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APPLYING HURST EXPONENT IN PAIR TRADING STRATEGIES

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Applying Hurst Exponent in Pair Trading Strategies

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Abstract: This research aims to seek an alternative approach to pair selection for the purpose of pair trading strategy. We try to build an effective pair trading strategy based on 103 stocks listed in NASDAQ-100 index. The dataset has daily frequency and covers the period from 01/01/2000 to 31/12/2018. In this study, Generalized Hurst Exponent, Correlation and Cointegration are employed to detect the mean-reverting pattern in the time series of linear combination of each pair of stock. The result show that Hurst method cannot outperform the benchmark, which implies that market is efficient. This result is quite sensitive to varying number of pairs traded and rebalancing period and less sensitive to financial leverage degree. Moreover, Hurst method is better than cointegration method but is not superior as compared to correlation method.

Keywords: generalized Hurst & Hurst exponent, algorithmic trading strategies, portfolio choice, mean-reversion strategy, pair trading, correlation & cointegration trading, efficient market hypothesis

JEL codes: C4, C14, C15, C52, C53, C58, G11, G12, G14

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1. Introduction

Trading strategy refers to a designed plan to achieve risk-adjusted returns by going long or short on an asset or portfolio of assets in the market. Trading strategies have become very popular amongst investors. The examples of them are: long/short equity, pair trading, momentum/contrarian trading, etc. Institutional investors usually build more complex trading models which are confidential and not disclosed to outsiders. Trading strategy can be based on fundamental analysis, in which the strategy is built on the expectation that price level will move to the predicted level according to fundamentals such as GDP growth, inflation level and other macroeconomics factors; or on technical analysis, in which the direction of prices is forecasted by inspection of past market data like prices and volume in order to identify price patterns and market trends in the financial world; or on both fundamental and technical analysis. Trading model is usually verified by back-testing, in which we build and apply the model on empirical past data in the in-sample period and verify its performance in out-of-sample period.

Trading strategy is often characterized by momentum or Mean-reverting pattern. Momentum refers to the fact that the prices are trending, so it is expected that prices with upward trend will continue to grow and prices with downward trend will keep falling. The slow spread of new information is the main source of Momentum effect. In other words, the price is pushed in the same direction as more people buy or sell a stock when a certain news, such as quarterly or annually earnings is announced. Another common source of Momentum effect is the incremental execution of a large order of institutional investors. However, it is hard to identify whether this source is the cause of observed Momentum due to the sophistication of execution system adopted by large brokerages. Herd-like behavior of investors which refers to the fact that investors make their own trading decisions based solely on the buying or selling decisions of others is also a source of Momentum. In contrast, Mean-reverting assumes that asset prices will be reverted to a reference price (usually mean price), so if the price is currently high, it is expected to decrease (revert back to the mean) and vice versa. In practice, stock prices on average are close to random walk, which makes it difficult to model. However, under some special condition and depending on time horizon, stock prices can exhibit some degree of trending and mean-reversion.

Determining the price patterns - mean-reverting or momentum, under certain conditions and for a certain period is the essential core of building a trading model. Hurst exponent, which was originally developed in hydrology for the practical matter of determining the optimum size

of dam for the Nile river by Harold Edwin Hurst, can help us classify the pattern of time series of prices under a certain time horizon. Hence, Hurst exponent can serve as a criterion to select either momentum or mean-reverting strategy. As such, this paper introduces a method based on Hurst exponent to effectively select pairs from a portfolio of stocks for pair trading strategy. As we know, pair trading takes advantage of the fact that the linear combination of 2 assets' prices may be stationary, hence we can apply mean-reverting strategy on the price spread of such pairs to make profit. Other popular methods to select pairs are correlation and cointegration, so we measure the correlation between the returns of 2 assets or check the cointegration between 2 times series of asset prices and decide which pairs should be selected. In this research, we compare the performance of all 3 methods to verify which method is superior to others.

In that context, the paper addresses two main hypotheses: (1) Does the Efficient Market Hypothesis work? i.e. whether Hurst Exponent can generate a trading strategy that can beat the market; and (2) Does the method to select pairs based on Hurst exponent outperform the methods based on correlation or cointegration? Additionally, based on these two main hypotheses, a few research questions are constructed: (1) Is the result obtained robust to varying number of pairs selected? (2) Is the result obtained robust to varying rebalancing period? (3) Is the result obtained robust to varying degree of financial leverage?

In order to answer the main hypotheses and research questions mentioned above, an empirical research is conducted based on 103 stocks listed in NASDAQ-100 updated until 31/12/2018. The period of this dataset is ranging from 01/01/2000 to 31/12/2018. The data is collected on daily basis. The pairs are formed from the pool of 103 stocks and try to apply different methods to select pairs that possibly have mean-reverting pattern. There are three selection methods that are applied in this paper: Hurst exponent, correlation and cointegration. The Hurst exponent is calibrated using Generalize Hurst Exponent (GHE) method. For examining correlation, the Pearson correlation coefficient is employed and for examining cointegration, Engle – Granger two-step method is implemented. Furthermore, the technique to generate buy/sell signal when we apply pair trading strategy on each pair of stocks is breakout volatility model, in which the upper and lower thresholds are calculated based on the exponential moving average and rolling standard deviation of the price spread for each pair. The performance of our strategy is evaluated based on five criteria: annualized return compounded, annualized standard deviation, maximum drawdown, information ratio and adjusted information ratio.

We expect that Hurst exponent can help build a trading model that can outperform the market. Besides, it is expected to generate a superior pair selection model as compared to correlation and cointegration method. On the other hand, as pair trading strategy has market neutrality feature, we expect its cumulative returns to be better than the market in recession period but worse than the market in expansion period. In addition, the returns gained by pair trading should be less volatile than benchmark returns i.e. its standard deviation and maximum drawdown will be lower than those of the market index.

The paper is structured as follows: Section II is an overview of literature about Hurst exponent, correlation, cointegration and Efficient Market Hypothesis. Section III describes all the details concerning the data set. Section IV explains the methodology of Hurst exponent calibration as well as the methods to select pairs based on Hurst exponent and also how the pair trading strategy is constructed and the technique to generate buy/sell signal for such strategy. Section V discusses the empirical results. Section VI conducts some sensitivity analysis to determine whether the result is robust to changes in initial assumptions. Section VII draws conclusions and suggests some extensions for future research.

2. Literature review

2.1 Hurst Exponent

Hurst exponent is a statistical measure which is used to study scaling properties in time series. The scaling property in time series refers to the patterns in financial asset prices which are repeated at different time scales and is studied in a number of past researches. In 1930s, the work of [Elliot](#) has pointed out the appearance of price patterns at different time horizon. The Brownian motion, originally proposed by [Bachelier \(1900\)](#), was one of the most popular models to study the scaling properties and has been developed and modified by several researchers such as the fractional Brownian motion ([Mandelbrot 1997](#), [Clark 1973](#)) or Lévy motion ([Mandelbrot 1967](#), [Fama 1965](#)).

Hurst exponent measures the long-term memory and fractality of a time series. It is known ([Peters, 1994](#)) that a Hurst value of 0.5 implies a Brownian random process; a Hurst value between 0.5 and 1 indicates trending (or momentum) behavior and a Hurst value between 0 and 0.5 indicates anti-trending (or mean-reverting) behavior of the time series. Especially, the more Hurst value is closer to 1, the higher the risk that there will be great and unexpected changes in the direction of time series. There are a number of techniques which can be exploited to study scaling properties such as the re-scaled range analysis (R/S analysis) and its

complement (Hurst et al., 1965); the modified R/S analysis (Lo, 1991); the detrended fluctuation analysis (DFA) (Ausloos, 2000), generalized Hurst exponent, etc. Hurst initially applied the R/S analysis to estimate long-term dependence of water levels in rivers and reservoirs. According to Matteo et al. (2005), this method can disclose the long-run correlation in random processes and help distinguish non-correlated from correlated time series. However, if the time series presents short memory, heteroskedasticity and multiple scale behavior, the original R/S approach proposed by Hurst may cause a problem and is also sensitive to outliers as it is calculated based on maxima and minima. Hence, several alternative approaches have been proposed, one of which is the modified R/S analysis by (Lo, 1991) that can detect long-term memory in the presence of short-term memory. Besides, the generalized Hurst exponent explores the scaling properties based on q^{th} order moments of distribution, and also includes in the algorithm many types of dependence in the data. Therefore this method is not affected by outliers and the type of dependence and can address the issues of R/S Analysis no matter it is long or short-term dependence.

There are a number of papers that present the application of Hurst exponent in finance. According to Matteo, et al. (2003), a powerful tool to describe and differentiate the scaling structure of different markets is the generalized Hurst exponent $H(q)$. The scaling structure of markets is an evolving quantity which can assist in differentiating different development stage among markets and portraying the overall fluctuation of market conditions. Morales, et al. (2012) used exponential smoothing method to compute weighted generalized Hurst exponent (wGHE) to study the scaling behavior in time of log-returns of bailed-out companies and found out that wGHE gives signal about the stability of a company and that we could assess risk by tracking its value in time and that an increase in multifractality signifies the occurrence of crisis. Qian and Rasheed (2004) analyzed the Hurst exponent in many rebalancing periods for Dow-Jones index. The results suggest that the periods with large Hurst exponent can be predicted more accurately than those with Hurst value closes to 0.5 (random series), which implies that stock markets are not always random – in some periods it can present strong persistence pattern which is predictable. Moreover, they noticed that Hurst exponent can be used as a guide for data selection before any forecasting as it gives us a measure for predictability. Kopeliovich (2013) uses Hurst exponent as a predictability measure for VIX index. It is observed that Hurst exponent of VIX index is stable and consistently less than 0.5 over the long period of time, which means that VIX index is a mean-reversion time series and can be predicted. Hence, they

apply various trading strategies on VIX index and the actual result corresponds to their expectation: it gains positive profit and outperforms the benchmark strategy.

