a specific parameted and targeted moment. Table 2 compares the model outcomes to targeted moments. Apart from the share of R&D expenditures in GDP, which is slightly lower in the model than in the data[23], all moments are replicated satisfactorily.

The lower part of the Table 2 reports also the comparison of two non-targeted model outcomes and their empirical counterparts. The "skilled wage premium" is obtained by calculating the ratio of weekly wages of supervisory employees to that of nonsupervisory and production employees, which corresponds relatively well with the model object. However, as in the data around 17% of employees are considered "skilled" (versus 10% in the model), the empirical measure of "skilled wage premium" is very likely to be biased downwards.

Admittedly, the share of profits in GDP predicted by the model is lower than in the data, even if the model's elasticity of substitution is relatively low compared to the usual parameter values. One reason for that may be that the model assumes that all establishments belong to a single industry, which lowers profitability compared to the situation of many industries with higher intra-industry but lower inter-industry elasticity of substitution.

[22] Acemoglu et al. (2013) estimate their model using SMM and obtain point a estimate of 7.8%. However, basing on other sources, they consider any number between 3.9% and 16.2% to be plausible.

[23] It should be noted that the Gross Domestic Product: Research and Development by BEA contains all R&D expenditures performed in the US economy, and not only those performed by private businesses.

Table 1: Calibration of the model parameters

| Parameter | Description | Value | Justification |
|---|---|---|---|
| | | Fixed | |
| $\beta$ | Discount factor | 0.99 | Standard (quarterly) |
| $\theta$ | Inverse of IES | 2 | Standard |
| $\kappa$ | Inverse of Frisch elasticity | $1-2\theta$ | Volatility of hours worked |
| $\sigma$ | Elasticity of substitution | 4 | Christopoulou and Vermeulen (2010) |
| $\rho_Z$ | Autocorr. of TFP process | 0.95 | Cooley and Prescott (1995) |
| $\sigma_Z$ | Std. dev. of TFP shock | 0.0055 | Match std. dev. of output |
| $s$ | Share of skilled workers | 10% | Ballpark estimate |
| | | Targeted | |
| $\iota$ | Innovative step size | 1.015 | Annual pc. GDP growth |
| $a$ | Incumbent R&D eff. | 10 | Expansions ≈ contractions |
| $a^e$ | Entrant R&D eff. | 10 | $a = a^e$ |
| $f$ | Incumbent labor req. | 1 | Share of R&D employment |
| $f^e$ | Entrant labor req. | 1 | Share of R&D in GDP |
| $\delta^{exo}$ | Exog. exit shock prob. | 0.02 | Exit rate |

Table 2: Long-run moments: comparison of model and data

| Description | Model | Data | Source |
|---|---|---|---|
| | Targeted | | |
| Annual pc. GDP growth | 2.02% | 2.08% | BEA, 1948q1-2016q2 |
| Relative share of expanding estabs. | 1.00 | 1.01 | BDM, 1992q3-2016q2 |
| Exit rate[a] | 3.07% | 3.07% | BDM, 1992q3-2016q2 |
| Share of R&D employment | 0.97% | 0.98% | NSF & CBP, 1964-2008 |
| Share of R&D in GDP | 2.07% | 2.23% | BEA, 1948q1-2016q2 |
| | Non-targeted | | |
| "Skilled wage premium" | 2.59 | 2.09 | BLS, 2006q1-2016q2 |
| Share of profits in GDP | 4.65% | 6.53% | BEA, 1948q1-2016q2 |

[a]Calculated from the data as the average between death and birth rates.

## 4.2 Model performance

As is standard in the business cycle literature, I present the comparison between the HP-filtered moments generated by the model and the data.

Data for the variables presented in the upper part of the table are based on the 1948q1-2016q2 sample. Output is based on Real Gross Domestic Product by BEA, Hours on Nonfarm Business Sector: Hours of All Persons[24] by BLS and Research and Development on

[24]Since in general the nonfarm business sector variables are more volatile than their economy-wide coun-

appropriately deflated Gross Domestic Product: Research and Development by BEA.

Data for variables presented in the lower part of the table are based on the 1992q3-2016q2 sample, covering 97 periods, and come from the BDM. All variables before filtering were divided by the US labor force.

Table 3: Business cycle moments: comparison of model and data

| | Standard deviation | | Correlation with $Y$ | | Autocorrelation | |
|---|---|---|---|---|---|---|
| Variable | Data | Model | Data | Model | Data | Model |
| Output | 1.58 | 1.58 | 1.00 | 1.00 | 0.82 | 0.77 |
| Hours | 1.36 | 0.73 | 0.86 | 0.99 | 0.89 | 0.72 |
| R&D | 2.36 | 2.95 | 0.32 | 0.98 | 0.89 | 0.68 |
| Establishments | 0.62 | 0.64 | 0.71 | 0.70 | 0.87 | 0.88 |
| Expansions | 2.84 | 1.22 | 0.82 | 0.89 | 0.75 | 0.91 |
| Contractions | 2.38 | 0.42 | -0.11 | -0.46 | 0.69 | 0.39 |
| Net Entry | 0.31 | 0.31 | 0.31 | 0.49 | 0.24 | 0.40 |

Model moments are based on 10000 simulated periods. Table 3 presents the set of moment statistics comparing model performance to data. Overall, the model is quite successful in capturing the cyclical characteristics of establishment dynamics.

The cyclical properties of the number of establishments are perfectly in line with the data. The model underpredicts the volatility of expansions and contractions, which may be caused by the fact that a significant share of US establishments does not adjust its employment quarter over quarter. It also underpredicts the volatility of hours worked, but this issue is common across the majority of business cycles literature. The model also slightly overpredicts the standard deviation of R&D expenditures, although as the next step shows, compared to R&D performed by private businesses, this statistic may well be too low.

As an additional exercise to confirm that the model is able to replicate the comovements of variables in the data, I obtain the values of productivity shocks hitting the economy so that the HP-deviations of model output and US GDP are identical[25]. In the next I run the model conditional on this particular history of shocks and compare the HP-deviations of model variables and their empirical counterparts. The result of this exercise are reported in Figure 8.

