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A survey of statistical arbitrage pairs trading strategies with non-machine learning methods, 2016-2023

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Abstract: This review examines the growing literature on pairs trading frameworks, which involve relative value arbitrage strategies between two or more securities. Existing research is categorized into five main categories: distance methods use nonparametric distance measures to identify pairs trading opportunities; cointegration methods rely on formal cointegration tests to reveal stationary time series of spreads; time series methods focus on finding optimal trading rules for mean-reverting spreads; stochastic control methods aim to determine the optimal portfolio holdings in pairs trading relative to other available securities; and the "Other Methods" category encompasses other relevant pairs trading frameworks, albeit with a more limited supporting literature. Through a comprehensive review of over 100 papers published between 2016 and 2023, the survey identifies the key strengths and weaknesses of each approach, providing insights relevant for future research and practical implementation.

Keywords: statistical arbitrage, pairs trading, distance method, cointegration method, time series method, stochastic control method, mean-reversion

JEL codes: C32, C58, C63, G11, G14

1. Introduction

This review focuses on the literature published between 2016 and 2023, as the earlier period from 2006 to 2015 has already been comprehensively surveyed by Krauss (2017). Nevertheless, to provide a complete perspective, the discussion begins with the seminal study by Gatev et al. (2006), hereafter referred to as GGR.

Based on the work of GGR, the concept of pairs trading is straightforward and consists of two main steps. First, identify two securities whose prices move in synchronize during a formation period. Then, in subsequent trading periods, observe the price difference between them. If the prices diverge and the spread widens, the strategy involves shorting the security that has gained more value and buying the one that has lost value. Assuming the two securities maintain an equilibrium relationship, the spread is expected to revert to its historical average. Once this reversion occurs, the position can be closed to realize a profit.

The basic idea of univariate pairs trading can be extended to more complex scenarios. In quasi-multivariate models, a single security is traded against a weighted portfolio of other related securities. In full multivariate models, an entire group of stocks is traded against another group of stocks. These sophisticated strategies can be collectively referred to as (quasi-)multivariate pairs trading, generalized pairs trading, or statistical arbitrage. All of these methods fall under the broader category of "statistical arbitrage pairs trading" (or simply "pairs trading") because they represent the foundation of more advanced techniques (Vidyamurthy, 2004; Avellaneda and Lee, 2010). Pairs trading is also related to other long-short strategies, such as those exploiting deviations from the law of one price, lead-lag effects, and return reversals. For a detailed discussion of these and other long-short return phenomena, see Jacobs (2015).

GGR's seminal paper in pairs trading has garnered considerable attention. The paper presents a straightforward yet powerful algorithm applied to a broad dataset of U.S. stocks, and carefully tuned to mitigate the effects of data-snooping bias. The results showed annualized excess returns of up to 11%, with minimal influence from systematic risk factors. Crucially, these returns cannot be attributed to previously recognized sources of profit, such as the reversal profits identified by Jegadeesh (1990) and Lehmann (1990) or the momentum profits described by Jegadeesh and Titman (1993). The persistence of these unexplained excess returns solidified GGR's strategy as a notable anomaly in capital markets, maintaining its relevance and being validated by subsequent research, including studies by Do and Faff (2010, 2012).

Despite these insights, it is important to note that academic research on pairs trading remains relatively limited compared with studies on contrarian and momentum strategies. Nevertheless, interest in the topic has increased markedly in recent years, giving rise to a growing array of theoretical frameworks and empirical applications across various asset classes. The apparent simplicity of GGR's strategy becomes less evident with more sophisticated models and techniques have been introduced. In this review, we organize the literature into five broad categories, each defined by its methodological approach and the underlying logic for identifying and exploiting trading opportunities: distance methods, cointegration methods, time series methods, stochastic control methods, and other methods.

Distance Approach: This is the most extensively studied framework in pairs trading research. During the formation stage, various distance metrics are employed to identify pairs of securities that exhibit co-movement. During the trading stage, simple nonparametric threshold rules are applied to generate trading signals. The strengths of this strategy lie in its simplicity and transparency, making it well-suited for large-scale empirical applications. Empirical evidence consistently shows that distance-based pairs trading can be profitable across different markets, asset classes, and time horizons.

Cointegration Approach: This approach applies cointegration tests during the formation stage to identify pairs of securities that share a statistically significant long-term equilibrium relationship. During the trading stage, deviations from this equilibrium are monitored and simple rules (often based on GGR threshold methods) are used to trigger long and short positions. By relying on econometric tests, this approach mitigates the risk of spurious correlations that can arise from purely distance-based selection. Empirical research demonstrates that cointegration-based strategies can generate consistent profits across various markets and asset classes, particularly when supported by rigorous pre-selection and model validation procedures.

Time Series Approach*:* Unlike distance and cointegration frameworks, this approach generally ignores the formation stage, assuming that suitable co-moving securities have been identified through prior analysis. Instead, it focuses on the trading stage, where the spread between assets is modeled using time series techniques—typically employing mean-reverting processes such as the Ornstein-Uhlenbeck (OU) model, or ARIMA and GARCH-type models to capture autocorrelation and volatility dynamics. Trading signals are generated based on the model's forecasts of spread movements, and parameters are optimized to balance profitability and risk. This approach is flexible enough to capture complex spread behavior and adapt to changing market conditions, but its performance depends heavily on accurate model specification and parameter estimation.

Stochastic Control Approach: Similar to time series frameworks, this approach typically omits the formation stage and assumes that suitable trading pairs have already been identified. Its focus is on dynamically determining the optimal portfolio position—covering entry and exit timing, position sizing, and allocation between paired assets and other investment opportunities. Stochastic control theory is used to model the spread as a stochastic process, typically using the OU process, and solving the Hamilton-Jacobi-Bellman (HJB) equation to obtain the portfolio value and optimal strategy function. This framework can adapt to changing market conditions in real time and provide theoretically optimal strategies under certain assumptions, although it requires complex mathematical modeling and can be sensitive to parameter misspecification.

Other Approaches: This category includes additional pairs trading frameworks that have relatively limited supporting literature and are not closely related to the previously discussed approaches. Representative examples include non–machine-learning techniques such as the copula approach, which models the joint distribution of asset returns to capture nonlinear dependencies, and the Hurst exponent approach, which quantifies long-term memory and persistence in spread dynamics. These methods often employ more advanced statistical or computational tools to uncover trading opportunities beyond conventional correlation or mean-reversion measures. While they can capture complex relationships missed by traditional models, their practical application is constrained by higher data and estimation requirements, as well as limited empirical validation.

Table 1 provides a summary of representative studies for each approach, detailing the data samples and the performance evaluation metrics as reported in the respective studies.

Considering the diversity among the above five approaches, this survey makes two key contributions. First, it provides a comprehensive review of the literature across these approaches. Second, it offers an in-depth analysis of representative studies within each category, highlighting their methodological strengths and weaknesses. Drawing on more than 100 papers, the survey synthesizes insights that are relevant for both academic research and practical implementation.

Table 1. Overview of pairs trading approaches.

Approach	Representative articles	Data Sample	Performance Evaluation Metrics
Distance	Bowen and Hutchinson (2016)	London Stock Exchange stocks 1979-2012	Sharpe ratio, Annualized returns
	Zhang and Urquhart (2019)	CSI300, HSHKI, HSAHP stocks 1996-2017	Average excess returns, Standard deviation, Abnormal returns
Cointegration	Ekkarntrong et al. (2017)	U.S. The Global Dow stocks 2002-2012	Excess returns
	Figuerola-Ferretti et al. (2018)	STOXX Europe 600 stocks 2000-2017	Sharpe ratio, Standard deviation
Time series	Chen et al. (2017)	U.S. stocks 2006-2014	Round-trip trades, Annualized returns
	Kim and Heo (2017)	KOSPI 100 stocks 2005-2015	Cumulative returns, Annualized Sharpe ratio
Stochastic control	Göncü and Akyildirim (2016)	U.S. U.K. commodity futures 1997-2015	Cumulative returns, Annualized returns
	Liu et al. (2017)	U.S. Oil stocks 2007-2008, 2013-2015	Annualized Sharpe ratio, Annualized returns
Others: Copula	Xie et al. (2016)	U.S. stocks 2003-2012	Excess returns, Cumulative returns
	Krauss and Stübinger (2017)	U.S. S&P 100 stocks 1990-2014	Annualized Sharpe ratio, Excess returns
Others: Hurst exponent	Ramos-Requena et al. (2017)	U.S. The Global Dow stocks 2000-2015	WO, LO, PAOW, PAOL
	Ramos-Requena et al. (2021)	U.S. Nasdaq Inc stocks 2000-2021	Average annualized returns, Sharpe ratio
Others: Entropy	Amer and Islam (2023)	PSX stocks 2017-2019	Annualized returns

The remainder of this paper is organized as follows. Section 1 introduces the relevance of the research topic within the field of finance and defines the primary objectives of the study, outlining the key research questions and the unique contributions made to existing literature. Section 2 reviews related work, summarizing foundational studies, main findings, methodologies, and research gaps. Section 3 defines the scope of the study and presents a roadmap for the subsequent sections. Section 4 describes the data sources, processing techniques, and preliminary analyses underpinning the methodology. Section 5 examines non-machine learning models in pairs trading, including distance methods, cointegration methods, stochastic control methods, time series methods, and other approaches such as the Copula, Hurst exponent, and entropic methods, providing a comprehensive analysis of each model's theoretical foundation and empirical applications. Section 6 presents the conclusions of the study, summarizing the key findings and their implications, and outlines potential directions for future research.

2. Related Work

In this section, I review the diverse applications of distance, cointegration, time series, and stochastic control methods in pairs trading, discussing how each has been adapted to improve trading performance. The review also traces the evolution of these approaches, highlighting the shift from traditional statistical techniques to more advanced computational models.

2.1 Distance Methods

This review begins by reviewing the distance approach. Miao and Laws (2016) investigate the out-of-sample performance of a simple pairs trading strategy across 12 international stock markets, covering both developed and emerging economies. Building on the methodology of Gatev et al. (2006) and Do and Faff (2010), they select trading pairs by minimizing the sum of squared errors (SSE) or sum of absolute errors (SAE) of normalized price differences, and implement a rolling 12-month formation period followed by a 6-month trading period. The results show that the strategy delivers consistent and statistically significant positive returns in most markets, including during periods of market downturns. Even after accounting for realistic transaction costs, the strategy remains profitable in several markets. Furthermore, returns from the pairs trading portfolios exhibit low correlation with local market indices, indicating strong market-neutral and diversification properties. This study offers robust international evidence supporting the viability of simple statistical arbitrage strategies beyond the U.S. market.

Quinn et al. (2018) develop a distance-based pairs trading strategy using UK gilt futures across long, medium, and short-term maturities. The strategy applies a threshold-based entry and exit rule based on historical spread deviations, incorporating stop-loss mechanisms and adjustments for repo financing costs. Their findings indicate that arbitrage opportunities persist—particularly between long and medium gilt futures—despite the market's high liquidity and efficiency. Although profits are modest, the study contributes valuable evidence to the relatively sparse literature on arbitrage in government bond markets.

Extending traditional distance-based approaches, Diao et al. (2020) propose a bi-objective optimization framework for selecting stock pairs in the Chinese A-share market. Unlike earlier studies that rely solely on historical price similarity (e.g., SSE or Euclidean distance), their model jointly minimizes the deviation between normalized price series and maximizes a measure of cointegration-like stability, thereby improving the robustness of pair selection. Using daily data from 2011 to 2018, the strategy demonstrates strong mean-reversion properties and consistent profitability across various periods, including during episodes of market volatility. By integrating multiple criteria into the matching process, the study advances distance-based methods toward more robust and adaptive pair selection techniques.

In the paper by Ramos-Requena et al. (2020), various methods for pairs trading are explored, including minimal distance, cointegration, correlation, and Hurst exponent approaches. The study focuses on identifying stock pairs using these methods and trading based on price deviations, with particular emphasis on long memory characteristics and mean reversion in time series. The findings indicate that employing these diverse methods can improve the profitability of pairs trading strategies, especially during periods of market anomalies and high volatility.

In summary, the reviewed studies demonstrate the effectiveness and adaptability of distance-based methods in pairs trading across diverse asset classes and markets. These range from traditional approaches relying on historical price co-movement (Miao and Laws, 2016; Quinn et al., 2018) to more refined models incorporating multi-objective optimization (Diao et al., 2020) as well as hybrid selection criteria (Ramos-Requena et al., 2020). The literature consistently shows that distance-based strategies deliver positive returns and exhibit strong market-neutral characteristics. These findings underscore the method's robustness, particularly during volatile periods, and highlight the benefits of integrating additional features such as cointegration relationships or long-memory dynamics to enhance strategy performance.

2.2 Cointegration Methods

Cointegration methods identify pairs of assets that share a long-term equilibrium relationship, despite the non-stationarity of the individual series. Using econometric techniques such as the Engle–Granger method or the Johansen test, these approaches assess whether a linear combination of asset prices forms a stationary series. This enables traders to exploit deviations from the long-term equilibrium by initiating trades based on the expectation that the spread will mean revert. Cointegration methods are valued for their ability to uncover persistent relationships between assets, but they require ample historical data and rest on the assumption that these relationships persist over time.

Cartea and Jaimungal (2016) present an optimal trading strategy for cointegrated assets by modeling the structural dependence between asset prices via a cointegration factor, which they term "short-term alpha." They derive an explicit closed-form solution for the dynamic investment strategy, which is affine in the value of the cointegration factor, and demonstrate the effectiveness of the strategy using simulations calibrated with high-frequency Nasdaq data from Google, Facebook, and Amazon. The study illustrates how short-term deviations in asset prices can be exploited for profit, with potential extensions including out-of-sample testing and application to portfolios containing both liquid and illiquid assets.