2.2 Correlation

The correlation is defined as a measure of the relationship between the changes of two or more financial variables over time. The most popular measure of correlation is Pearson correlation coefficient which measure the linear correlation between 2 variables and takes the value between -1 and +1, with -1 means perfectly negatively correlation; +1 means perfectly positively correlation and 0 means no correlation. Correlation plays a key role in Capital Asset Pricing Model (CAPM) as it indicates the potential for the diversification of the portfolio and such high diversification between assets improves the return/risk ratio.

The Correlation has a wide range of application, one of which is in portfolio management and risk assessment. [Lee \(2003\)](#) applies [Spurgin et al. \(2000\)](#) model, which is a model of correlation shifts with market return as independent variable, on 31 real estate market segments in the UK. The results show both significant negative and positive correlation shifts in different market segments. Depending on this piece of information, fund managers can effectively make decision on portfolio allocation based only on expected market volatility and can improve overall portfolio performance during different stages of real estate cycle. [Kalotychou, et al. \(2009\)](#) estimates the value of correlation dynamics in asset allocation using mean-variance analysis. They tested the correlation-timing framework models against industry correlation and found out how superior statistical properties of multivariate conditional correlation models account for different aspects of the correlation dynamics. They noticed that optimizing portfolios based on multivariate conditional correlation models enhances investment performance and make significant economic gains which are not deteriorated by transaction cost. On the other hand, the research by [García-Álvarez and Luger \(2011\)](#) assesses performance of different multivariate dynamic correlation models applied by an active portfolio manager on a portfolio of international equities for the purpose of rebalancing it on a daily basis. The results show that there is a considerable fluctuation of international correlations day to day, and the dynamic correlation models present stable parameter estimates overtime. Unfortunately, the portfolio performance based on these models is not statistically better than that based on the naive diversification benchmark strategy. Besides, it also points out that including risk-free security in the portfolio is crucial regardless of the level of market volatility.

Another application of correlation is in trading strategy. [Meissner \(2015\)](#) shows the analysis of opportunities and limitations of six correlation trading strategies which is most popular in financial practice, including Empirical Correlation Trading, Pairs Trading, Multi-asset Options, Structured Products, Correlation Swaps, and Dispersion trading. Most strategies associate with trading an underlying asset, so we have to control for the levels and volatilities of the underlying, except for pairs trading and variance dispersions which are pure correlation plays, hence they make correlation trading simpler. According to [Riedinger \(2017\)](#), the two factors that influence profitability of pair trading strategy via the return per trade and trading frequency are pair volatility and correlation. There is a trade-off between the improvement of return per trade and trading frequency, that is, high pair volatility and low correlation are favorable for the return per trade but unfavorable for the trading frequency. Hence, the total returns of pair trading depend on the selection of pair volatility and correlation levels, which should be optimized stock selection process to maximize the returns

2.3 Cointegration

Cointegration, roughly speaking, describes the long-run relationship between two or more non-stationary but integrated at the same order time series, so we expect that any deviations from this long-run relationship are non-stationary. In a formal definition ([Engle & Granger, 1987](#)), time series x_t and y_t are cointegrated of order $d; b$ (where $d \geq b > 0$) if: (1) both time series are integrated of order d and (2) there exists a linear combination of these variables, $a_1x_t + a_2y_t$, which is integrated of order $d - b$. It can be formally expressed as $x_t; y_t \sim CI(d; b)$ and vector $[a_1, a_2]$ is called cointegrating vector. A typical example for 2 cointegrated time series is a stock market index and its associated future contracts, both are integrated at order one $I(1)$ but their linear combination forms a stationary time series $I(0)$. Cointegration is one of the crucial characteristics in time series analysis. A time series often have either deterministic or stochastic trends. Typical example of stochastic time series which also integrated of order 1 is US macroeconomic time series such as GNP, wages, employment, etc.

[Yule \(1926\)](#) first introduced and analyzed the concept of spurious or non-sense regression in 1926. [Granger and Newbold \(1974\)](#) showed that using linear regression on non-stationary time series data is a dangerous approach that could lead to spurious correlation – untrue correlation. The paper by Granger and Engle in 1987 established the cointegrating vector approach and coined the term “cointegration”. [Granger and Newbold \(1974\)](#) presented that detrending the integrated $I(1)$ series does not solve the problem of spurious correlation and that

checking for cointegration is a superior alternative as cointegration only exists if there exists a genuine relationship between two $I(1)$ series. If we run a regression model on two $I(1)$ time series to test their relationship, it's possible for the regression to generate spurious correlation. Usually, the procedure for testing hypotheses concerning the relationship between non-stationary time series is to run ordinary least square (OLS) regression on differenced data. However, with the possible presence of cointegration between two non-stationary variables, this method is biased.

There are 3 methods for cointegration test: Engle – Granger two-step method; Johansen test and Phillips–Ouliaris cointegration test. According to the Engle-Granger method, residuals are extracted from the static regression run on two non-stationary variables, then the residuals are tested for stationarity using ADF or KPSS test. We expect that the residuals will be stationary if the cointegration between two time series exists. The major issue in Engle-Granger method stays in the choice of dependent and independent variable when we run the regression to extract residuals as it may lead to inconsistent conclusions ([Armstrong, 2001](#)). This issue may be addressed by alternative tests such as Phillips-Ouliaris and Johansen's. The [Phillips – Ouliaris \(1990\)](#) residual-based unit root test is an enhancement over Engle – Granger test. Before 1987, the assumption underlying cointegration tests is that the regression residuals are independent from common variance, which is rarely true in practice ([Chaovalitwongse et al., 2010](#)). Phillips-Oularis test resolves this issue by taking into account supplementary variability. The result generated from this test is also robust to the choice of dependent and independent variables. Besides, Johansen's cointegration test also allows for avoiding the selection of dependent – independent variables and can detect multiple cointegrating vectors in case there are more than two time series to be tested. In this research, the Engle – Granger method is employed to recognize cointegration. The detail of this method is discussed later in the methodology section.

Cointegration is usually employed in trading strategy. [Rad, et al. \(2016\)](#) examines the performance of three pair trading strategies – the distance, cointegration and copula methods – on the US equity market in the period 1962 – 2014 with dynamic trading costs. They find that the monthly return of distance method is slightly higher than that of cointegration method, but the Sharpe ratio of cointegration method is a bit higher than that of distance method. On a risk-adjusted basis, the performance of these two strategies are the same over full sample period. The performance of copula method is not as good as the other two methods on both economic and risk-adjusted basis, but it shows better result for its unconverged trades. Moreover, all

strategies perform well during high volatile periods, but the cointegration method proved to be superior during turbulent market conditions. [Caldeira and Moura \(2013\)](#) proposed pair trading strategy for stocks of Sao Paulo stock exchange. The strategy is expected to make profit based on the cointegration between each pair of stocks. They apply cointegration test on all possible pair combinations to select the pairs that have long-term relationship, as the spread of those pairs will have mean-reverting pattern – a key factor to make pair trading strategy profitable. The result show that the cumulative new profit for out-of-sample was 189.29%, with annual mean of 16.38%. In addition, pair trading strategy exhibits low volatility level and insignificant correlation to market index Ibovespa, which verifies the market neutrality feature of this strategy. [Alexander and Dimitriu \(2002\)](#) demonstrate several trading strategies based on cointegration including classic index tracking strategy, long-short equity market neutral strategy and combination of these two strategies. According to the result, index tracking strategy significantly outperforms market index in recession period such as Russian crisis (1998) or the burst of technology bubble (2000 – 2002). The return of long-short market neutral strategy shows insignificant correlation with market return and it sometimes gains better Sharpe ratio than the market. However, the profit of long-short market neutral strategy is easily harmed by transaction costs. Furthermore, the combination of index tracking and long-short strategy can significantly enhance the individual characteristic of each basic strategy.

2.4 Efficient Market Hypothesis

Efficient Market Hypothesis ([Fama, 1970](#)) states that markets are efficient in the sense that the current stock prices reflect completely all currently available information that could anticipate future prices, i.e. there is no information hidden that could be used to predict future market development. This model is based on the assumption that market changes can be represented by a normal distribution. The underlying implication of Efficient Market Hypothesis is that investors cannot consistently outperform the market on a risk-adjusted basis by employing any complex stock selection or market timing process. Investors can only achieve higher returns by chance or by investing in riskier assets. Only when the advancement in technology allows computers to work faster and the comparison of prices of hundreds of stocks became quick and efficient in 1960s, the efficient market hypothesis started to be popular. F.A. Hayek (1945) pointed out that market was very effective in collecting information dispersed among individual within society. This is because individual traders contribute to the efficiency of market prices by acquiring and acting on the private information which they can earn profit from. Hence, market prices only shift in response to news and reflect all available information.

The random walk hypothesis, which claims that stock market prices is a random walk process and cannot be predicted, has a close connection with EMH. There are several empirical researches that indicated that US stock prices and other financial time series followed random walk process in the short-terms although it may be predicted in the long-terms. However, the reason for this predictability is due to rational time-varying risk premia or behavioral reason still remains in debate. Moreover, professional investors in general cannot beat the market and keep earning excess returns (Cowles and Jones, 1937).

There are three forms of market hypothesis: weak-form efficiency, semi-strong-form efficiency and strong-form efficiency. The main difference between those forms lies in the assumptions on how market works. In the weak-form efficiency, we cannot forecast future prices based on historical prices. Applying technical analysis techniques on investment strategies cannot help investors consistently earn excess returns, although some fundamental analysis may still generate excess returns. There is no autocorrelation in stock price time series and hence no patterns in asset price, which implies that stock prices follow random walk process. The future price movement is determined totally by information outside the price series itself. However, a number of academic literature study the momentum effect identified by Jegadeesh and Titman (1993), which is an anomaly phenomenon in price patterns. Momentum effect refers to the fact that the trend in prices is persistent, that is stock prices that go up (go down) in that past 3 to 12 months will continue to keep this trend in the next 3 to 12 months. Hence, investors can long increasing stocks and/or short decreasing stock to earn positive risk-adjusted returns. This momentum strategy produces strong evidence against the weak-form efficiency of market hypothesis as it is based completely on the past prices to build profitable investment. Besides, stock returns of many countries, industry returns and equity market indices present momentum effect in their patterns. Fama also accepted momentum effect as an anomaly premier.