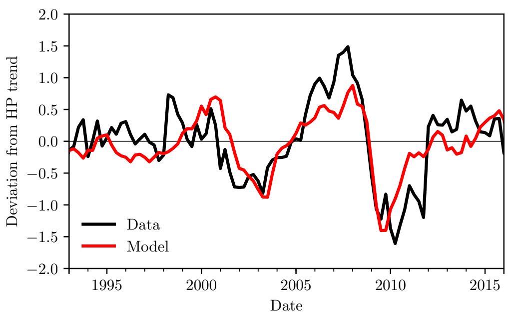
The performance of the model is very satisfactory as all the predicted movements match the data quite closely. The right hand column contains graphs where the volatility of the deviations is much bigger in the data than in the model and two scales are introduced. In the panel (e), instead of using the Research and Development series by BEA, I use

terparts, the reported standard deviation of HP-deviations in hours is normalized by dividing by the ratio of volatility of Nonfarm Business Sector: Real Output by BLS and Real Gross Domestic Product.

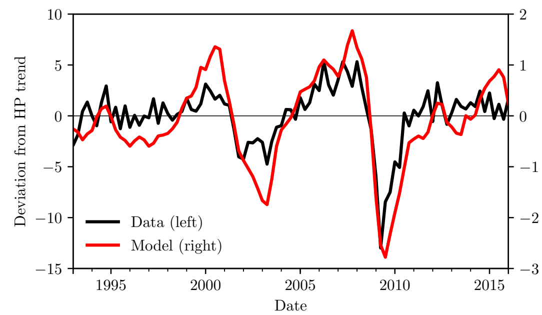
[25]In this step I construct a first-order approximation of the model and use Dynare version 4.4.3 to obtain the shock history via the estimation step.

the appropriately deflated Industrial R&D series by the National Science Foundation as it tracks specifically the R&D performed in the private business sector, although unfortunately is available only at the yearly frequency and with a significant lag. While the model captures very well the dynamics of R&D in the vicinity of the dot-com bubble, it overpredicts the reduction in R&D at the beginning of the Great Recession.

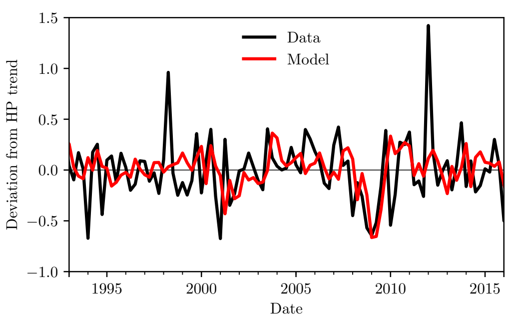
Figure 8: Hodrick-Prescott trend deviations: comparison of model and data (%)