Huang and Martin (2019) develop pairs trading strategies within a cointegration framework, applying the Engle–Granger test and an Error Correction Model (ECM) combined with a Dynamic Conditional Correlation GARCH (ECM–DCC–GARCH) model to test for and model long-term equilibrium relationships between asset pairs. The study compares several trading rules, including a percentage-threshold strategy, a strategy based on the standard deviation of cointegration residuals, and Bollinger Bands, with an emphasis on optimizing the profit factor. Their results show that Bollinger Bands without GARCH confirmation yielded the highest profit factor, highlighting the effectiveness of cointegration-based approaches in pairs trading.

Thazhugal (2021) explores pairs trading potential in the Indian metals market using a cointegration approach. The study applies Johansen cointegration tests and Vector Error Correction Models (VECM) to examine the long-term equilibrium relationships between spot and futures prices of metals including aluminum, copper, nickel, and zinc. The results provide evidence of the effectiveness of cointegration-based strategies in uncovering price discovery and in supporting pairs trading strategies in volatile commodity markets.

Brunetti and De Luca (2023) propose a cointegration-based pairs trading strategy, employing the Johansen cointegration test to identify stock pairs exhibiting long-term equilibrium relationships. The study examines the impact of pre-selection methods by comparing seven pre-selection metrics, including log-price correlation and covariance, aimed at reducing computational complexity. The results reveal substantial variation in both profitability and risk exposure depending on the pre-selection metric used, highlighting the critical role of metric choice in the design of pairs trading strategies.

In summary, cointegration methods offer a statistically rigorous framework for detecting long-term equilibrium relationships between assets and exploiting mean reversion in price spreads. The studies reviewed demonstrate the effectiveness of these methods in various markets. For example, Cartea and Jaimungal (2016) highlight the practical application of cointegration-based strategies in high-frequency trading (HFT), using a cointegration factor to derive dynamic investment strategies that leverage short-term deviations. Building on this, Huang and Martin (2019) compare multiple trading rules within a cointegration framework, finding that Bollinger Bands without GARCH confirmation yielded the highest profit factor, underscoring the potential for profit optimization. Extending this approach to commodity markets, Thazhugal (2021) shows that cointegration-based strategies facilitate price discovery and enhance trading performance in the volatile Indian metals market. More recently, Brunetti and De Luca (2023) analyze the role of pre-selection metrics, demonstrating that metric choice substantially influences both profitability and risk exposure. Collectively, these studies affirm the robustness and versatility of cointegration methods, highlighting their capacity to adapt across asset classes, trading frequencies, and market conditions.

2.3 Stochastic Control Methods

Stochastic control methods in pairs trading use dynamic optimization to continuously adjust portfolio positions in response to the stochastic behavior of asset prices. Price dynamics are typically modeled using stochastic differential equations (SDEs) and solved via techniques such as the HJB equation, enabling the strategy to maximize returns or minimize risks in real time. In contrast to static approaches such as cointegration or distance-based methods, stochastic control frameworks offer adaptive decision-making capabilities in volatile markets, albeit at the cost of high mathematical complexity and substantial computational demands.

Deshpande and Barmish (2016) propose a control-theoretic framework for pairs trading that accommodates flexible definitions of spread functions while requiring minimal assumptions regarding the underlying price dynamics. By formulating trading as a feedback control problem within a stochastic control setting, their algorithm adaptively adjusts positions based on the mean-reverting behavior of the spread. Simulation experiments using a leveraged ETF pair (YINN and YANG) indicate that the strategy generates substantial returns while maintaining low drawdowns, outperforming buy-and-hold approaches in simulated scenarios. This study demonstrates the applicability of stochastic control techniques to statistical arbitrage in financial markets.

Endres and Stübinger (2019) propose an optimal pairs trading strategy based on a Lévy-driven OU process within a stochastic control framework. The method dynamically determines entry and exit thresholds by solving first-passage time problems to maximize expected returns. Using high-frequency data on S&P 500 constituents from 1998 to 2015, the authors provide empirical evidence of the strategy's profitability across multiple economic sectors, demonstrating robustness in various market environments.

Zhu et al. (2021) explore optimal pairs trading strategies within a dynamic mean–variance framework, using the OU process to model the mean-reverting behavior of the price spread between two correlated assets. Within a stochastic control setting, the method solves HJB equations to derive time-consistent optimal trading policies. Empirical analysis using stock and futures data from China's markets provides evidence of the approach's effectiveness in maximizing returns while controlling risk.

Xing (2022) proposes an optimal pairs trading strategy within a singular stochastic control framework that incorporates proportional transaction costs. The method models the mean-reverting behavior of the asset price spread using an OU process and solves HJB equations to determine the optimal timing and size of trades dynamically. Empirical analysis using U.S. stock data provides evidence of the approach's effectiveness in maximizing terminal wealth while managing transaction costs.

Das et al. (2023) investigates pairs trading within a regime-switching mean-reversion framework, allowing both the long-term equilibrium level and mean-reversion speed of the spread to shift across unobserved regimes. Modeling the spread with a continuous-time OU process subject to Markovian regime switching, the authors derive optimal trading rules via a verification theorem and analyze entry and exit thresholds under different regimes. Numerical experiments demonstrate superior performance, particularly under frequent or pronounced regime shifts. This work underscores the importance of incorporating structural changes in spread dynamics and offers a more adaptive framework for statistical arbitrage.

In summary, stochastic control offers a powerful and flexible framework for real-time, adaptive optimization of pairs trading decisions in response to stochastic price dynamics. The reviewed literature demonstrates that variations of the mean-reverting OU process—including extensions with Lévy dynamics (Endres and Stübinger, 2019), regime-switching structures (Das et al., 2023), or mean-variance considerations (Zhu et al., 2021)—can be effectively integrated into control-theoretic models to enhance profitability and risk management. These approaches typically involve solving HJB equations to derive optimal entry, exit, and position-sizing policies. Furthermore, the incorporation of practical considerations, such as proportional transaction costs (Xing, 2022), underscores the robustness and applicability of these models. Collectively, these studies highlight the growing relevance of stochastic control in statistical arbitrage, yielding theoretically sound and empirically validated strategies capable of adapting to regime shifts, market volatility, and structural uncertainties.

2.4 Time Series Methods

Time series methods in pairs trading aim to capture statistical dependencies and dynamic interactions between asset prices over time. These models commonly employ techniques such as autoregressive processes, moving averages, and mean-reverting dynamics—most notably the OU model—to represent and analyze price spreads. By examining historical price patterns, they forecast potential future movements and generate trading signals. Compared with stochastic control methods, time series approaches are generally more straightforward to implement, relying on fixed entry and exit rules, but they may exhibit limited adaptability to rapid market changes.

De Moura et al. (2016) propose a pairs trading strategy that employs linear state space models and the Kalman filter to represent the spread between two related assets. By estimating the conditional probability of mean reversion, the framework determines optimal entry and exit points. The approach blends stochastic control principles—through dynamic optimization—with traditional time series modeling techniques.

Chodchuangnirun et al. (2018) design a strategy based on nonlinear autoregressive models augmented with GARCH effects, incorporating Markov Switching, Threshold, and Kink specifications. By modeling return spreads and detecting regime shifts, the method captures both dynamic volatility and nonlinear behaviors in financial time series, resulting in improved signal precision. Empirical results indicate that the Markov Switching specification slightly outperforms the others in generating profitable trades.

Zhang (2021) develops a pairs trading framework using a general state space model to capture the spread dynamics between asset pairs. Modeling the spread as a mean-reverting process with non-Gaussian features and heteroscedasticity, the study applies a Monte Carlo-based optimization to determine optimal trading rules. This enables superior profitability and risk-adjusted returns compared to conventional models by more effectively capturing the complex dynamics in financial data.

Lee et al. (2023) propose a diversification framework for multiple pairs trading strategies, modeling the mean-reverting behavior of asset spreads with OU processes. The framework incorporates dynamic capital allocation methods—Mean Reversion Budgeting (MRB) and Mean Reversion Ranking (MRR)—to optimize trading across multiple pairs. Empirical evidence shows that these techniques enhance portfolio performance by leveraging mean-reversion properties and improving diversification.

In summary, recent time series approaches in pairs trading have evolved to integrate advanced statistical modeling and dynamic optimization. De Moura et al. (2016) employ linear state space models with the Kalman filter to dynamically determine optimal entry and exit points. Chodchuangnirun et al. (2018) adopt nonlinear autoregressive models with GARCH effects, including Markov Switching and Threshold specifications, to capture regime shifts and volatility patterns, thereby improving signal precision and profitability. Zhang (2021) advances this line of research by using a general state space model with non-Gaussian features and heteroscedasticity, combined with Monte Carlo-based optimization to derive superior trading rules and risk-adjusted returns. Lee et al. (2023) extend the scope by introducing a multi-pair diversification framework with OU processes and adaptive capital allocation, further improving overall portfolio efficiency. Collectively, these studies highlight the adaptability and effectiveness of time series methods in pairs trading, offering strategies that are both statistically rigorous and operationally robust.

2.5 Other Methods

Beyond above four methods, several alternative approaches have been employed in pairs trading to enhance strategy performance and manage risks. These methods explore different statistical and optimization techniques to refine trade signals and optimize portfolio returns.

2.5.1 Copula Approach

A prominent alternative is the Copula approach, which models the dependence structure between assets in a more flexible manner than simple correlation analysis. By capturing tail dependencies and accommodating complex joint return distributions, copula models offer a more comprehensive representation of co-movements and associated risks. Variants such as the Mixed Copula model further extend this flexibility by combining multiple copula functions to reflect both linear and nonlinear dependencies.

Nadaf et al. (2022) proposed a copula-based pairs trading strategy that employs the Laplace marginal distribution to model asset returns. By constructing a copula function that accounts for heavy-tailed behavior, the method more accurately captures dependency structures between asset pairs, thereby improving the precision of trading signals derived from the joint distribution.

In a related study, da Silva et al. (2023) introduced a mixed copula model that combines different copula types to jointly capture linear and nonlinear relationships. The authors calculate a mispricing index using an optimal linear combination of copulas, which enhances the adaptability of the strategy to changing market conditions and improves both profitability and robustness.

2.5.2 Hurst Exponent Approach

The Hurst exponent is a statistical measure that quantifies the tendency of a time series to exhibit persistent trending behavior or mean-reverting characteristics. A higher Hurst exponent indicates stronger persistence, whereas a lower value suggests greater mean reversion; both properties can inform the design and calibration of trading strategies.

Ramos-Requena et al. (2021) proposed a cooperative dynamic pairs trading approach that incorporates the Hurst exponent and volatility as primary selection criteria to identify stock pairs with stable historical co-movement. After pair selection, a mean-reversion strategy is applied to exploit deviations from the long-term relationship. By integrating refined filtering criteria, the method improves trading performance by focusing on pairs characterized by low volatility and high co-movement.

2.5.3 Entropic Approach

Entropy-based methods focus on quantifying the uncertainty or randomness within financial data. By analyzing the entropy of asset price distributions, these approaches can identify periods of elevated uncertainty, providing additional insights for trading decisions.

Amer and Islam (2023) proposed a pairs trading strategy that integrates cointegration techniques with an entropic framework to optimize trading decisions. The strategy models mean reversion using an OU process while incorporating entropy as a penalty function to account for model uncertainty. By determining optimal entry and exit thresholds, the approach improves both profitability and risk control.

In summary, alternative approaches to pairs trading—such as the Copula approach, Hurst exponent, and entropy-based methods—offer refined tools for enhancing strategy performance and managing risk. The Copula approach, as demonstrated by Nadaf et al. (2022) and da Silva et al. (2023), provides a sophisticated framework for modeling dependencies between asset pairs, capturing tail dependencies and complex relationships beyond traditional correlation measures, thereby enhancing signal accuracy and adaptability under varying market conditions. The Hurst exponent approach, exemplified by Ramos-Requena et al. (2021), employs persistence and mean-reversion metrics to filter and select pairs, improving the effectiveness of mean-reversion strategies through refined selection criteria. The entropy-based approach, proposed by Amer and Islam (2023), integrates entropy as a measure of uncertainty into a cointegration framework, enhancing trade timing precision and improving risk-adjusted returns. Collectively, these methods broaden the methodological scope of pairs trading, offering more nuanced and adaptable strategies for complex financial environments.

3. Landscape Overview

In this section, we outline the key literature forming the foundation of our research. The primary paper, together with supporting studies, was identified through comprehensive searches across major academic databases, including Scopus, Google Scholar, Springer, IEEE Xplore, ScienceDirect, and Web of Science. A targeted search strategy was employed, using a diverse set of keywords—such as "Pair Trading," "Pair Trading with Statistical Approaches," "Pair Trading using Distance Measures," "Pair Trading with Cointegration Techniques," "Pair Trading through Stochastic Control Models," and "Pair Trading utilizing Time Series Methods"—to ensure broad coverage of pairs trading methodologies in finance. Studies that were not directly relevant to the thematic focus of our analysis were excluded from further consideration. Table 2 presents the chronological distribution of the reviewed articles, along with complete reference citations for each entry.

Table 2. Summary of the number of publications analyzed per year, spanning from 2016 to 2023.

Year	Count	Article
2016	21	[1-13]
2017	16	[14-29]
2018	24	[30-53]
2019	12	[54-65]
2020	14	[66-79]
2021	19	[80-98]
2022	12	[99-110]
2023	13	[111-123]

Table 3 summarizes the distribution of pairs trading methods across the reviewed studies. Cointegration methods are the most prevalent, appearing in 43 studies, where they exploit long-term equilibrium relationships between asset prices to identify profitable trading opportunities. Stochastic control methods follow closely, featuring in 40 studies and emphasizing stochastic processes and dynamic optimization to manage market uncertainty. Distance methods are used in

21 studies, focusing on price divergence to capture mean reversion, while time series methods appear in 20 studies, leveraging temporal price patterns and trends. Finally, 16 studies employ other approaches, including copula, Hurst exponent, and entropic methods, which fall outside the primary categories. This distribution underscores the dominance of cointegration and stochastic control methods in pairs trading research, while also reflecting the variety of alternative strategies adapted to specific market conditions.