In semi-strong form efficiency, it states that share prices quickly and unbiasedly reflect the publicly available information, so no one can earn excess return by trading on this piece of information. Thus, the market will absorb all public relevant information and adjust share prices to the new equilibrium accordingly to reflect the shift in supply and demand induced by the new information. The implication under semi-strong form efficiency is that we cannot earn excess return by applying either fundamental or technical analysis techniques.

In strong-form efficiency, all public and private information are reflected in share prices, so no one can make profit from that information. If someone holds a private information and is aware that the market price is currently under-estimated, they will act on this piece of information by purchasing the stocks, which will drive the price up to a new level according to supply – demand law. At this point, they will exit from the position as there is not motivation to continue buying and the stock price will stay stable at the new equilibrium.

Inefficient Market Hypothesis ([Shleifer, 2000](#)) was developed later as some anomalies in the market cannot be explained by efficient market hypothesis. The work by Shleifer describes an alternative approach – the behavioral finance – to study how the financial market works. This approach suggests that investors may experience some stress and anxiety due to the overload of information amount while making investment decision, hence causing them to make irrational decisions and leading to the deviation from the norms of the stock price movements. This concept can explain some anomalous phenomenon in market behaviors such as short-term momentum, long-term reversal, weekend anomaly and value premium.

More recent attempts showing that the market is inefficient analyze the topic from the perspective of algorithmic investment systems. In such approaches, the buy/sell signals were based on both classic investment techniques enriched with new asset classes ([Latoszek and Ślepaczuk, 2020](#), [Jablecki et al., 2015](#)) and machine learning methods ([Kijewski and Ślepaczuk, 2020](#)) showing that it is possible to build investment systems with abnormal risk-weighted returns.

3. Data Description

The data set includes 103 stocks of companies listed in NASDAQ 100 (updated until December 2018) and the NASDAQ 100 index itself, which is used as the benchmark for our strategy. The data frequency is daily. The period is from 01/01/2000 to 31/12/2018, so the sample size for each stock is 4778 observations, in total making 496 912 observations for the whole data set. The data for each stock was carefully investigated by graph and cleaned for outliers if any. Basic statistics of all the stocks were also gathered for cleaning data purpose.

3.1 Problem with survivorship bias

Our data set includes only all the stocks which are listed in NASDAQ 100 index on 31/12/2018 because we cannot obtain the full constituents of NASDAQ 100 during the period 2000 – 2018 easily or it is too costly to get access to such information. Hence all the stocks which are delisted

during the period 2000 – 2018 are excluded from the data set, which may induce a problem called survivorship bias. In this paper, as we cannot fix this problem due to lack of data and information, we discuss the problem here and evaluate its impact on our result.

Survivorship bias is defined as “*the logical error of concentrating on the people or things that made it past some selection process and overlooking those that did not, typically because of their lack of visibility*”. In other words, a historical database suffers from survivorship bias if it does not include stocks that disappeared because of bankruptcy, delisting, merger or acquisition. Hence, only surviving stocks stay in the database, leading to the overestimation of historical performance when we backtest a strategy based on such database. (Ernest Chan, 2008). In particular, testing of mean-reverting strategy type such as pair trading can be vulnerable to the effects of survivorship bias. This is because mean-reverting strategy would short very-high-price stocks and long very-low-price stocks, but these stocks are likely to be either acquired (for high valued stocks) or go bankrupt (for low valued stocks), hence leading to loss in both cases. However, if the historical database is characterized by survivorship bias, these stocks may not be included in that database, thus overestimating the test performance.

There are a number of papers analyzing the impact of survivorship bias, especially on hedge fund performance. Lo (2001) shows the analysis of impact of survivorship bias, stating that entire cross section of funds is influenced and the impact is magnified over time, hence it can result to significant misleading for investors that are not aware of this fact when building an optimal portfolio of hedge fund. Brown, et al. (1999) estimated the impact of survivorship bias on offshore hedge funds and recorded an annual attrition rate of 14% over the period from 1989 to 1995. The result shows that survivorship bias leads to 3% higher in annual returns, which agrees with the estimation by Fung and Hsieh (2000) on TASS database¹ in the period 1994 – 1998. On the other hand, Ackermann, et al. (1999) evaluated only 0.2% survivorship bias. The gap in these estimates may be due to the difference in structure of databases and time horizon (Liang, 2000). According to Barry (2003), the impact of survivorship bias on TASS database during the period 1994 – 2001 is 3.8%, higher than the estimates by Fung and Hsieh (2000) due to three extra years of return data.

In the context of this paper, the exact effect of survivorship bias on the strategy performance cannot be characterized in details. However, when we compare the result with the benchmark, we can approximately evaluate whether the strategy really outperforms the

¹ TASS database is a recognized database that contains monthly hedge fund return data.

benchmark by looking at the difference in the risk-adjusted returns. Moreover, we can be sure that the results of this research would be worse in terms of annual risk-adjusted returns when survivorship bias would be omitted.

4. Methodology

4.1 GHE methodology for calibration of Hurst Exponent

GHE, a generalization of original method introduced by [Hurst \(1951\)](#), is one of the most popular technique for Hurst exponent calibration. It is an effective tool that applies the scaling of q^{th} – order moments of the increments distribution in order to investigate the scaling properties of time series ([Barabasi and Vicsek, 1991](#), [Mandelbrot, 1997](#)). Compared to Rescaled Range (R/S) analysis which takes into account the maxima/minima of data, it is less sensitive to the outliers. The Generalized Hurst Exponent verifies whether some statistical properties of data adjust with both number of observations and the time resolution.

To do this, we first define a time series $X(t)$ with $t = 1, 2, \dots, k$. The statistic $K_q(\tau)$ is calculated according to the following formula:

$$K_q(\tau) = \frac{\text{Mean}(|X(t + \tau) - X(t)|^q)}{\text{Mean}(|X(t)|^q)} \quad (1)$$

where τ can vary between 1 and τ_{\max} , while τ_{\max} is usually chosen as a quarter of the length of the series.

The GHE characterizes the scaling properties of time series $X(t)$ and hence is associated with the scaling behavior of the statistic $K_q(\tau)$. Given the power-law, the GHE is calculated as:

$$K_q(\tau) \propto \tau^{qH(q)} \quad (2)$$

where \propto denotes direct proportionality. Given two variables x and y , y is directly proportional to x if there is a non-zero constant k such that $y = kx$.

The GHE method is appealing for the fact that all of the information about scaling properties are reflected in the only one measure $H(q)$, which makes the analysis and interpretation of Hurst exponent simpler. The Hurst exponent is involved with different characteristics for each value of q . In specific, $H(q = 1)$, which should be close to the original Hurst exponent H , illustrates the scaling properties of absolute deviation of time series. On the other hand, the value of $H(q = 2)$ scales with the autocorrelation function of the increments

($C(t, \tau) = \langle X(t + \tau)X(t) \rangle$) and has connection with the power spectrum and is crucial if we are interested in investigating long-range dependence ([Ramos-Requena, et al., 2017](#))

In this paper, we choose $q = 1$ to calculate the Generalized Hurst Exponent as we are not concerned about the multi-fractal feature of GHE.

4.2 Pair Selection based on Hurst Exponent

From the 103 stocks of NASDAQ-100, we form pairs of stock, so the total number of possible pairs is 5253 pairs. For each possible pair, we calculate the spread according to the formula:

$$\text{Spread} = \ln(P_A) - h * \ln(P_B) \quad (3)$$

where P_A and P_B are the prices of stock A and B; and

$$h = \text{std}(\ln(\text{ret_A})) / \text{std}(\ln(\text{ret_B}))$$

where std is standard deviation and ret_A and ret_B are the log returns of stock A and B.

When we buy the pair, it means that for each share of A that we buy, we short sell h shares of B. According to the above formula to estimate h , we will have the same volatility for both the position in A and in B.

We then calculate the Generalize Hurst Exponent at $q = 1$ for each spread and select N pairs of stock whose spread has the lowest Hurst Exponent, which indicates the strong mean reversion characteristic of the spread - a key condition for our pair trading strategy to make profit. We can vary the number of pairs parameters (N) to estimate the robustness of our result. The selection of pair will be made every 6 months, so at the beginning of each 6 month, we conduct the calculation of GHE based on the data of past one-year (past 252 observations) and select 10 ($N = 10$) pairs with lowest Hurst exponent to be traded during that 6 months. This process will be repeated from the beginning of 2002 until the end of 2018

4.3 Pair selection based on Correlation

We employ the Pearson correlation to gain the correlation for the two series of log returns. The Greek letter ρ (rho) is commonly used to represent the Pearson's correlation coefficient of a population. Given a pair of random variables (X, Y), the formula for ρ is:

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} \quad (4)$$

or

$$\rho_{X,Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y} \quad (5)$$

where μ_X and μ_Y are the mean of X and Y and σ_X and σ_Y are their standard deviation.

The Pearson sample correlation coefficient is commonly represented as r_{XY} . We can get a formula for r_{XY} by replacing the estimates of sample covariances and variances into the formula above. Given paired data $\{(x_1, y_1), \dots, (x_n, y_n)\}$ consisting of n pairs, r_{XY} is defined as:

$$r_{X,Y} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}} \quad (6)$$

In this context, we would like to use the correlation formula for sample. Similar as pair selection based on Hurst exponent, we will conduct calculation of correlation based on the data of past one year (past 252 observations) at the beginning of each 6 months during the period 2002 – 2018. The 10 pairs that have highest correlation would be used in pair trading strategy during that 6 months.