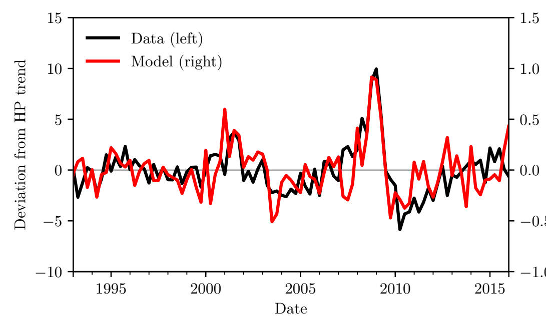

(a) Establishments



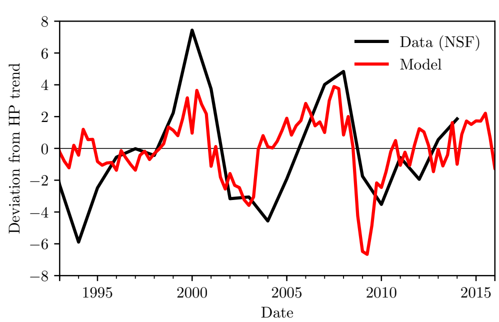

(b) Expansions



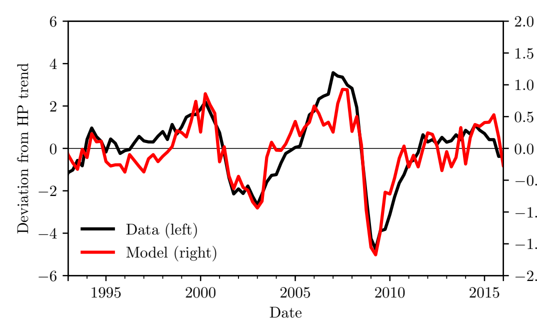

(c) Net Entry




(d) Contractions




(e) Research & Development

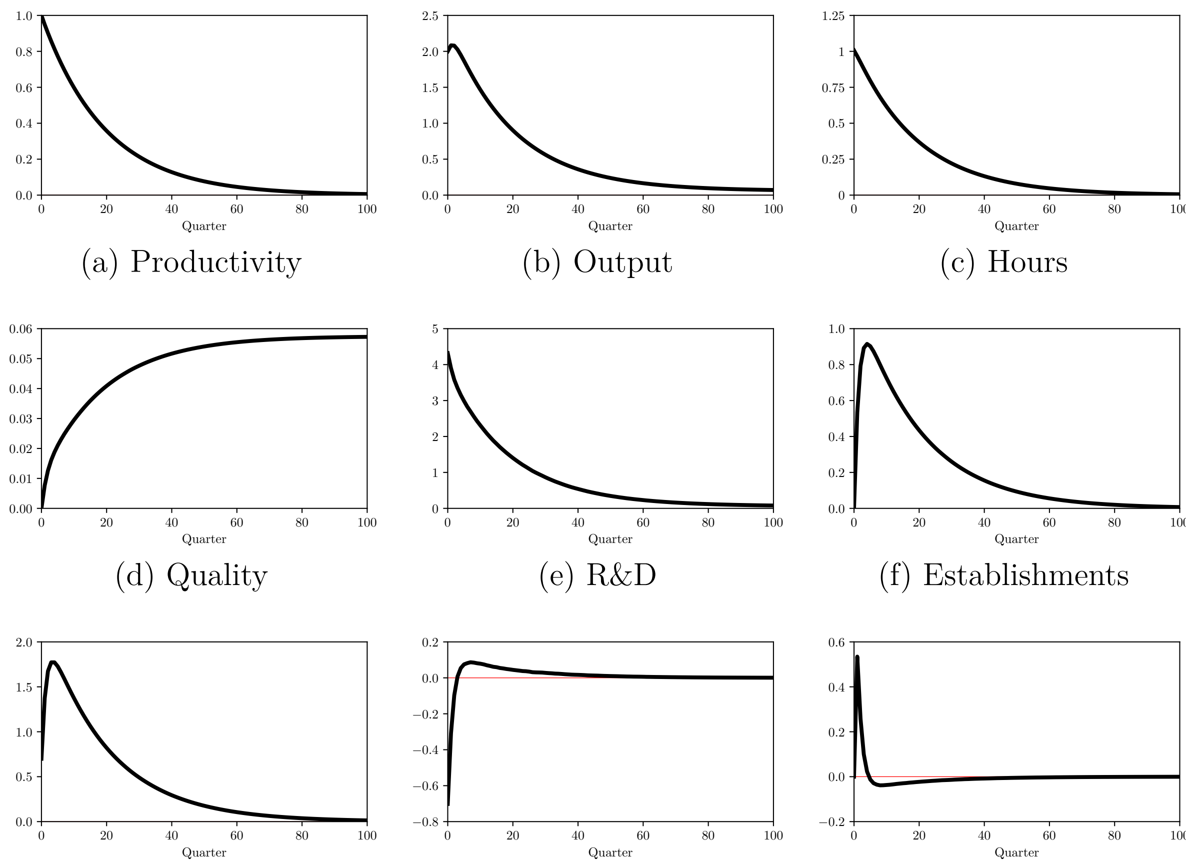



(f) Hours

## 4.3 Long shadows of temporary shocks

Figure 9 presents the impulse response function to a 1% productivity shock. A temporary increase in productivity (a) boosts output (b) directly, but also indirectly via an increase in hours worked (c) and the mass of active establishments (f), which gets bigger due to elevated net entry (i). The mass of expanding establishments (g) increases while the mass of contracting establishments (h) decreases on impact, while after a while increases due to bigger mass of active establishments. Last but not least, a positive shock to the goods producing sector increases R&D expenditures (e), which results in an increase of aggregate quality index (d) above its trend. Notice well that even if the shock eventually dissipates, the effects of the shock due to a level shift in quality, and in consequence the balanced growth path, remain. Almost 6% of the shock is translated to the level shift in the BGP, and around 2/3 of this effect is already in place 5 years after the shock.

Figure 9: Impulse response functions to 1% productivity shock (%)




(a) Productivity (b) Output (c) Hours

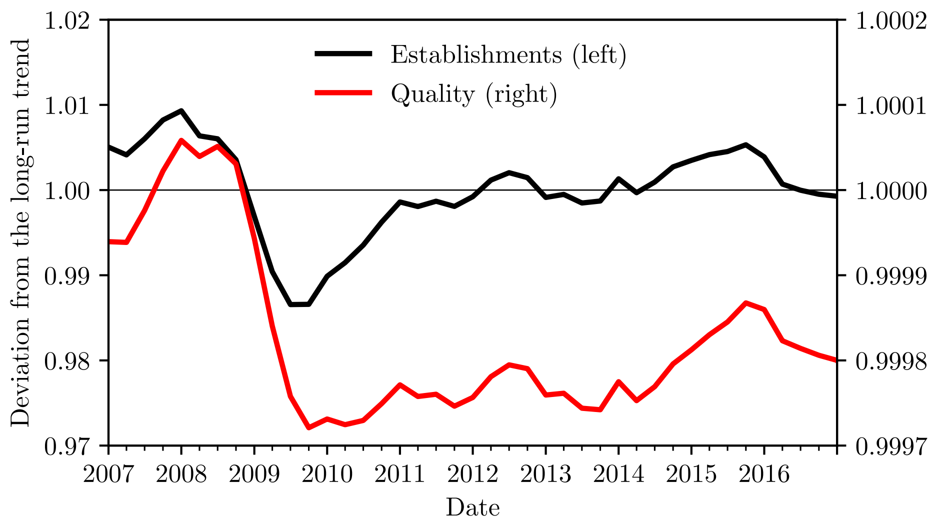
(d) Quality (e) R&D (f) Establishments

(g) Expansions (h) Contractions (i) Net Entry

### 4.3.1 Great Recession

Figure 10 tells an interesting story about the aftermath of the Great Recession. Following the negative shocks that hit the economy in 2008 and 2009, the number of active etablishments decreased due to elevated exits and depressed entries. However, in the following years the number of establishments experienced a rebound and by 2012 it has returned to the long-run trend and remains now in its vicinity. The aggregate quality index however experienced also a fall in 2008 and 2009, but it has not recovered much since 2010 and remains consistently depressed. While the quantitative shift is negligible, it is most likely the result of the models' shortcoming in generating enough volatility in expansions and contractions. The shift in the actual quality index is likely to be much more substantial and, coupled with a downward shift in the US employment, may be a key determinant of the seemingly permanent downward shift in the path of US GDP.

Figure 10: Deviation of establishments and quality from the long-run trend




# 5 Welfare analysis

The issue of quantifying welfare effects of business cycles has been recognized at least since Lucas (1987). His original assessment, basing on the variance of consumption, puts the cost of business cycles at roughly 1/20th of a percent of consumption. Although economic intuition suggests that those costs are probably much higher, the subsequent literature had limited success in generating large welfare effects of business cycles, see Lucas (2003) and Barlevy (2004).

When the aggregate quality process is endogenous, which gives rise to hysteresis, the welfare costs of business cycles are likely to be larger than in the case when aggregate quality follows an exogenous trend. Moreover, since the trend "productivity" growth can now be influenced, it naturally creates space for potentially welfare-improving policies.

## 5.1 Welfare cost of business cycles

The welfare cost of business cycles can be readily assessed using the consumption equivalent transformation. It denotes a lifetime percentage change in the consumption path of an agent which makes her indifferent across living in two distinct states of the world. The equivalent, denoted with $\mu$, can be computed via the following procedure:

$$W(\mu) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\left((1+\mu)\, c_t\right)^{1-\theta}}{1-\theta} + u_t^n \right] = (1+\mu)^{1-\theta} U_0^c + U_0^n = U_0^{BGP} \tag{60}$$

$$\mu = \left( \frac{U_0^{BGP} - U_0^n}{U_0^c} \right)^{\frac{1}{1-\theta}} - 1 \tag{61}$$

where $u_t^n$ denotes the instantaneous disutility of labor, and $U_0^c$ and $U_0^n$ denote expected lifetime utility from consumption and expected lifetime disutility of labor, respectively.

Table 4 presents the comparison of expected welfare in three states of the world: non-stochastic (BGP) and two stochastic: one in which growth is exogenous, i.e. where the R&D activity is not sensitive to the shocks, and the one with the endogenous growth subject to business cycles mechanism. In line with the literature, the cost of business cycles, expressed in consumption equivalent terms, is very small if the growth rate is exogenous, since the path of output returns to the balanced growth path reasonably quickly. However, if endogenous growth rates react to shocks, generating level shifts in the balanced growth path, the expected output path becomes much more volatile and can deviate from the BGP significantly. This increased uncertainty in outcomes translates to an almost 100-fold increase in the consumption equivalent, making the number two orders of magnitude higher than in the exogenous growth case.