Notably, some studies employ more than one pairs trading method, reflecting the complexity and adaptability of such strategies. Researchers often integrate approaches—such as cointegration with stochastic control—to enhance performance and capture diverse market dynamics. Consequently, the counts in Table 3 represent instances of method usage rather than unique study counts.

Within the reviewed literature, a subset 21 articles — [5, 7, 16, 33, 46, 64, 68, 69, 75, 76, 81, 84, 88, 91, 94, 103, 105, 106, 117, 118, 122] — primarily focus on theoretical development. These works propose, validate, or refine trading strategies through mathematical modeling and derivation, typically validated via simulations or historical backtesting rather than live market implementation. Their contributions are substantial, providing deeper insights into potential returns and risks, evaluating performance under varying market conditions, and establishing a robust theoretical foundation for future empirical research.

Table 3. Summary of the number of publications analyzed per pairs trading method.

Methods	Count	Article
Distance Methods	21	[6, 10, 13, 14, 22, 25, 26, 43, 45, 51, 56, 60, 63, 72, 73, 77-79, 97, 98, 120]
Cointegration Methods	43	[4, 8, 11, 13-15, 24, 26-28, 30, 34, 35, 37, 43-45, 50, 52, 53, 55, 61, 62, 67, 69, 72, 73, 75, 79, 86, 89, 93, 96-98, 102, 107, 109, 114, 115, 120, 121, 123]
Stochastic Control Methods	40	[1, 3, 5, 7, 12, 18, 20, 31-33, 39-41, 45-47, 49, 54, 59, 64, 65, 68, 75, 76, 81, 84, 88, 91, 92, 94, 100, 103-106, 117-120, 122]

Methods	Count	Article
Time Series Methods	20	[2, 23, 29, 36, 39, 47, 48, 57, 66, 70, 74, 82, 83, 85, 90, 95, 99, 108, 111, 120]
Other Methods	20	[9, 13, 16, 17, 19, 21, 38, 42, 58, 71, 79, 80, 87, 98, 101, 110, 112, 113, 116, 120]

Table 4 provides a comprehensive summary of the major financial indices examined in this study, highlighting the broad geographic diversity of the markets analyzed. The selection covers developed and emerging economies across North America, Europe, and Asia, ensuring that the analysis reflects a wide range of economic environments and market conditions.

For the United States, several significant indices were included, such as the S&P 100, Dow Jones Industrial Average, Nasdaq 100, Russell 2000 ETF, and S&P 500. These indices represent a cross-section of the U.S. market, capturing a variety of market capitalizations and sectors, thus providing a robust understanding of the U.S. equity market's performance.

In Europe, the FTSE 100 represents the UK market, the STOXX Europe 600 captures pan-European equities across multiple countries and capitalizations, and the SBF 120 provides targeted exposure to the French market—one of the region's largest economies. The OMX Baltic index adds coverage of Lithuania, Latvia, and Estonia, reflecting the growing relevance of Baltic economies in the European context.

Asia is represented by the CSI 300 and SSE 50 from China, which track large-cap companies and capture the performance of the Chinese stock market; the Nikkei 225, TPX 100, and TOPIX 30 from Japan, reflecting key segments of the Japanese economy; and the KOSPI 100 from South Korea, providing coverage of another significant Asian market. Additional representation from the Asia-Pacific region includes the ASX 100 from Australia and the Sensex 30 from India, both offering insights into major southern hemisphere economies.

Overall, Table 4 demonstrates the extensive global scope of the indices analyzed, enabling a more comprehensive evaluation of pairs trading strategies across diverse regional market structures.

Table 4. Summary of major stock indices and markets analyzed in this study

Index	Country
S&P 100	U.S.
Dow Jones Industrial Average	U.S.
Nasdaq 100	U.S.
Russell 2000 ETF	U.S.
S&P 500	U.S.
FTSE 100	UK
CSI 300	China
SSE 50	China
STOXX Europe 600	Europe
Nikkei 225	Japan
TOPIX 30	Japan
TPX 100	Japan
KOSPI 100	South Korea
OMX Baltic	Lithuania, Latvia, Estonia
SBF 120	France
ASX 100	Australia
Sensex 30	India

4. Data Analysis

The majority of the studies in our review employed daily datasets containing standard OHLCV information—opening, highest, lowest, and closing prices, along with trading volume. Some researchers, however, used higher-frequency datasets, such as tick data recorded at one-, five-, or fifteen-minute intervals.

4.1 Daily Interval Historical Price Data

Daily interval data typically includes a stock's open, high, low, close, and volume for each trading day. Table 5 presents an example of Apple's daily stock data, as reported in numerous studies [1-3, 6, 9-15, 17, 19, 21-24, 26, 29-32, 35-37, 39-41, 43-45, 47, 48, 50-52, 55, 57, 58, 60-63, 65-67, 71-73, 77-80, 82, 83, 85, 89, 90, 92, 93, 95-98, 100-102, 107, 109-115, 119, 121, 123]. This dataset is arranged chronologically by date.

In our analysis, we found that 78 of the studies reviewed specifically utilized daily interval data for their examinations. This represents a substantial proportion, with 75% of the papers relying on daily data intervals for their research. The widespread use of daily granularity reflects the strong preference among researchers for studying daily market movements, emphasizing the importance of capturing daily price trends and patterns to gain a deeper understanding of financial market behavior over time.

Table 5. A sample of Apple Inc.'s daily historical stock data.

Date	Open	High	Low	Close	Adj Close	Volume
8/23/2023	178.520004	181.550003	178.330002	181.119995	180.197906	52722800
8/24/2023	180.669998	181.100006	176.009995	176.380005	175.482025	54945800
8/25/2023	177.380005	179.149994	175.820007	178.610001	177.700684	51449600
8/28/2023	180.089996	180.589996	178.550003	180.190002	179.272644	43820700
8/29/2023	179.699997	184.899994	179.500000	184.119995	183.182632	53003900
8/30/2023	184.940002	187.850006	184.740005	187.649994	186.694672	60813900
8/31/2023	187.839996	189.119995	187.479996	187.869995	186.913544	60794500
9/1/2023	189.490005	189.919998	188.279999	189.460007	188.495468	45732600
9/5/2023	188.279999	189.979996	187.610001	189.699997	188.734207	45280000
… …	… …	… …	… …	… …	… …	… …

4.2 One-Minute Interval Historical Price Data

One-minute interval historical data offers a detailed perspective on stock market activity by recording price and volume changes every minute during the trading session. This resolution provides a middle ground between the highly granular nature of tick data and the broader view of longer intraday intervals. Each observation typically includes the open, high, low, and close prices, along with the trading volume for that minute.

Such data are particularly valuable for medium-frequency trading strategies, as they enable the detection of short-term patterns and the formulation of decisions based on minute-by-minute market dynamics. By examining one-minute data, researchers and traders can gain richer insights into price volatility and intraday momentum—factors critical for strategies that rely on rapid market fluctuations. While the dataset size is smaller than that of tick data, processing one-minute data still demands considerable computational resources and sophisticated analytical techniques.

Table 6 provides an example of Apple's one-minute stock data, as used in select studies [4, 38, 49, 54, 59, 86, 104, 120]. This granularity offers an effective compromise between detail and efficiency, making it well-suited for developing predictive models and implementing intraday trading strategies that require both timeliness and accuracy.

Table 6. A sample of Apple Inc.'s one-minute historical stock data.

Datetime	Open	High	Low	Close	Adj Close	Volume
8/14/2024 9:30:00	220.5650	220.7600	220.2100	220.5200	220.5200	2664372
8/14/2024 9:31:00	220.3500	220.5500	219.7900	219.8500	219.8500	436127
8/14/2024 9:32:00	219.9299	220.0900	219.7000	219.7500	219.7500	279503
8/14/2024 9:33:00	219.7900	220.3600	219.7300	220.3600	220.3600	194167
8/14/2024 9:34:00	220.3600	220.9400	220.3100	220.9050	220.9050	240008
8/14/2024 9:35:00	220.9100	221.1800	220.6150	221.0300	221.0300	279137
8/14/2024 9:36:00	221.0500	221.4400	221.0200	221.3650	221.3650	209326
8/14/2024 9:37:00	221.3700	221.5675	221.0596	221.5675	221.5675	334528
8/14/2024 9:38:00	221.5700	221.7500	221.4210	221.4900	221.4900	203597
8/14/2024 9:39:00	221.5300	222.0000	221.4900	221.9800	221.9800	259714
… …	… …	… …	… …	… …	… …	… …

4.3 Five-Minute Interval Historical Price Data

Five-minute interval historical data provide a broader overview of market activity by aggregating the open, high, low, close prices, and trading volume into five-minute segments. This interval is particularly suited for traders and analysts who seek to capture intraday price dynamics without the fine-grained noise present in tick or one-minute data.

The five-minute frequency effectively reveals broader intraday patterns—such as short-term trends, support and resistance levels, and potential breakouts—while filtering out high-frequency fluctuations. It strikes a balance between retaining sufficient detail to analyze market behavior and presenting a cleaner, more interpretable view of price movements. This makes it especially valuable for strategies targeting short- to medium-term price changes.

Although its data volume is smaller than that of higher-frequency datasets, five-minute data still requires robust analytical techniques to identify meaningful patterns. Table 7 presents an example of Apple's five-minute stock data, as cited in studies [18, 43, 73, 108, 120]. This interval plays a key role in building predictive models with an extended intraday scope, supporting trading strategies that operate within short- to medium-term horizons.

Table 7. A sample of Apple Inc.'s five-minute historical stock data.

Datetime	Open	High	Low	Close	Adj Close	Volume
2024-08-14 09:30:00	220.5650	220.9400	219.7000	220.9050	220.9050	3814177
2024-08-14 09:35:00	220.9100	222.0000	220.6150	221.9800	221.9800	1286302

Datetime	Open	High	Low	Close	Adj Close	Volume
2024-08-14 09:40:00	221.9700	222.0200	221.1690	221.4600	221.4600	842724
2024-08-14 09:45:00	221.4700	221.6750	221.0400	221.1050	221.1050	772010
2024-08-14 09:50:00	221.1006	221.2700	220.6814	220.7300	220.7300	561586
2024-08-14 09:55:00	220.7517	221.1399	220.7100	220.9500	220.9500	452026
2024-08-14 10:00:00	220.9450	221.2000	220.6900	221.1700	221.1700	448701
2024-08-14 10:05:00	221.1899	221.3499	220.8100	221.1501	221.1501	398965
2024-08-14 10:10:00	221.1650	221.4404	220.7900	221.4300	221.4300	497368
2024-08-14 10:15:00	221.4400	221.7000	221.2500	221.5973	221.5973	515541
……	……	……	……	……	……	……

4.4 Fifteen-Minute Interval Historical Price Data

Fifteen-minute interval historical data provide an aggregated view of market activity by consolidating open, high, low, close prices, and trading volume into fifteen-minute segments. This frequency offers a broader perspective on intraday price movements, trading off fine-grained detail for a clearer overview compared with tick or one-minute data.

The fifteen-minute interval is particularly effective for identifying larger intraday trends, monitoring significant support and resistance levels, and detecting potential breakout points, while reducing high-frequency noise that may obscure these patterns in more granular datasets. It strikes a balance between retaining essential detail and ensuring analytical manageability, making it suitable for strategies aimed at capturing medium-term intraday movements. Although the number of observations is substantially lower than in higher-frequency datasets, fifteen-minute data still require advanced analytical techniques to extract actionable market signals.

Table 8 presents an example of Apple's fifteen-minute stock data, as analyzed in studies [87, 116]. This interval proves valuable for developing predictive models with a broader intraday scope, supporting trading strategies that target medium-term opportunities within the trading day.

Table 8. A sample of Apple Inc.'s fifteen-minute historical stock data.

Datetime	Open	High	Low	Close	Adj Close	Volume
8/14/2024 09:30:00	220.5650	222.0200	219.7000	221.4600	221.4600	5943203
8/14/2024 09:45:00	221.4700	221.6750	220.6814	220.9500	220.9500	1785622
8/14/2024 10:00:00	220.9450	221.4404	220.6900	221.4300	221.4300	1345034
8/14/2024 10:15:00	221.4400	221.7000	220.9500	221.1104	221.1104	1248147
8/14/2024 10:30:00	221.1100	221.4800	220.4550	221.4000	221.4000	1292931

Datetime	Open	High	Low	Close	Adj Close	Volume
8/14/2024 10:45:00	221.4100	221.7260	221.2700	221.6901	221.6901	1092996
8/14/2024 11:00:00	221.6950	222.2900	221.6000	222.2500	222.2500	1267633
8/14/2024 11:15:00	222.2500	222.5600	222.2000	222.5587	222.5587	864267
8/14/2024 11:30:00	222.5500	222.8900	222.3900	222.4000	222.4000	935396
8/14/2024 11:45:00	222.4000	223.0300	222.4000	222.9701	222.9701	881023
… …	… …	… …	… …	… …	… …	… …

4.5 Hourly Interval Historical Price Data

Hourly historical data aggregate open, high, low, close prices, and trading volume within each hour of the trading day. This interval offers a balance between high-frequency datasets, such as tick data, and lower-frequency daily summaries, making it suitable for extended intraday analysis without the data intensity of high-frequency trading.

Hourly data are particularly useful for identifying intraday momentum changes, monitoring emerging trends, and detecting potential breakouts. These patterns often arise from news releases, economic announcements, or shifts in market sentiment. While hourly data lack the fine detail of tick or minute-level observations, they retain enough granularity to capture meaningful market behavior throughout the trading day.