4.4 Pair selection based on Cointegration

Similar to Hurst exponent and Correlation, we select pair based on Cointegration between 2 time series of prices. We employ Engle – Granger two-step approach to test for cointegration. The pair selection is conducted every 6 months and 10 pairs with the lowest KPSS test statistics, which indicates higher chance of cointegration between 2 time series, would be used to trade during that 6 months. The detail of Engle – Granger approach is as follows:

Step 1: check whether 2 time series is integrated at the same order d using Augmented Dickey-Fuller test. For stock prices, they are normally integrated at order 1.

$$X, Y \sim I(d) \quad (7)$$

The ADF test is applied to the model:

$$\Delta y_t = \alpha + \beta t + \omega \cdot y_{t-1} + \sum_{i=1}^k \delta_i \Delta y_{t-i} + \varepsilon_t \quad (8)$$

The null hypothesis is that the series is non-stationary and alternative hypothesis is that the series is stationary:

$$H_0 : \omega = 0 \text{ (non-stationary)}$$

$$H_1 : \omega < 0 \text{ (stationary)}$$

The test statistic is calculated as follow:

$$DF_{\tau} = \frac{\hat{\omega}}{SE(\hat{\omega})} \quad (9)$$

As this is a lower-tail test, we reject the null when test statistic is lower than critical value (which is normally -2.86 for 5% significance level)

Step 2: Estimation of the cointegrating vector using standard OLS regression and testing whether residuals obtained in the regression are stationary using KPSS test.

$$Y_t = \alpha + \beta X_t + \varepsilon_t \quad (10)$$

$$\rightarrow \varepsilon_t = Y_t - \alpha - \beta X_t \quad (11)$$

$$\rightarrow \text{Check: } \varepsilon_t \sim I(0)$$

KPSS stationary test is quite similar to ADF test but more powerful. The most important difference between KPSS and ADF test stays in the null hypothesis. For KPSS test, the null hypothesis is that the series is stationary and the alternative hypothesis is that the series is non-stationary. Hence, it's an upper-tail test and we reject the null when our test statistic is higher than the critical value, which is normally 0.463 for 5% significance level.

4.5 Trading strategies Criteria

We use breakout volatility for each series of spread to generate signal to go long or short the pair. The upper and lower threshold are calculated as follow:

$$\text{Upper threshold} = \text{EMA}_n(\text{spread}) + m * \text{rolling_std}_k(\text{spread}) \quad (12)$$

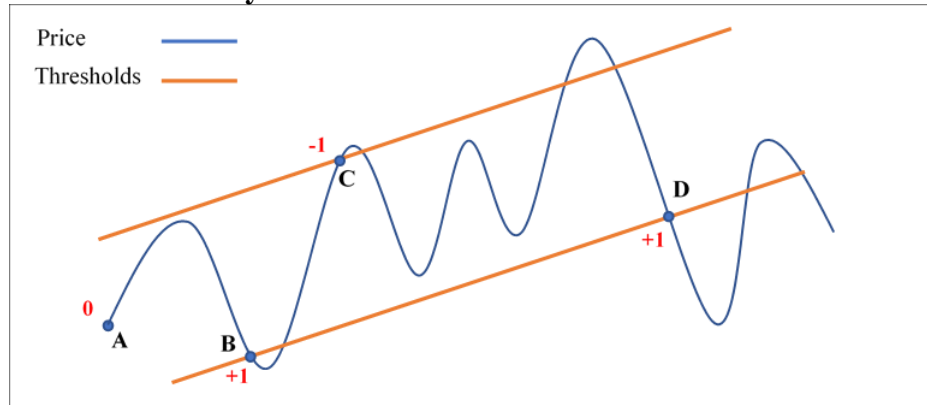
$$\text{Lower threshold} = \text{EMA}_n(\text{spread}) - m * \text{rolling_std}_k(\text{spread}) \quad (13)$$

where EMA_n is the rolling Exponential Moving Averages of the spread of each pair with certain window size n ranging from 10 to 180 (days), rolling_std_k is the rolling standard deviation of

the spread with certain window size k ranging from 10 to 120 (days), multiplier m takes a certain value ranging from 0.5 to 3.

Note that the window size of EMA (n), the window size of rolling standard deviation (k) and multiplier m are 3 parameters that will be optimized i.e. the set of parameters that give the best performance on historical data up to the moment of selection will be chosen. This optimization method would prevent our results from suffering from forward looking or look-ahead bias. Forward looking bias refers to the fact that we use the data or information from the future to backtest or analyze the current situation. Forward looking bias may artificially inflate our test result of trading strategy. In the context of this research, if we optimize parameters for breakout volatility model based on the whole sample of the dataset and then apply the optimized parameters to deduct the performance of our strategy, our result will be inaccurate as it suffers from look-ahead bias. To avoid this situation, we will use the data available up to the moment of selection to optimize parameters i.e. we use data of the first rebalancing period to get optimized parameters and apply these parameters for calculating performance statistics of second rebalancing period, then we use data of the first and second rebalancing period to get optimized parameters and apply these parameters for the third period, and so on.

For pair trading strategy, we will short the recent winners (the price is increasing) and long the recent losers (the price is decreasing) as we believe that their prices will behave according to mean-reverting pattern. It means that the current high price will go down and the current low price will go up in the future. In general, the rule for the entry and exit is as follow: if our position is being flat at time $t-1$, we will short the pair if the spread reaches upper threshold or long the pair if the spread reaches lower threshold at time t ; if our position is being long at time $t-1$, we only switch to short position if the spread reaches the upper threshold at time t ; if our position is being short at time $t-1$, we only switch to long position if the spread reaches the lower threshold at time t . Figure 1 illustrates the rule of breakout volatility model as described above. We can see that initially at point A, our position is being flat (0), then we switch to long position (+1) when the price reaches lower threshold at point B, then we switch to short position (-1) when the price reaches upper threshold at point C, and we just that keep short position and only switch to long position when the price reaches lower threshold at point D.

Figure 1. Breakout volatility model illustration

Note: Point C and D indicate sell and buy signal based on the breakout volatility channel rule.

The initial investment is assumed to be \$10 000 at the beginning of rebalancing period. This investment is divided equally for N pairs, so each pair takes up $\$10\,000/N$. In our case, we have 10 pairs, so the amount of \$1000 is invested in each pair. The transaction is assumed to be 0.01% of value of the pair. So, the transaction cost for trading 1 pair is $0.01\% * (\text{Price_A} + h * \text{Price_B})$. Moreover, we can also take advantage of financial leverage to improve the returns of pair trading strategy. So, instead of investing $\$10\,000/N$, we can invest $\$10\,000/(N/2)$ or $\$10\,000/(N/4)$ in each pair by borrowing money. The financial leverage can help us magnify the positive returns, but it also magnifies the negative returns if we make loss. Moreover, the double or quadruple times of degree of financial leverage will definitely make the equity curve more volatile.

For the benchmark, we just use buy & hold strategy on NASDAQ-100 index in the period 2002 – 2018 and compare our strategy's performance with this benchmark.

4.6 Summary of assumptions made for the purpose of trading strategy

Before investigating the result, we would like to describe the list of assumptions made in our methodology. It will give us a clear picture of the factors that may have impacts on our result and based on that we can then test the sensitivity of our results against the change of these factors.

Table 1. The list of initial assumptions and optimized parameters

Parameters	Assumptions/parameters
Window size of EMA	To be optimized: { 10, 20, 45, 60, 90, 120, 150, 180 }
Window size of rolling std	To be optimized: { 10, 20, 45, 60, 90, 120 }
Multiplier m	To be optimized: { 0.5, 1, 1.5, 2, 2.5, 3 }
Rebalancing period	6 months
Number of pairs	10 pairs
Initial investment	\$ 10 000
Degree of Financial Leverage	100% (investment in each pair = Initial investment/N with N = no. of pairs)
Spread of each pair	$\ln(P_A) - h * \ln(P_B)$ (refer to section IV.2)
Transaction cost	$0.01\% * (Price_A + h * Price_B)$ for trading 1 pair

Note: All optimized parameters were set on in-sample window

4.7 Performance statistics

4.7.1 Annualized Return Compounded (ARC)

Based on the methodology presented in [Kość et al. \(2019\)](#) we calculated the following performance statistics enabling us to evaluate tested algorithmic strategies.

The Annualized Return Compounded is expressed as percentage (%) and calculated as:

$$ARC = \left(\prod_{t=1}^N (1 + R_t) \right)^{252/N} - 1 \quad (14)$$

where:

- R_t is the percentage rate of return
- N is the sample size

4.7.2 Annualized Standard Deviation (ASD)

The Annualized Standard Deviation is expressed as percentage (%) and calculated in the standard way:

$$ASD = \sqrt{252} \times \sqrt{\frac{1}{N-1} \sum_{t=1}^N (R_t - \bar{R})^2} \quad (15)$$

where:

- R_t is the percentage rate of return
- \bar{R} is the average rate of return
- N is the sample size

4.7.3 Maximum Drawdown (MD)

The Maximum Drawdown is the difference between the global maximum and the global minimum of the equity curve. Time order is important here, which means the global minimum must occur after the global maximum. It is expressed as percentage (%) and calculated as:

$$MD(S)_{t_1}^{t_2} = \max_{(x,y) \in \{[t_1, t_2]^2: x \leq y\}} \frac{S_x - S_y}{S_x} \quad (16)$$

where:

- S is the price process
- $[t_1, t_2]$ is the period between time t_1 and t_2

4.7.4 Information Ratio (IR*)

Information Ratio is simply the ratio of annualized return compounded (ARC) and annualized standard deviation (ASD):

$$IR^* = \frac{ARC}{ASD} \quad (17)$$

4.7.5 Adjusted Information ratio (IR**)

Adjusted Information Ratio is similar to Information Ratio, but it also takes into account Maximum Drawdown as one of the risk factors:

$$IR^{**} = \frac{ARC^2 \times \text{sign}\{ARC\}}{ASD \times MD} \quad (18)$$

where:

