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PORTFOLIO CONSTRUCTION FROM THE
PERSPECTIVE OF A POLISH INVESTOR.

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Does the inclusion of exposure to volatility into diversified portfolio improve the investment results? Portfolio construction from the perspective of a Polish investor.

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Abstract: The main goal of this research is to analyse the investment benefits from an incorporation of the volatility exposure to the diversified portfolio from the perspective of a Polish investor. Volatility, treated as a new asset class, may improve the performance of the portfolio due to its negative correlation with most types of assets. This topic has been widely investigated for the United States and Europe whereas Polish market appears to be not heavily researched and this study may fill this gap. The research covers the period from October 2010 to July 2018 and is performed on the daily close prices. To construct the portfolios, the analysis uses the mean-variance framework and the naïve diversification approach. The comparison of risk-adjusted returns between investments with and without volatility exposure enables to answer the research question about an improvement of the results by the addition of a non-standard asset to the diversified portfolios. The VXX is considered as the proxy for volatility as it is the most popular ETN which follows the volatility index derivatives with the given maturity. To test the robustness of the results, the portfolios are constructed with a broad range of different parameters and assumptions imposed on the optimization procedure.

Keywords: Volatility, asset class, portfolio optimization, Polish market, Markowitz portfolio, naïve diversification

JEL codes: C4, C45, C61, C15, G14, G17

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1 Introduction

Volatility as an asset class is a relatively new concept in the financial theory but due to its rapid development and new investment possibilities, an interest of academia and professionals has aroused remarkably. In the previous years, volatility has been considered as one of the features of the investment which corresponds to the risk related to the uncertain outcomes. An introduction of the VIX index in 1993 by the Chicago Board Options Exchange, which aim was to measure the market's expectations of volatility implied from the options' quotations, and the launch of derivatives based on the VIX eleven years later, are the key dates for the new investment possibilities. Even though the VIX methodology has changed during the last 25 years, its core characteristics remain the same – the turbulences and concerns about markets' dynamics were followed by the high values of the volatility index (VIX is commonly known as a “fear index”). A negative correlation with most markets attracted a large number of investors who were interested in the non-standard assets.

The VIX index is not directly investable and investor who wants to obtain the exposure to volatility needs to use derivatives. Although it is technically possible to directly replicate the VIX – it requires an extremely sophisticated strategy that involves high transactional costs. The figures confirm a great increase in interest of volatility derivatives since the introduction. According to the CBOE data, the average daily turnover of VIX futures has grown from 462 in 2004 to almost 300,000 in 2018. A similar pattern was observed in the case of VIX options – a daily volume of traded contracts has risen from 23,500 in 2006 up to over 650,000 in 2018. As an American volatility index has been gaining popularity, another stock exchanges have introduced similar products like European (VSTOXX), Japanese (Nikkei Stock Average Volatility Index) or German (VDAX).

The popularity of volatility indices and their derivatives has brought a broad range of investment opportunities to exploit. The popular examples, which commonly appear in the literature, are the following:

- volatility risk premium (VRP), which benefits from the historically persisting difference between implied and realized volatility;
- trading on VIX futures term structure and finding out arbitrage possibilities;
- portfolio diversification and portfolio hedging which is possible thanks to the negative correlation of volatility index with the market, especially during the turbulence times (conditional correlation).

The paper's aim is to focus on portfolio diversification and the benefits associated with it.

The studies of American and European markets are the popular investigations' goal. The development and progress of volatility instruments have encouraged the researchers to explore the subject of this new asset class. A broad range of the academic papers, which examine the investment possibilities with the VIX, VSTOXX or other volatility indices, has been already widely documented. Due to their non-standard characteristics of negative correlation with the market, trading volatility is a good option for extra profits, and an effective way of hedging or diversifying the portfolio. The main goal of this paper is to analyse the diversification opportunities with volatility instruments from the Polish market perspective. The simulation is devoted to the hypothetical investor who operates in Poland and has a diversified portfolio of Polish assets, but at the same time has also access to the global markets. The research seems to be interesting as most of the papers in the literature focus on the American and European markets only.

The analysis focuses on diversification benefits of adding volatility exposure to the portfolio containing a wide range of different assets. The research tries to answer the question whether a hypothetical investor that has a diversified portfolio can improve his efficiency, measured by the risk-adjusted return, by an inclusion of volatility exposure. The research presents a comparison of two types of portfolios – the first one containing volatility exposure and the second one with the standard type of assets only. To test the hypothesis about the beneficial effect of the volatility exposure to the diversified portfolio, the following research questions were formulated:

- What are the results of the Markowitz portfolio optimization?
- Does the naïve diversification method perform better or worse than the Markowitz?
- What is the sensitivity of the results to the chosen parameters?

The answers for the above questions may lead to a rejection or a confirmation of the hypothesis.

Warsaw Stock Exchange (WSE) has a relatively short history and the Polish financial market is still underdeveloped compared to western Europe countries, which results in the illiquid derivatives market and no volatility instruments at all. As a consequence, the abovementioned investor cannot invest in volatility implied from WIG20 option and cannot have direct exposure to Polish volatility. However, Ślepaczuk and Zakrzewski (2008) in their paper have proposed VWIG20 – volatility index for WIG20 index which is equivalent to the VIX, VSTOXX and other volatility indices. They presented that an introduction of such index would be a beneficial improvement for the Polish financial market and would boost the derivatives sector and provide new investment alternatives. The VIX, as the most popular

volatility index that has the broadest offer of derivatives and ETFs, is an intuitive solution to overcome a lack of the Polish alternatives. On the one hand, the standard asset classes, like equity or bonds, exhibit common characteristics and move in the same direction, no matter where geographically located they are. On the other hand, the Polish market may be vulnerable to the shocks that are not observable on the American market and the VIX inclusion will not diversify the portfolio efficiently. The goal of this research is to look closer into this matter. The VXX, an Exchange Traded Note which provides exposure to the short-term VIX futures, will serve as a proxy of volatility in this analysis. Thanks to its high liquidity and long history of quotations, it allows for robust results.

The paper has the following structure. Section 2 is dedicated to the literature review of the subject. Next chapter describes the methodology, i.e. what kind of methods were performed, which parameters were used and what assumptions were established. Section 4 contains the description of data – what kind of assets were used, their characteristics, features and summary. The following chapter shows the results of the research and the sensitivity analysis is presented to check the robustness of the outcomes. Section 6 summarizes the research, draws conclusions and presents a way for further development and improvement. The appendix contains more detailed tables with the results of the research.

2 Literature review

Volatility commonly refers to the risk associated with the investments on the financial markets. The uncertain outcomes lead to many risk management solutions and applications. Correct measurement of volatility and ability to forecast its future value are an important part of the business for banks, insurance companies, investment funds and other financial institutions. The academia followed this tendency providing many researches devoted to the risk management and volatility forecasting.

Introduction of the VIX futures, options and other derivatives allowed the investors to look on volatility from a different perspective – as a new asset class. The academic papers dedicated to the investment possibilities and portfolio diversification with volatility instruments are not as popular as the previous subject but this stream is still developing as academia provides new research ideas and results. This section presents empirical work contributing to this matter.

Szadło (2009) focused his analysis on the diversification impact of the VIX derivatives during the last financial crisis in 2008 and its long-term benefits. He divided his research into two periods: the first one begins with an introduction of the VIX options in March 2006 and finishes at the end of 2008; the second one is particularly devoted to the last financial crisis and

consists of August-December 2008 period. The research was performed on many asset classes, like equity, bond, real estate, commodities, hedge funds, private equity, etc. He established different portfolio types (e.g. only equity, 60% equity – 40% bonds and their alternatives) and added volatility exposure via VIX options and futures (2,5% or 10% portfolio share for futures and 1% or 3% weight for options) to them. Once the portfolio was constructed, no rebalancing was done until the end of the analysis period. The comparison of different portfolios led to the conclusion that the VIX was an effective hedging instrument during the market turmoil of 2008. The long-term relationship is, however, less efficient and provided poor investment results due to the negative expected return of the VIX derivatives.

Briere et al. (2010) focused their analysis on European markets and possible ways of hedging portfolios via volatility instruments. They analysed both global European equity perspective (via MSCI EMU index) and single countries of core (Germany and France) and peripheral (Portugal, Ireland, Greece and Spain) of the eurozone. The authors performed portfolio optimization and compared the results of adding long volatility exposure via VSTOXX and VIX futures. The authors used modified conditional Value at Risk as a measure of risk, instead of standard deviation. Such approach takes into account not only returns and standard deviation, but also higher order moments (skewness and kurtosis) which may better reflect the distribution of volatility index. The research covered the period from 1999 to 2010. As the futures data are available only since 2004 for the VIX and 2009 for the VSTOXX, the authors estimated futures price via average linear relationship between the spot price of the volatility indices and derivatives. The authors concluded that the incorporation of the volatility exposure to the European indices had a positive impact on their performance and was an effective way of hedging. The effect was stronger for the VSTOXX futures than the VIX derivatives as the European volatility index futures better covered the risk associated with the eurozone equity downturns than American ones. In addition, the authors suggested that steeper VIX futures term structure incurs higher costs of carry than the VSTOXX.

Alexander and Korovilas (2011) focused their analysis on difficulties related to portfolio diversification with volatility instruments caused by high rolling costs of futures contracts. The research was conducted on SPY (an ETF on S&P 500) and 3 series of the VIX futures contracts: 1-month, 3-month and the Longest Traded series. To reduce the negative impact of high volatility when the contracts are close to the expiration, the roll over is done 5 days before the maturity of the VIX futures. The research was done via the Markowitz framework and its extension – the Black-Littermann model that tries to overcome the disadvantages of the Markowitz model by including investors' opinion about the future market movement into the

optimization process. Initially, the authors were analysing portfolio diversification with VIX futures ex-post. They used the mean-variance framework to look for the tangency portfolio (i.e. the portfolio with the highest Sharpe Ratio) and also its generalised Sharpe Ratio (GSR) extension, which includes the fact that most of the financial assets are non-normally distributed. The results confirmed the usefulness of volatility exposure but mainly during markets' turbulence, like the banking crisis in 2008 or the eurozone debt crisis in 2011. In ex-ante analysis of the Markowitz framework, the authors emphasize the dependency of results on the in-sample data. However, the conclusions remain similar as VIX futures perform well during the large drops. The investor's views in the Black-Littermann model were presented in two ways: the first one used the static expectations, i.e. including the latest historical data, and the second was a perfect foresight, which took into account actually realised returns (it can be treated as ex-post analysis). The results confirm that the VIX exposure deteriorates returns in the long-term. The authors concluded that volatility instruments should be left for the speculative purpose rather than used by the long-term investors, like mutual and pension funds. The VIX term structure is in contango most of the time, which makes that procedure of rolling the position very expensive, especially for the short-term maturity contracts.

In the following paper, Alexander and Korovilas (2012) made extensive research about the Exchange Traded Notes (ETN) on VIX futures. A high cost of carry, related to rolling over the futures contract, significantly decreases the performance of the ETNs based on the VIX, e.g. VXX, which is the most popular product within the class. A difference between the investment results of the VXX (ETN on short-term contracts) and VXZ (ETN on mid-term VIX futures) emphasizes the importance of rolling costs. To limit their negative impact, the authors were analysing the investment results of the XVIX and XVZ ETNs which are the combination of the mid-term VIX futures (VXZ) and the inverse of the short-term contracts (VXX). It provides exposure for the volatility index but reduces the negative impact of rolling the futures position. The authors noticed that the XVIX performs well when the market is tranquil and the term structure of VIX futures is in contango, while the XVZ achieves better results during the market's downturn. It led them to exploit the volatility instruments' strategy even more and use the mix of both abovementioned ETNs. The CVIX is a static combination of 25% of the XVIX and 75% of the XVZ as the market is approximately in backwardation for one-fourth of the time and in three-fourths is in contango. The CVZ presents a dynamic allocation and holds the XVZ when the slope of the term structure is positive and the XVIX when the slope is negative. A general conclusion of the investment results is that the presented strategy may achieve very

good results, perform well in every market condition and be an interesting alternative for a long-term investor who seeks for an alternative solution to its portfolio.