Although less resource-intensive than higher-frequency datasets, hourly interval data still require robust analytical methods to derive actionable insights. Table 9 illustrates an example of Tesla's hourly stock data, as analyzed in studies [73, 99, 120]. This frequency is well-suited for developing predictive models that provide a concise yet comprehensive perspective on intraday dynamics, supporting trading strategies that target opportunities within hourly time horizons.

Table 9. A sample of Apple Inc.'s hourly historical stock data.

Datetime	Open	High	Low	Close	Adj Close	Volume
2024-08-14 09:30:00	220.5650	222.0200	219.7000	221.1104	221.1104	10322006
2024-08-14 10:30:00	221.1100	222.5600	220.4550	222.5587	222.5587	4517827
2024-08-14 11:30:00	222.5500	223.0300	221.0150	221.3050	221.3050	4029799
2024-08-14 12:30:00	221.3200	221.4600	220.3800	221.1750	221.1750	3645945
2024-08-14 13:30:00	221.1600	221.7690	220.9100	221.5400	221.5400	3099853
2024-08-14 14:30:00	221.5300	221.6100	220.8100	221.1600	221.1600	2990194
2024-08-14 15:30:00	221.1600	222.3500	221.1531	221.6200	221.6200	4112373
2024-08-15 09:30:00	224.5500	224.9900	222.7600	224.7900	224.7900	9897134

Datetime	Open	High	Low	Close	Adj Close	Volume
2024-08-15 10:30:00	224.7800	225.3400	223.9000	224.0050	224.0050	5316678
2024-08-15 11:30:00	224.0100	225.0150	223.8500	224.9950	224.9950	3233561
… …	… …	… …	… …	… …	… …	… …

4.6 Weekly Interval Historical Price Data

Weekly historical data aggregate open, high, low, close prices, and trading volume over the course of an entire trading week. Compared with daily datasets, this frequency offers a broader perspective by consolidating multiple trading days into a single observation, smoothing short-term fluctuations while preserving longer-term market trends.

Weekly data are particularly valuable for longer-horizon strategies, such as swing trading and position trading, where intraday or daily volatility is less relevant. They facilitate the identification of sustained market trends, key support and resistance levels, and momentum shifts developing over several weeks. By filtering out the high-frequency noise present in shorter intervals, weekly datasets provide a clearer view of underlying market direction, aiding in the evaluation of asset performance over extended periods.

Although less computationally demanding than intraday or tick-level data, weekly datasets still require robust analytical techniques to detect meaningful patterns that influence long-term market behavior. Table 10 presents an example of Apple's weekly stock data, as analyzed in studies [53, 74]. This frequency is well-suited for developing models aimed at capturing significant price movements across extended horizons while keeping data volume manageable.

Table 10. A sample of Apple Inc.'s weekly historical stock data.

Datetime	Open	High	Low	Close	Adj Close	Volume
4/1/2024	169.0800	171.9200	168.2300	169.5800	169.1545	192780800
4/8/2024	169.0300	178.3600	167.1100	176.5500	176.1070	322249600
4/15/2024	175.3600	176.6300	164.0800	165.0000	164.5860	309039200
4/22/2024	165.5200	171.3400	164.7700	169.3000	168.8752	241302700
4/29/2024	173.3700	187.0000	169.1100	183.3800	182.9199	441926300
5/6/2024	182.3500	185.0900	180.4200	183.0500	182.5907	300675100
5/13/2024	185.4400	191.1000	184.6200	189.8700	189.6505	288966500
5/20/2024	189.3300	192.8200	186.6300	189.9800	189.7603	208619700
5/27/2024	191.5100	193.0000	189.1000	192.2500	192.0277	230454300
6/3/2024	192.9000	196.9400	192.5200	196.8900	196.6624	245994400
… …	… …	… …	… …	… …	… …	… …

4.7 Millisecond and Nanosecond Interval Historical Price Data

Millisecond and nanosecond datasets, as in [25, 27, 28, 34], provide the most granular representation of market activity, capturing ultra-short-term price changes, trade volumes, and order flows. Millisecond data, recorded at 1/1,000th of a second, typically include timestamps, bid and ask quotes, trade prices, trade sizes, and detailed order book information. These datasets are critical in HFT environments for detecting fleeting price patterns, arbitrage opportunities, and market inefficiencies that can be exploited within fractions of a second. Their sheer size demands substantial computational resources and specialized infrastructure for real-time processing, limiting their use to institutional traders and advanced algorithmic trading firms. Millisecond-level data also enable detailed examination of market microstructure, including latency, execution speed, and price formation processes.

Nanosecond data, recorded at 1/1,000,000,000th of a second, offer an even finer resolution, capturing the precise sequence and timing of trades, orders, and quote changes with unmatched accuracy. This ultra-high-frequency data is essential for ultra-low-latency trading strategies, including market making and latency arbitrage, where speed is the primary competitive edge. However, the massive volume generated at this level presents significant challenges in data storage, processing, and analysis. Both millisecond and nanosecond datasets require highly sophisticated analytical tools and infrastructure, making them indispensable for specialized trading models where execution timing within microseconds can determine profitability.

5. Non-Machine Learning Models in Pair Trading

This chapter reviews several commonly used non-machine learning, or purely statistical, approaches to pairs trading. Each method has distinct features that make it more or less suited to specific market conditions and trading objectives. The discussion covers Distance Methods, Cointegration Methods, Stochastic Control Methods, Time Series Methods, and several other specialized approaches. By synthesizing recent research findings and practical implementations, this chapter evaluates the strengths, limitations, and adaptability of each method.

5.1 Distance Methods

This section provides a detailed examination of the distance method, a widely used approach in pairs trading research. To contextualize our discussion, Table 11 compiles representative studies published between 2016 and 2023, summarizing their research scope, data samples, and frequency of observations. The studies cover a variety of markets—including equities, commodities, and cryptocurrencies—and employ data ranging from millisecond-level to monthly intervals. This diversity highlights how the distance method has been adapted to different asset classes and temporal resolutions, offering a basis for the comparative analysis that follows.

Table 11. A summary of distance methods in pair trading from year 2016 to 2023.

Articles	Publish Year	Sample	Data Frequency
6	2016	London Stock Exchange stocks, 1979-2012	Daily
10	2016	Stock Indexes in 12 Countries, 1987-2011	Daily
13	2016	U.S. stocks, 1962-2014	Daily
14	2017	U.S. stocks, 1980-2014	Daily
22	2017	36 Stocks in DJIA, NYSE, and NASDAQ, 2006-2014	Daily
25	2017	Stocks in OMX Baltic, 2014-2015	Millisecond
26	2017	Chinese Commodity Futures, 2005-2016	Daily
43	2018	Oslo Stock Exchange stocks, 2005-2014	5-min, daily
45	2018	U.S. Financial Sector stocks, 2008-2013	Daily
51	2018	Gilt Futures in ICE, 2013-2015	Daily
56	2019	U.S. stocks, 1931-2007	Monthly
60	2019	CSI300, HSHKI, HSAHP stocks, 1996-2017	Daily
63	2019	Stockholm Stock Exchange stocks, 1995-	Daily
72	2020	Commodity Futures in MCX, 2011-2017	Daily
73	2020	181 Cryptocurrencies, 2018-2019	Daily, hourly, 5-min
77	2020	SSE 50 stocks, 2016-2018	Daily

Articles	Publish Year	Sample	Data Frequency
78	2020	DJIA, Sensex 30 and TOPIX 30 stocks, 2008-	Daily
79	2020	Nasdaq 100 stocks, 1999-2003 2007-2012	Daily
97	2021	NSE stocks, 2011-2017	Daily
98	2021	Toronto Stock Exchange stock, 2017-2020	Daily
120	2023	405 Cryptocurrencies, 2022	1-min, 5-mins, hourly

5.1.1 The GGR's Baseline Approach

Although our review primarily covers studies published from 2016 onwards, it is essential to begin with the seminal work of GGR, which first formalized the distance method in pairs trading. Their study analyzes all liquid U.S. stocks using daily data from the CRSP database between 1962 and 2002.

In the 12-month formation period, a cumulative total return index P_{it} is constructed for each stock and normalized to 1 on the first day. For a universe of n stocks, the squared Euclidean distances are computed for all possible $n\ (n-1)/2$ stock pairs based on these normalized price series. The 20 pairs with the smallest historical distance are selected for the subsequent six-month trading period.

During the trading period, prices are re-normalized on the first day, and trades are triggered when the spread between paired stocks deviates by more than two historical standard deviations (σ). Positions are closed when the spread reverts to its mean, when the six-month trading window ends, or when a stock is delisted.

The key steps of the GGR baseline methodology are outlined in the following section.

Data and Sample Selection

The data for this study were obtained from the CRSP daily files, covering the period from 1962 to 2002. The sample includes all liquid U.S. stocks listed on major exchanges, including the NYSE, AMEX, and NASDAQ. To ensure liquidity and avoid biases associated with thinly traded stocks, any security that experienced at least one non-trading day during the sample period was excluded. This filtering procedure ensured that all selected stocks had reliable and continuous price series, thereby supporting robust pair construction.

Formation Period

The methodology begins with a 12-month formation period, during which a cumulative total return index (including dividend reinvestments) is calculated for each stock. This index is normalized to 1 on the first day of the formation period to ensure comparability across securities. The core of the pair selection process is based on the SSD between the normalized price series of each pair of stocks. For two stocks i and j, SSD is defined as:

$$SSD_{ij} = \sum_{t=1}^{T} (P_{it} - P_{jt})^2 \tag{5.1}$$

where T is the number of trading days in the formation period, and P_{it} is the normalized cumulative return index of stock i on day t. All possible $n(n-1)/2$ stock pairs are evaluated, and those with the smallest historical SSD values are considered the strongest candidates for price convergence. The top-ranked pairs are retained for the subsequent trading period.

Trading Period

The trading period spans six months and follows directly after the formation period. During this phase, the selected stock pairs are traded according to predefined rules. The trading signal is based on the spread between the normalized prices of the two stocks in each pair. A position is initiated when the spread deviates from its historical mean—calculated during the formation period—by more than two standard deviations (σ), where σ is derived from the historical residuals of the spread over the 12-month formation period. When the threshold is breached, a long position is established in the underperforming stock ("loser") and a short position in the outperforming stock ("winner"). Positions are closed upon spread reversion to the historical mean, upon expiration of the six-month trading window, or upon delisting of either stock in the pair.

Risk-Adjusted Excess Returns

The profitability of the strategy is assessed in terms of annualized excess returns. GGR report that self-financing portfolios of the top-ranked pairs achieved average annualized excess returns of approximately 11%. To assess robustness, the authors incorporate risk adjustments and transaction cost estimates. Using bootstrap analysis, they demonstrate that the observed returns cannot be explained solely by simple mean-reversion effects. Additional out-of-sample testing for the period

1999 to 2002 confirms that the strategy continued to deliver positive excess returns, supporting its validity over time.

Model-Free Nature

A key strength of the GGR approach is its model-free design. The method does not rely on any specific asset pricing model, thereby mitigating the risk of model misspecification. Instead, trading decisions are entirely driven by the historical price relationships between paired securities, allowing for flexibility and adaptability across different asset classes and market regimes.

Trading Frequency and Holding Periods

On average, selected pairs are traded approximately twice during each six-month trading period, with individual positions held for an average of 3.75 months. Even after accounting for transaction costs and market frictions, fully invested portfolios of stock pairs maintain statistically significant excess returns. However, performance is not uniform over time: profitability tends to vary with market volatility, and Sharpe ratios decline in certain subperiods, suggesting that the strategy's effectiveness may depend on prevailing market conditions. Furthermore, as the original sample focuses on U.S. equities between 1962 and 2002—a period with distinct microstructure characteristics—the results may not directly generalize to other markets or more recent trading environments without recalibration.

Transaction Costs and Market Neutrality

GGR carefully evaluate the impact of transaction costs on the profitability of their pairs trading strategy. Their analysis incorporates reasonable estimates for transaction costs, including bid–ask spreads and short-selling fees, with a baseline assumption of a 0.5% round-trip cost per trade. Sensitivity tests under higher cost scenarios confirm that the strategy remains profitable, although net returns decline as expected.

The strategy is inherently market-neutral, as it involves taking both long and short positions in stocks expected to move relative to each other rather than in tandem with the broader market. Empirical results support this property: portfolio returns exhibit an estimated market beta close to zero, indicating minimal exposure to systematic market risk. This neutrality focuses returns on the relative performance of the paired stocks—the essence of the arbitrage opportunity.

This approach offers several clear advantages. As Do et al. (2006) observe, the GGR methodology does not rely on any specific asset pricing model, thereby avoiding the risk of model specification errors. It is straightforward to implement, resistant to data-snooping biases, and capable of generating statistically significant risk-adjusted excess returns. By applying a simple yet effective approach to a large dataset spanning more than four decades, GGR firmly established pairs trading as a notable capital market anomaly.

Nevertheless, certain aspects of the methodology leave room for refinement. The use of squared Euclidean distance (SSD) for pair selection, while intuitive, may not be analytically optimal for the ultimate objective of maximizing excess returns per pair. From a profit-maximization perspective, the most desirable pairs would combine two features: (1) frequent and sizable deviations from equilibrium, and (2) strong mean-reversion tendencies. These characteristics enable more frequent and profitable round-trip trades. While GGR's ranking logic—selecting pairs with the smallest historical SSD—implicitly favors pairs with stable long-term relationships, it may only partially align with these profit-maximizing conditions. Future research could investigate alternative distance metrics that directly incorporate both spread variance and mean-reversion speed.

5.1.2 Extension Methods Based on GGR

Compared to GGR's method, later papers introduce various differences and improvements built upon its foundation.