- $\text{sign}\{ARC\}$ is the sign of annualized return compounded and can take values of 0, -1 or +1

5. Empirical Results and Discussion

The performance of each method for pair trading strategy and benchmark are presented in Table 2. Looking at the result, we can see that pair trading strategy in general does not outperform benchmark strategy. In particular, the correlation method generates the highest Information Ratio (IR*) among 3 methods with 0.531, the second one is the Hurst method with IR* of 0.374 and the last one is the cointegration method with IR* of 0.075, while the IR* of benchmark is 0.609, which is higher than all of 3 methods. Although benchmark strategy gives the highest Annualized Return Compounded (ARC) with 13.36%, its Annualized Standard Deviation (ASD) is also the highest with 21.96%. For the 3 methods applied in pair trading strategy, Hurst method generates the highest ARC with 3.14% while correlation and cointegration method have lower ARC with 2.97% and 0.06% respectively. Three tested methods generate a much lower ASD compared to the benchmark with 5.60% for correlation, 8.37% for Hurst and 8.44 for cointegration method. In terms of adjusted Information Ratio (IR**), we see a slightly different result. The correlation method with IR** of 0.197 can beat the market with IR** of 0.151. Hurst method and cointegration still cannot outperform the benchmark with IR** of 0.078 and 0.001 respectively. What makes the difference in result is that in IR**, we take into account Maximum Drawdown as a measure of risk beside ASD. We can see that the MD of benchmark (53.71%) is significantly higher than the MD of other 3 methods. Among 3 methods, the correlation generates the lowest MD with 7.99%, following the Hurst method with 15.02% and cointegration method with 36.66%.

Table 2. Performance of all methods and benchmark

Method	ARC (%)	ASD (%)	IR*	MD (%)	IR**
Hurst method	3.14	8.37	0.374	15.02	0.078
Correlation method	2.97	5.60	0.531	7.99	0.197
Cointegration method	0.06	8.44	0.075	36.66	0.001
Buy & Hold (NASDAQ-100)	13.36	21.96	0.609	53.71	0.151

Note: ARC: Annualized Return Compounded; ASD: Annualized Standard Deviation; MD: Maximum Drawdown; $IR^* = ARC/ASD$: Information Ratio; $IR^{**} = (ARC^2 * \text{sign}(ARC))/(ASD * MD)$: adjusted Information Ratio. Bold font indicates the best results in case of each performance statistics.

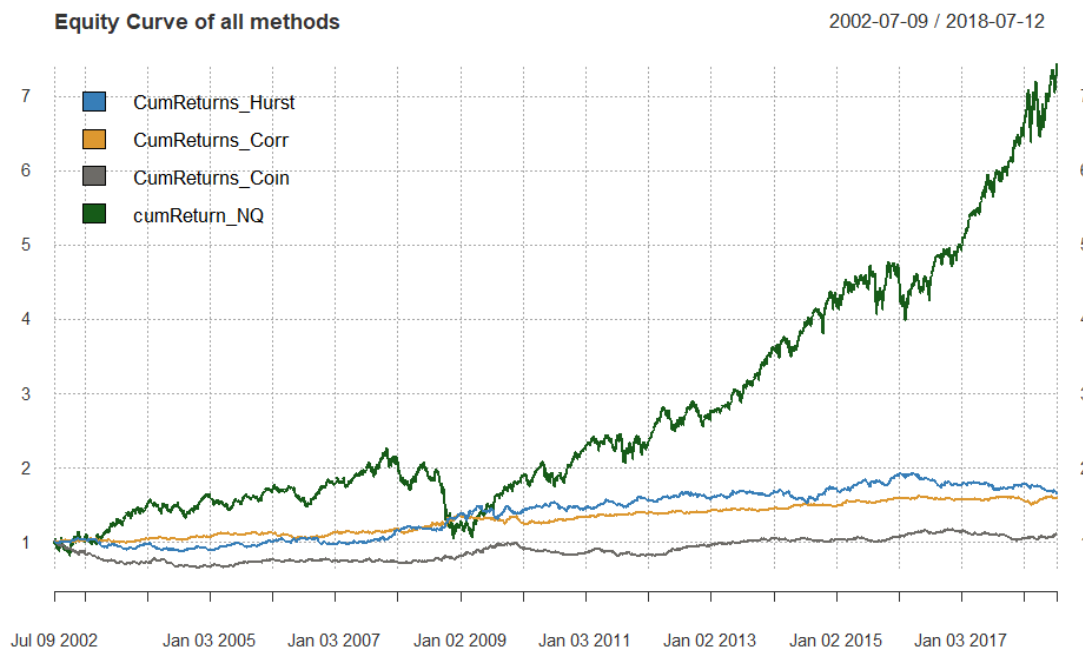
In general, this result implies that we cannot build a trading strategy that can beat the market in terms of risk-adjusted returns although correlation method may beat the market if we

additionally take into account MD as a measure of risk. This result refers to the first hypothesis of this paper, and we can conclude that the market is efficient according to this result.

On the other hand, we should also consider the fact that our result may be inflated due to survivorship bias. Besides, comparing the performance of 3 methods, we can see that Hurst method cannot outperform correlation but performs better than cointegration method. The cumulative returns of all 3 methods and benchmark are illustrated in Figure 2.

Figure 2 gives us more information of how each strategy performs in different market condition. We can see that in the bear market condition (market downturn), benchmark performs worse than Hurst and correlation methods, but still better than cointegration method. In bull market condition, benchmark performs the best, while correlation and Hurst methods perform better than cointegration method though still lower than benchmark. Besides, we can see that during the whole rebalancing period, pair trading strategy in general generates much lower volatility of returns compared to the benchmark.

Figure 2. Equity curve of all methods and benchmark



Note: The result of pair trading strategy with signals based on Hurst exponent, correlation and cointegration methods compared with the results of buy & hold strategy for NASDAQ-100. The trading period started on 01/01/2002 and ended on 31/12/2018. The number of pairs traded was 10. The rebalancing period was 6 months. Degree of financial leverage = 100%.

6. Robustness Test

In this section, we would verify how the result we obtained above is robust to varying number of pairs traded, varying rebalancing period and varying degree of financial leverage. In the

result above, we trade 10 pairs during the period of 6 months with the initial investment of \$10 000. To check the sensitivity of this result, we change the number of pairs traded to 5, 20 and 30 pairs, the rebalancing period to 3 months and 1 year and the financial leverage degree to 200% and 400% in each pair.

6.1 Varying number of pairs traded

Table 3 presents the performance of all three methods with 5 pairs, 10 pairs (the original number of pairs), 20 pairs and 30 pairs. As we can see in Table 3, with 5 pairs, the performance of Hurst and Cointegration method cannot beat the market with ARC of -1.55% and -4.84%; ASD of 10.30% and 11.77%; IR* of -0.150 and -0.411 and IR** of -0.004 and -0.035 respectively, while correlation method performs quite well and can beat the market with IR* of 0.811 and IR** of 0.399 (ARC = 5.89%, ASD = 7.25% and MD = 11.98%). With 20 pairs, no method can beat the market. Especially, only Hurst method can make positive profit with IR* of 0.176 and IR** of 0.012. Both correlation and cointegration methods lead to negative profit with IR* of -0.356 and -0.130 and IR** of -0.021 and -0.004 respectively. With 30 pairs, we can observe that the performance of all 3 methods improve more compared to other number of pairs. In particular, correlation method works well with IR* of 0.712 and IR** of 0.168, which outperforms the market. Hurst and cointegration methods' performance, though better, are still lower than the benchmark with IR* of 0.421 and 0.203 and IR** of 0.078 and 0.009 respectively.

From all of these results, we can conclude that all three methods seem to be sensitive to the number of pairs traded. However, it is not clear whether the performance of all three methods improves with the greater number of pairs. As we can see, Hurst and cointegration method perform better with 10 and 30 pairs and worse with 5 and 20 pairs, while correlation method works especially well with 5 and 30 pairs and quite poor with 20 pairs.

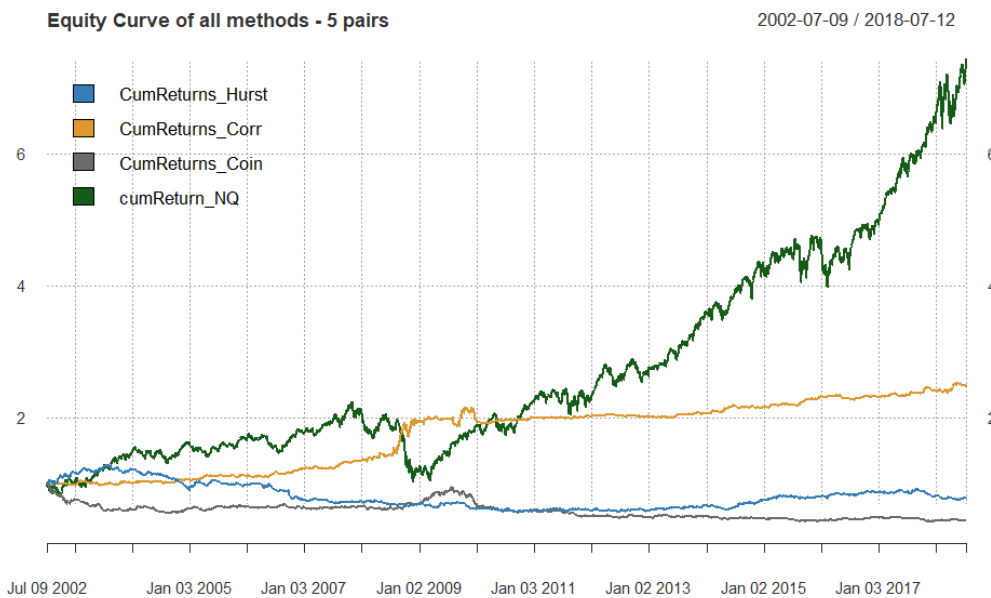
The result we obtained here does not correspond with the result in other papers, such as [Alexander and Dimitriu \(2002\)](#) where they state that cointegration method's performance is superior as compared to the market; or in [Rad, et al. \(2016\)](#) where they claim that the cointegration method is the superior strategy during turbulent market conditions. Similarly, [Ramos-Requena, et al. \(2017\)](#) found that Hurst method performs better than the correlation method. However, our result, together with the results in other papers, confirms that pair trading strategy performs well and better than the market in the period of market crash, and confirms the market neutral characteristics of pair trading strategy.