Stanescu and Tunaru (2013) added volatility exposure to the portfolios from American and European markets. They used the S&P500 and Euro Stoxx 50 as the proxies for equity markets and Barclays Aggregated bond index which represents the debt instruments. They constructed equity only and the equity-bond portfolios, and added volatility index futures in a similar way as Szado (2009). In addition, they divided the American sample into the pre-crisis period (2004-2007), post-crisis time (2008-2012) and separated 2008 year when the serious turbulences hit the financial markets. Due to a shorter time of the VSTOXX futures quotations (introduced in 2009), the European market analysis has no sub-samples. The authors rebalanced portfolios weekly and assumed no transaction costs. The results of the analysis proved the authors' hypothesis that adding volatility instruments to the portfolio produces not only effective hedging but also contributes to the higher risk-adjusted return, especially during the markets' downturns. The authors also performed a statistical arbitrage strategy on the VIX-VSTOXX futures differences. They forecasted the futures price via GARCH models which signalled entries and exits of the trades. The results showed that statistical arbitrage on the volatility index futures' difference may provide profits over the period of analysis.

Jablecki et al. (2015) analysed the impact of VIX futures on the portfolio diversification via the Markowitz framework and its extension – Black-Littermann model. The analysis covered the period from 2006 to 2013. The first part focused on the portfolio which consisted of the American equity, global bonds and commodity instruments. Initially, the authors selected the range of target returns, which varied from 0 to twice the average historical return in the given period. The efficient portfolio was calculated for every target return and one with the highest Sharpe ratio was finally selected. Unless the Sharpe ratio was positive for any portfolio, the weights were set to equal 0. 60, 120, 180 and 240 trading days were selected as a *memory* parameter on which weights were calculated. The constructed portfolios were invested for 1, 2, 4 and 8 weeks and after that period the portfolio was established again, with different weights. The authors also included the possibility of short selling the assets. The results depended on the selected parameters: long investment horizon (8 weeks) and long memory (240 trading days) produced poorer results. However, the possibility of a short selling generated higher profits for portfolios with the VIX. Analysing the results year by year led to the conclusion that VIX has a beneficial effect during markets' downturns for all parameters, whereas the performance during other periods depends on the selected criteria.

The second part was devoted to the Black-Littermann model. This part contained ten different equity indices from developed and emerging countries. Rebalancing was done every week and the last 13, 26 and 52 weeks were taken into account as a *memory* parameter. The investor's views for the future price were equal to the previous period return. The portfolio with the highest Sharpe Ratio according to the optimization methodology was chosen for the next week investment. The general conclusions were again ambiguous and considerably depended on the selected scenario. VIX futures inclusion, however, had a positive effect during the financial crisis (2008) and markets' decrease (2011) which confirmed the findings of the mean-variance analysis. These results suggested that the volatility index futures may be a powerful risk management tool, but it requires attentive selection and proper research.

Caloiero and Guidolin (2017) optimized portfolios consisting of the American assets and volatility exposure. Instead of VIX futures or options, the authors used Exchange Traded Products and checked how their performance affected investment profitability. The authors performed the optimization via three different types of investors' utility function – mean-variance, power and negative exponential utility function. In addition, they compared the results between the portfolios without volatility exposure, with the VIX index and with the VXX ETN as one of the most popular volatility Exchange Traded Product. The research was divided into two parts. The first one was devoted to the ex-post analysis where the authors checked whether the efficient portfolios contained volatility exposure. The second part simulated the potential investment possibilities – initially, the portfolio was constructed basing on the first 56 weeks and then the data was monthly updated and rebalancing was done. The analysis used weekly data, the long-only portfolio was constructed and the time period ranged between January 2009 and February 2016. The authors showed that the VIX index lays on the efficient frontier and improves the investment results when assessing ex-post. Nonetheless, the VXX remains to be an important part of the portfolio only for special preferences and assumptions of an investor. Results of the simulated investment strategies presented similar conclusions: the VIX index tended to be a valuable part of the portfolio, whereas the VXX provided poorer results and decreased the performance (however, there existed minor examples of the profitable VXX applications). The authors concluded that negative average returns of volatility ETN deteriorates portfolio performance. Even though the VXX tracks VIX quite accurately in terms of correlation, the volatility ETN needs to be managed with prudence.

3 Methodology

The main goal of the analysis was to compare the performance of the portfolios with and without any volatility exposure. An Information Ratio, which is a ratio of annual compounded return to the annual standard deviation of daily returns, was chosen as a decisive criterion.

$$\text{ARC (Annual Return Compounded)} = 252 * \frac{1}{N} \sum_{t=1}^N R_t \quad (3.1)$$

where R_t , is the percentage daily return on day t .

$$\text{ASD (Annual Standard Deviation)} = \sqrt{252} * \sqrt{\frac{1}{N-1} \sum_{t=1}^N (R_t - \bar{R})^2} \quad (3.2)$$

where $\bar{R}$ is the average of all n -returns.

$$\text{IR1 (Information Ratio)} = \frac{\text{ARC}}{\text{ASD}} \quad (3.3)$$

A higher value of the IR of the portfolio which includes the VXX (an ETF on short-term VIX futures) would lead to the conclusion that adding volatility exposure improves the efficiency of the investments thanks to the diversification benefits and vice versa. Risk-adjusted measures are the popular indicators of the investment profitability and intuitive comparison could be made between portfolios with different parameters. In addition, to reflect better the assets' behaviour during the period of analysis, comparisons of maximum drawdowns and information ratio weighted by maximum drawdown were done for descriptive statistics purposes (detailed values are presented in section 4).

$$\text{MD (Maximum Drawdown)} = \max_{\tau \in [0, T]} (\max_{t \in [0, T]} R_t - R_\tau) \quad (3.4)$$

$$\text{Information Ratio 2 (IR2)} = \text{sign}(\text{ARC}) * \frac{\text{ARC}^2}{(\text{ASD} * \text{MD})} \quad (3.5)$$

The research methodology is divided into two parts. The first one uses the breakthrough Markowitz framework to calculate the efficient frontier, i.e. the combination of the assets that have the highest expected return for a given level of risk or the risk factor is the lowest for a given level of return. Next, the efficient frontier is searched through the point with the highest return to risk ratio, where risk is associated with a standard deviation. Finally, the shares of each asset are extracted from the chosen point on the efficient frontier and investment is done according to the calculated weights.

The mean-variance analysis is performed on the past returns of assets. Historical returns and variance-covariance matrix determine the weights of the portfolio. A length of a chosen period is a key parameter for the estimated results and could affect the outcomes – this

parameter is called *memory* for the purpose of the analysis. Historical data from the last 15, 30, 60 and 120 trading days were selected for each case. The length of the investment depends on the value of another parameter – called *rebalancing*. It determines how often weights are calculated and new portfolios are constructed. The research considers three frequencies of rebalancing: weekly, monthly and quarterly. After the end of each period, the Markowitz analysis is performed one more time and the investment with a different share of assets is done again.

The first part of the research is performed on four different scenarios, each assuming different weight constraints imposed on the portfolio. The basic one considers the “long only” portfolio – weights are restricted from 0% to 100% interval. To reduce the impact of one asset on the whole portfolio performance, the second scenario assumes the limitations of the upper bound to 25%. Such approach may be useful when historical data are not good predictors for future performance which results in too large share of one asset within the portfolio. The third scenario allows for the short-selling, i.e. the situation when weights may have negative values and the investor could benefit from decreasing the value of the asset. The weight interval for this case ranges from -100% to 100%. Similarly to the second case, the weights in the last scenario are restricted from -25% to 25%. A sum of every asset share equals to 100% in all cases. Such broad range of parameters and different scenarios allows for robust sensitivity analysis and durable conclusions.

The Markowitz portfolio has laid the foundations for the modern financial theory and serves as a benchmark in many studies. The limitations of this framework have been widely documented, e.g. unrealistic assumptions about the market microstructure (for example no taxes or transactional costs) or taking into account only past performance and assuming a normal distribution of the assets’ returns. The aim of the second part of the research is not to overcome these disadvantages, but rather to present another way of investing. The methodology is similar to that used by Szado (2009), where he applied a naïve diversification. That approach suggests to impose a fixed value on weights in the portfolio at the beginning of each period. At the end of each period, the weights come back to the previous values, established before. Similarly to the previous part, rebalancing is done on weekly, monthly and quarterly basis.

Even though the second approach seems to be less sophisticated as it does not involve any mathematical formula – it is a popular concept of investing in a mutual or retirement funds. What is more, DeMiguel, Garlappi and Uppal (2007) showed that the naïve diversification approach does not systematically underperform when compared to the mean-variance framework. The authors concluded that more attention should be paid on the estimation of

assets' moments and thus reducing the estimation errors which is not the case for the naïve approach. In addition, naïve diversification reduces the transaction costs, limits the extreme position of one asset and is a sensible choice for the investors who are oriented for the passive investments and prefer to follow the market trend. (What is more, Benartzi and Thaler (2001) showed that most individuals follow the simple rule of thumb and evenly distribute the welfare across different asset classes.) Although such framework may not be the most effective one (the portfolio may not lay on the efficient frontier when assessed ex-post), the comparison of results of the naïve diversification with the Markowitz approach tests the robustness of the previous outcomes and may lead to different conclusions.

To assess the effectiveness of adding the volatility exposure to a diversified portfolio, the naïve approach compares the risk-adjusted return between the portfolios with 1%, 3% and 5% shares of the VXX and without volatility instrument at all. Instead of dividing asset shares for $1/N$ weight each, as the previously mentioned researchers did, the aim of the constructed portfolios were to mimic the mutual fund performance which operates on the Polish market but also has a limited access to the global investment opportunities. Based on this assumption, the following four types of scenarios were tested – each with different portfolios' characteristics. The first one represents the risk-averse approach to investing – with high shares of debt and money market instruments, but remaining a broad range of different asset classes in the portfolio, same as in the Markowitz framework. The second scenario is dedicated to a more aggressive strategy with a relatively higher value of equity and having the same type of assets as in the first scenario (the detailed description of used components is presented in the section Data). Both portfolios have a stable ratio between Polish and global assets – 80% and 20%, respectively. The third and the fourth type of portfolios simulate the investment fund which is devoted to the only-equity strategy. These results may differ from the two previous scenarios as the stocks are stronger correlated with the volatility instrument. The third scenario exhibits higher exposure to the Polish assets, while the fourth is more geographically diversified – WIG20 share is equal to 70% and 40%, respectively. The added exposure of the VXX in each scenario is distributed in such a way that the ratio of Polish and global assets remains stable. Table 10 in the appendix presents the weights for each portfolio and table 1 summarizes the assumptions about the tested scenarios.

Table 1. Summary of the assumptions for tested scenarios

Methodology	Scenario	Assumptions
Markowitz	First	A long only strategy - the weight of single assets may vary between (0%:100%)
	Second	A long only strategy with an upper bound limitation to 25% - (0%:25%)
	Third	An allowance for short selling - the weights are restricted to (-100%:100%) range
	Fourth	An allowance for short selling with an upper and lower bound limitations - (-25%:25%)
Naive	First	A diversified portfolio with high share of safe assets, i.e. money market and debt instruments
	Second	A diversified portfolio with relatively higher share of more aggressive assets, i.e equity
	Third	An equity only portfolio with 70% of shares dedicated to the Polish market
	Fourth	An equity only portfolio which is more geographically diversified (40% for the Polish equity and 60% for global indices)

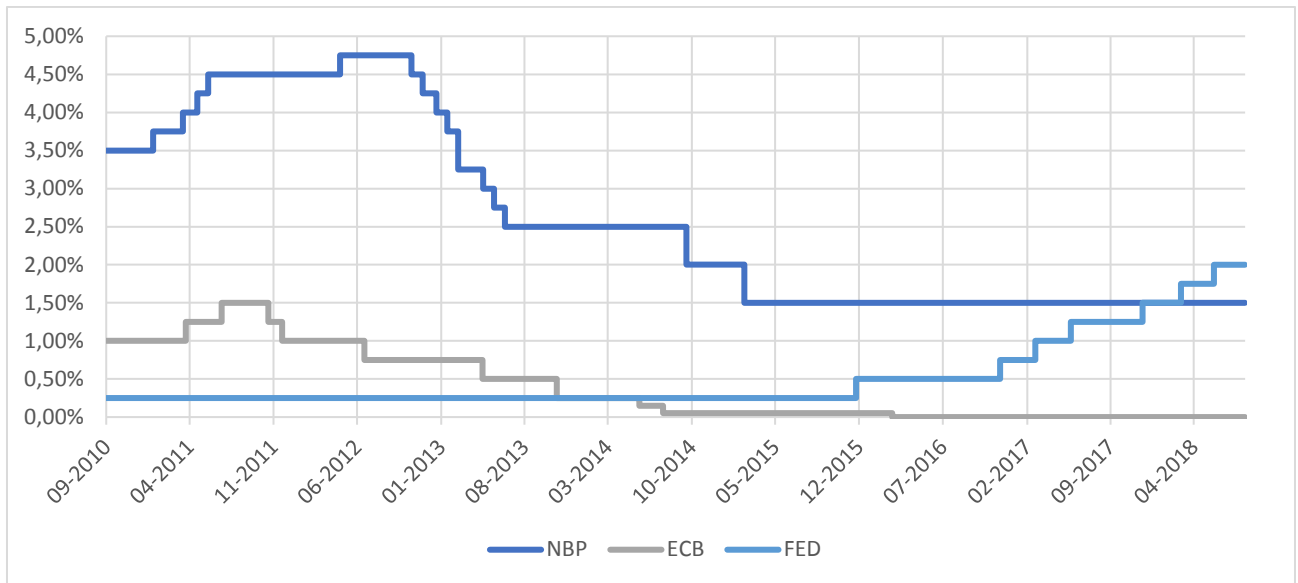
Note: The table presents the assumptions of the tested scenarios for each methodology.