Table 4: Welfare cost of business cycles

| State of the world | Welfare | $U^c$ | $U^n$ | Consumption equivalent |
|---|---|---|---|---|
| Nonstochastic (BGP) | -122.26 | -168.06 | 45.80 | – |
| Stochastic with exogenous growth | -122.37 | -168.19 | 45.83 | 0.06% |
| Stochastic with endogenous growth | -129.89 | -175.73 | 45.83 | 4.54% |

## 5.2 Sensitivity analysis

Before I analyze the effects of policies, I discuss how the key model statistics – aggregate growth rate, the level shift in the BGP after 5 and 25 years, and the expected social welfare – depend on the model parameters. For most of the parameters I will consider a 10% change in a parameter value. The exception to this operation are the discount factor $\beta$, where I increase it from 0.99 to 0.995 and productivity shock autocorrelation $\rho_Z$ which is increased from 0.95 to 0.97. Table 5 documents the effects of the changes in parameter values on the aggregate ouctomes and welfare.

Both the growth rate along the BGP, as well as in the stochastic equilibrium, are higher when: innovative step is larger, incumbents' R&D efficacy is higher, incumbents' fixed cost is higher, exogenous exit shock probability is higher, and households are more patient. The reason why an economy with higher incumbents' fixed cost grows faster stems from the fact that it is populated by fewer establishments and each of them can employ more R&D labor. An economy with higher exit shock probability grows faster since the mass of entrants, who perform more radical innovations than the incumbents, is higher. The economy grows slower when the elasticity of substitution is higher, as the gains from innovation are smaller.

The shift in level of BGP due to a shock is higher when: innovative step is larger, entrants' R&D efficacy is higher, exogenous exit shock probability is higher, both the productivity autocorrelation and volatility are higher, and households are more patient and more risk-averse. Again, higher exit shock probability results in higher population of entrants, and a higher entry rate in response to a positive shock. In this model, higher risk aversion amplifies the reaction in skilled labor, which results in stronger reaction of aggregate quality. The shift in level of BGP is smaller when incumbents' R&D efficacy is higher, fixed costs are higher, and elasticity of substitution is higher. Higher R&D efficacy of incumbents increases their share of R&D employment at the expense of entrants who have innovative advantage, thus dampening the effect of a shock.

As it turns out, the share of skilled workers in the economy does not influence the aggregate growth rate nor the cyclical properties of the model, although it affects the population of active establishments in the economy.

When discussing welfare issues, I exclude the parameters that directly influence the social welfare function, as comparing across different utility functions is infeasible. The last column of Table 5 includes the consumption equivalent between the world with a changed parameter and the baseline world. Slightly counterintuitively, positive consumption equivalent signals welfare deterioration. A general pattern that emerges from comparing aggregate outcomes andchanges in the welfare is that the larger the shifts in the BGP, the lower the welfare, even if the aggregate growth rates are higher. This underscores the importance of the welfare costs stemming from living in a fundamentally more uncertain world, compared to the one where "productivity" reverts to an exogenous trend.

Accordingly, higher innovative step size and higher standard deviation of shocks are strongly welfare deteriorating. On the other hand, higher entry barriers also have a slightly negative effect on welfare, but this effects stems from slightly lower aggregate growth rates, rather than from the volatility channel. Higher probability of an exit shock, higher elasticity of substitution, higher incumbent's fixed costs and higher R&D efficacy all bring welfare improvements. Usually those improvements are a result of lowered volatility of the economy, although in the case of exit shock probability an increase in aggregate growth rates dominates the effect of slightly elevated volatility.

Two conclusions can be drawn from this exercise. First, the quantitative impact of a shock on the shift in BGP is relatively robust with respect to changes in specific parameters. The parameters which have the strongest impact on the quantitative effect – elasticity of substitution, productivity process autocorrelation and elasticity of intertemporal substitution – were all taken from previous literature and did not depend on the specifics of calibration. Second, there is ample scope for policy intervention as policies limiting the economy's volatility are expected to generate nontrivial welfare effects.

Table 5: Sensitivity analysis

| | $\gamma^{BGP}$ | $\gamma$ | $\Delta Q_{20}$ | $\Delta Q_{100}$ | $U^{BGP}$ | $U$ | $U^c$ | $U^n$ | $\mu^a$ |
|---|---|---|---|---|---|---|---|---|---|
| Baseline | 2.02 | 2.04 | 2.25 | 3.15 | -122.26 | -129.89 | -168.06 | 45.80 | – |
| $\iota$ | 2.16 | 2.19 | 2.33 | 3.28 | -127.88 | -136.22 | -184.22 | 48.00 | 3.56% |
| $a$ | 2.06 | 2.09 | 2.17 | 3.03 | -120.87 | -128.04 | -173.35 | 45.31 | -1.06% |
| $a^e$ | 2.02 | 2.04 | 2.26 | 3.16 | -121.52 | -129.11 | -174.67 | 45.55 | -0.45% |
| $f$ | 2.06 | 2.08 | 2.21 | 3.07 | -119.78 | -126.92 | -171.82 | 44.90 | -1.70% |
| $f^e$ | 2.02 | 2.04 | 2.22 | 3.12 | -122.67 | -130.29 | -176.28 | 45.99 | 0.22% |
| $\delta^{exo}$ | 2.09 | 2.11 | 2.28 | 3.20 | -115.34 | -122.31 | -165.46 | 43.15 | -4.38% |
| $\sigma$ | 1.68 | 1.71 | 1.86 | 2.64 | -119.14 | -126.01 | -173.87 | 47.86 | -2.19% |
| $\rho_Z = 0.97$ | 2.02 | 2.04 | 2.24 | 4.03 | -122.26 | -129.85 | -175.71 | 45.86 | -0.03% |
| $\sigma_Z$ | 2.02 | 2.05 | 2.48 | 3.47 | -122.26 | -131.50 | -177.34 | 45.84 | 0.91% |
| $\beta = 0.995$ | 2.08 | 2.11 | 2.29 | 4.14 | | | | | |
| $\theta$ | 2.00 | 2.04 | 3.66 | 6.61 | | | | | |
| $\kappa$ | 2.02 | 2.04 | 1.79 | 3.25 | | | | | |
| $s$ | 2.02 | 2.04 | 2.25 | 3.15 | | | | | |

[a]The lower the number, the higher the welfare gain

## 5.3 Policy experiments

The conclusions of the previous sections also have implications for the optimal industrial policy. In particular, encouraging entry by either lowering entry barriers or by subsidizing entrants' R&D yields positive welfare effects in the stochastic equilibrium. Additionally, direct subsidies to incumbents are welfare deteriorating, while R&D subsidies boost welfare.