Table 3. Performance of all methods and benchmark - 5, 10, 20 and 30 pairs

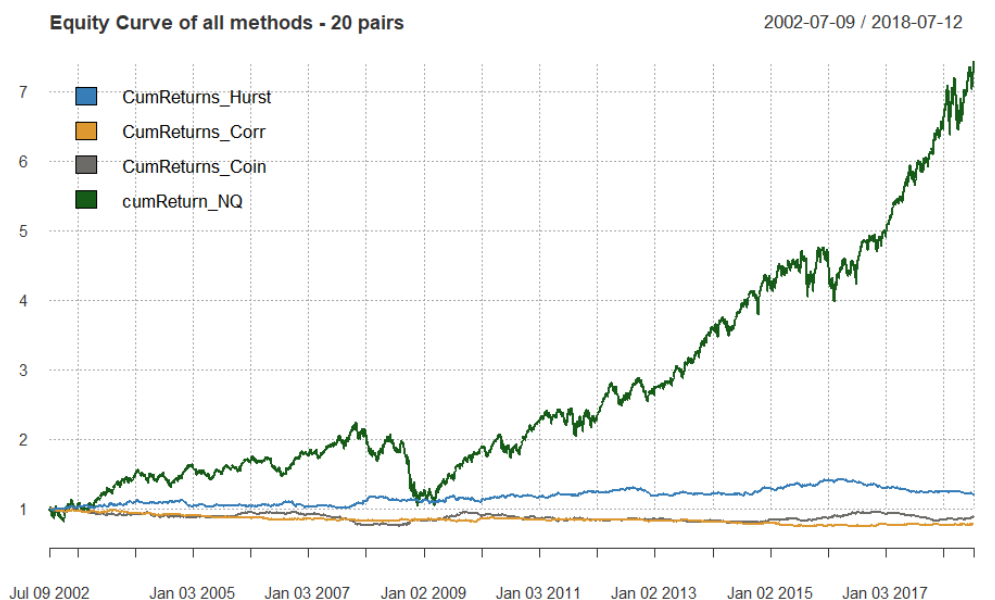
Method	No. of pairs	ARC (%)	ASD (%)	IR*	MD (%)	IR**
Hurst method	5	-1.55	10.30	-0.150	55.06	-0.004
Correlation method	5	5.89	7.25	0.811	11.98	0.399
Cointegration method	5	-4.84	11.77	-0.411	57.31	-0.035
Hurst method	10	3.14	8.37	0.374	15.02	0.078
Correlation method	10	2.97	5.60	0.531	7.99	0.197
Cointegration method	10	0.06	8.44	0.075	36.66	0.001
Hurst method	20	1.11	6.28	0.176	16.61	0.012
Correlation method	20	-1.53	4.29	-0.356	26.12	-0.021
Cointegration method	20	-0.08	5.89	-0.130	28.06	-0.004
Hurst method	30	2.41	5.72	0.421	13.00	0.078
Correlation method	30	3.08	4.33	0.712	13.10	0.168
Cointegration method	30	1.00	4.95	0.203	23.12	0.009
Buy & Hold (NASDAQ-100)	—	13.36	21.96	0.609	53.71	0.151

Note: ARC: Annualized Return Compounded; ASD: Annualized Standard Deviation; MD: Maximum Drawdown; $IR^* = ARC/ASD$: Information Ratio; $IR^{**} = (ARC^2 * \text{sign}(ARC))/(ASD * MD)$: adjusted Information Ratio. Bold font indicates the best results in case of each performance statistics.

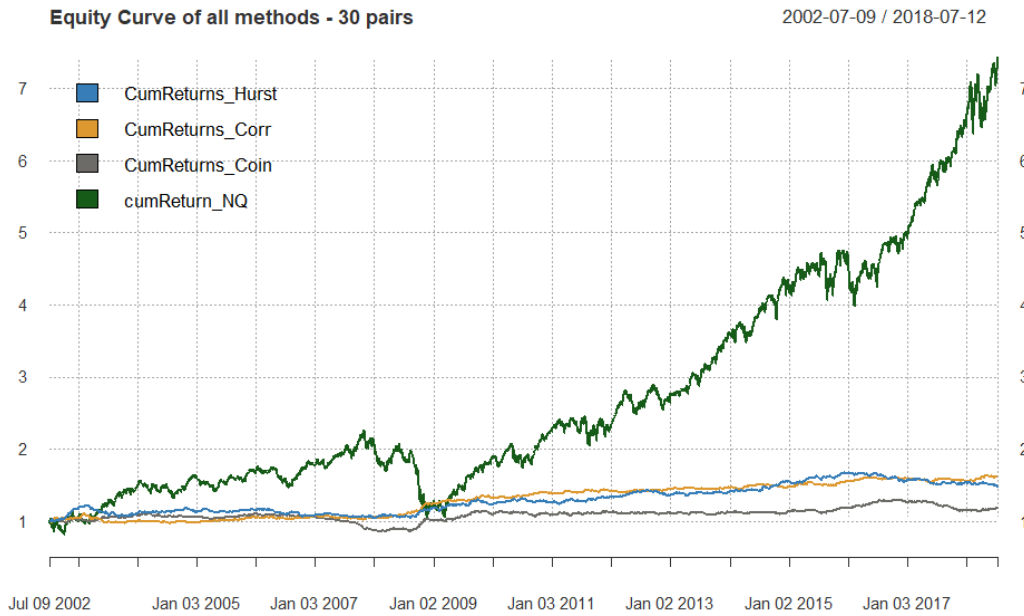
The equity curves of all methods and the benchmark with 5, 20 and 30 pairs are plotted in Figure 3, 4 and 5 respectively. In general, they all show that pair trading strategy with all methods (Hurst, correlation and cointegration) work roughly the same or poorer compared to the benchmark in market downturn (except for the case with 5 pairs for the correlation method) and always worse in expansion stage. Besides, with 5 pairs, the correlation method's performance seems to be outstanding compared to cointegration and Hurst method in both bull and bear market condition, and it works much better than the market in crisis period. With 20 and 30 pairs, no method can outperform the benchmark in any market conditions. Cointegration method seems to perform poorer than the other two methods for any number of pairs.

Figure 3. Equity curve of all methods and benchmark - 5 pairs

Note: The result of pair trading strategy with signals based on Hurst exponent, correlation and cointegration methods compared with the results of buy & hold strategy for NASDAQ-100. The trading period started on 01/01/2002 and ended on 31/12/2018. The number of pairs traded was 5. The rebalancing period was 6 months. Degree of financial leverage = 100%.

Figure 4. Equity curve of all methods and benchmark - 20 pairs

Note: The result of pair trading strategy with signals based on Hurst exponent, correlation and cointegration methods compared with the results of buy & hold strategy for NASDAQ-100. The trading period started on 01/01/2002 and ended on 31/12/2018. The number of pairs traded was 20. The rebalancing period was 6 months. Degree of financial leverage = 100%.

Figure 5. Equity curve of all methods and benchmark - 30 pairs

Note: The result of pair trading strategy with signals based on Hurst exponent, correlation and cointegration methods compared with the results of buy & hold strategy for NASDAQ-100. The trading period started on 01/01/2002 and ended on 31/12/2018. The number of pairs traded was 30. The rebalancing period was 6 months. Degree of financial leverage = 100%.

6.2 Varying rebalancing period

Table 4 demonstrates the performance of all three methods with rebalancing period of 3 months, 6 months (the original rebalancing period) and 1 year, and the number of pairs traded is fixed at 10 pairs for all 3 methods. As we can see in Table 4, with 3 months, only the performance of correlation method (with $IR^* = 0.527$ and $IR^{**} = 0.212$) outperforms the benchmark (with $IR^* = 0.470$ and $IR^{**} = 0.092$), while Hurst method, though gain positive result, cannot beat the market with $IR^* = 0.180$ and $IR^{**} = 0.011$ and cointegration method gains negative result with $IR^* = -0.035$ and $IR^{**} = -0.0003$. With 1-year rebalancing period, the correlation method has the highest performance with $IR^* = 0.357$ and $IR^{**} = 0.075$, following the Hurst method with $IR^* = 0.051$ and $IR^{**} = 0.001$, and cointegration method has lowest performance with $IR^* = -0.148$ and $IR^{**} = -0.006$. Hence, we can see that no method can beat the market (with $IR^* = 0.623$ and $IR^{**} = 0.250$) with 1-year rebalancing period.

From all of these results, we can observe that all three methods are quite sensitive to different rebalancing period, but the correlation method seems to be more stable than the other two as it keeps earning positive profit with different rebalancing periods, though it cannot beat the market with longer rebalancing period (6 months and 1 year). The performance of Hurst

method drops significantly with 1-year and 3-month rebalancing period as compared to 6-month period. For cointegration method, only 6-month rebalancing period earns positive profit while 3-month and 1-year period earn negative profit. The result we obtained here again does not agree with the result in [Alexander and Dimitriu \(2002\)](#) or in [Rad, et al. \(2016\)](#) or [Ramos-Requena, et al. \(2017\)](#) as we mentioned above.