All calculations were performed in the local currencies. During the period of analysis, the interest rates in Poland were higher than in the United States and Europe, because of central banks policies and their Quantitative Easing programme (except the negligible short period in 2018 when FED's reference rate grew above the NBP's interest rate). Thanks to a positive difference between the Polish and European and American interest rates, the beneficial hedge could be made with a forward instrument. The forward exchange rate of Polish zloty to euro or USD was lower than the spot price what made it possible to not incur additional costs related to hedging of exchange rates.

Transaction costs are included for all scenarios to mirror the reality of the investments. The portfolio is charged with the transaction costs every time the portfolio is rebalanced. Moreover, for the first and the last day of investments, when the portfolio is launched and liquidated, the costs are included. The value of the transaction costs is set to 0.25% of the value of the trade. To include the costs into the portfolio performance, the Turnover Ratio is multiplied by the value of transaction costs and then subtracted from daily return. The Turnover Ratio shows how much the portfolio shares changed. This value varies between 0% (the weights remained the same) to twice the range of possible asset shares (which is a total reallocation of the assets). For example, the Turnover Ratio for a long-only portfolio varies between 0% and 200%.

$$\text{Turnover Ratio (TR)} = \sum_{i=1}^N |w_{i,t} - w_{i,t-1}| \quad (3.6)$$

where $w_{i,t}$ is a weight of i th asset on a t -day.



Source: <https://www.federalreserve.gov>, <https://www.ecb.europa.eu>, <https://www.nbp.pl>

Note: The plot presents the changes of reference interest rates of Polish, European and American central banks. The figures cover the period from September 2010 to August 2018.

Figure 1. The plot of interest rates of NBP, ECB and FED

All calculations were performed in R Studio environment.

4 Data

The analysis is performed on the daily close prices and covers the period since the end of September 2010, when an ETF tracking the WIG20 index was introduced, to July 2018. All of the constituents presented below are either the ETFs or mutual investment funds quotations which allows for easy calculation of all capital gains and management fees. In addition, it mirrors the real-world investments and makes the analysis robust. The data were downloaded from stooq.com, yahoo! finance and the ETFs' providers webpages. The following ten types of assets were selected to represent the diversified portfolio:

- WIG20 – an index, which gathers the 20 biggest companies on the Warsaw Stock Exchange. It is represented by Lyxor WIG20 UCITS ETF. Source: <https://stooq.pl/q/d/?s=etfw20l.pl>
- UniKorona Dochodowy (previously known as UniKorona Pieniężny) – the Union Investment fund which represents the Polish money market. Source: <https://stooq.pl/q/?s=3961.n>

- NN Obligacji – a fund that serves as a proxy for the Polish treasury market. Managed by the NN Investment Partners. Source: <https://www.nntfi.pl/fundusze-inwestycyjne/fundusze-obligacji/nn-obligacji>
- S&P500 – an index of the 500 largest US companies. It is represented by SPDR S&P500 ETF. Source: <https://finance.yahoo.com/quote/SPY/>
- MSCI Europe – an index that covers the behaviour of large and medium stocks from 15 developed countries in Europe. The ETF is called iShares Core MSCI Europe and is run by BlackRock. Source: <https://www.ishares.com/uk/individual/en/products/251861/ishares-msci-europe-ucits-etf-acc-fund>
- FTSE Pacific – an index of all cap equities located in the Pacific area (i.e. Japan, Australia, Hong Kong, South Korea and Singapore). The ETF is managed by Vanguard. Source: <https://finance.yahoo.com/quote/VPL/history?p=VPL>
- FTSE G7 Government Bond – an index that measures the performance of sovereign bonds of Group-of-Seven countries. It is represented by iShares Global Govt Bonds UCITS ETF run by BlackRock. Source: <https://www.ishares.com/ch/individual/en/products/251753/ishares-global-government-bond-ucits-etf>
- S&P GSCI – an index that serves as a benchmark for the commodity markets. iShares S&P GSCI Commodity–Indexed Trust, which is managed by BlackRock, tries to mimic the index behaviour. Source: <https://stoq.pl/q/g/?s=gsg.us>
- Dow Jones Real Estate – an index that represents the performance of a real estate investment trusts (REIT) and the other companies related directly or indirectly to a real estate investments. An ETF is provided by SPDR. Source: <https://finance.yahoo.com/quote/RWO>
- VXX – an ETN which provides the exposure for short-term VIX futures. Serves as a proxy for volatility. Source: <https://finance.yahoo.com/quote/VXX/>

Table 2 presents a correlation matrix of the assets used in the analysis. VXX has a strong negative correlation with foreign equities (S&P500, MSCI Europe and FTSE Pacific), commodity and real estate components as was expected. Polish assets are less prone to movements of the VXX as correlation with the NN Obligacji fund equals to -0.14, the money-market fund has no common co-movement with volatility, while the WIG20 index's correlation

with the VXX amounts to -0.32. The above observations may be an initial suggestion for diversification benefits of adding volatility exposure to the portfolio.

Table 2. Correlation matrix

Correlation matrix	WIG20	UniKorona Dochodowy	NN Obligacji	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Government Bond	S&P GSCI	Dow Jones Real Estate	VXX
WIG20	1									
UniKorona Dochodowy	0,0249	1								
NN Obligacji	0,1704	0,1326	1							
S&P 500	0,3821	0,0013	0,1264	1						
MSCI Europe	0,5188	0,0171	0,2054	0,6201	1					
FTSE Pacific	0,3710	0,0010	0,1655	0,8245	0,6062	1				
FTSE G7 Government Bond	-0,0519	0,0592	0,2185	-0,1359	-0,2302	-0,0637	1			
S&P GSCI	0,2448	-0,0087	0,0813	0,4211	0,3320	0,4192	-0,0190	1		
Dow Jones Real Estate	0,3253	0,0210	0,2400	0,8132	0,5204	0,7745	0,0473	0,3495	1	
VXX	-0,3200	0,0051	-0,1430	-0,8297	-0,5119	-0,7020	0,1078	-0,3201	-0,6613	1

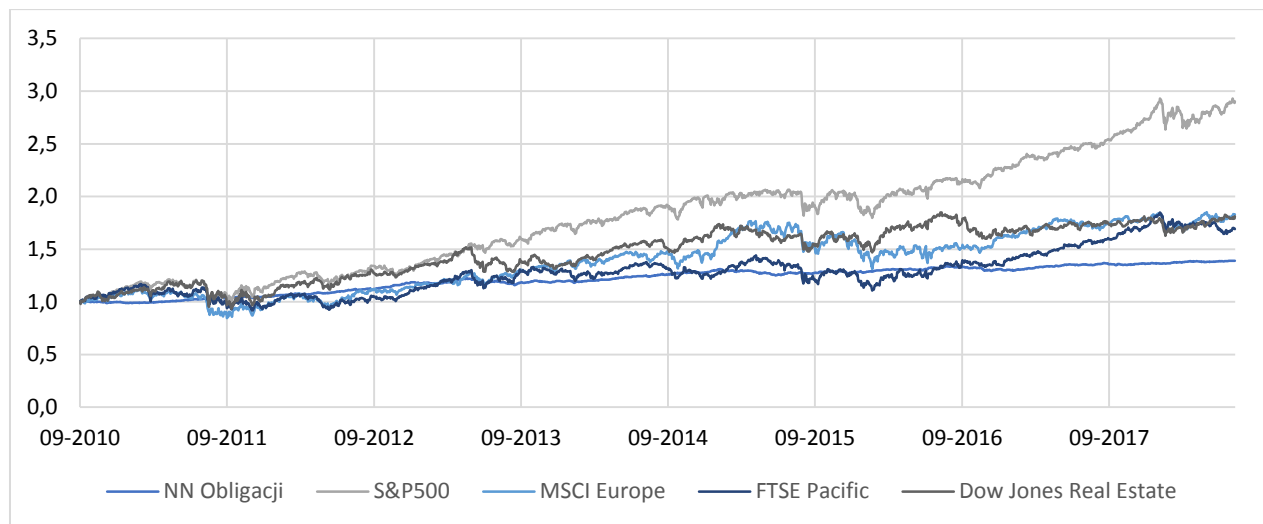
Note: The table presents the correlation matrix for the assets used in the research. The calculation were prepared for the period of September 2010 – July 2018 for the simple daily returns. Bolded observations indicate semi-strong and strong correlations: $|p| \geq 0.33$

Table 3 displays a summary of the assets performance and basic descriptive statistics during the period of analysis while figure 1 and 2 display equity line of each asset. Taking into account the Information Ratio 1, UniKorona Dochodowy exhibits the best result which amounts to 2.54. Although its return is not large, an extremely small value of the standard deviation contributes to the high IR1. The second best result is obtained by the NN Obligacji fund which IR equals to 1.5 – while the return is subtly higher than the money market fund, the risk factor has almost doubled. S&P500 has the highest annualized compounded return over the analysed period (approximately 14%) and relatively good risk-adjusted return which equals to 1. Real estate, European and Pacific equities have performed decent – their IR1s vary around 0.45, whereas global bonds and Polish equities annualized returns are close to 0. A serious decrease in oil prices has resulted in poor performance of the S&P GSCI while the VXX has lost almost all its value. The reason for that situation is the fact that the ETN was launched at the beginning of 2009 when the financial crisis hit the markets and high values of the VIX were observed. Then, the volatility has dropped which brought intense decreases of the VXX quotations. The following years were not profitable for the volatility ETN as well. The annual returns for every year of analysis, except 2018, were negative. Most of the assets are negatively skewed, whereas values of the kurtosis vary, but the majority is above the level of 3 which stands for normal distribution benchmark. UniKorona Dochodowy exhibits a dramatically high value of kurtosis and skewness due to the one extreme event, which differed much from the general feature of the money market fund – stable growth and low volatility.