The above results fall in line with those obtained by the endogenous growth literature in deterministic settings[26]. The value added of my approach is in considering the effects of countercyclical subsidies. Such subsidies do not affect significantly the aggregate growth rates directly, but can act as moderators of business cycle fluctuations and via the volatility channel affect welfare.

As neither the productivity shock nor the aggregate quality level are observable, I set up a scheme where subsidy reacts to variables that are observable almost in real time: the level of output $Y$ and mass of active establishments $M$. Both of those schemes yield

[26]See e.g. Acemoglu et al. (2013).

similar outcomes[27]. The subsidy/tax is financed by lump sum taxes/transfers levied on the households. As the fixed and R&D costs are in effect incomes of skilled workers, the subsidy/tax does not redirect the resources from the private economy, and merely affects the incentives of entrants and incumbents. Therefore, one must take the results with a grain of salt as implementabe tax/subsidy schemes would introduce additional distortions.

Table 6 documents the effects of applying countercyclical subsidies to the model economy. A symmetric subsidy/tax lowers the effective costs of skilled labour in the recession and increases those costs during expansions, therefore counteracting the cyclical effects on the skilled workers' wage. Similarly as in the constant subsidy case, subsidizing either entrants' R&D efforts or lowering the costs of entry generates positive welfare effects.

The situation is however radically different in the case of incumbents: while it is welfare improving to implement a constant subsidy to incumbents' R&D efforts, countercyclical subsidies are welfare deteriorating. This effects is the result of competition for skilled labor between entrants and incumbents: subsidy to incumbents' R&D diverts resources from entrants where they are needed the most: in times when entry is already depressed. This is also evidenced by increased impact of shock on the level shift in BGP compared to the baseline.

On the other hand, countercyclical subsidies to the fixed cost of incumbents does increase welfare, whereas constant subsidies were welfare deteriorating. The reason is that while permanent subsidies discourage entry, maintain lower productivity establishments and thus divert resources from more efficient use, countercyclical subsidies prevent inefficient exits and thus reduce significantly the volatility of the economy. In fact, those subsidies generate the strongest positive welfare effects. This conclusion, if supported by further studies, gives justification for policies designed to support existing establishments in the severe downturns.

Table 6: Effects of countercyclical subsidies

| | $\Delta Q_{20}$ | $\Delta Q_{100}$ | $U$ | $U^c$ | $U^n$ | $\mu^a$ |
|---|---|---|---|---|---|---|
| Baseline | 2.25 | 3.15 | -129.89 | -175.73 | 45.83 | – |
| 0.5% subidy if $Y$ is 1% below trend | | | | | | |
| $f$ | 0.91 | 1.22 | -128.84 | -174.66 | 45.82 | -0.60% |
| $a$ | 3.03 | 4.42 | -133.16 | -179.00 | 45.84 | 1.86% |
| $f^e$ | 2.14 | 2.99 | -129.67 | -175.51 | 45.83 | -0.13% |
| $a^e$ | 2.22 | 3.10 | -129.83 | -175.66 | 45.83 | -0.04% |
| 0.5% subidy if $M$ is 1% below trend | | | | | | |
| $f$ | 1.69 | 2.33 | -129.64 | -175.47 | 45.83 | -0.14% |
| $a$ | 2.58 | 3.71 | -130.99 | -176.82 | 45.84 | 0.62% |
| $f^e$ | 2.24 | 3.12 | -129.74 | -175.57 | 45.83 | -0.09% |
| $a^e$ | 2.25 | 3.15 | -129.85 | -175.68 | 45.83 | -0.03% |

[a]The lower the number, the higher the welfare gain

[27]Since the volatility of establishments is lower than that of output, it is natural that the quantitative estimates will be lower in the case of establishment subsidy.

# 6 Conclusions

As documented by Comin and Gertler (2006), Barlevy (2007) and Anzoategui et al. (2016), expenditure on R&D is volatile and procyclical. In this paper I have presented an endogenous growth model, featuring monopolistically competitive, heterogeneous establishments that endogenously decide on the intensity of R&D. The model is consistent with the abovementioned facts and generates predictions on the strength of the impact of business cycle fluctuations on the endogenous growth rates of the economy.

The results suggest that the mechanism governing innovation dynamics generates hysteresis effects of temporary shocks on the BGP level, translating almost 6% of the strength of a shock to the level shift of the BGP. This observation, coupled with other "missing generation of firms" effects, as identified by Siemer (2014) and Messer et al. (2016), urges to reassess the previous estimates of the welfare costs of business cycles.

I find that the welfare effects of business cycles are nontrivial and of two orders of magnitude higher than in the models with exogenous growth. Considerable welfare effects and the potential to influence endogenous growt rates makes scope for policy inervention. In line with existing endogenous growth literature, e.g. Acemoglu et al. (2013), I find that subsidizing incumbents is welfare deteriorating, while subsidizing entry and R&D investments is welfare improving. These results do not carry over to the case of sountercyclical subsidies, where I find that subsidizing incumbents' R&D expeditures is strongly welfare deteriorating, while subsidizing incumbents' operation costs becomes welfare enhancing. This conclusion, if supported by further studies, gives justification for policies designed to support existing establishments in the severe downturns.

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# A Appendix

## A.1 Formula for real operating profit

Real operating profit:

$$\begin{aligned}\pi_t^o(i) &= p_t(i)\,y_t(i) - w_t^u n_t^u(i) - w_t^s f \\ &= p_t(i)\,y_t(i) - mc_t(i)\,y_t(i) - w_t^s f \\ &= p_t(i)\,y_t(i) - \frac{\sigma-1}{\sigma} p_t(i)\,y_t(i) - w_t^s f \\ &= \left(1 - \frac{\sigma-1}{\sigma}\right) p_t(i)\,Y_t p_t(i)^{-\sigma} - w_t^s f \\ &= \frac{1}{\sigma} Y_t \left(\frac{\sigma}{\sigma-1}\frac{w_t^u}{Z_t q_t(i)}\right)^{1-\sigma} - w_t^s f \\ &= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t \left(w_t^u\right)^{1-\sigma} Z_t^{\sigma-1} q_t(i)^{\sigma-1} - w_t^s f \end{aligned} \tag{A.1}$$