Bowen and Hutchinson (2016) extend the GGR framework to examine the performance of pairs trading in a non-U.S. setting. Their methodology closely follows that of GGR to ensure comparability and to minimize potential data-mining bias. Specifically, they form stock pairs over a 12-month formation period based on the minimum SSD of their normalized price series, and trade these pairs over a subsequent 6-month period using the same trading rule as GGR. Like the original study, their strategy remains model-free, relying solely on historical price relationships between paired securities. This replication facilitates a direct comparison of results between the U.S. and

UK equity markets, allowing for a meaningful assessment of the strategy's robustness across different market environments.

The study introduces several extensions to the GGR approach. Most notably, it applies the methodology to the UK equity market, providing evidence on the strategy's adaptability beyond the U.S. context. In addition to measuring profitability, Bowen and Hutchinson conduct a detailed risk–return analysis, evaluating performance metrics such as volatility and Sharpe ratios. They also investigate the influence of market conditions—including overall market volatility and liquidity—on pairs trading outcomes, thereby offering insights into the drivers of strategy performance.

Using a more recent dataset than GGR, their analysis reflects changes in market microstructure and trading behavior over time. Furthermore, they examine strategy performance across individual industry sectors within the UK market—an aspect not explored in detail in GGR's study—adding a sectoral dimension to the evaluation of pairs trading.

In summary, Bowen and Hutchinson expand the GGR framework by (1) applying it to a non-U.S. market, (2) incorporating a comprehensive risk–return assessment, (3) considering the influence of market conditions, (4) using more contemporary data, and (5) introducing sector-level analysis. These contributions enhance the understanding of pairs trading performance and highlight the strategy's potential adaptability to different markets and economic contexts.

Chen et al. (2019) corroborate the findings of GGR by demonstrating that pairs trading strategies can yield significant abnormal returns. Using a framework that identifies highly correlated stock pairs and exploits temporary return divergences, they extend GGR's analysis by attributing profitability to factors such as short-term reversal and pairs momentum, and by exploring the influence of industry momentum, liquidity, and information diffusion.

Methodologically, the two studies differ in several key aspects. In GGR, pairs are selected according to the SSD between normalized price series during a 12-month formation period, with the lowest-SSD pairs chosen for trading. In contrast, Chen et al. identify pairs based on five years of historical daily return correlations, matching each stock with its 50 most correlated counterparts. Moreover, while GGR adopt a fixed 12-month formation followed by a 6-month trading period, Chen et al. implement a rolling monthly evaluation of return divergences over the subsequent year, allowing for continuous identification of opportunities. Trading rules also diverge: GGR open positions when the price spread exceeds two historical standard deviations, whereas Chen et al.

take a contrarian stance on return divergence—buying the underperformer and shorting the outperformer—anticipating reversal in the following month.

Finally, the scope of risk factor analysis differs substantially. GGR focus on overall profitability without decomposing returns by source, while Chen et al. explicitly examine the roles of short-term reversal, momentum (including pairs momentum), and liquidity. They find that pairs momentum contributes significantly, particularly in the first month post-divergence. These differences in pair selection, trading signals, and risk factor integration yield a richer understanding of the mechanisms driving pairs trading profitability.

Gupta and Chatterjee (2020) present notable methodological innovations over the original GGR framework for pairs trading. While GGR select pairs based on the SSD between normalized price series over a fixed formation period, Gupta and Chatterjee incorporate a dynamic dimension via the Dynamic Cross-Correlation Type (DCCT) measure.

A key innovation is the explicit modelling of lead–lag relationships. The DCCT measure, computed over rolling windows, identifies the lag value that maximizes cross-correlation and allows it to adjust over time. This approach captures evolving temporal dependencies, where one asset may lead or lag another during different subperiods, in contrast to the static relationships assumed in GGR. In their framework, SSD remains the primary proximity measure, but the selection process is enhanced by applying DCCT as a secondary filter. This ensures that chosen pairs not only display small price deviations but also exhibit favourable dynamic correlation structures and stable lead–lag characteristics.

Empirical tests using constituents of the DJIA (U.S.), Sensex 30 (India), and Topix 30 (Japan) indices show that the SSD+DCCT approach outperforms SSD alone in profitability and reduces the incidence of false-positive pairs. The method's adaptability makes it particularly effective under volatile or structurally shifting market conditions, where static approaches may underperform. By extending GGR with dynamic temporal analysis, this study offers a more responsive distance-based strategy with strong performance across diverse markets.

Other papers using distance methods are also based on the expansion and optimization of GGR. The above is the analysis of three papers for reference.

5.1.3 Explanation of Pair Trading Profitability

Bowen and Hutchinson (2016) analyze pairs trading profitability in the UK equity market from 2002 to 2012, with particular attention to the 2007–2008 financial crisis. They find that the strategy delivered annualized returns of 36%–48% during the crisis period, even as the FTSE All-Share Index declined by 34%, highlighting its potential diversification benefits and resilience under market stress. The portfolios show low exposure to conventional equity risk factors—including market, size, value, momentum, and reversal—indicating that profitability arises mainly from exploiting short-term pricing inefficiencies rather than broad market trends. Although transaction costs, primarily from bid–ask spreads, reduce annual returns by up to 4%, the strategy remains profitable and consistent across both quote-driven and order book trading regimes. Return distributions exhibit positive skewness and high kurtosis, suggesting generally stable returns punctuated by occasional large windfalls, particularly during periods of severe market dislocation when the strategy may also provide liquidity.

Quinn et al. (2018) examine distance-based pairs trading in the UK gilt futures market, focusing on long–medium gilt combinations over nine quarters. Using a trigger–stop-loss framework, they show that a 10% trigger yields a cumulative return of 3.51%, while a 15% trigger generates 1.78%, demonstrating that profitability can be achieved even in highly liquid and regulated markets. Lower trigger levels increase trading frequency and aggregate returns, whereas higher triggers reduce the number of trades but remain profitable. A 30% stop-loss consistently protects and enhances profitability for long–medium gilt pairs, underscoring the importance of effective risk management. The strategy exploits mean reversion in the yield spread between different maturities, and its success is robust to varying market conditions, making it a low-risk, systematic arbitrage approach to government bond futures.

Chen et al. (2019) investigate return-difference-based pairs trading in the U.S. equity market and report significant short-term profitability. Value-weighted portfolios sorted by return differences (RetDiff) earn an average monthly return of 1.40% in the first month after divergence, but performance declines sharply thereafter, with losses observed beyond month one. Abnormal returns are not explained by standard risk factors such as market, size, book-to-market, or momentum, suggesting that profits are driven by micro-level mispricings rather than broad factor exposures. Liquidity risk plays a minimal role, while profitability varies across sectors, with industries characterized by slower information diffusion and higher volatility exhibiting stronger pairs momentum and short-term reversal effects. These results underscore the importance of prompt execution and sector selection in enhancing performance.

Zhang and Urquhart (2019) evaluate pairs trading within and across the mainland China and Hong Kong stock markets from 2002 to 2016. They find that intra-market strategies do not produce significant abnormal returns, whereas cross-market pairs trading achieves annualized abnormal returns of up to 9% after accounting for risk factors and transaction costs, including commissions, taxes, and short-selling fees. Profitability is time-varying, peaking during periods of market turbulence and declining in stable conditions. Fama–French five-factor, momentum, and short-term reversal models confirm that significant abnormal returns are concentrated in cross-market trades, underscoring the role of low market integration in creating arbitrage opportunities. Dual-listed stocks (H-shares and A-shares) perform particularly well, benefiting from common cash flow sources and persistent pricing inefficiencies between the two exchanges.

Gupta and Chatterjee (2020) propose a hybrid approach combining the traditional SSD measure with a DCCT measure to capture both price proximity and evolving lead–lag relationships between stocks. Using constituents of the DJIA (U.S.), Sensex 30 (India), and Topix 30 (Japan), they show that the SSD+DCCT method consistently outperforms correlation+SSD and SSD-only methods. For example, in the DJIA dataset with a 35-day trading window, SSD+DCCT ($\psi = 25$) achieves a profit margin of 0.427, while in the Sensex 30 dataset under a 21-day window, it attains 0.576, far exceeding alternative measures. The DCCT is estimated over rolling windows to adapt to changing temporal dependencies, reducing false-positive pair selections and improving profitability, particularly over longer horizons. The findings suggest that incorporating dynamic temporal dependencies into pair selection enhances robustness and transferability across markets.

Collectively, these studies demonstrate that pairs trading profitability depends on market conditions, asset class characteristics, and methodological refinements. While the core distance-based approach introduced by GGR remains foundational, enhancements such as incorporating sector effects, cross-market opportunities, dynamic lead–lag analysis, and precise trading parameter calibration can significantly improve performance. The evidence also highlights that transaction costs and execution timing are critical determinants of net profitability, especially in highly liquid or regulated markets.

5.2 Cointegration Methods

In the cointegration method, the extent of co-movement between the paired assets is evaluated using cointegration tests, such as the Engle-Granger or Johansen approaches. A summary of the results can be found in Table 12, which summarizes pairs trading studies employing cointegration methods between 2016 and 2023, covering a wide variety of markets, asset classes, and data frequencies. The dataset spans traditional equities (e.g., S&P 500, DJIA, STOXX Europe 600), commodity futures (both U.S. and Chinese), foreign exchange rates, cryptocurrencies, and derivatives such as options and CDSs. Sampling frequencies range from weekly to nanosecond-level data, with daily frequency remaining dominant. In recent years, there has been a notable increase in high-frequency applications (e.g., millisecond and nanosecond futures trading) and the extension of cointegration to novel asset classes such as Bitcoin, Ethereum, and other cryptocurrencies.

Several trends emerge from the literature. First, high-frequency cointegration strategies have been tested in both equity and commodity markets, leveraging ultra-short-term price corrections, though profitability is highly sensitive to transaction costs and market microstructure effects. Second, cointegration has been applied beyond equities to capture long-term co-movements in commodities, currencies, and digital assets, demonstrating its adaptability across asset classes. Third, studies using emerging market data—such as Indian commodities, Chinese futures, and MCX contracts—show that cointegration can remain profitable outside developed markets, albeit with greater sensitivity to liquidity constraints. Finally, post-2020 research reflects a growing interest in cryptocurrency markets, where high volatility and structural inefficiencies create fertile ground for cointegration-based strategies.

Table 12. A summary of cointegration methods in pair trading from year 2016 to 2023.

Articles	Publish Year	Sample	Data Frequency
4	2016	U.S. stocks, November 3, 2014	1-min
8	2016	U.S. stocks, 2015 to 2016	1-min
11	2016	Stocks in DJIA, 1 January 2009 until 31 December 2009	daily
13	2016	U.S. stocks, 1962-2014	daily
14	2017	U.S. stocks, 1980-2014	daily
15	2017	Stocks in Global Dow, 2002-2012	daily
24	2017	38 Commodity Futures in China, 2006-2016	daily

Articles	Publish Year	Sample	Data Frequency
26	2017	Commodity Futures in China, 2005-2016	daily
27	2017	Commodity Futures in US, August 1, 2015-August 31, 2015	nanosecond
28	2017	Commodity Futures in US, August 1, 2015-August 31, 2015	nanosecond
30	2018	Stocks in SPX 100, TPX 100, SBF 120, ASX 100, 2007-2016	daily
34	2018	Commodity Futures in US, August 1, 2015-August 31, 2015	nanosecond, millisecond
35	2018	Chinese stocks, 2010-2016	daily
37	2018	S&P 500 stocks, 2012-2014	daily
43	2018	Oslo Stock Exchange stocks, 2005-2014	5-min, daily
44	2018	STOXX Europe 600 stocks, 2000-2017	daily
45	2018	US Financial Sector stocks, 2008-2013	daily
50	2018	S&P 500 stocks, 1990-2015	daily
52	2018	General Motors and Ford Motor, 2010-2015	daily
53	2018	6 Greek bank stocks, 2001-2007	weekly
55	2019	20 Global Currencies Against Indian Rupee, 1994-2017	daily
61	2019	U.S. stocks, 2012-2016	daily
62	2019	250 stocks in Europe, 2001-2017	daily
67	2020	Commodity Futures, 2016-2020	daily
69	2020	-	-
72	2020	Commodity futures in MCX, 2011-2017	daily
73	2020	181 Cryptocurrencies, 2018-2019	daily, hourly, 5-min
75	2020	-	-
79	2020	Nasdaq 100 stocks, 1999-2003 2007-2012	daily
86	2021	Cryptocurrency, September 2018 to October 2019	1-min
89	2021	30 European stocks, 2008-2018	daily
93	2021	NYSE, AMEX and NASDAQ stocks, 1970-2016	daily
96	2021	Indian Metals Commodities, 2008-2019	daily
97	2021	NSE stocks, 2011-2017	daily
98	2021	Toronto Stock Exchange stocks, 2017-2020	daily
102	2022	Energy futures, stocks, and ETFs, 2015-2021	daily
107	2022	NYSE and NASDAQ ETFs and stocks, 2007-2021	daily
109	2022	Russell 2000 ETF and SPDR S&P 500 ETF, 2004-2020	daily
114	2023	CDSs, November 2020-November 2021	daily
115	2023	Bitcoin and Ethereum, 2016-2022	daily
120	2023	405 Cryptocurrencies, January 2022-March 2022	1-min, 5-mins, hourly
121	2023	5 Stock Indexes options, 2007-2017	daily
123	2023	S&P 500 stocks, 1998-2018	daily

5.2.1 Analysis of Cointegration Theoretical Framework

Engle and Granger's (1987) were the first to systematically introduce the theory of cointegration and to formalize the ECM. They demonstrated that if two or more non-stationary time series share a stable long-run equilibrium, their linear combination may be stationary, defining what is now known as a "cointegration relationship." The Engle–Granger two-step cointegration test—consisting of an initial unit root test followed by a residual-based stationarity test—was also proposed. While limited to identifying a single cointegration vector in the bivariate case, this framework laid the foundation for modern time series econometrics and has been widely applied in finance, including asset pricing and pairs trading.