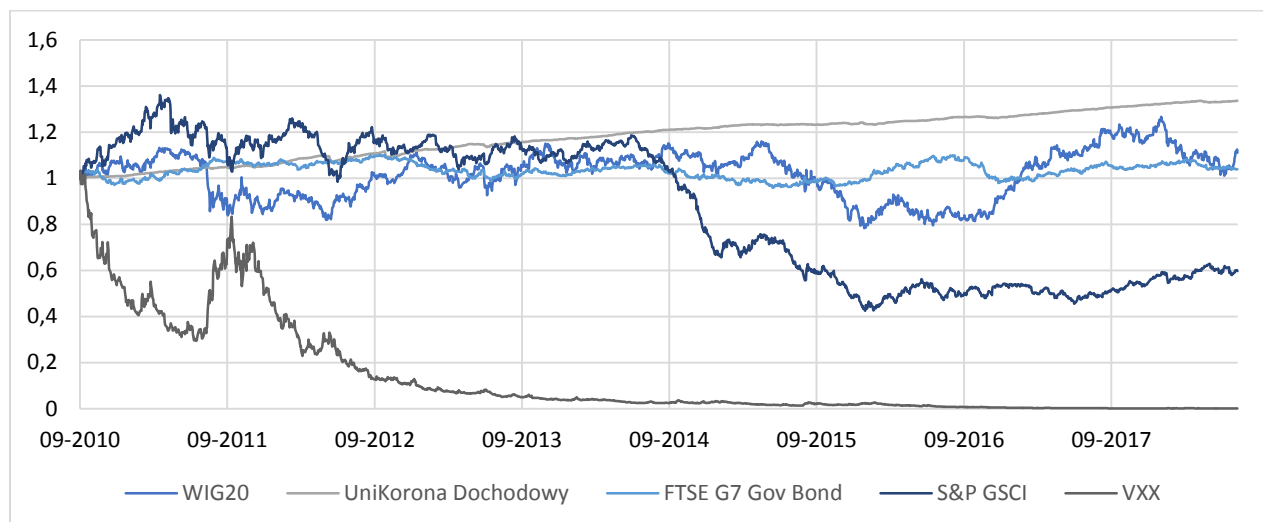
Table 3. Assets performance and descriptive statistics

	WIG20	UniKorona Dochodowy	NN Obligacji	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Government Bond	S&P GSCI	Dow Jones Real Estate	VXX
ARC	1,31%	3,64%	4,15%	14,08%	7,77%	6,70%	0,47%	-6,19%	7,63%	-54,25%
ASD	18,83%	1,43%	2,76%	13,94%	15,85%	15,93%	6,30%	18,52%	15,67%	63,38%
IR1	0,07	2,54	1,50	1,01	0,49	0,42	0,07	-0,33	0,49	-0,86
MD	32,47%	2,58%	5,44%	18,61%	25,90%	23,23%	13,02%	68,78%	22,30%	99,86%
IR/MD	0,22	98,45	27,64	5,43	1,89	1,81	0,57	-0,49	2,18	-0,86
IR2	0,00	3,58	1,15	0,76	0,15	0,12	0,00	-0,03	0,17	-0,47
Skewness	-0,19	-18,18	-0,52	-0,51	-0,34	-0,43	-0,16	-0,18	-0,37	1,28
Kurtosis	4,54	477,71	4,72	5,12	3,68	4,13	1,58	2,10	10,20	6,26

Note: ARC – annualized return compounded, ASD – annualized standard deviation, IR1 and IR2 – Information Ratios, presented in section 3, MD – maximum drawdown. All the statistic were prepared for the period of September 2010 – July 2018 for the simple daily returns.



Note: The plot presents the equity line for 5 assets with the highest returns. The figures were prepared for the period of September 2010 – July 2018 for the simple daily returns.

Figure 2. The equity line for 5 assets with the highest returns

Note: The plot presents the equity line for 5 assets with the lowest returns. The figures were prepared for the period of September 2010 – July 2018 for the simple daily returns.

Figure 3. The equity line for 5 assets with the lowest returns

5 Results

The results presented in this section provide a deep analysis of the portfolios' performance. The broad range of parameters and assumptions contribute to many alternative variants. The annual return compounded (ARC), the annualized Information Ratio, the average of transaction costs and the average of assets weights are calculated for each portfolio. All presented figures below are net values.

5.1 Markowitz Portfolio

Tables 4 and 5 display the results of the annual return compounded (ARC) and the annualized Information Ratio for all selected assumptions and specifications. The results of the first scenario are unambiguous – the inclusion of the VXX to the diversified portfolio brings a decrease of the returns and IRs in all situations, regardless of the parameters selection. The portfolios without VXX have the highest values of the risk-adjusted returns for the memory of 120 days and weekly and monthly rebalancing. On the other hand, when comparing the portfolios which included volatility exposure, quarterly reallocation and short memory tend to perform better than other parameters. Nevertheless, the inclusion of VXX into the optimization process does not produce better results of Information Ratio in any case.

Tables 11 and 12 in the appendix present the average of the asset shares for the first scenario. Both types of portfolios are dominated by money market fund which average weights vary between 62% and 80%. NN Obligacji is the second most often chosen asset within the portfolios which average weight amounts to approximately 10%-12%. Remaining assets have much smaller impact on the portfolio performance, around which S&P500 and FTSE G7 Government Bond index appear to have relatively greater average weights than others. Polish equity's share is insignificant (approximately 1% in most cases) due to its poor risk-adjusted return over the analysis horizon. An important observation is that the shorter the *memory* parameter and the less often the frequency of rebalancing, the portfolio is less dominated by the UniKorona Dochodowy and remaining assets increase their weights. It could be explained by the fact of more stable weights across different time periods when longer historical data are taken into consideration and the impact of any extreme events does not influence the results too much. The average share of the VXX varies between 1% and 4% – it is higher for the quarterly rebalancing. An inclusion of volatility exposure to the portfolio results in a bigger share of the S&P500 and smaller for the global government bonds which is an intuitive fact and was expected based on the correlation matrix.

Table 4. Annual return compounded (ARC) of the Markowitz portfolio with and without the VXX exposure

Rebalancing on	Memory	1st scenario		2nd scenario		3rd scenario		4th scenario	
		no VXX	with VXX	no VXX	with VXX	no VXX	with VXX	no VXX	with VXX
Weeks	15	-3,23%	-6,27%	-2,27%	-6,73%	-13,39%	-17,82%	-6,62%	-7,63%
	30	-0,57%	-5,28%	-1,44%	-4,22%	-3,19%	-5,80%	-4,06%	-4,89%
	60	1,79%	1,05%	1,00%	0,47%	-0,34%	-2,67%	0,55%	1,67%
	120	3,48%	0,68%	1,89%	0,28%	2,41%	1,55%	0,27%	1,26%
Months	15	1,27%	-4,15%	2,28%	-1,10%	-0,90%	-2,16%	-0,92%	-1,18%
	30	1,92%	-2,16%	2,18%	-0,79%	0,85%	0,20%	0,84%	0,84%
	60	1,91%	0,94%	3,86%	2,43%	2,64%	2,33%	3,51%	4,80%
	120	3,86%	-0,57%	2,96%	1,29%	3,94%	3,54%	2,25%	3,04%
Quarters	15	4,67%	1,71%	3,27%	2,17%	2,40%	0,63%	2,14%	1,14%
	30	3,71%	2,21%	3,10%	2,49%	2,56%	1,43%	1,44%	1,14%
	60	0,90%	0,42%	3,35%	1,28%	2,65%	3,36%	4,17%	2,33%
	120	2,97%	-0,47%	2,66%	0,32%	3,12%	3,51%	2,77%	2,80%

Note: The table presents the ARC during the period of analysis. The 1st scenario assumes that one asset share may range between 0% and 100%, the 2nd scenario limits one asset weight upper-bound to 25%, the 3rd scenario allows for short-selling and relax the previous limitation (a single asset shares vary between -100% and 100% and the 4th scenario limits lower and upper bound to -25% and 25%, respectively. *Memory* parameter shows how many historical trading days were used to establish an efficient frontier, whereas *Rebalancing on* parameter presents the frequency of the portfolio reallocation. The bolded numbers indicate portfolios, which include the VXX and have higher values of ARC.

Table 5. Annualized Information Ratio of the Markowitz portfolio with and without the VXX exposure

Rebalancing on	Memory	1st scenario		2nd scenario		3rd scenario		4th scenario	
		no VXX	with VXX	no VXX	with VXX	no VXX	with VXX	no VXX	with VXX
Weeks	15	-0,6919	-0,9116	-0,4500	-1,2052	-2,5005	-2,3043	-1,0876	-1,2250
	30	-0,1875	-0,7269	-0,2889	-0,7994	-1,0675	-1,3716	-0,7098	-0,8489
	60	0,5437	0,2220	0,2070	0,1070	-0,1343	-0,7010	0,1017	0,3364
	120	2,1868	0,1120	0,3826	0,0558	1,4066	0,7054	0,0494	0,2447
Months	15	0,2520	-0,6712	0,4142	-0,1968	-0,2095	-0,3499	-0,1567	-0,1906
	30	0,5512	-0,3874	0,4054	-0,1524	0,2682	0,0493	0,1376	0,1451
	60	0,5070	0,2607	0,7670	0,5607	1,1131	0,6945	0,6440	0,9880
	120	2,3226	-0,0740	0,5795	0,2474	2,2021	1,4928	0,4027	0,5526
Quarters	15	0,7499	0,4348	0,5797	0,4745	0,4962	0,0913	0,3779	0,1992
	30	1,3705	0,2077	0,5808	0,3852	1,0407	0,2333	0,2572	0,1673
	60	0,1659	0,0764	0,6165	0,2746	0,9990	0,8198	0,7180	0,4064
	120	1,3905	-0,0325	0,4642	0,0558	1,3314	1,1550	0,4419	0,4712

Note: The table presents the average Information Ratio (return to standard deviation ratio) during the period of the analysis. The 1st scenario assumes that one asset share may range between 0% and 100%, the 2nd scenario limits an asset weight upper-bound to 25%, the 3rd scenario allows for short-selling and relax the previous limitation (asset shares vary between -100% and 100%) and the 4th scenario limits lower and upper bound to -25% and 25%, respectively. *Memory* parameter shows how many historical trading days were used to establish an efficient frontier, whereas *Rebalancing on* parameter presents the frequency of the portfolio reallocation. The bolded numbers indicate portfolios, which include the VXX and have higher values of IR.

The second scenario, which assumes a limitation of upper bound share to 25% leads to a similar conclusion – all parameters provide poorer results for the portfolio with the VXX than without, in terms of returns and IR. The portfolios which include volatility ETN follow the same pattern as in the first scenario, i.e. the performance of the portfolios with the memory of

60 trading days are the best among the weekly and monthly rebalancing. On the other hand, the portfolios estimated on the last 15 days have the strongest figures among quarterly rebalancing. In comparison to the previous scenario, the portfolios with the VXX and upper bound limitation improve results for monthly and quarterly rebalancing, whereas the difference of first and second scenarios for portfolios without volatility exposure does not follow any pattern. Nevertheless, a general conclusion remains the same that adding volatility does not enhance the overall performance.

Average asset weights for the second scenario are presented in table 13 and 14 in the appendix. An upper bound limitation of a single asset share provides more diversified portfolios, but the UniKorona Dochodowy remains the most important component with an average weight equals to over 24%. Similarly to the previous scenario, the NN Obligacji, S&P500 and FTSE G7 Government Bond index play an important role within the portfolio performance. The VXX share slightly increased, but its weight does not exceed 5% in any case. What is more, an incorporation of volatility exposure follows similar pattern as previously – an addition of the VIX is followed by an increase of American equity and a decrease of the global bonds. In contrary to the previous scenario, the asset weights are more stable across different *memory* or *rebalancing* parameters.

The third scenario allows for short-selling and the weight of each asset may vary from -100% to 100%. The ARC of portfolio with the VXX presents better results than without volatility exposure in only 2 out of 12 cases (for quarterly rebalancing and the memory of 60 and 120 trading days), whereas the Information Ratio is higher only for weekly portfolio reallocation and 15 trading days of the memory (however, the performance of this portfolio is extremely poor). Both types of portfolios exhibit two similar patterns – longer memory and less frequent rebalancing provide better results in terms of absolute returns and the Information Ratio.

The portfolios in the third scenario are even more dominated by the UniKorona Dochodowy than in the first scenario as its weights vary around 82%. Remaining assets follow a similar order as previously – NN Obligacji, the American equity and global bonds have the greatest weights around the leftover components. Dow Jones Real Estate, commodity market and the Polish and Asian equity have on average negative weights, but their values do not exceed -5%. The VXX share decreases in comparison to the first and the second scenario and ranges between 0% and 1%. These average figures, however, may be neutralised by positive and negative values which results in a value close to 0.

The results for the last scenario much differ from the previous cases. The portfolios with the VXX have higher Information Ratio than those without the ETN on VIX index in 6 out of 12 situations, mostly when the *memory* parameter equals to 60 and 120 trading days. In addition, volatility exposure improves the ARC of the portfolio in 5 out of 12 cases. Nonetheless, the comparison of the third and the fourth scenario clearly shows that the limitation of the upper and lower bound of single asset weights decrease the overall performance of portfolios.