Aggregate price index:

$$\begin{aligned}P_t &= \left[\int_0^{M_t} P_t(i)^{1-\sigma}\,\mathrm{d}i\right]^{\frac{1}{1-\sigma}} \\ &= \left[M_t \int_0^\infty P_t(q)^{1-\sigma}\,\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{1-\sigma}} \\ &= M_t^{\frac{1}{1-\sigma}} \left[\int_0^\infty \left(\frac{\sigma}{\sigma-1}\frac{P_t w_t^u}{Z_t q}\right)^{1-\sigma} \mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{1-\sigma}} \\ &= M_t^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1}\frac{P_t w_t^u}{Z_t} \left[\left[\int_0^\infty q^{\sigma-1}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}\right]^{-1} \\ &= M_t^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1}\frac{P_t w_t^u}{Z_t Q_t}\end{aligned} \tag{A.2}$$

Real unskilled wage:

$$w_t^u = \frac{\sigma-1}{\sigma} Z_t Q_t M_t^{\frac{1}{\sigma-1}} \tag{A.3}$$

Real operating profit again:

$$\begin{aligned}\pi_t^o(i) &= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t \left(w_t^u\right)^{1-\sigma} Z_t^{\sigma-1} q_t(i)^{\sigma-1} - w_t^s f \\ &= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} Y_t \left(M_t^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} Z_t Q_t\right)^{1-\sigma} Z_t^{\sigma-1} q_t(i)^{\sigma-1} - w_t^s f \\ &= \frac{1}{\sigma}\frac{Y_t}{M_t}\left(q_t(i)/Q_t\right)^{\sigma-1} - w_t^s f \\ &= \frac{1}{\sigma}\frac{Y_t}{M_t}\phi_t(i) - w_t^s f\end{aligned} \tag{A.4}$$

## A.2 Formula for aggregate output

Relative labor input:

$$\frac{y_t(i)}{y_t(j)} = \frac{y_t p_t(i)^{-\sigma}}{y_t p_t(j)^{-\sigma}} = \left(\frac{\frac{\sigma}{\sigma-1}\frac{w_t^u}{z_t q_t(i)}}{\frac{\sigma}{\sigma-1}\frac{w_t^u}{z_t q_t(j)}}\right)^{-\sigma} = \left(\frac{q_t(i)}{q_t(j)}\right)^{\sigma}$$

$$\frac{Z_t q_t(i)\, n_t^u(i)}{Z_t q_t(j)\, n_t^u(j)} = \left(\frac{q_t(i)}{q_t(j)}\right)^{\sigma}$$

$$\frac{n_t^u(i)}{n_t^u(j)} = \left(\frac{q_t(i)}{q_t(j)}\right)^{\sigma-1} \longrightarrow n_t^u(i) = \left(\frac{q_t(i)}{Q_t}\right)^{\sigma-1} n_t^u(Q_t)$$

where $n_t^u(Q_t)$ is the level of employment by establishment with quality level equal to the aggregate quality index.

Aggregate labor input:

$$\begin{aligned}
N_t^u &= \left[\int_0^{M_t} n_t^u(i)\,\mathrm{d}i\right] \\
&= \left[M_t \int_0^{\infty} n_t^u(q)\,\mu_t(q)\,\mathrm{d}q\right] \\
&= M_t\left[\int_0^{\infty}\left(\frac{q_t(i)}{Q_t}\right)^{\sigma-1} n_t^u(Q_t)\,\mu_t(q)\,\mathrm{d}q\right] \\
&= M_t Q_t^{1-\sigma} n_t^u(Q_t)\left[\int_0^{\infty} q_t(i)^{\sigma-1}\,\mu_t(q)\,\mathrm{d}q\right] \\
&= M_t Q_t^{1-\sigma} n_t^u(Q_t)\, Q_t^{\sigma-1} \\
&= M_t n_t^u(Q_t)
\end{aligned}$$

$$n_t^u(Q_t) = \frac{N_t^u}{M_t} \tag{A.5}$$

Aggregate output:

$$
\begin{aligned}
Y_t &= \left[\int_0^{M_t} y_t\left(i\right)^{\frac{\sigma-1}{\sigma}} \,\mathrm{d}i\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M_t \int_0^{\infty} y_t\left(q\right)^{\frac{\sigma-1}{\sigma}} \mu_t\left(q\right) \mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} \left[\int_0^{\infty} \left(Z_t q n_t^u\left(q\right)\right)^{\frac{\sigma-1}{\sigma}} \mu_t\left(q\right) \mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}} Z_t \left[\int_0^{\infty} \left(q \left(\frac{q}{Q_t}\right)^{\sigma-1} \frac{N_t^u}{M_t}\right)^{\frac{\sigma-1}{\sigma}} \mu_t\left(q\right) \mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{\sigma}{\sigma-1}-1} Z_t Q_t^{1-\sigma} N_t^u \left[\int_0^{\infty} \left(q_t^{\sigma}\right)^{\frac{\sigma-1}{\sigma}} \mu_t\left(q\right) \mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}} \\
&= M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\sigma} N_t^u \left[\left[\int_0^{\infty} q_t\left(i\right)^{\sigma-1} \mu_t\left(q\right) \mathrm{d}q\right]^{\frac{1}{\sigma-1}}\right]^{\sigma} \\
&= M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\sigma} N_t^u Q_t^{\sigma} \\
&= Z_t Q_t M_t^{\frac{1}{\sigma-1}} N_t^u \qquad \text{(A.6)}
\end{aligned}
$$

## A.3 Aggregate quality evolution

Aggregqte quality index at the end of period $t$:

$$Q_t = \left[\int_0^\infty q^{\sigma-1}\mu_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}} = \left[\frac{1}{1-G_t\left(q_{t-1}^*\right)}\int_{q_{t-1}^*}^\infty q^{\sigma-1}g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}} \tag{A.7}$$

The aggregate quality level after exits and innovation resolution but before entry:

$$\begin{aligned} Q_t^* &= \left\{\frac{1}{1-G_t\left(q_t^*\right)}\left[(1-\alpha_t)\int_{q_t^*}^\infty q^{\sigma-1}g_t(q)\,\mathrm{d}q + \alpha_t\int_{q_t^*}^\infty \left(\iota^{\frac{1}{\sigma-1}}q\right)^{\sigma-1}g_t(q)\,\mathrm{d}q\right]\right\}^{\frac{1}{\sigma-1}} \\ &= \left[(1-\alpha_t+\alpha_t\iota)\frac{1}{1-G_t\left(q_t^*\right)}\int_{q_t^*}^\infty q^{\sigma-1}g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}} \end{aligned} \tag{A.8}$$