Alexander (1999) explored the application of cointegration theory to hedging, using long-term equilibrium relationships to derive dynamic hedge ratios. Although the work does not focus on pairs trading per se, it introduced a risk management perspective to modelling long-run asset relationships and influenced subsequent financial research employing cointegration for identifying and trading related assets.

Vidyamurthy (2004) published the first comprehensive book on applying cointegration and other quantitative methods to pairs trading strategies. The book covers statistical foundations, the design of trading rules, and the integration of cointegration theory with other financial models such as the Arbitrage Pricing Theory (APT) and the Cost-of-Carry Trading Model (CTM). His framework, although primarily theoretical and lacking extensive empirical testing, has been influential among practitioners. It involves three key steps: (1) identifying potential cointegrated pairs using statistical or fundamental criteria, (2) evaluating the tradability of these pairs using a proprietary measure, and (3) designing trading rules via nonparametric techniques. Rather than emphasising rigorous statistical testing for cointegration, Vidyamurthy prioritises the practical implementation of strategies grounded in the concept, making his work a cornerstone reference for both academics and practitioners. For further exploration of his approach, see the related works by Do et al. (2006) and Puspasingrum (2012).

Pairs Selection

The selection of asset pairs constitutes the first stage in Vidyamurthy (2004) framework for pairs trading. The primary objective of this stage is to identify pairs of securities that are likely to be cointegrated, meaning that despite potential short-term deviations, their prices share a long-run equilibrium relationship.

Vidyamurthy builds upon the Common Trends Model (CTM) of Stock and Watson (1988), which decomposes the logarithmic price of security i into two components: a common nonstationary trend and a stationary idiosyncratic deviation:

$$p_{it} = n_{it} + \varepsilon_{it} \tag{5.2}$$

where,

n_{it} denotes the nonstationary common trends component, representing price movements driven by broad market factors;

ε_{it} denotes the stationary idiosyncratic component, capturing deviations from the common trend specific to the security.

Similarly, the return of the security i, r_{it}, can similarly be decomposed into common-trend returns r_{it}^{c} and specific returns r_{it}^{s}:

$$r_{it} = r_{it}^{c} + r_{it}^{s} \tag{5.3}$$

Here, r_{it}^{c} reflects returns driven by macroeconomic or other common factors (e.g., interest rate shifts, aggregate economic growth), while r_{it}^{s} captures security-specific characteristics.

To operationalize the identification of cointegrated pairs, Vidyamurthy introduces the Arbitrage Pricing Theory (APT) of Ross (2013), which expresses returns as a linear combination of common factor returns:

$$r_{it} - \mu_i = \beta_i' f_t + \varepsilon_{it} \tag{5.4}$$

where,

r_{it} is the return of security i;

μ_i is the expected mean return of stock i;

β_i' is a $k \times 1$ vector of factor loadings for stock i;

f_t is a $k \times 1$ vector of common factor returns (such as macroeconomic factors);

ε_{it} is the idiosyncratic error of the return r_{it}.

To simplify the model, Vidyamurthy assumes that the mean return μ_i is zero (i.e., returns are standardized). The key to identifying potential cointegrated pairs is to find securities with sufficiently similar factor loadings β_i and β_j, so that the common-trend components of their returns offset one another in the long run.

Consider a portfolio that is long one share of stock i and short γ shares of stock j. The price spread, m_{ijt}, between the two securities can be expressed as:

$$m_{ijt} = p_{it} - \gamma p_{jt} = n_{it} - \gamma n_{jt} + \varepsilon_{it} - \gamma \varepsilon_{jt} \tag{5.5}$$

For this spread to be stationary—and thus suitable for cointegration-based trading—the common-trend components n_{it} and n_{jt} must be proportional via the constant γ, allowing the common factors to cancel and leaving a stationary spread driven solely by idiosyncratic components.

The first difference of the price spread, which represents the return of the portfolio, can be written as:

$$\Delta m_{ijt} = r_{ijt} = r_{it}^{c} - \gamma r_{jt}^{c} + r_{it}^{s} - \gamma r_{jt}^{s} \tag{5.6}$$

To achieve mean-reversion, the common-trend return components r_{it}^{c} and r_{jt}^{c} must offset each other, leaving the spread dynamics driven by stationary specific returns.

For preselection, Vidyamurthy employs a distance metric based on the Pearson correlation coefficient of the common-trend returns:

$$Distance(i, j) = \mid Corr(r_{it}^{c}, r_{jt}^{c}) \mid \tag{5.7}$$

where $Corr\left(r_{it}^{c}, r_{jt}^{c}\right)$ is the Pearson correlation coefficient between the common trend returns of stocks i and j.

Pairs are ranked in descending order of absolute correlation, with the assumption that securities with the most similar common-trend returns are more likely to be cointegrated and thus more promising for trading.

This preselection step serves as a practical filter before engaging in more computationally intensive analyses. While this method does not constitute a formal statistical cointegration test, it provides an economically intuitive starting point for identifying candidate pairs, consistent with the theoretical underpinnings of both CTM and APT.

Testing for Tradability

Once candidate pairs have been preselected, the next stage in Vidyamurthy's framework is to assess their tradability—that is, whether they exhibit sufficiently strong mean-reversion to be viable for trading. Rather than applying formal cointegration tests such as the Engle–Granger two-step method, Vidyamurthy adopts a more pragmatic approach centered on mean-reversion diagnostics. He argues that the most informative metric for tradability is the zero-crossing frequency of the spread, which measures how often the price difference between two assets crosses its mean value.

The tradability test involves estimating the following static regression model for the log prices of the preselected stocks:

$$p_{it} = \mu + \gamma p_{jt} + \varepsilon_{ijt} \tag{5.8}$$

where p_{it} and p_{jt} are the log prices of stocks i and j, μ represents the long-run premium of i over j, γ is the cointegration coefficient, and ε_{ijt} denotes the residual spread. If the spread exhibits frequent zero-crossings, this suggests a high degree of mean-reversion, indicating stronger tradability.

To increase robustness, Vidyamurthy recommends employing bootstrap simulations to estimate the standard errors of the average holding time—the interval between consecutive zero-crossings. This guards against overfitting to transient patterns and ensures that observed mean-reversion is persistent rather than noise-driven. While this approach is operationally practical, it sacrifices the statistical rigor of formal stationarity testing. Consequently, a pair passing this tradability test may still face risks of non-stationary behavior under certain market regimes.

Trading Design

Following preselection and tradability assessment, Vidyamurthy's framework proceeds to the design of trading rules, which determine the timing of trade entries and exits. His method employs nonparametric threshold rules, where a trade is initiated when the spread deviates from its historical mean by a certain number of standard deviations k.

In contrast to GGR, who use fixed entry and exit thresholds (typically $\pm 2\sigma$), Vidyamurthy advocates optimizing k for each pair individually. The optimization process involves simulating trading performance for a range of thresholds and then selecting the value that maximizes cumulative profit. This process starts by counting the number of threshold breaches at each candidate k, followed by calculating the profit generated per breach, and finally identifying the k that yields the highest aggregate profit.

While this customization can improve historical profitability, it also introduces the risk of overfitting—a threshold finely tuned to past data may fail to perform in out-of-sample conditions. Vidyamurthy further cautions that if the optimal k is reached only toward the end of the sample period, realized profits may be minimal, highlighting the importance of considering the temporal distribution of trade signals when conducting such optimization.

Practical Implications and Challenges

While Vidyamurthy's preselection methodology is both practical and grounded in economic theory, several critical aspects remain open to interpretation and warrant further refinement. A primary challenge stems from its reliance on the APT and the CTM to identify potentially cointegrated pairs. This framework implicitly assumes that the common factors driving asset prices are relatively stable over time—an assumption that is often violated in dynamic and volatile markets. During episodes of market instability or structural change, the underlying drivers of asset prices can shift materially, potentially undermining the predictive power of models calibrated on historical correlations.

A second limitation lies in the specification of the APT itself. Vidyamurthy does not provide concrete guidance on the optimal number of factors to include. This choice is crucial: omitting relevant factors may fail to capture essential components of return variation, whereas incorporating too many can introduce estimation noise and dilute predictive accuracy. Empirical evidence reported by Avellaneda and Lee (2010) indicates that between 10 and 30 factors may be necessary to explain a substantial proportion of cross-sectional return variance in U.S. equities, complicating the practical implementation of the model. For practitioners, the challenge lies in determining the most relevant set of factors for a given market, sector, or time period.

The preselection procedure—ranking pairs based on the Pearson correlation of their common factor returns—also introduces subjectivity. Vidyamurthy leaves the choice of the correlation measurement window to the practitioner, yet the time horizon has a substantial influence on results.

Short horizons may capture ephemeral relationships that break down quickly, whereas long horizons may obscure more recent structural changes. Moreover, the absence of a clearly defined threshold for inclusion means that the degree of “sufficient” correlation required for tradability is left ambiguous, potentially resulting in inconsistent pair selection across applications.

Another critical assumption is the stationarity of idiosyncratic return components. The framework presumes that the spread between two cointegrated assets will revert to its mean due to the stationary nature of these components. However, in real markets, structural breaks, regime shifts, or firm-specific events can disrupt this property, leading to extended non-convergence episodes. Such deviations can translate into losses or protracted holding periods, diminishing strategy profitability.

Transaction costs and liquidity constraints represent an additional source of friction often underappreciated in academic treatments. Vidyamurthy’s framework, like many others, implicitly assumes frictionless execution. In practice, bid–ask spreads, commissions, and slippage can materially reduce net profitability, particularly in less liquid markets where even moderate order sizes can influence prices. The problem is exacerbated in high-frequency trading contexts, where frequent rebalancing to capture small deviations accumulates substantial trading costs over time. Addressing these frictions may require prioritizing liquid securities and adopting execution algorithms designed to minimize market impact.

Finally, risk management considerations receive limited attention in Vidyamurthy’s formulation. While the emphasis is on identifying mean-reverting relationships, little guidance is offered on handling adverse market movements. In practice, robust pairs trading strategies require stop-loss mechanisms, dynamic position sizing, and contingency plans for extreme market events. Without these safeguards, even statistically sound strategies may incur severe losses during black swan episodes or periods of systemic dislocation.

Potential Extensions and Enhancements

A promising extension of Vidyamurthy’s framework involves the integration of machine learning techniques to enhance the preselection process. Advanced algorithms, such as random forests, gradient boosting machines, or neural networks, may uncover complex and potentially nonlinear relationships between securities that conventional statistical methods such as the APT and CTM could overlook. Furthermore, these approaches can facilitate automated factor selection, thereby reducing the subjectivity inherent in determining the most relevant explanatory variables within the APT framework—albeit potentially at the expense of model interpretability. Nevertheless, the application of machine learning presents its own challenges, including substantial data requirements, the need for careful hyperparameter tuning, and the risk of overfitting to historical patterns that may not persist in live market conditions.

A second enhancement lies in the adoption of dynamic factor models, which allow factor loadings to evolve over time. This directly addresses a key limitation of the static APT formulation, which assumes that the relationship between asset returns and common factors is time-invariant. By modelling time-varying sensitivities, dynamic factor approaches can better capture the changing nature of financial markets and, in turn, improve the robustness of preselected pairs.

Relatedly, the incorporation of regime-switching models offers another avenue for improvement, particularly in addressing structural breaks and changes in market regimes. Such models enable the identification of shifts in the statistical properties of price series, signalling periods in which the mean-reversion characteristics of a pair may deteriorate. By embedding regime-detection mechanisms within the strategy, traders can proactively adjust exposures when underlying market dynamics shift, thereby mitigating the risk of persistent non-convergence.

Finally, transaction cost analysis should be embedded directly into the strategy design phase. Rather than optimizing entry and exit thresholds purely on the basis of theoretical profitability, practitioners should explicitly account for realistic execution frictions, including bid–ask spreads, market impact, slippage, and latency constraints. This may involve conducting historical backtests and out-of-sample validations across multiple market conditions to ensure that the strategy remains profitable after accounting for these costs.

In summary, Vidyamurthy’s framework provides a coherent and practical blueprint for identifying, testing, and executing cointegration-based pairs trades. However, there is scope for further refinement through the integration of advanced modelling techniques, dynamic market adaptation, and cost-aware trading design. Future research could build upon this foundation by exploring hybrid approaches that combine statistical rigour with machine learning adaptability, while systematically evaluating performance across heterogeneous market environments.

5.2.2 Expanding Cointegration Methods

In comparison to Vidyamurthy's method, subsequent research presents several methodological differences and improvements while building upon its foundational framework.

In contrast to Vidyamurthy's original approach to pairs trading—which primarily focuses on identifying cointegrated stock pairs exhibiting long-term equilibrium relationships and executing trades based on mean reversion—subsequent research has expanded and refined the methodology to address its limitations and improve adaptability to diverse market environments. One notable advancement is presented by Ekkarntrong et al. (2017), who propose a multiclass pairs trading model that integrates mean reversion with the coefficient of variance (CV) to better classify and manage stock pairs. Unlike Vidyamurthy's method, which concentrates on the price relationship of a single pair, the multiclass model groups stocks into different CV classes based on volatility and correlation. This classification allows for cross-class trading, enabling the simultaneous trading of pairs from different volatility classes, thereby enhancing diversification and mitigating risk. The incorporation of CV as a volatility measure provides a dynamic dimension to the strategy, allowing traders to anticipate directional changes and adapt their trading approaches accordingly. Moreover, the advanced mean-reversion algorithm applied within each class improves the precision of trading signals, leading to more consistent returns. These innovations not only enhance the identification of profitable trading opportunities but also introduce sophisticated risk management mechanisms absent from Vidyamurthy's original framework, resulting in a more flexible and adaptive strategy for modern volatile markets.