The average weights for the fourth scenario are positive in almost all cases. The general order is followed from the previous scenarios – the UniKorona Dochodowy, NN Obligacji, S&P500 and FTSE G7 government bonds indices have the highest shares across the portfolios. However, the relative values in comparison to the second scenario (when upper bound was also 25%, but there was no possibility for short selling) increased in favour of the Polish and Asian equities, global commodity and real estate market. The share of the VXX varies around 1,5%-3% and is considerably higher for a shorter memory.

An analysis of the portfolio performance constructed via the Markowitz framework exhibits clear pattern – weekly rebalancing produces poor investment results. It is extremely visible for a short memory (15 trading days) parameter when the average returns are considerably negative. An analysis of transaction costs may be a useful tool to understand such weak portfolio performance. Table 6 shows the average transaction costs in the percentage term. In order to make an intuitive comparison between different frequencies, the figures are annualized, i.e. weeks, months and quarters are multiplied by 52, 12 and 4, respectively. The general conclusion is that more frequent rebalancing produces higher expenses and reduces investment profitability. What is more, the lengthen of the *memory* parameter results in lower transaction costs which may be explained by more stable weights and smaller Turnover Ratio. A short range of historical data could bring extraordinary asset selection in the Markowitz portfolio which makes rebalancing more expensive. There is also a relatively substantial difference between the first and the third scenario. An allowance for the short selling produces higher transaction costs, which was expected as Turnover Ratio range doubles. By contrast, the upper and the lower bound limitation of asset shares in the second and the fourth scenario does not systematically reduce the transaction costs in comparison to their respective benchmarks (the first and the third scenario).

Table 6. Annualized average of transaction costs of the Markowitz portfolio

		1st scenario		2nd scenario		3rd scenario		4th scenario	
Rebalancing on	Memory	no VXX	with VXX	no VXX	with VXX	no VXX	with VXX	no VXX	with VXX
Weeks	15	6,79%	7,14%	5,73%	6,29%	17,85%	21,84%	10,19%	10,68%
	30	3,84%	4,29%	3,92%	4,22%	7,10%	9,05%	6,99%	7,15%
	60	2,59%	2,67%	2,84%	2,78%	4,06%	5,11%	4,50%	4,41%
	120	1,10%	1,20%	1,96%	1,85%	1,69%	2,28%	3,05%	2,86%
Months	15	2,94%	3,01%	2,30%	2,53%	5,13%	6,54%	3,77%	3,79%
	30	1,96%	2,02%	1,95%	1,99%	2,80%	3,52%	3,15%	3,03%
	60	1,35%	1,38%	1,36%	1,34%	1,81%	2,29%	2,11%	2,01%
	120	0,71%	0,75%	0,92%	0,84%	0,88%	1,35%	1,48%	1,38%
Quarters	15	1,05%	0,97%	0,69%	0,77%	1,55%	2,29%	1,11%	1,23%
	30	0,68%	0,74%	0,70%	0,70%	0,87%	1,51%	1,21%	1,25%
	60	0,84%	0,87%	0,72%	0,72%	1,11%	1,38%	1,09%	1,02%
	120	0,44%	0,47%	0,54%	0,52%	0,56%	0,80%	0,89%	0,81%

Note: The table presents the annualized average of transactional costs during the period of analysis. The 1st scenario assumes that one assets share may range between 0% and 100%, the 2nd scenario limits an asset weight upper-bound to 25%, the 3rd scenario allows for the short-selling and relax the previous limitation (asset shares vary between -100% and 100%) and the 4th scenario limits the lower and upper bound to -25% and 25%, respectively. Memory parameter shows how many historical trading days were used to establish an efficient frontier, whereas *Rebalancing on* parameter presents the frequency of the portfolio reallocation.

A long-term analysis of the portfolio selection with the VXX showed that the inclusion of volatility exposure does not systematically improve the performance of the portfolios (except minor examples). The addition of the VIX ETN to the process of portfolio optimization via the Markowitz framework has worsened the Information Ratio when compared to those portfolios without volatility instruments. Even though the VXX has performed poorly during the period of analysis, there may exist sub-periods when the incorporation of volatility exposure could bring profitable results. Tables 19-22 in the appendix present the year by year analysis of the differences between the Information Ratios for both types of portfolio. Positive values of this difference would indicate the beneficial effect of the VXX inclusion in the given sub-period.

The results for the first scenario follow a similar trend as the analysis for the whole period. The results of the portfolio without volatility exposure are better in 78% of cases. An inclusion of the VXX to optimization procedure produced more profitable portfolios in terms of IR only for the last quarter of 2010 and 2018 (but solely on weekly and monthly frequency of rebalancing). Minor examples of better performance are observable in almost every year, but they do not follow any pattern. The numbers for the second scenario have slightly improved as the 39 out of 105 cases have higher IR when the VXX was included. The risk-adjusted return was higher for 2010, 2011, 2014 and 2018, mostly no matter which parameters (i.e. the memory or the frequency of rebalancing) were chosen. The yearly analysis for the third scenario exhibits similar percentage of higher Information Ratio for the portfolios which include the VXX as for

the second scenario (approximately 40%). However, the pattern is not as clear as previously. The portfolios with volatility exposure have decreased their relative performance for 2011 and scarcely have higher IRs. 2014 presents decent results, but only for a weekly and monthly rebalancing, whereas Information Ratio for the portfolios with the ETN on the VIX in 2015 is higher only for quarterly frequency. In addition, portfolios without the VXX and rebalanced weekly are much worse, while results for months and quarters of rebalancing are ambiguous. The ratio of positive values of the difference for the fourth scenario is the highest among all assumptions and is equal to 48,5%. Similarly to the second case, higher Information Ratios were obtained by the portfolios with volatility exposure in 2011, 2014, 2015 and 2018. However, quarterly rebalancing for 2014 and 2018 provided negative IR differences again.

The general conclusions from the portfolio optimization via the Markowitz framework are unambiguous. The inclusion of the VXX into the optimization procedure led to the deterioration of the investment performance, in terms of both annualized average returns and annualized Information Ratio. Even though the assumption of a range of a single asset from -25% to 25% produced relatively better results, compared to the other scenarios – the Information Ratios were higher only for 50% of cases what cannot provide any durable conclusions about the investment profitability with volatility instruments. Nevertheless, there exist sub-periods when the differences between portfolios with and without the VXX are positive. It may indicate on special treatment of volatility instrument with precaution and proper investment tools.

5.2 Naive Diversification

Tables 7 and 8 present the results of annual return compounded and Information Ratio for a naïve diversification approach. The clear pattern may be observed for all scenarios – higher volatility exposure results in a decrease in profitability. The tables confirm that the negative average return of volatility ETN deteriorates the efficiency of the portfolios. Together with the increase of the VXX share, the portfolios' results are worsening which forms a visible trend. The first and the second scenario have similar values of the ARC, with almost 0% for the portfolios when a share of the VXX equals to 5% and up to approximately 3,5% when portfolios do not contain any volatility exposure. In contrary, the first scenario, which assumes a safer approach, outperforms the second one in terms of the Information Ratio. Taking into account the third and the fourth scenario, the results are much better when the foreign assets have larger share. Both annualized returns and Information Ratio have higher values for the fourth scenario. Nevertheless, the lower the share of the VXX, the better the overall performance.

Table 7. Annual return compounded (ARC) for portfolios with fixed weights

Scenario	Rebalancing on	No VXX	VXX 1%	VXX 3%	VXX 5%
First	Weeks	3,33%	2,67%	1,37%	0,06%
	Months	3,34%	2,68%	1,38%	0,08%
	Quarters	3,38%	2,81%	1,68%	0,54%
Second	Weeks	3,46%	2,82%	1,55%	0,22%
	Months	3,48%	2,83%	1,55%	0,23%
	Quarters	3,53%	2,98%	1,89%	0,73%
Third	Weeks	3,80%	3,14%	1,85%	0,51%
	Months	3,83%	3,16%	1,85%	0,50%
	Quarters	3,86%	3,31%	2,21%	1,06%
Fourth	Weeks	6,30%	5,62%	4,31%	2,95%
	Months	6,33%	5,65%	4,31%	2,94%
	Quarters	6,36%	5,80%	4,68%	3,50%

Note: The table presents the annualized average of returns for portfolios with fixed weights. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values. The detailed description of scenarios and their assumptions are included in the Methodology section.

Table 8. Annualized Information Ratio for portfolios with fixed weights

Scenario	Rebalancing on	No VXX	VXX 1%	VXX 3%	VXX 5%
First	Weeks	0,7078	0,6092	0,3351	0,0146
	Months	0,7157	0,6180	0,3407	0,0180
	Quarters	0,7275	0,6509	0,4167	0,1257
Second	Weeks	0,3930	0,3323	0,1941	0,0288
	Months	0,3970	0,3363	0,1970	0,0298
	Quarters	0,4044	0,3558	0,2407	0,0967
Third	Weeks	0,2448	0,2073	0,1274	0,0364
	Months	0,2466	0,2089	0,1282	0,0361
	Quarters	0,2487	0,2188	0,1538	0,0766
Fourth	Weeks	0,4677	0,4344	0,3592	0,2641
	Months	0,4707	0,4376	0,3624	0,2661
	Quarters	0,4732	0,4495	0,3938	0,3177

Note: The table presents the annualized average of Information Ratio for portfolios with fixed weights. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values. The detailed description of scenarios and their assumptions are included in the Methodology section.

The more often frequency of rebalancing results in slightly poorer performance of the portfolios which finds the explanation in table 9 where annualized transaction costs are displayed. Even though the average of Turnover Ratio is smaller for weekly rebalancing, the annualized figures show that the more frequent rebalancing contributes to higher transaction costs on a yearly basis. What is more, a rise of the VXX share in portfolio results in higher

transaction costs too. It may be explained by the fact that the VXX is one of the most volatile assets within the portfolio what increases the Turnover Ratio and costs, as a consequence.

Table 9. An annualized average of the transactional costs for portfolios with fixed weights

Scenario	Rebalancing on	No VXX	VXX 1%	VXX 3%	VXX 5%
First	Weeks	0,16%	0,16%	0,18%	0,19%
	Months	0,11%	0,11%	0,12%	0,13%
	Quarters	0,08%	0,08%	0,09%	0,10%
Second	Weeks	0,19%	0,19%	0,21%	0,23%
	Months	0,12%	0,12%	0,13%	0,14%
	Quarters	0,09%	0,09%	0,10%	0,10%
Third	Weeks	0,15%	0,16%	0,18%	0,20%
	Months	0,10%	0,11%	0,12%	0,13%
	Quarters	0,08%	0,08%	0,09%	0,10%
Fourth	Weeks	0,17%	0,18%	0,20%	0,23%
	Months	0,12%	0,12%	0,13%	0,14%
	Quarters	0,09%	0,09%	0,10%	0,10%

Note: The table presents the annualized values of transactional costs for portfolios with fixed weights. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values. The detailed description of scenarios and their assumptions are included in the Methodology section.

Tables 23-26 in the appendix present the Information Ratio on a yearly basis for each scenario. The detailed analysis of each sub-period may provide some extra information about the portfolio performance and its more effective management. The results for the first scenario show that the incorporation of the VXX reduces risk-adjusted return for all years except 2011, 2014 and 2018. The results for the second, the third and the fourth scenario adhere to the following pattern: the volatility exposure is profitable in 2018 as the share of the VXX equal to 5% brings the highest risk-adjusted return (in most of the cases it reduces losses instead of providing extra positive return). Similarly, the best IR in 2011 is observable for a quarterly rebalancing when the VXX is included, but the value remains negative. For all remaining cases, volatility instruments do not improve the yearly performance by increasing the Information Ratio.

6 Conclusions

The introduction of the VIX index and its derivatives allowed for new investment possibilities. A negative correlation with most of the markets encouraged investors for seeking and exploiting the new strategies which include exposure to volatility. The non-standard characteristics of the

VIX index were considered to be beneficial for long-term investors who are looking for better portfolio diversification. However, the detailed research showed that it is not always valid and significantly depends on the markets' conditions. The global financial crisis in 2008 and the euro debt crisis in 2011 are good examples when investors may benefit from this new asset class as the volatility index jumped significantly and provided impressive profits. Nonetheless, the long-term tendency for volatility instruments is characterized by negative expected return caused by very high average roll yields which deteriorate the overall profits.