Aggregate quality index in $t+1$ after entry:

$$\begin{aligned} Q_{t+1} &= \left\{\frac{1-\alpha_t+\alpha_t\iota}{1-G_t\left(q_t^*\right)}\left[\left(1-\frac{M_t^e}{M_{t+1}}\right)\int_{q_t^*}^\infty q^{\sigma-1}g_t(q)\,\mathrm{d}q + \frac{M_t^e}{M_{t+1}}\int_{q_t^*}^\infty \left(\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{\sigma-1}}q\right)^{\sigma-1}g_t(q)\,\mathrm{d}q\right]\right\}^{\frac{1}{\sigma-1}} \\ &= \left[(1-\alpha_t+\alpha_t\iota)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right)\frac{1}{1-G_t\left(q_t^*\right)}\int_{q_t^*}^\infty q^{\sigma-1}g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}} \end{aligned} \tag{A.9}$$

Transformed aggregate growth rate $\eta_t$:

$$\begin{aligned} \eta_t &= \left(\frac{Q_{t+1}}{Q_t}\right)^{\sigma-1} \\ &= \left\{\frac{\left[(1-\alpha_t+\alpha_t\iota)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right)\frac{1}{1-G_t\left(q_t^*\right)}\int_{q_t^*}^\infty q^{\sigma-1}g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}}{\left[\frac{1}{1-G_t\left(q_{t-1}^*\right)}\int_{q_{t-1}^*}^\infty q^{\sigma-1}g_t(q)\,\mathrm{d}q\right]^{\frac{1}{\sigma-1}}}\right\}^{\sigma-1} \\ &\approx (1-\alpha_t+\alpha_t\iota)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right) \end{aligned} \tag{A.10}$$

where if the distribution is invariant with respect to the cutoff points $q_{t-1}^*$ and $q_t^*$ (as is the case with Pareto and other power-law distributions) then the above relationship holds with equality.

## A.4 Labor market equilibrium

Unskilled consumption-labor choice:

$$
\begin{aligned}
(1-s)\,\psi_t^u\,(n_t^u)^\kappa &= w_t^u c_t^{-\theta} \\
(1-s)\left(\psi^u Q_t^{1-\theta}\right)(n_t^u)^\kappa &= \left(\frac{\sigma-1}{\sigma} Q_t Z_t M_t^{\frac{1}{\sigma-1}}\right)\left(Q_t Z_t M_t^{\frac{1}{\sigma-1}}\,(1-s)\,n_t^u\right)^{-\theta} \\
(1-s)\,\psi^u\,(n_t^u)^{\kappa+\theta} &= \frac{\sigma-1}{\sigma} Z_t^{1-\theta} M_t^{\frac{1-\theta}{\sigma-1}}\,(1-s)^{-\theta} \\
n_t^u &= \left[\frac{\sigma-1}{\sigma} Z_t^{1-\theta} M_t^{\frac{1-\theta}{\sigma-1}}\,(1-s)^{-\theta-1}/\psi^u\right]^{\frac{1}{\kappa+\theta}}
\end{aligned}
\tag{A.11}
$$

Skilled consumption-labor choice:

$$
\begin{aligned}
s\psi_t^s\,(n_t^s)^\kappa &= w_t^s c_t^{-\theta} \\
s\left(\psi^s Q_t^{1-\theta}\right)(n_t^s)^\kappa &= \left(\omega_t Q_t Z_t M_t^{\frac{1}{\sigma-1}} N_t^u\right)\left(Q_t Z_t M_t^{\frac{1}{\sigma-1}} N_t^u\right)^{-\theta} \\
s\psi^s\,(n_t^s)^\kappa &= \omega_t\left(Z_t M_t^{\frac{1}{\sigma-1}} N_t^u\right)^{1-\theta} \\
n_t^s &= \left[\omega_t\left(Z_t M_t^{\frac{1}{\sigma-1}} N_t^u\right)^{1-\theta}/\,(s\psi^s)\right]^{\frac{1}{\kappa}}
\end{aligned}
\tag{A.12}
$$

## A.5 Shape of the relative quality distribution

Consider the situation of incumbents with relative quality levels $\phi > \phi_t^*$ such that they will all choose the same success probability $\alpha$. Therefore, for all of them:

$$
\phi_{t+1} = \begin{cases} \iota\phi_t/\eta_t & \text{with probability } \alpha_t \\ \phi_t/\eta_t & \text{with probability } 1-\alpha_t \end{cases}
\tag{A.13}
$$

Define the countercumulative distribution of relative establishment quality by $H_t(\varphi) = \mathrm{P}(\phi_t > \varphi)$. The equation for motion of $H$ is given by:

$$
H_{t+1}\left(\frac{\varphi}{\zeta_t}\right) = \alpha_t H_t\left(\frac{\eta_t\varphi}{\iota}\right) + (1-\alpha_t)\,H_t\left(\frac{\varphi}{\iota}\right) \qquad \text{for } \varphi > \phi_t^*
\tag{A.14}
$$

where $\zeta_t \equiv [\alpha_t(\iota-1)+1]/\eta_t$ takes into account that an incumbent in expectation "slides down" in the relative quality distribution due to presence of entry advantage.

Conjecture that the countercumulative distribution takes the power-law form $H_t(\varphi) = \Gamma\varphi^{-k}$. Under this conjecture, the ergodic distribution satisfies:

$$\begin{aligned} \frac{\Gamma}{\left(\frac{\varphi}{\zeta_t}\right)^k} &= \frac{\alpha_t\Gamma}{\left(\frac{\eta_t\varphi}{\iota}\right)^k} + \frac{(1-\alpha_t)\,\Gamma}{(\eta_t\varphi)^k} \\ \zeta_t^k &= \frac{\alpha_t}{\left(\frac{\eta_t}{\iota}\right)^k} + \frac{(1-\alpha_t)}{(\eta_t)^k} \\ (\zeta_t\eta_t)^k &= \iota^k\alpha_t + (1-\alpha_t) \end{aligned} \tag{A.15}$$

Only two values of $k$ can satisfy the above equation: $k = 0$ and $k = 1$. The first case is a degenerate one where the distribution collapses to a point mass, which we disregard. Therefore, the only other possibility is that the BGP distribution is Pareto with shape parameter $k = 1$ and scale parameter $\Gamma = \phi^*$:

$$\mathrm{P}(\varphi \leq \phi) = 1 - \frac{\phi^*}{\phi} \tag{A.16}$$

Note however, that this conclusion only applies to the right of $\phi^*$, that is, we know that the distribution posesses a Pareto right tail. As the establishments located to the left of $\phi^*$ will exit anyway and do not invest in R&D, the effects on aggregate growth rate from this portion of the distribution are negligible and I opt to approximate the "true" relative quality distribution with Pareto distribution for the entire support.