Table 4. Performance of all methods and benchmark - 3 months, 6 months and 1 year

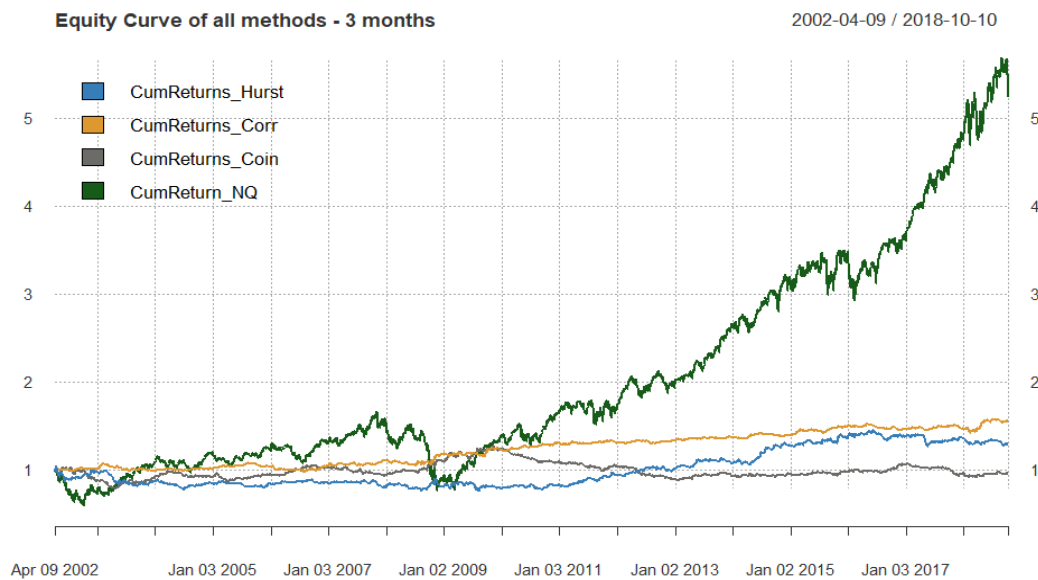
Method	Rebalancing period	ARC (%)	ASD (%)	IR*	MD (%)	IR**
Hurst method	3 months	1.63	9.04	0.180	26.62	0.011
Correlation method	3 months	2.72	5.16	0.527	6.79	0.212
Cointegration method	3 months	-0.28	8.11	-0.035	3.12	-0.0003
Buy & Hold (NASDAQ-100)	—	10.55	22.45	0.470	53.71	0.092
Hurst method	6 months	3.14	8.37	0.374	15.02	0.078
Correlation method	6 months	2.97	5.60	0.531	7.99	0.197
Cointegration method	6 months	0.06	8.44	0.075	36.66	0.001
Buy & Hold (NASDAQ-100)	—	13.36	21.96	0.609	53.71	0.151
Hurst method	1 year	0.40	7.93	0.051	26.06	0.001
Correlation method	1 year	1.77	4.98	0.357	8.49	0.075
Cointegration method	1 year	-1.47	9.93	-0.148	3.42	-0.006
Buy & Hold (NASDAQ-100)	—	12.95	20.79	0.623	53.71	0.150

Note: ARC: Annualized Return Compounded; ASD: Annualized Standard Deviation; MD: Maximum Drawdown; $IR^* = ARC/ASD$; Information Ratio; $IR^{**} = (ARC^2 * \text{sign}(ARC))/(ASD * MD)$: adjusted Information Ratio. Bold font indicates the best results in case of each performance statistics.

Figure 6 and 7 illustrate the equity curves of all strategies and benchmark with 3-month and 1-year rebalancing period respectively. For 3-month rebalancing period, correlation and cointegration methods outperform the benchmark while Hurst method performs roughly the same as the benchmark in bear market condition. For 1-year rebalancing period, all three

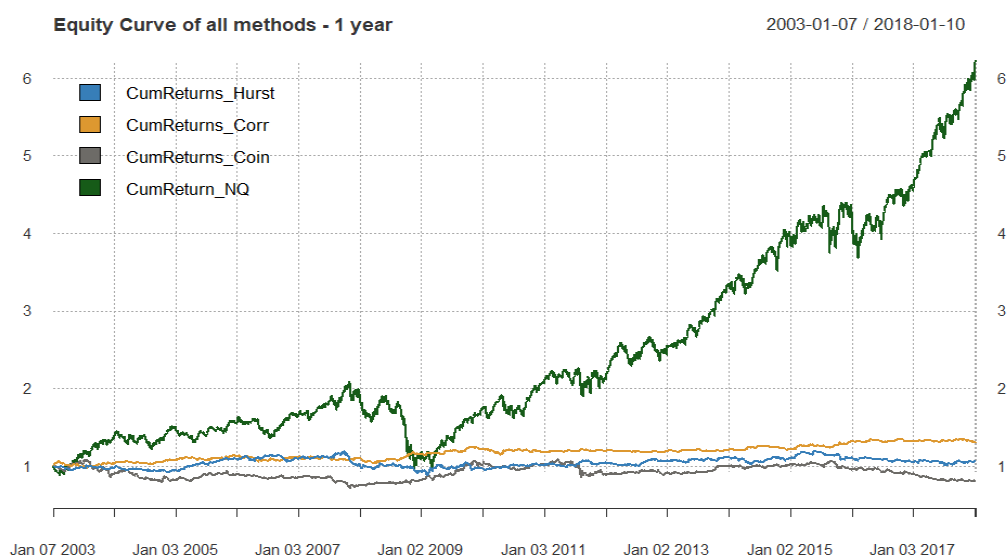
methods perform the same or a little bit lower than the benchmark in bear market condition. In the bull market condition, no method can beat the market for both 3-month and 1-year rebalancing period. However, in general, correlation method seems to perform better than the other two methods in all market conditions.

Figure 6. Equity curve of all methods and benchmark - 3 months



Note: The result of pair trading strategy with signals based on Hurst exponent, correlation and cointegration methods compared with the results of buy & hold strategy for NASDAD-100. The trading period started on 01/01/2002 and ended on 31/12/2018. The number of pairs traded was 10. The rebalancing period was 3 months. Degree of financial leverage = 100%.

Figure 7. Equity curve of all methods and benchmark - 1 year



Note: The result of pair trading strategy with signals based on Hurst exponent, correlation and cointegration methods compared with the results of buy & hold strategy for NASDAD-100. The trading period started on 01/01/2002 and ended on 31/12/2018. The number of pairs traded was 10. The rebalancing period was 1 year. Degree of financial leverage = 100%.

6.3 Varying degree of financial leverage

Table 5 shows the result for varying degree of financial leverage, so instead of invest \$10 000/N in each pair, we try to invest \$10 000/(N/2) and \$10 000/(N/4) in each pair, taking into account that we can earn higher return with reasonable level of volatility. The number of pairs is fixed at 10 and rebalancing period is fixed at 6 months. With \$10 000/(N/2) investment in each pair, the IR^* of correlation, Hurst and cointegration methods are 0.327, 0.513 and 0.069 respectively, which cannot beat the market with $IR^* = 0.609$. We can see that with double degree of financial leverage, we gain almost double ARC for each method (5.47% for Hurst, 5.69% for Correlation and 1.17% for Cointegration), but we also get almost double ASD (16.71% for Hurst, 11.10% for Correlation and 17.13% for Cointegration), in total making our IR^* not improve at all. In terms of IR^{**} , only Correlation method can beat the market with $IR^{**} = 0.188$, both Hurst and cointegration method with IR^{**} of 0.062 and 0.001 cannot beat the market. Similar to IR^* , we get almost double ARC but also double ASD and MD for each method, overall making IR^{**} not better.

With \$10 000/(N/4) investment in each pair, the Information Ratio (IR^*) of correlation method can outperform the market with 0.631, which is mainly because correlation method can earn higher ARC with lower ASD compared to the market. For Hurst method, the IR^* of 0.260 cannot beat the market and is also lower than the original performance (without financial leverage) as it earns 4 times ARC (8.66%) but also has more than 4 times ASD (33.35%). For cointegration, it has negative profit with ASD of -3.14%, which is the cost of employing financial leverage as our loss is amplified as compared to not using financial leverage, together with very high ASD (38.43%), in total leading to IR^* of -0.082. In terms of adjusted Information Ratio (IR^{**}), correlation method can 2 times outperform the benchmark with IR^{**} of 0.305 as its MD (28.40%) is as half of the MD of benchmark (53.71%), while Hurst and cointegration method still perform poorer with IR^{**} of 0.043 and -0.003 respectively as their MD (53.34% for Hurst and 91.59% for cointegration) are almost the same or even higher than that of benchmark.

From the analysis of the result in Table 5, we can see that the return of all methods improves significantly with higher degree of financial leverage, but it also results in higher standard deviation and maximum drawdown. Overall, only correlation method can outperform the market while Hurst and cointegration methods cannot. Hence, we can say that the result we gain is less sensitive to the degree of financial leverage than to the number of pairs or

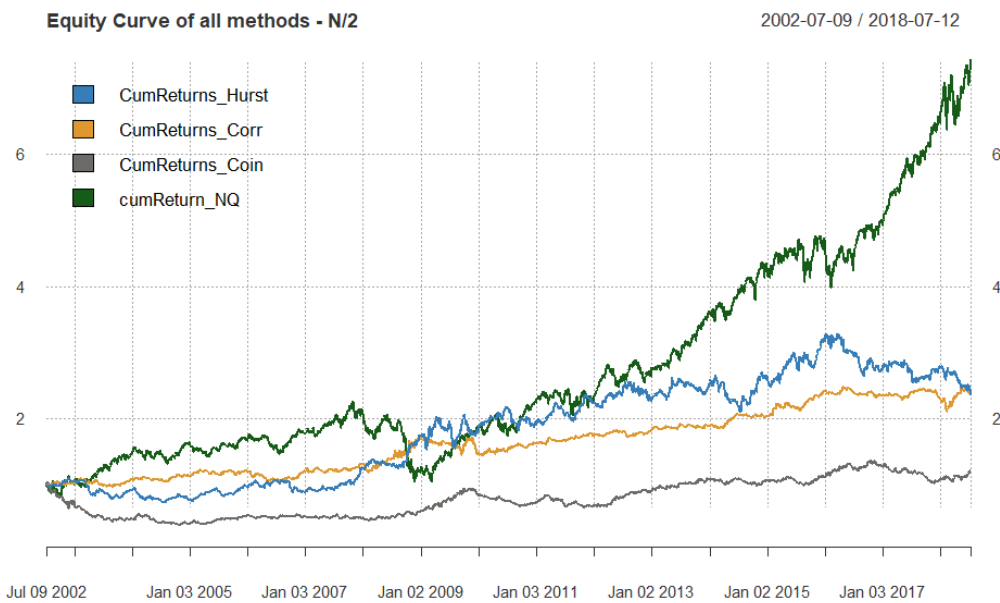
rebalancing period as we discussed above. This result again does not correspond with other papers as we mentioned above where they state that cointegration method is superior as compared to the market (Alexander and Dimitriu, 2002) or Hurst method can outperform correlation method (Ramos-Requena, et al. 2017).