The aim of the research was to construct the portfolios via two different frameworks (the mean-variance of Markowitz and the naïve diversification) and compare the risk-adjusted return between the portfolios with and without the volatility exposure. As many studies evaluated the diversification issues from the American and European perspective, this analysis focused on the Polish investor viewpoint, i.e. hypothetical portfolio contained different assets from the Polish capital market. To benefit from volatility as an asset class, that investor had to look for the foreign opportunities as the Warsaw Stock Exchange does not provide necessary instruments. The VXX was chosen as the proxy for volatility due to its long data availability and a common recognition as the most popular volatility Exchange Traded Note. The final portfolio contained a set of different asset classes from various locations and the assessment of the results' differences between the portfolios with and without volatility exposure was done.

The general conclusions are mostly unambiguous. Polish investor faces similar problems as the American, European and global ones as the portfolios which included the volatility exposure in optimization procedure performed generally worse than those without the VXX in the long-term. The only example when the overall performance was relatively better is the portfolio optimized via the Markowitz framework when a short-selling is allowed and the weight of a single asset varies only from -25% to 25%. However, these results were only better when compared to other scenarios as the Information Ratio was higher in only 50% of cases which cannot produce any durable conclusions. The results for naïve diversification approach confirmed the outcomes of Markowitz procedure – the VXX inclusion worsened the investment profitability. What is more, the results in the second approach were poorer for all parameters. The year by year analysis, however, showed that some improvements in the portfolio outcomes may be achieved. The higher Information Ratios for the portfolios with volatility exposure were observed in 2011, 2014 and 2018, but it much depended on selected criteria (i.e. the frequency of rebalancing and the length of the memory parameter).

A chosen time period is a crucial factor for the above results. The analysis covers the data from the end of 2010 until the mid of 2018 when no severe decrease took place. The euro debt

crisis in 2011, a serious decrease in oil price in 2014 or the Brexit referendum in 2016 are the examples when the markets were hit by uncertainty. However, the scale of these events was much less significant, when compared to the banking crisis of 2008 or the dot-com bubble, and the scepticism of the investors about the market's perspective was only temporary. Most of the equity assets exhibited steady growth, stimulated by economic expansion and monetary policy. The results confirm that the volatility exposure in the portfolio may be profitable in sub-periods (e.g. 2011, 2018) but most profits are deteriorated in the long-term unless significant financial crash appears. The prediction of the financial crisis, however, seems to be an extremely difficult task that goes beyond the aim of a passive long-term investor of a diversified portfolio.

Another important factor that affected the obtained results is related to VIX futures market and its relation to the underlying. As Alexander and Korovilas (2012) have shown, keeping permanent volatility exposure to VIX futures incurs high rolling costs due to the upward slope of the VIX futures term structure by most of the time. The VXX invests in short-term contracts and as a result, is particularly exposed to the high costs of carry. This aspect is linked with a durable difference between an implied and realized volatility (volatility risk premium-VRP). The investors tend to overshoot implied volatility when compared to ex-post realized value. As Asensio (2013) presented, the VRP of VIX futures is permanent (and particularly intensive in the post-crisis period) and especially visible for the short-term contracts. The author concluded that higher interest in volatility instruments, by both professional and individual investors, results in greater mispricing as more participants are exposed to the forecast bias. It is also mentioned that VIX futures are treated as insurance for unfavourable market's movement which may push the VRP even more. Summarizing the volatility risk premium may contribute to the poorer investment results for the long-term investors who are interested in a long volatility exposure.

Further research can be dedicated to improving the portfolio optimization procedure. This analysis has not taken into account higher order moments of returns or the downside risk. In addition, different functions to maximize can be chosen to consider investor preferences. What is more, following the idea proposed by Ślepaczuk and Zakrzewski (2006) of an introduction the volatility index on WIG20 options, the potential diversification benefits may be much higher with the VWIG20 index instead of VIX index. The Polish volatility index would be much more negatively correlated with the Polish assets what could provide lowering investment risk.

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Appendix

Table 10. Asset shares in naïve diversification strategy

Scenario	WIG20	UniKorona	NN Obligacji	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Bond	S&P GSCI	DJ Real Estate	VXX
First	20,00%	40,00%	20,00%	2,00%	2,00%	2,00%	10,00%	2,00%	2,00%	0,00%
	20,00%	40,00%	20,00%	1,80%	1,80%	1,80%	10,00%	1,80%	1,80%	1,00%
	20,00%	40,00%	20,00%	1,50%	1,50%	1,50%	9,50%	1,50%	1,50%	3,00%
	20,00%	40,00%	20,00%	1,20%	1,20%	1,20%	9,00%	1,20%	1,20%	5,00%
Second	40,00%	20,00%	20,00%	4,00%	4,00%	4,00%	4,00%	2,00%	2,00%	0,00%
	40,00%	20,00%	20,00%	3,80%	3,80%	3,80%	3,80%	1,90%	1,90%	1,00%
	40,00%	20,00%	20,00%	3,50%	3,50%	3,50%	3,50%	1,50%	1,50%	3,00%
	40,00%	20,00%	20,00%	3,00%	3,00%	3,00%	3,00%	1,50%	1,50%	5,00%
Third	70,00%			10,00%	10,00%	10,00%				0,00%
	70,00%			9,50%	9,75%	9,75%				1,00%
	70,00%			9,00%	9,00%	9,00%				3,00%
	70,00%			8,00%	8,50%	8,50%				5,00%
Fourth	40,00%			20,00%	20,00%	20,00%				0,00%
	40,00%			19,50%	19,75%	19,75%				1,00%
	40,00%			19,00%	19,00%	19,00%				3,00%
	40,00%			18,00%	18,50%	18,50%				5,00%

Note: *Scenario* parameter indicates a type of constructed portfolio. Detailed information is presented in the Methodology section.

Table 11. Average asset share for the first scenario – portfolio without the VXX

Rebalancing on	Memory	WIG20	UniKorona	NN Obligacji	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Bond	S&P GSCI	DJ Real Estate
Weeks	15	1,74%	68,40%	10,44%	2,99%	2,91%	1,17%	7,87%	2,52%	1,96%
	30	1,10%	73,22%	10,87%	3,54%	1,69%	0,52%	5,93%	2,13%	1,00%
	60	1,10%	75,40%	12,02%	4,06%	1,09%	0,40%	3,91%	1,48%	0,55%
	120	0,38%	79,66%	14,05%	1,24%	0,44%	0,52%	3,20%	0,28%	0,23%
Months	15	2,03%	65,69%	9,98%	2,39%	3,34%	2,96%	9,78%	2,19%	1,63%
	30	0,97%	74,04%	10,10%	3,34%	1,38%	0,44%	5,79%	2,38%	1,57%
	60	0,80%	76,91%	10,92%	2,39%	1,78%	0,44%	3,86%	2,49%	0,41%
	120	0,33%	79,23%	14,19%	1,28%	0,45%	0,62%	3,39%	0,30%	0,22%
Quarters	15	1,87%	62,22%	9,60%	0,60%	7,03%	4,27%	11,34%	1,83%	1,24%
	30	0,93%	73,91%	9,44%	0,68%	2,19%	0,48%	9,42%	1,62%	1,34%
	60	1,53%	72,49%	12,89%	2,23%	1,68%	0,53%	4,32%	3,76%	0,56%
	120	0,36%	77,20%	13,78%	1,39%	0,54%	0,76%	5,39%	0,40%	0,17%

Note: *Memory* parameter shows how many historical trading days were used to establish the efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

Table 12. Average asset share for the first scenario – portfolio with the VXX

Rebalancing on	Memory	WIG20	UniKorona	NN Obligacji	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Bond	S&P GSCI	DJ Real Estate	VXX
Weeks	15	1,56%	65,86%	10,32%	5,63%	2,76%	1,60%	5,72%	2,34%	2,14%	2,07%
	30	1,09%	70,07%	10,33%	6,31%	1,74%	0,72%	3,75%	2,11%	1,82%	2,05%
	60	1,11%	73,47%	11,43%	6,01%	1,11%	0,47%	3,03%	1,44%	0,88%	1,05%
	120	0,39%	77,26%	13,85%	3,20%	0,45%	0,53%	2,17%	0,27%	0,41%	1,46%
Months	15	1,92%	64,46%	9,84%	4,77%	2,34%	3,31%	5,66%	1,88%	2,77%	3,05%
	30	0,98%	70,82%	9,62%	5,48%	1,47%	0,63%	3,64%	2,50%	2,68%	2,18%
	60	0,84%	74,97%	10,37%	4,63%	1,05%	0,50%	3,04%	2,51%	1,00%	1,09%
	120	0,34%	77,06%	13,73%	3,31%	0,46%	0,64%	2,02%	0,30%	0,37%	1,76%
Quarters	15	2,13%	61,80%	10,86%	2,89%	3,87%	4,08%	6,87%	1,83%	3,03%	2,64%
	30	0,92%	68,04%	9,77%	2,71%	2,23%	0,74%	4,20%	3,01%	4,05%	4,33%
	60	1,58%	70,58%	11,41%	4,46%	1,69%	0,57%	3,18%	3,76%	1,72%	1,05%
	120	0,37%	74,89%	13,48%	3,12%	0,59%	0,81%	2,25%	0,41%	0,29%	3,78%

Note: *Memory* parameter shows how many historical trading days were used to establish the efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

Table 13. Average asset share for the second scenario – portfolio without the VXX

Rebalancing on	Memory	WIG20	UniKorona	NN Obligacj	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Bond	S&P GSCI	DJ Real Estate
Weeks	15	5,50%	24,57%	19,97%	9,25%	7,03%	4,39%	16,14%	6,54%	6,61%
	30	5,16%	24,67%	20,67%	9,95%	6,70%	3,98%	16,49%	6,32%	6,07%
	60	4,79%	24,88%	21,42%	11,05%	6,14%	4,43%	15,88%	5,27%	6,14%
	120	3,44%	24,97%	21,87%	11,96%	6,08%	4,18%	16,62%	3,52%	7,37%
Months	15	5,42%	24,23%	20,07%	8,22%	6,68%	5,49%	17,21%	6,62%	6,07%
	30	5,30%	24,73%	20,23%	8,67%	6,80%	4,13%	16,63%	7,04%	6,47%
	60	4,70%	24,91%	21,22%	10,87%	6,27%	4,60%	15,95%	5,85%	5,63%
	120	3,16%	24,97%	21,65%	12,20%	5,96%	4,38%	17,09%	3,74%	6,86%
Quarters	15	5,04%	25,00%	20,68%	7,47%	5,76%	6,57%	18,02%	5,04%	6,42%
	30	5,26%	24,26%	23,30%	8,23%	7,07%	4,10%	15,07%	6,72%	5,99%
	60	4,82%	25,00%	22,16%	11,53%	5,47%	4,90%	14,13%	6,55%	5,44%
	120	3,30%	25,00%	21,32%	12,39%	5,67%	4,67%	15,93%	4,63%	7,10%

Note: Memory parameter shows how many historical trading days were used to establish the efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

Table 14. Average asset share for the second scenario – portfolio with the VXX

Rebalancing on	Memory	WIG20	UniKorona	NN Obligacj	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Bond	S&P GSCI	DJ Real Estate	VXX
Weeks	15	4,40%	24,33%	18,53%	12,51%	6,24%	4,71%	13,29%	4,85%	7,17%	3,99%
	30	4,19%	24,22%	19,74%	13,70%	5,90%	4,27%	13,54%	4,70%	6,06%	3,68%
	60	3,95%	24,75%	20,92%	14,52%	5,55%	4,35%	13,34%	4,08%	5,62%	2,93%
	120	3,20%	24,96%	21,75%	15,91%	5,37%	3,92%	14,30%	2,07%	5,43%	3,09%
Months	15	4,52%	23,84%	18,90%	11,01%	5,68%	5,16%	14,27%	5,40%	7,15%	4,07%
	30	4,34%	24,19%	18,79%	12,11%	5,69%	4,39%	14,10%	5,44%	7,04%	3,89%
	60	4,00%	24,57%	20,56%	13,85%	5,73%	4,67%	13,77%	4,71%	5,25%	2,89%
	120	2,92%	24,97%	21,62%	15,94%	5,26%	4,03%	15,01%	2,32%	4,85%	3,08%
Quarters	15	4,60%	24,71%	19,45%	9,51%	4,31%	6,53%	15,49%	4,60%	7,29%	3,51%
	30	3,94%	23,44%	20,55%	12,84%	5,47%	3,95%	13,29%	5,11%	6,67%	4,74%
	60	3,84%	24,70%	21,10%	14,42%	5,24%	5,25%	12,55%	4,72%	5,49%	2,68%
	120	3,12%	25,00%	21,40%	15,71%	5,59%	3,64%	14,12%	3,47%	4,77%	3,18%