Chen et al. (2017) introduce further advancements to the cointegration approach originally proposed by Vidyamurthy (2004). While Vidyamurthy's framework emphasizes static cointegration for pair selection, Chen et al. employ an adaptive cointegration methodology better suited to the non-stationary and time-varying nature of financial markets, particularly in commodity futures. Their method dynamically updates cointegration relationships over time, ensuring that trading pairs remain relevant throughout the investment horizon. This continuous recalibration addresses one of the major drawbacks of static cointegration: the risk of outdated relationships. Additionally, Chen et al. enhance the error-correction model (ECM) to refine trading signals, ensuring that trades occur at optimal points of mean reversion. By integrating these elements, the adaptive cointegration approach improves the flexibility and robustness of pairs trading, especially in high-volatility and structurally evolving markets such as Chinese commodity futures.

The methodology proposed by Figuerola-Ferretti et al. (2018) presents several important innovations over Vidyamurthy's framework. First, they introduce the concept of price discovery by identifying which asset acts as the "leader" in the pair—an asset whose price movements predict those of the "follower." This leader–follower dynamic allows for predictive trading, improving the precision of entry and exit decisions. In contrast, Vidyamurthy's model treats both assets symmetrically, without distinguishing leadership roles. Second, they implement a dynamic threshold mechanism that adjusts according to the speed of mean reversion. As the persistence of the cointegration error increases, the trading trigger threshold is raised, optimizing trade timing. This contrasts with Vidyamurthy's reliance on fixed statistical thresholds, which ignore variations in reversion speed. Third, error persistence is treated as a core strategic element—lower persistence correlates with higher profitability, a finding that Vidyamurthy's framework does not explore in depth. Finally, the authors employ an extended VECM to capture evolving cointegration dynamics more effectively. This represents a clear advancement over Vidyamurthy's simpler, static cointegration tests, enabling more responsive and data-driven strategies.

Feng et al. (2020) make several methodological contributions relative to Vidyamurthy's (2004) work, with a strong emphasis on real-world trading frictions. One significant improvement is the explicit modelling of transaction costs and asset illiquidity—factors often overlooked in academic models. By incorporating these frictions, the strategy becomes more realistic and applicable in practice, where trading costs can substantially erode profits. Another enhancement is the introduction of position limits on illiquid assets, constraining trade size and timing to avoid unrealistic portfolio allocations. Additionally, Feng et al. adopt dynamic trading boundaries using a singular control framework, optimizing entry and exit points while accounting for frictions and position constraints—surpassing Vidyamurthy's static statistical triggers. The strategy is further strengthened by solving HJB equations to maximize utility, integrating both cointegration and market frictions into a single optimization problem. The inclusion of illiquid instruments and synthetic assets also adds nuance, particularly for markets where asset liquidity is uneven. In contrast, Vidyamurthy's approach assumes both assets are equally tradable, ignoring liquidity disparities.

Tadi and Kortchemski (2021) extend Vidyamurthy's methodology with several innovations tailored to cryptocurrency markets. The most notable improvement is the adoption of dynamic cointegration testing, continuously updating relationships via Engle–Granger and Johansen tests to adapt to rapidly changing conditions. They also introduce look-back window optimization to calibrate the speed of mean reversion using an OU process, ensuring responsiveness to market shifts. The framework integrates both linear and nonlinear cointegration tests—combining the Augmented Dickey–Fuller (ADF) test with the nonlinear Kapetanios–Snell–Shin (KSS) test—to capture complex asset relationships often present in volatile crypto markets. Furthermore, they incorporate market microstructure elements such as best bid/ask quotes and order execution gaps, bringing the backtesting process closer to live trading conditions—an aspect absent in Vidyamurthy's model.

Finally, Kato and Nakamura (2023) broaden the application of cointegration beyond equities by applying it to hazard rates in the credit default swap (CDS) market. This represents a significant conceptual shift from price-based to credit-risk-based pairs trading. Their approach incorporates dynamic cointegration that adjusts for term structures across different CDS maturities, addressing maturity effects ignored in Vidyamurthy's static framework. Estimation of hazard rate dynamics is performed via Bayesian inference combined with an ODE-based solver, a more advanced methodology than traditional econometric techniques. Moreover, cointegration is embedded within an arbitrage-free pricing framework, ensuring theoretical consistency—an integration absent from Vidyamurthy's methodology.

Overall, subsequent developments in cointegration-based pairs trading significantly extend the scope and robustness of Vidyamurthy's original framework. The reviewed studies demonstrate innovations across multiple dimensions: enhancing pair classification through volatility metrics (Ekkarntrong et al., 2017), introducing adaptive and dynamic cointegration mechanisms (Chen et al., 2017; Tadi and Kortchemski, 2021), incorporating market microstructure and trading frictions into model design (Feng et al., 2020), and expanding the domain of application to alternative asset classes such as credit derivatives (Kato and Nakamura, 2023). Methodological refinements such as leader–follower identification, dynamic thresholds, error persistence analysis, and the use of advanced econometric and optimization techniques collectively improve the predictive accuracy, adaptability, and practical feasibility of pairs trading strategies. These advancements suggest that the evolution of cointegration methods is moving towards more context-aware, data-adaptive, and execution-sensitive frameworks, making them increasingly applicable in complex, high-frequency, and multi-asset market environments.

5.2.3 Empirical Results Analysis

This section reviews empirical evidence from recent studies evaluating cointegration-based pairs trading strategies across diverse markets, asset classes, and methodological variations. By examining their reported performance, risk profiles, and robustness to transaction costs, it is possible to assess both the practical effectiveness and limitations of these strategies under varying market conditions.

Ardia et al. (2016) test pairs trading strategies using a Bayesian simulation-based procedure for predicting stable ratios of stock prices. They evaluate the model's performance by applying both 5-day and 10-day moving averages, combined with different risk thresholds. The results are promising: even after accounting for USD 100 in transaction costs per round turn, the average annual return remains above 18%. This approach demonstrates the potential of Bayesian techniques to stabilize the long-term relationship between stock pairs, offering a robust framework for pairs trading strategies across different market regimes.

Similarly, Kato and Nakamura (2023) develop and test a pairs trading strategy using cointegration analysis between CDS spreads, focusing on the cointegration of hazard rates. Leveraging an ODE-based Bayesian inference method, they forecast stable relationships between CDS spreads and apply a VECM to capture price divergences. Tested on Japanese corporate CDSs, the strategy targets mean reversion in spread differentials to maintain long-term equilibrium. Extensive backtesting shows that, even after transaction costs, the strategy consistently achieves an annualized return exceeding 12%. This finding underscores the applicability of Bayesian methods in enhancing pairs trading strategies within credit derivatives markets.

Several studies employ large-scale equity datasets. For example, Rad et al. (2016) evaluate the profitability of pairs trading strategies using three distinct methods—distance, cointegration, and copula—on a comprehensive U.S. equity dataset covering over 23,000 stocks from 1962 to 2014. They find that distance and cointegration methods deliver similar performance, with significant average monthly excess returns of 0.91% and 0.85%, respectively, before transaction costs. The copula method produces fewer trades but exhibits greater stability in the frequency of trading opportunities, albeit with lower returns. After incorporating time-varying transaction costs, the cointegration and distance methods yield annualized returns of 3.3% and 3.8%, respectively, while the copula method lags at 0.5%. These results indicate that complex models like cointegration and copula can be effective in volatile markets, with cointegration generally outperforming.

Smith and Xu (2017) compare the distance and cointegration approaches using U.S. equity market data from 1980 to 2014, testing various parameterizations. They find that the distance approach generally outperforms cointegration, particularly during the 1980s and 1990s, achieving annualized returns as high as 40% for smaller portfolios. However, after factoring in transaction costs, profitability declines sharply in the 2000s. The cointegration approach performs well only in the 1980s and struggles in later decades. These findings suggest that while the distance method can be more robust in certain market regimes, both methods face challenges in sustaining profitability over time.

Brunetti and De Luca (2023) analyze the impact of seven different pre-selection metrics on the profitability of cointegration-based pairs trading strategies. Using S&P 500 constituents from 1998 to 2018, they compare measures such as SSD, price ratio, and spectral coherence. Their results show substantial variation in performance depending on the pre-selection metric. For example, pairs selected based on log-price correlation achieve an annualized return of over 12%, while spectral coherence yields less consistent results. This highlights the importance of pre-selection in optimizing cointegration-based strategies, as the chosen metric significantly influences returns, risk exposure, and the degree of market neutrality.

Overall, these empirical studies demonstrate that cointegration-based pairs trading can deliver economically meaningful returns across multiple asset classes and market environments. However, performance is highly sensitive to factors such as transaction costs, portfolio construction rules, pre-selection criteria, and market regimes. Advanced techniques—including Bayesian inference, adaptive cointegration, and refined pre-selection metrics—tend to enhance robustness and profitability, particularly in volatile or specialized markets. In contrast, simpler approaches may yield strong results in certain historical periods but often face difficulties in maintaining consistent performance in evolving market conditions. These findings underscore the importance of methodological adaptability and parameter calibration when implementing cointegration strategies in practice.

5.3 Stochastic Control Methods

This section provides a comprehensive review of studies applying stochastic control techniques to pairs trading. Table 13 summarizes the key literature from 2016 to 2023, detailing the asset classes examined, sample periods, and data frequencies. The compilation covers a wide range of markets, including equities, ETFs, commodity futures, and cryptocurrencies, across both developed and emerging markets. Data frequencies vary substantially, from high-frequency intervals of one minute and five minutes to daily observations, reflecting the adaptability of stochastic control frameworks to different market microstructures.

The table highlights several notable patterns. First, while early studies (2016–2018) predominantly focus on daily equity and ETF datasets—particularly U.S. and Japanese stocks—later research increasingly incorporates high-frequency data, especially in the context of intraday trading for S&P 500 constituents and cryptocurrencies. Second, the scope of assets has expanded beyond equities to include commodity futures (e.g., energy and precious metals) and credit-related instruments, suggesting a diversification of stochastic control applications. Third, recent years (2021–2023) show greater attention to non-U.S. markets, including Chinese A-shares and energy futures, indicating the growing relevance of stochastic control methods in emerging market contexts.

Table 13. A summary of stochastic control methods in pair trading from year 2016 to 2023.

Articles	Publish Year	Sample	Data Frequency
1	2016	Direxion ETF, 2011-2015	daily
3	2016	Commodity Futures, 1997-2015	daily
5	2016	-	-
7	2016	-	-
12	2016	U.S. stocks, 2007-2014	daily
18	2017	U.S. Oil Company stocks, June 2013-April 2015 July 2007-December 2008	5-min
20	2017	Cryptocurrency in Bitstamp, BTC-e, itBit, January 2014-June 2016	1-min
31	2018	-	-
32	2018	-	-
33	2018	-	-
39	2018	Nikkei 225 stocks, 2012-2016	daily
40	2018	-	-

Articles	Publish Year	Sample	Data Frequency
41	2018	82 Stock Pairs, 2010-2015	daily
45	2018	U.S. Financial Sector stocks, 2008-2013	daily
46	2018	-	-
47	2018	Nikkei 225 stocks, 2011-2016	daily
49	2018	S&P 500 stocks, 1998-2015	1-min
54	2019	S&P 500 stocks, 1998-2015	1-min
59	2019	S&P 500 stocks, 1998-2015	1-min
64	2019	-	-
65	2019	U.S. stocks, 2012-2017	daily
68	2020	-	-
75	2020	-	-
76	2020	-	-
81	2021	-	-
84	2021	-	-
88	2021	-	-
91	2021	-	-
92	2021	Chinese stocks, 2012-2016 Futures au1612 and au1702, February 2016-August 2016	daily
94	2021	-	-
100	2022	6 Stock Pairs, 2014-2015	daily
103	2022	-	-
104	2022	Chinese Energy Futures, January 2020-November 2021	1-min
105	2022	-	-
106	2022	-	-
117	2023	-	-
118	2023	-	-
119	2023	SSE and SZSE stocks, 2019-2022	daily
120	2023	405 Cryptocurrencies, January 2022-March 2022	1-min, 5-min, hourly
122	2023	-	-

5.3.1 Structure of Ornstein-Uhlenbeck Process Application

In the context of pairs trading, stochastic control methods provide a rigorous framework for constructing and optimizing trading strategies under uncertainty. Among these, the OU process represents one of the most widely adopted models for mean-reverting strategies. By modeling the spread between two assets as a continuous-time mean-reverting stochastic process, the OU framework offers an analytically tractable basis for predicting the tendency of the spread to revert toward its long-term equilibrium following deviations. This property aligns closely with the core principle of pairs trading, which seeks to exploit temporary mispricings between economically related assets to achieve market-neutral arbitrage.

As indicated in Table 2, more than half of the 40 reviewed studies employing stochastic control methods between 2016 and 2023 incorporate some form of the OU process. Jurek and Yang (2007) remain a foundational reference in this domain, presenting one of the earliest and most influential treatments of the finite-horizon optimal control problem in pairs trading based on an OU model.

Application of the OU Process: Insights from Jurek and Yang

Jurek and Yang (2007) made a significant contribution to the field of arbitrage trading by modeling the price differential between two similar assets—commonly referred to as a spread—using a mean-reverting OU process. This modeling choice is particularly well suited for pairs trading strategies, which are based on the premise that economically linked assets will exhibit a stable long-term relationship. However, short-term market forces can drive temporary deviations in their prices, creating arbitrage opportunities.

The OU process is effective for capturing the dynamics of such spreads, as it models the tendency of the price differential to revert to a long-term average over time. This mean-reversion property aligns directly with the principles of pairs trading, where temporary price deviations are expected to self-correct. The mathematical formulation employed by Jurek and Yang specifies the spread S_t as evolving according to the following stochastic differential equation (SDE):

$$dS_t = \theta(\mu - S_t)dt + \sigma dW_t \tag{5.9}$$

Here, S_t represents the spread at time t, μ is the long-term mean to which the spread reverts, and θ denotes the speed of mean reversion, with larger values indicating faster adjustment. The parameter σ measures the volatility of the spread, introducing randomness through the Wiener process dW_t. This OU framework reflects the expectation that, although the spread may fluctuate due to market factors, it will eventually return to its historical mean. Such dynamics make the OU process a natural and effective tool for arbitrageurs seeking to exploit temporary mispricings.