Table 5. Performance of all methods and benchmark - N/2, N/4 and N

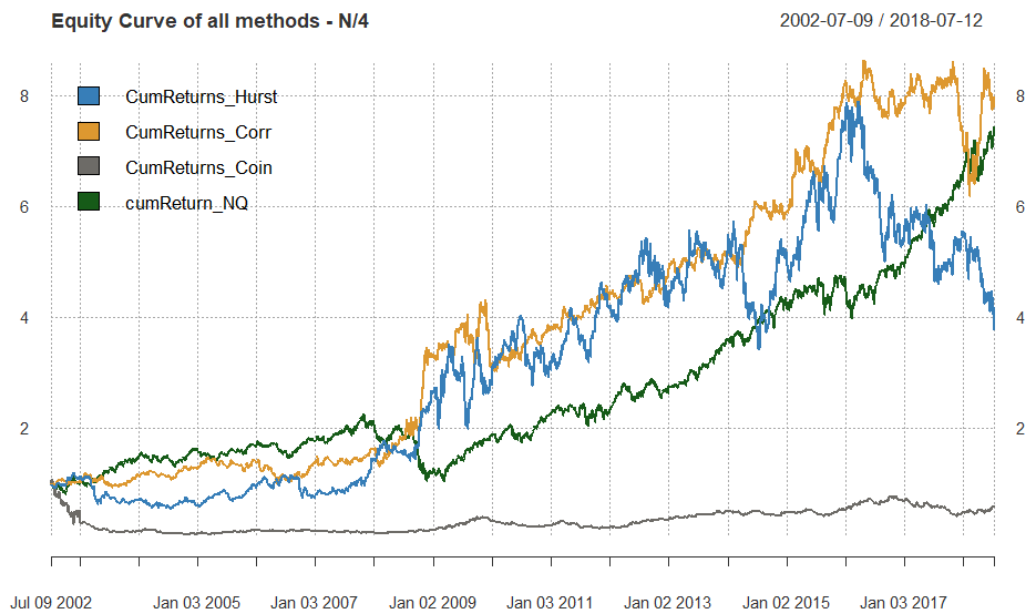
Method	DFL	ARC (%)	ASD (%)	IR*	MD (%)	IR**
Hurst method	\$10000/(N/4)	8.66	33.35	0.260	52.24	0.043
Correlation method	\$10000/(N/4)	13.76	21.79	0.631	28.40	0.305
Cointegration method	\$10000/(N/4)	-3.14	38.43	-0.082	91.59	-0.003
Hurst method	\$10000/(N/2)	5.47	16.71	0.327	28.74	0.062
Correlation method	\$10000/(N/2)	5.69	11.10	0.513	15.52	0.188
Cointegration method	\$10000/(N/2)	1.17	17.13	0.069	62.73	0.001
Hurst method	\$10000/N	3.14	8.37	0.374	15.02	0.078
Correlation method	\$10000/N	2.97	5.60	0.531	7.99	0.197
Cointegration method	\$10000/N	0.06	8.44	0.075	36.66	0.001
Buy & Hold (NASDAQ-100)	—	13.36	21.96	0.609	53.71	0.151

Note: ARC: Annualized Return Compounded; ASD: Annualized Standard Deviation; MD: Maximum Drawdown; IR* = ARC/ASD: Information Ratio; IR** = $(ARC^2 * \text{sign}(ARC)) / (ASD * MD)$: adjusted Information Ratio. Bold font indicates the best results in case of each performance statistics.

Figure 8 and 9 plots the equity curves of all three methods and the benchmark for double degree of financial leverage and quadruple degree of financial leverage respectively. Both of the graphs show that in recession period, Hurst and correlation methods perform better than the benchmark while cointegration performs poorer. In expansion period, all three methods perform lower than the benchmark for double degree of financial leverage, but Hurst and correlation method performs significantly better for quadruple degree of financial leverage. However, as we indicated above, their equity curves are characterized by much higher volatility, which is shown clearly in Figure 9.

Figure 8. Equity curve of all methods and benchmark - N/2

Note: The result of pair trading strategy with signals based on Hurst exponent, correlation and cointegration methods compared with the results of buy & hold strategy for NASDAQ-100. The trading period started on 01/01/2002 and ended on 31/12/2018. The number of pairs traded was 10. The rebalancing period was 6 months. Degree of financial leverage = 200%.

Figure 9. Equity curve of all methods and benchmark - N/4

Note: The result of pair trading strategy with signals based on Hurst exponent, correlation and cointegration methods compared with the results of buy & hold strategy for NASDAQ-100. The trading period started on 01/01/2002 and ended on 31/12/2018. The number of pairs traded was 10. The rebalancing period was 6 months. Degree of financial leverage = 400%.

7. Conclusion

This paper aims to explore an alternative method of pair selection for pair trading strategy. The main hypotheses of this paper are (1) whether the Efficient Market Hypothesis work i.e. whether Hurst exponent can generate a trading strategy that outperforms the market; and (2) whether Hurst exponent is a superior pair selection model for pair trading strategy as compared to correlation and cointegration method. Based on these two hypotheses, the research questions are constructed as follow: whether our results are robust to (1) varying number of pairs selected; (2) varying rebalancing period; and (3) varying degree of financial leverage.

The dataset used for this empirical research is 103 stocks listed in NASDAQ-100 (updated until 31/12/2018) and the NASDAQ-100 index itself. The data is collected on daily basis over the period from 01/01/2000 to 31/12/2018. The pairs are established from the pool of these 103 stocks. We take advantage of Hurst exponent, which can be used to classify the pattern of time series (mean-reverting, momentum or random walk). Based on that, we select the pairs with the lowest Hurst exponent, indicating mean-reverting pattern, which is a key condition to be profitable in pair trading strategy. Generalized Hurst exponent methodology with first moment ($q = 1$) is applied to calibrate Hurst exponent for each pair. For correlation method we apply the Pearson correlation coefficient to estimate the correlation between 2 series of log returns and for cointegration method we employ Engle – Granger two-step approach to test for cointegration between 2 time series of prices. To generate buy and sell signals for pair trading strategy, volatility breakout model is applied with upper and lower thresholds calculated based on the exponential moving average and rolling standard deviation of the time series. The window size of exponential moving average, the window size of rolling standard deviation and multiplier m are parameters to be optimized i.e. the set of parameters that generate the best performance on in-sample data will be selected. The performance of Hurst method is compared with the benchmark (buy & hold NASDAQ-100 index) as well as with the correlation and the cointegration methods in terms of annualized return compounded, annualized standard deviation, maximum drawdown, Information Ratio and adjusted Information Ratio.

Overall, the result shows that Hurst method cannot generate a strategy that can outperform the market and its performance is superior as compared to cointegration method but it is not as compared to correlation method. This result does not correspond with the result of some other papers in the past, which states that cointegration method is superior compared

to other methods. On the other hand, the result implies that market is efficient, so we cannot build a trading strategy and earn excess return on the market, which corresponds with the efficient market hypothesis, though the question of whether market is efficient or not still remains debated in literature. Moreover, pair trading strategy in case of all methods can sometimes outperform the market in recession stage but cannot in the expansion stage, which confirms the market neutrality of pair trading strategy. However, our result is quite sensitive to different number of pairs traded and rebalancing period and less sensitive to the financial leverage degree. In specific, it is not clear whether Hurst method's performance improves with greater number of pairs or longer rebalancing period. On the other hand, it is clear that all methods' returns and volatility is greater with higher degree of financial leverage, hence the information ratio does not improve, except for the correlation method with quadruple degree of financial leverage.

To conclude, from the obtained result, we can state that the market is efficient in general (first hypothesis) and Hurst method of pair selection is superior as compared to the cointegration method but is not as compared to correlation method (second hypothesis). Furthermore, the obtained result is sensitive to varying number of pairs (first research question) and varying rebalancing period (second research question) and less sensitive to varying degree of financial leverage (third research question).

There are some limitations of this paper which can be improved in future work. First, we should mention the impact of survivorship bias due to the lack of full historical data. It may inflate the performance of all methods, especially when we employ mean-reverting pattern in pair trading strategy. This issue may be addressed if we get a full constituent of NASDAQ-100 index over the period 2000 – 2018. Second, we use Pearson correlation coefficient to calculate the correlation between 2 series of log returns, which may suffer some limitations. In specific, Pearson correlation only measures linear relationship, but in practice, most of stock price movements have non-linear dependencies. Besides, correlation coefficient varies with different time frames and hence lead to different estimates of correlation parameters. Regression is also sensitive to outliers, so spurious correlations can occur ([Meissner, 2015](#)). To solve this issue, we can try to use other conditional correlation models such as dynamic conditional correlation (DCC) model of [Engle \(2002\)](#) or the time-varying correlation (TVC) model of [Tse and Tsui \(2002\)](#). Third, we apply Engle – Granger two-step approach to test for cointegration of 2 time series. However, the major issue in Engle-Granger method stays in the choice of dependent and independent variable when we run the regression to extract residuals as it may lead to

inconsistent conclusions (Armstrong, 2001). This issue may be addressed by alternative tests such as Phillips-Ouliaris and Johansen's.

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