Note: Memory parameter shows how many historical trading days were used to establish the efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation

Table 15. Average asset share for the third scenario – portfolio without the VXX

Rebalancing on	Memory	WIG20	UniKorona	NN Obligacj	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Bond	S&P GSCI	DJ Real Estate
Weeks	15	-0,12%	77,61%	10,17%	6,26%	2,28%	0,28%	3,42%	-0,02%	0,11%
	30	-0,17%	84,18%	9,81%	5,91%	0,31%	-1,21%	1,36%	0,36%	-0,55%
	60	0,08%	83,78%	11,57%	7,22%	0,04%	-1,50%	0,95%	-0,25%	-1,88%
	120	-0,28%	82,71%	14,65%	4,95%	0,06%	-0,57%	1,00%	-1,09%	-1,43%
Months	15	-0,64%	82,71%	6,19%	5,65%	1,37%	1,54%	4,06%	-0,35%	-0,53%
	30	-0,43%	85,55%	10,95%	5,87%	0,31%	-1,85%	-0,41%	0,38%	-0,37%
	60	-0,26%	84,81%	11,35%	6,62%	0,38%	-1,73%	1,10%	-0,43%	-1,83%
	120	-0,40%	82,70%	14,56%	4,92%	0,14%	-0,27%	0,99%	-1,07%	-1,59%
Quarters	15	0,63%	87,65%	-1,00%	-0,87%	0,14%	3,41%	5,69%	0,02%	4,34%
	30	-0,81%	88,13%	9,78%	4,69%	1,53%	-2,55%	-0,39%	-0,93%	0,55%
	60	-0,06%	83,64%	12,53%	8,20%	0,30%	-2,87%	0,82%	-1,00%	-1,55%
	120	-0,53%	81,52%	15,52%	5,45%	0,16%	-0,21%	0,93%	-1,18%	-1,65%

Note: Memory parameter shows how many historical trading days were used to establish the efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation

Table 16. Average asset share for the third scenario – portfolio with the VXX

Rebalancing on	Memory	WIG20	UniKorona	NN Obligacj	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Bond	S&P GSCI	DJ Real Estate	VXX
Weeks	15	-1,51%	75,68%	14,17%	7,70%	1,34%	-0,62%	3,62%	0,04%	-0,40%	-0,01%
	30	-0,47%	82,23%	11,22%	9,03%	-0,70%	-2,07%	0,29%	-0,31%	0,19%	0,60%
	60	-0,08%	81,17%	13,57%	9,02%	-0,02%	-1,72%	0,61%	-1,12%	-1,75%	0,33%
	120	-0,43%	80,64%	16,60%	7,35%	0,08%	-1,38%	0,25%	-1,60%	-1,81%	0,31%
Months	15	-1,62%	75,99%	13,03%	6,05%	1,63%	2,44%	4,45%	-1,07%	-0,90%	-0,01%
	30	-1,47%	84,29%	10,10%	8,19%	-0,74%	-1,84%	-0,01%	0,74%	0,13%	0,61%
	60	-0,45%	82,12%	12,43%	8,99%	0,38%	-2,00%	0,05%	-1,40%	-0,82%	0,70%
	120	-0,60%	80,47%	16,82%	7,13%	0,01%	-1,10%	-0,03%	-1,60%	-1,48%	0,37%
Quarters	15	-2,85%	75,36%	9,05%	0,63%	3,25%	6,43%	4,63%	-1,92%	3,81%	1,62%
	30	-4,32%	87,60%	7,88%	9,59%	-0,44%	-1,84%	-1,07%	1,23%	0,25%	1,10%
	60	-0,35%	81,86%	13,92%	11,25%	0,47%	-5,01%	-2,29%	-2,52%	1,67%	1,00%
	120	-0,73%	79,95%	18,11%	9,00%	-0,08%	-2,18%	-0,69%	-2,15%	-1,66%	0,42%

Note: Memory parameter shows how many historical trading days were used to establish the efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation

Table 17. Average asset share for the fourth scenario – portfolio without the VXX

Rebalancing on	Memory	WIG20	UniKorona	NN Obligacj	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Bond	S&P GSCI	DJ Real Estate	VXX
Weeks	15	2,22%	24,74%	22,83%	12,18%	6,66%	3,26%	19,42%	3,84%	4,86%	
	30	2,09%	24,86%	23,34%	13,38%	6,87%	1,87%	19,92%	3,53%	4,13%	
	60	1,84%	24,96%	23,19%	15,60%	7,63%	0,79%	20,24%	2,67%	3,07%	
	120	0,81%	24,98%	24,63%	17,56%	8,75%	-0,12%	20,96%	-0,01%	2,44%	
Months	15	2,71%	24,73%	22,48%	9,58%	5,47%	4,91%	20,99%	4,56%	4,56%	
	30	1,53%	24,91%	23,84%	11,21%	6,32%	1,68%	19,98%	5,06%	5,47%	
	60	1,46%	24,98%	23,12%	15,02%	7,97%	1,09%	20,48%	3,11%	2,79%	
	120	0,91%	24,97%	24,59%	16,88%	8,69%	0,28%	21,16%	0,10%	2,43%	
Quarters	15	3,95%	25,00%	24,83%	7,01%	0,69%	9,34%	20,37%	4,11%	4,70%	
	30	1,27%	24,74%	24,86%	12,55%	5,44%	1,99%	20,93%	2,99%	5,23%	
	60	1,25%	25,00%	24,99%	15,99%	7,38%	1,88%	18,65%	4,70%	0,16%	
	120	-0,37%	25,00%	24,20%	18,51%	9,56%	0,79%	20,81%	-0,32%	1,81%	

Note: Memory parameter shows how many historical trading days were used to establish the efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation

Table 18. Average asset share for the fourth scenario – portfolio with the VXX

Rebalancing on	Memory	WIG20	UniKorona	NN Obligacj	S&P 500	MSCI Europe	FTSE Pacific	FTSE G7 Bond	S&P GSCI	DJ Real Estate	VXX
Weeks	15	1,33%	24,36%	22,07%	16,25%	6,09%	4,52%	16,52%	1,18%	5,17%	2,51%
	30	1,47%	24,75%	23,15%	16,62%	6,05%	2,26%	18,03%	1,71%	4,07%	1,88%
	60	1,62%	25,00%	23,93%	17,10%	6,95%	1,19%	18,55%	1,11%	3,21%	1,34%
	120	1,41%	25,00%	24,56%	19,38%	7,90%	0,57%	19,58%	-2,48%	2,21%	1,86%
Months	15	1,19%	23,98%	22,04%	14,37%	4,54%	6,34%	17,45%	2,34%	5,30%	2,46%
	30	0,53%	24,76%	23,14%	16,70%	5,17%	2,42%	17,45%	3,50%	4,24%	2,11%
	60	1,24%	25,00%	23,90%	17,50%	6,64%	2,19%	18,24%	1,80%	2,06%	1,44%
	120	1,04%	25,00%	24,62%	19,23%	7,48%	0,98%	20,12%	-2,30%	2,13%	1,70%
Quarters	15	1,10%	24,91%	23,46%	11,69%	1,57%	9,13%	18,03%	1,22%	5,95%	2,95%
	30	-0,64%	24,76%	23,82%	18,98%	3,48%	2,25%	16,52%	4,01%	4,05%	2,77%
	60	1,67%	25,00%	24,79%	17,88%	6,26%	2,62%	17,02%	2,75%	0,71%	1,29%
	120	0,10%	25,00%	24,30%	20,77%	8,15%	0,57%	20,06%	-2,06%	1,62%	1,49%

Note: Memory parameter shows how many historical trading days were used to establish the efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

Table 19. Yearly differences of IR between portfolios with and without the VXX for the first scenario

Rebalancing on	Memory	2010	2011	2012	2013	2014	2015	2016	2017	2018
Weeks	15	0,2181	-2,2830	0,2107	-0,2982	-0,4283	-0,3314	-0,3414	-0,6258	0,5557
	30	0,0739	-1,0344	-0,4522	-1,2326	-0,8700	0,4709	-1,2905	-0,7364	0,8049
	60	-0,0896	0,2242	-0,2110	-0,5367	-0,2743	-0,1034	-1,9250	-0,0277	0,4136
	120	NA	-0,5276	-0,2636	-0,4097	-0,0739	-1,2836	-0,4242	-0,0879	0,3203
Months	15	-2,3963	-0,5428	-1,4921	-1,5669	0,2315	-0,1492	-0,4108	-0,0796	0,7454
	30	0,0333	0,0169	-0,2702	-2,0004	0,3227	-0,8034	-0,4247	-0,1605	0,4146
	60	0,0354	-0,6070	-0,0804	-0,9344	-0,2372	-0,1876	-0,3963	0,0102	0,0236
	120	NA	-0,5402	-0,2263	-0,1522	-0,1363	-1,1116	-0,5842	-0,0562	-0,1396
Quarters	15	-0,0721	0,0744	0,1457	-1,6843	-0,7598	-0,2798	-0,2194	-0,0229	-0,1628
	30	0,0026	-1,1874	-0,0476	-2,2285	-0,7999	1,0696	-0,7745	-0,0003	-0,4082
	60	0,0354	-0,3993	-0,0805	-0,8469	-0,5463	-0,0265	-0,1127	-0,0032	-0,0578
	120	NA	-0,4510	-0,4582	-0,1107	-0,0477	-0,2574	-0,3005	-0,0217	-0,0158

Note: Values above 0, i.e. when portfolio with the VXX has higher Information Ratio than without, are bolded. As the period of analysis starts at the end of September 2010, it is not possible to establish portfolio based on the last 120 trading days in the first sub-period, which results in NA values in the third column. Memory parameter shows how many historical trading days were used to establish the efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

Table 20. Yearly differences of IR between portfolios with and without the VXX for the second scenario

Rebalancing on	Memory	2010	2011	2012	2013	2014	2015	2016	2017	2018
Weeks	15	-1,7391	-0,6910	-1,0848	-0,8193	0,2335	-1,0760	-1,5108	-1,2514	1,7729
	30	0,0163	0,1563	-1,5499	-1,3867	-0,3118	-0,1356	-1,3463	-0,7461	2,5854
	60	-0,0463	1,3415	-0,7305	-0,9360	0,5943	-0,4265	-0,6111	-0,2192	0,6183
	120	NA	1,3029	-1,1398	-0,3235	0,3401	-0,4377	-0,4376	-0,0845	0,9074
Months	15	-5,0145	0,7928	-1,8124	-1,5234	-0,0913	-0,3351	-0,7465	-0,1429	0,0628
	30	0,0044	0,5194	-2,0935	-1,9528	0,2568	-0,4807	-0,1860	-0,1704	1,0822
	60	0,0172	0,9668	-0,4850	-1,1956	0,0132	-0,6460	-0,0241	0,0205	0,0319
	120	NA	1,2856	-0,9791	0,0146	0,2865	-0,4251	-0,4271	-0,0647	0,6103
Quarters	15	-0,0117	2,0543	-0,2023	-1,9610	0,3050	-0,3588	-0,6892	-0,0331	-0,3537
	30	-0,0050	0,8084	-0,1667	-1,3535	0,5350	1,1203	-0,4613	0,0011	-0,3385
	60	0,0172	-0,4356	-0,8377	-0,7354	0,0344	-0,4909	0,0504	0,0118	-0,4532
	120	NA	0,2124	-1,3429	-0,3951	0,2018	-0,3904	-0,9123	-0,0596	0,4305

Note: Values above 0, i.e. when portfolio with the VXX has higher Information Ratio than without, are bolded. As the period of analysis starts at the end of September 2010, it is not possible to establish portfolio based on the last 120 trading days in the first sub-period, which results in NA values in the third column. Memory parameter shows how many historical trading days were used to established efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

Table 21. Yearly differences of IR between portfolios with and without the VXX for the third scenario