## A.6 Solution of incumbents' problem along the BGP

First order and envelope conditions of incumbents:

$$B = \frac{\omega}{a} \frac{1}{(1-\alpha)^2} \frac{1}{\xi} \frac{\eta}{\iota - 1}$$
$$B = \left( \frac{1}{\sigma M} - \frac{\omega}{a} \frac{\alpha}{1-\alpha} \right) + \xi B \frac{\alpha(\iota - 1) + 1}{\eta}$$

Use $\zeta = [\alpha(\iota - 1) + 1] / \eta$ to simplify the envelope condition:

$$B = \frac{\frac{1}{\sigma M} - \frac{\omega}{a} \frac{\alpha}{1-\alpha}}{1 - \xi \zeta} \tag{A.17}$$

### A.6.1 "Partial equilibrium" approach

Rework the first order condition:

$$B = \frac{\omega}{a} \frac{1}{(1-\alpha)^2} \frac{1}{\xi} \frac{\eta}{\iota - 1} = \frac{\omega}{a} \frac{1}{\xi} \frac{1}{1-\alpha} \frac{\eta}{\iota - [\alpha(\iota - 1) - 1]} = \frac{\omega}{a} \frac{1}{\xi} \frac{1}{1-\alpha} \frac{\eta}{\iota - \zeta \eta}$$

Equate both expressions for $B$:

$$\begin{aligned}
\frac{\frac{1}{\sigma M} - \frac{\omega}{a} \frac{\alpha}{1-\alpha}}{1 - \xi \zeta} &= \frac{\omega}{a} \frac{1}{\xi} \frac{1}{1-\alpha} \frac{\eta}{\iota - \zeta \eta} \\
\frac{1}{\sigma M} - \frac{\omega}{a} \frac{\alpha}{1-\alpha} &= \frac{\omega}{a} \frac{1 - \xi \zeta}{\xi} \frac{1}{1-\alpha} \frac{\eta}{\iota - \zeta \eta} \quad \left| \quad \cdot \frac{a}{\omega}(1-\alpha) \right. \\
\frac{1}{\sigma M} \frac{a}{\omega} (1-\alpha) - \alpha &= \frac{1 - \xi \zeta}{\xi} \frac{\eta}{\iota - \zeta \eta} \\
\frac{1}{\sigma M} \frac{a}{\omega} - \alpha \left( 1 + \frac{1}{\sigma M} \frac{a}{\omega} \right) &= \frac{1 - \xi \zeta}{\xi} \frac{\eta}{\iota - \zeta \eta} \\
\alpha &= \frac{\frac{1}{\sigma M} \frac{a}{\omega} - \frac{1 - \xi \zeta}{\xi} \frac{\eta}{\iota - \zeta \eta}}{1 + \frac{1}{\sigma M} \frac{a}{\omega}}
\end{aligned} \tag{A.18}$$

Solution exists if:

$$\alpha \in [0, 1] \iff \frac{1}{\sigma M} \frac{a}{\omega} \geq \frac{1 - \xi \zeta}{\xi} \frac{\eta}{\iota - \zeta \eta} \tag{A.19}$$

### A.6.2 "Explicit solution" approach

Equate both expressions for $B$:

$$\begin{aligned}
\frac{\frac{1}{\sigma M} - \frac{\omega}{a}\frac{\alpha}{1-\alpha}}{1-\xi\zeta} &= \frac{\omega}{a}\frac{1}{(1-\alpha)^2}\frac{1}{\xi}\frac{\eta}{\iota-1} \\
\frac{1}{\sigma M} - \frac{\omega}{a}\frac{\alpha}{1-\alpha} &= \frac{\omega}{a}\frac{1-\xi\zeta}{\xi}\frac{1}{(1-\alpha)^2}\frac{\eta}{\iota-1} \\
\frac{1}{\sigma M} - \frac{\omega}{a}\frac{\alpha}{1-\alpha} &= \frac{\omega}{a}\frac{1-\xi\zeta}{\xi}\frac{1}{(1-\alpha)^2}\frac{\eta}{\iota-1}
\end{aligned}$$

$$\frac{1}{\sigma M}\frac{a}{\omega}(1-\alpha)^2\xi(\iota-1) - \alpha(1-\alpha)\xi(\iota-1) = \left(1 - \xi\frac{\alpha(\iota-1)+1}{\eta}\right)\eta$$

Define:

$$X \equiv \frac{1}{\sigma M}\frac{a}{\omega}\xi(\iota-1) \tag{A.20}$$

$$\begin{aligned}
X\left(1-2\alpha+\alpha^2\right) - \left(\alpha-\alpha^2\right)\xi(\iota-1) &= \eta - \xi(\iota-1)\alpha - \xi \\
\alpha^2(X+\xi(\iota-1)) + \alpha(-2X - \xi(\iota-1) + \xi(\iota-1)) + (X+\xi-\eta) &= 0 \\
\alpha^2(X+\xi(\iota-1)) + \alpha(-2X) + (X+\xi-\eta) &= 0
\end{aligned}$$

Solve:

$$\alpha = \frac{X \pm \sqrt{X^2 - (X+\xi(\iota-1))(X+\xi-\eta)}}{(X+\xi(\iota-1))} \tag{A.21}$$

Existence of real roots is assured:

$$\begin{aligned}
X^2 - (X+\xi(\iota-1))(X+\xi-\eta) &> 0 \\
X^2 - \left(X^2 + X(\xi-\eta) + X\xi(\iota-1) + \xi(\iota-1)(\xi-\eta)\right) &> 0 \\
-(\underbrace{X(\xi\iota-\eta)}_{<0} + \underbrace{\xi(\iota-1)}_{>0}\underbrace{(\xi-\eta)}_{<0}) &> 0
\end{aligned} \tag{A.22}$$

since $\iota > \xi\iota > \eta > 1 > \xi$.

Uniqueness is also almost certain. Consider :

$$\begin{aligned}
\frac{X+\sqrt{X^2-(X+\xi(\iota-1))(X+\xi-\eta)}}{(X+\xi(\iota-1))} &> 1 \\
X+\sqrt{X^2-(X+\xi(\iota-1))(X+\xi-\eta)} &> X+\xi(\iota-1) \\
\sqrt{-(X(\xi\iota-\eta)+\xi(\iota-1)(\xi-\eta))} &> \xi(\iota-1) \quad | \quad .^2 \\
-(\underbrace{X(\xi\iota-\eta)}_{<0}+\underbrace{\xi(\iota-1)}_{>0}\underbrace{(\xi-\eta)}_{<0}) &> \xi^2(\iota-1)^2 \approx 0 \qquad \text{(A.23)}
\end{aligned}$$

We know from Equation A.22 that the LHS of above equation is positive. The above inequality is likely to hold since $(\iota-1)^2 \approx 0$.

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