Rebalancing on	Memory	2010	2011	2012	2013	2014	2015	2016	2017	2018
Weeks	15	2,5350	-1,0630	-0,1610	-0,4715	0,6006	0,3640	0,8519	-0,0206	0,6232
	30	0,4358	-0,6254	0,0791	-1,1454	-0,0806	-0,3939	0,5156	-0,4851	0,9003
	60	0,0235	0,4157	-0,8258	-0,6708	0,2540	0,1624	-0,2940	0,0725	0,1166
	120	NA	-0,0700	-0,0377	-0,2684	0,0719	-1,7112	-0,4027	-0,1148	0,2651
Months	15	-0,5229	0,0056	-0,4907	-1,2940	0,3832	-0,2560	0,7657	-0,8157	0,5159
	30	-0,3208	0,0188	0,3722	-0,0271	0,1273	-0,3378	-0,1660	-0,3720	-0,4794
	60	0,0732	-0,2421	-0,1797	-0,2941	0,1282	-0,1750	-0,4128	-0,0745	-0,0071
	120	NA	-0,6669	-0,1216	-0,1070	0,0096	-0,7360	-1,1011	-0,0660	0,0017
Quarters	15	0,3712	-1,8074	-0,2100	-0,2532	-0,1501	-0,6771	0,2776	-0,3157	0,1632
	30	-0,2450	-0,3767	0,4966	-1,5145	-0,6109	0,1863	-0,7637	0,0099	-1,2379
	60	0,0732	-1,3095	-0,1516	-0,3250	-0,2362	1,2640	-0,7187	-0,1294	-0,1505
	120	NA	-0,6851	-0,2911	-0,2186	0,1118	0,8869	-0,9294	0,0418	0,1810

Note: Values above 0, i.e. when portfolio with the VXX has higher Information Ratio than without, are bolded. As the period of analysis starts at the end of September 2010, it is not possible to establish portfolio based on the last 120 trading days in the first sub-period, which results in NA values in the third column. Memory parameter shows how many historical trading days were used to established efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

Table 22. Yearly differences of IR between portfolios with and without the VXX for the fourth scenario

Rebalancing on	Memory	2010	2011	2012	2013	2014	2015	2016	2017	2018
Weeks	15	-0,6691	-0,3982	-0,4605	-0,7607	0,8592	0,3403	-0,2319	-0,5581	1,0518
	30	-0,3083	0,4264	-0,2198	-1,2213	0,0841	0,2312	-0,7870	-0,4083	1,1461
	60	2,9884	1,2170	-0,3108	-0,0451	0,5335	0,1110	-0,5215	0,1547	0,8696
	120	NA	1,7183	-1,2747	0,0073	0,7227	0,3414	0,0487	-0,0691	-0,0657
Months	15	-2,8274	1,3453	-1,0986	-0,4819	0,5177	0,2017	0,0553	-0,7074	0,8748
	30	-1,0239	0,8989	-0,5414	-0,9699	0,4918	-0,2038	-0,0565	-0,0378	0,1023
	60	2,6155	1,6443	0,2111	-0,1372	0,4196	-0,1367	-0,2552	0,1557	0,6071
	120	NA	1,9655	-1,2465	0,1787	0,3097	0,1527	-0,2265	-0,0311	0,1939
Quarters	15	-0,2212	1,6826	-0,6129	-1,3035	-0,0588	0,2302	-0,6292	-0,1025	-0,0819
	30	1,5810	1,0765	0,2159	-0,8814	-0,5181	0,6556	0,0400	-0,0663	-1,2786
	60	2,6155	0,3424	-0,9576	-0,4205	-0,6966	-0,3328	0,1433	-0,1527	-0,8883
	120	NA	0,4051	-0,8779	-0,2408	0,1758	0,1503	-0,1253	-0,1732	0,2074

Note: Values above 0, i.e. when portfolio with the VXX has higher Information Ratio than without, are bolded. As the period of analysis starts at the end of September 2010, it is not possible to establish portfolio based on the last 120 trading days in the first sub-period, which results in NA values in the third column. Memory parameter shows how many historical trading days were used to established efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

Table 23. Yearly values of Information Ratio for portfolio with fixed weights. First scenario

Rebalancing on	VXX Share	2010	2011	2012	2013	2014	2015	2016	2017	2018
Weeks	No VXX	2,5310	-0,2102	2,5277	0,4705	0,7936	-0,7370	0,7207	3,0946	-0,1794
	VXX 1%	2,1058	-0,1850	2,3497	0,2300	0,8173	-0,8287	0,5497	2,7529	-0,0568
	VXX 3%	1,0346	-0,1231	1,7607	-0,2908	0,7796	-0,9669	0,1361	1,8578	0,1869
	VXX 5%	-0,2604	-0,0396	0,9799	-0,7886	0,6355	-0,9877	-0,2925	0,8931	0,3329
Months	No VXX	2,5341	-0,2229	2,5448	0,4792	0,7919	-0,7514	0,7183	3,0844	-0,1652
	VXX 1%	2,1338	-0,1938	2,3792	0,2436	0,7920	-0,8363	0,5417	2,7399	-0,0821
	VXX 3%	1,1462	-0,1181	1,8283	-0,2669	0,7020	-0,9550	0,1107	1,8397	0,0906
	VXX 5%	-0,0622	-0,0160	1,0817	-0,7620	0,5186	-0,9504	-0,3257	0,8801	0,1962
Quarters	No VXX	2,5435	-0,2019	2,5335	0,4519	0,8127	-0,7444	0,7521	3,0048	-0,1667
	VXX 1%	2,2073	-0,0928	2,3918	0,2299	0,8078	-0,8685	0,5741	2,6993	-0,0482
	VXX 3%	1,4028	0,1842	1,9010	-0,2449	0,7073	-1,0750	0,1461	1,9204	0,1744
	VXX 5%	0,3988	0,4816	1,2088	-0,7159	0,5142	-1,1481	-0,2948	1,0629	0,2926

Note: In each sub-period and each frequency of rebalancing, 4 different portfolios are considered: with 1%, 3%, and 5% of the VXX share and without volatility exposure at all (VXX Share parameter). The portfolio with the highest Information Ratio in each year is bolded. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values.

Table 24. Yearly values of Information Ratio for portfolio with fixed weights. Second scenario

Rebalancing on	VXX Share	2010	2011	2012	2013	2014	2015	2016	2017	2018
Weeks	No VXX	2,6051	-0,5712	2,0335	0,2562	0,3434	-0,7443	0,5193	2,4322	-0,3512
	VXX 1%	2,3557	-0,5756	1,9180	0,1229	0,3376	-0,7929	0,4259	2,2497	-0,2904
	VXX 3%	1,7718	-0,5818	1,6249	-0,1570	0,3239	-0,8781	0,2175	1,8353	-0,1562
	VXX 5%	1,1154	-0,5789	1,2503	-0,4634	0,2759	-0,9640	-0,0085	1,3558	-0,0196
Months	No VXX	2,6046	-0,5861	2,0457	0,2656	0,3387	-0,7527	0,5169	2,4303	-0,3408
	VXX 1%	2,3692	-0,5919	1,9383	0,1342	0,3200	-0,7989	0,4201	2,2456	-0,3005
	VXX 3%	1,8249	-0,5992	1,6652	-0,1412	0,2787	-0,8789	0,2029	1,8276	-0,2057
	VXX 5%	1,2204	-0,5932	1,3142	-0,4434	0,2040	-0,9582	-0,0326	1,3454	-0,1026
Quarters	No VXX	2,6131	-0,5772	2,0367	0,2481	0,3510	-0,7487	0,5404	2,3793	-0,3416
	VXX 1%	2,4160	-0,5447	1,9432	0,1239	0,3293	-0,8141	0,4425	2,2138	-0,2823
	VXX 3%	1,9650	-0,4640	1,7014	-0,1331	0,2801	-0,9354	0,2251	1,8458	-0,1492
	VXX 5%	1,4714	-0,3563	1,3837	-0,4165	0,1995	-1,0574	-0,0098	1,4236	-0,0176

Note: In each sub-period and each frequency of rebalancing, 4 different portfolios are considered: with 1%, 3%, and 5% of the VXX share and without volatility exposure at all (VXX Share parameter). The portfolio with the highest Information Ratio in each year is bolded. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values.

Table 25. Yearly values of Information Ratio for portfolio with fixed weights. Third scenario

Rebalancing on	VXX Share	2010	2011	2012	2013	2014	2015	2016	2017	2018
Weeks	No VXX	2,7638	-0,7261	1,6188	0,2693	0,1155	-0,6604	0,4187	2,2344	-0,4565
	VXX 1%	2,5962	-0,7326	1,5409	0,1853	0,1046	-0,6880	0,3642	2,1213	-0,4258
	VXX 3%	2,2438	-0,7429	1,3671	0,0131	0,0840	-0,7461	0,2506	1,8798	-0,3543
	VXX 5%	1,8555	-0,7547	1,1668	-0,1717	0,0535	-0,8024	0,1255	1,6156	-0,2788
Months	No VXX	2,7587	-0,7277	1,6195	0,2798	0,1133	-0,6587	0,4198	2,2418	-0,4538
	VXX 1%	2,6001	-0,7368	1,5470	0,1960	0,0941	-0,6854	0,3631	2,1264	-0,4357
	VXX 3%	2,2693	-0,7519	1,3855	0,0248	0,0567	-0,7420	0,2440	1,8803	-0,3879
	VXX 5%	1,9071	-0,7674	1,1988	-0,1585	0,0093	-0,7970	0,1127	1,6117	-0,3338
Quarters	No VXX	2,7608	-0,7300	1,6199	0,2768	0,1169	-0,6550	0,4305	2,2207	-0,4519
	VXX 1%	2,6279	-0,7188	1,5558	0,1964	0,0953	-0,6922	0,3728	2,1163	-0,4230
	VXX 3%	2,3536	-0,6897	1,4119	0,0336	0,0529	-0,7701	0,2526	1,8959	-0,3519
	VXX 5%	2,0538	-0,6566	1,2432	-0,1405	0,0006	-0,8469	0,1211	1,6571	-0,2749

Note: In each sub-period and each frequency of rebalancing, 4 different portfolios are considered: with 1%, 3%, and 5% of the VXX share and without volatility exposure at all (VXX Share parameter). The portfolio with the highest Information Ratio in each year is bolded. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values.

Table 26. Yearly values of Information Ratio for portfolio with fixed weights. Fourth scenario

Rebalancing on	VXX Share	2010	2011	2012	2013	2014	2015	2016	2017	2018
Weeks	No VXX	3,0720	-0,5847	1,5810	1,0471	0,2833	-0,3332	0,4912	2,7951	-0,2106
	VXX 1%	2,9198	-0,5896	1,5093	0,9664	0,2792	-0,3584	0,4347	2,6833	-0,1675
	VXX 3%	2,5923	-0,5972	1,3428	0,7895	0,2724	-0,4164	0,3106	2,4214	-0,0623
	VXX 5%	2,2106	-0,6083	1,1390	0,5751	0,2512	-0,4797	0,1618	2,0967	0,0534
Months	No VXX	3,0612	-0,5857	1,5827	1,0664	0,2815	-0,3325	0,4933	2,8073	-0,2061
	VXX 1%	2,9161	-0,5934	1,5157	0,9867	0,2671	-0,3545	0,4346	2,6934	-0,1817
	VXX 3%	2,6077	-0,6068	1,3606	0,8130	0,2380	-0,4055	0,3044	2,4262	-0,1169
	VXX 5%	2,2526	-0,6238	1,1705	0,6028	0,1925	-0,4617	0,1469	2,0951	-0,0442
Quarters	No VXX	3,0637	-0,5881	1,5838	1,0586	0,2854	-0,3237	0,5080	2,7738	-0,2010
	VXX 1%	2,9425	-0,5717	1,5265	0,9812	0,2684	-0,3590	0,4477	2,6697	-0,1633
	VXX 3%	2,6890	-0,5316	1,3925	0,8152	0,2342	-0,4389	0,3158	2,4307	-0,0672
	VXX 5%	2,4002	-0,4872	1,2256	0,6173	0,1830	-0,5264	0,1586	2,1405	0,0391

Note: In each sub-period and each frequency of rebalancing, 4 different portfolios are considered: with 1%, 3%, and 5% of the VXX share and without volatility exposure at all (VXX Share parameter). The portfolio with the highest Information Ratio in each year is bolded. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values.



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