Addressing Horizon Risk and Divergence Risk

In their model, Jurek and Yang identify two critical risks encountered by arbitrageurs: horizon risk and divergence risk.

Horizon risk refers to the uncertainty over whether the spread will revert to its mean within the arbitrageur's limited investment horizon. Institutional traders, such as fund managers, typically operate under fixed timelines—such as a fiscal quarter or year—and thus cannot wait indefinitely for convergence to occur. If the spread fails to revert within this period, positions may need to be closed at a loss. This risk is particularly pronounced in finite-horizon models, where trade profitability depends heavily on timing.

Divergence risk, in contrast, concerns the possibility that the spread may widen further before ultimately converging. Although the OU process ensures mean reversion in the long run, it offers no guarantee against substantial short-term divergence. This risk is often quantified by the variance of the running maximum of the spread, which measures how far it can move away from the mean before reversing. Traders must incorporate this consideration into their strategies to avoid significant interim losses that could erode capital before convergence is realized.

Development of the Optimal Dynamic Strategy

Jurek and Yang's principal contribution lies in formulating an optimal dynamic strategy for arbitrage trading based on the OU process. This strategy continuously adjusts the allocation of capital between the mispriced assets and a risk-free asset, taking into account both the current level of the spread and the time remaining until the trading horizon ends.

A key insight from their approach is the concept of time-varying allocation. As the investment horizon shortens, the strategy adopts a more conservative stance. With less time available for the spread to revert to its mean, the likelihood of horizon-related losses increases, prompting a gradual reduction in the optimal allocation to the mispricing as the end date approaches.

The strategy is also highly responsive to the magnitude of the spread. When the spread deviates markedly from its mean, it may recommend a larger position to take advantage of the anticipated mean reversion. However, if the divergence becomes excessively large, the strategy advises reducing the position to mitigate the elevated risk of further divergence. This reflects sound risk management principles, in which potential gains are carefully weighed against the increased likelihood of interim losses.

Moreover, Jurek and Yang incorporate intertemporal hedging demands to account for uncertainties in future market conditions. Unlike simpler pairs trading rules that focus solely on exploiting current mean-reversion opportunities, their model adjusts allocations to manage the risk of future divergence. This forward-looking component ensures that the strategy not only captures present mispricings but also remains resilient to potential shifts in the spread's dynamics over time.

Hedging Demands and Position Adjustments

A distinctive feature of Jurek and Yang's framework is the decomposition of the optimal position into two components: the myopic demand and the intertemporal hedging demand. The myopic demand represents the immediate response to the current mispricing, emphasizing the exploitation of present arbitrage opportunities by increasing position size when the spread deviates significantly from its long-term mean. In contrast, the intertemporal hedging demand is a forward-looking adjustment that incorporates the risks associated with future changes in the spread and the time remaining in the investment horizon. For instance, if the spread widens substantially as the horizon approaches, the hedging demand may recommend reducing the position size to limit exposure to further divergence, even if the current opportunity appears profitable. This dual-demand structure enables a more sophisticated approach to arbitrage trading, allowing position sizes to be calibrated not only to current market conditions but also to anticipated future risks.

Comparison with Threshold-Based Rules

To assess the effectiveness of their dynamic strategy, Jurek and Yang compare it with a simpler threshold-based rule, a common approach in pairs trading. The threshold-based rule entails opening a position when the spread deviates from its mean by more than a predetermined number of standard deviations and closing the position once it reverts. Their findings show that the OU-based strategy offers several notable advantages over this static approach. First, it enables dynamic positioning by continuously adjusting the position size in response to changes in the risk–return profile, making it more adaptable to evolving market conditions than fixed threshold triggers. Second, it explicitly incorporates key risks—such as divergence risk and horizon risk—that are typically ignored in threshold-based strategies. By accounting for these risks, the OU-based framework achieves superior risk-adjusted returns, effectively managing downside exposure while still capturing profitable opportunities.

Empirical Testing and Results

Jurek and Yang evaluate their strategy using empirical data, notably the well-known Royal Dutch and Shell shares (Siamese twin shares). The results indicate that the dynamic OU-based strategy consistently outperforms the threshold rule in terms of risk-adjusted returns, particularly when the spread exhibits strong mean-reversion.

For example, the Sharpe ratio—a measure of risk-adjusted performance—improves markedly under the optimal strategy. In the case of Royal Dutch–Shell pairs, the Sharpe ratio ranges from 0.50 to 0.61, depending on the specific calibration of risk aversion. These findings highlight the strategy's ability to achieve superior performance by effectively balancing risk and return, especially in markets characterized by pronounced mean-reversion.

Limitations and Future Research Directions

Although Jurek and Yang's model offers notable advantages, it is not without limitations. A key drawback is its exclusion of transaction costs. The model assumes a frictionless market, enabling continuous rebalancing; however, in practice, frequent trading would generate substantial transaction costs, which could erode profitability. In contrast, simpler threshold-based rules, which involve fewer trades, may prove more cost-effective in real-world settings.

Moreover, the model could be further enhanced by testing its robustness across larger datasets and diverse market environments, including periods of heightened volatility or financial crises. Such extensions would provide deeper insights into the strategy's performance under varying conditions and could reveal areas where adjustments to the framework might be necessary.

The framework itself builds upon and extends prior research, notably the work of Boguslavsky and Boguslavskaya (2004), who developed an optimal investment strategy for a single risky asset following an OU process under power utility, and Boguslavsky and Boguslavskaya (2004), who also addressed a stochastic control problem for pairs trading using the OU process. By focusing on non-myopic arbitrageurs, incorporating intertemporal hedging demands, and adopting a dynamic portfolio allocation approach, Jurek and Yang's contribution provides substantial value for understanding optimal trading strategies in real-world financial markets.

5.3.2 Empirical Research based on Ornstein-Uhlenbeck Process

Building on the theoretical framework of the OU process discussed earlier, a number of empirical studies have explored its applications in various financial markets, often enhancing the model to address specific market characteristics. These works demonstrate how incorporating features such as non-Gaussian innovations, regime-switching dynamics, jump-diffusion components, or stochastic volatility can improve the realism and profitability of OU-based pairs trading strategies.

Göncü and Akyildirim (2016) extend the traditional OU process by introducing a Lévy process with generalized hyperbolic (GHYP) distributed marginals to model the spread between commodity pairs. Using daily data from January 2007 to December 2014 across multiple commodity futures markets, their model captures empirical features of commodity spreads—such as pronounced peaks and fat tails—that are often observed in real-world data but are poorly represented by Gaussian models. The Lévy process accounts for the non-Gaussian nature of the spreads, while the GHYP distribution captures the skewness and kurtosis inherent in the data. The authors derive optimal trading thresholds by maximizing expected profits from spread positions, and their strategy, applied to pairs such as crude oil and natural gas, yields an average annual return of 15.3%, significantly outperforming traditional Gaussian-based models. This advancement results in a more accurate representation of market behavior and a more robust pairs trading strategy in commodity futures markets.

Liu et al. (2017) adopt a novel approach by applying a doubly mean-reverting process to model spreads between stock pairs, using high-frequency intraday data from the NYSE and NASDAQ between January 2015 and December 2016. Their model combines two mean-reverting processes to capture both the long-term trend and short-term fluctuations of the spread: the long-term trend follows an OU process, while short-term deviations are modeled conditionally. This structure enables the strategy to exploit temporary market inefficiencies that may be missed by daily-data-based models, which typically assume static relationships between asset pairs. By leveraging high-frequency data, the model dynamically adjusts positions in response to rapidly changing market conditions. Applied to technology and financial sector stocks, the strategy achieves an average annualized return of 18.2%, outperforming standard daily-frequency pairs trading models and demonstrating the potential of exploiting short-term mispricings in high-frequency trading environments.

Suzuki (2018) introduces an OU-based model with a regime-switching mechanism that allows dynamic transitions between long, short, and neutral (square) positions. This extends the traditional binary long/short OU framework by allowing traders to remain neutral during periods of uncertainty. Using real-world data from "stub" pairs—such as parent and subsidiary companies—the optimal switching points between regimes are determined through Monte Carlo simulations. Empirical results show that this approach achieves significantly higher Sharpe ratios (around 0.69 on average) than classical two-sigma strategies, even after accounting for transaction costs. This added flexibility improves risk-adjusted returns and enhances the model's robustness under complex market conditions.

Luo et al. (2023) propose an enhanced OU process incorporating both jump-diffusion dynamics and regime-switching to model spreads between Chinese energy futures contracts. Using minute-level data from January 2, 2020, to November 30, 2021, across five major contracts (fuel oil, thermal coal, coke, crude oil, and coking coal), their model captures the mean-reverting nature of spreads while accounting for large, sudden price jumps. The study compares several pairs trading strategies, including minimum distance and classical cointegration, against their advanced OU models. Results show that the three-regime-switching OU model (3RS-OUM) achieves an annualized return of 167.11%, and the two-regime-switching OU model (2RS-OUM) achieves 101.66%, both far exceeding the performance of traditional approaches. This highlights the value of incorporating jump-diffusion and regime-switching when operating in volatile high-frequency markets.

Zhang and Xiong (2023) further extend the OU process by integrating a fast-mean-reverting stochastic volatility model, specifically the Scott model, to better reflect the dynamic nature of volatility in pairs trading. Here, volatility itself follows its own OU process, addressing the limitations of constant or deterministic volatility assumptions. Using data from the Chinese stock market—particularly in clean energy and coal energy sectors—the authors find that their enhanced strategy outperforms constant-volatility models, achieving a 57% win rate, an average profit of

0.14% per trade, and a Sharpe ratio of 3.58 in out-of-sample testing. These results demonstrate the effectiveness of incorporating stochastic volatility into OU-based strategies.

Overall, these empirical studies illustrate the adaptability of the OU process framework. By integrating features such as non-Gaussian innovations, high-frequency adjustments, regime-switching, jump-diffusion, and stochastic volatility, researchers have significantly improved the robustness and profitability of pairs trading strategies across diverse asset classes and market conditions. These findings reinforce the relevance of the OU process as a foundational tool while highlighting the importance of tailoring its structure to capture the specific statistical and dynamic properties of the targeted spreads.

5.3.3 Theoretical Research based on Ornstein-Uhlenbeck Process

As shown in Table 12, 22 out of the 40 papers are theoretical studies, accounting for more than half of the total. This highlights the importance of reviewing these works in detail, as they form the conceptual foundation for many empirical applications discussed in the previous section. The following studies represent key theoretical developments in pairs trading models based on the OU process, each extending the framework to incorporate more realistic market features and rigorous optimization techniques.

Ngo and Pham (2016) extend the traditional pairs trading model by formulating the problem as an optimal switching problem with three distinct regimes: a flat position (no holdings), a long position (buying the underpriced asset and selling the overpriced one), and a short position (selling the underpriced asset and buying the overpriced one). The spread between the two cointegrated assets is modeled as an OU process, with transactions triggered when the spread deviates sufficiently from its long-term mean. The authors use a viscosity solutions approach to determine the optimal switching boundaries between regimes, ensuring smooth-fit conditions for the value functions. This approach offers a mathematically rigorous method for deriving trading rules, improving upon traditional threshold-based strategies that rely primarily on empirical calibration.

Bai and Wu (2018) model the spread between two highly correlated stocks using a regime-switching mechanism within the OU framework. Specifically, they adopt a Markov-modulated Ornstein–Uhlenbeck (MMOU) process, in which key parameters—including the mean reversion rate, long-term mean, and volatility—change dynamically according to regimes determined by an underlying Markov chain. The authors derive closed-form solutions to a double boundary stopping time problem, enabling the optimization of entry and exit points for pairs trades under varying market conditions. This extension allows the mean-reversion dynamics to adapt across different market states, enhancing the flexibility and responsiveness of OU-based strategies to real-world fluctuations.

Holý and Černý (2022) adopt an OU process to model the spread between two cointegrated assets, building on Bertram's original framework for pairs trading. They enhance the strategy by incorporating a risk-bounded constraint, which limits the variance of profit per time unit, thus addressing both regulatory requirements and practical risk management considerations. This transforms the optimization problem from an unconstrained maximization of expected profit into one that balances profitability with explicit risk limits. The study also examines the impact of parameter misspecification, quantifying the losses that can arise when the OU parameters—such as mean reversion speed and volatility—are inaccurately estimated.

Xie et al. (2023) also follow Bertram's approach, assuming the spread follows a stationary Gaussian–Markov process in continuous time with mean-reverting behavior. Their key contribution lies in extending the framework to include constraints on the volatility of the expected profit per time unit, ensuring the strategy remains within predefined risk boundaries. They provide solutions to the resulting optimization problem, even in non-convex cases, and analyze the consequences of parameter estimation errors based on finite samples. This approach ensures that the strategy not only maximizes expected returns but also maintains compliance with specified risk thresholds, making it more applicable to real-world trading environments.

In summary, these theoretical studies advance the OU process framework by incorporating optimal switching, regime-dependent dynamics, and explicit risk constraints. They also address practical issues such as parameter estimation errors, making the resulting strategies more robust and applicable to real trading scenarios. Collectively, they bridge the gap between idealized mean-reversion models and the complexities of real-world financial markets, providing a solid theoretical foundation for empirical implementations.

5.4 Time Series Methods

This section provides a detailed review of the time series approach to modeling mean reversion in pairs trading, focusing on methodologies that do not rely on cointegration. Table 14 summarizes 20 representative studies published between 2016 and 2023, detailing their data samples and observation frequencies. The research covers a wide range of asset classes, including equities from major indices such as the DJIA, KOSPI, Nikkei 225, and CSI 300, sector-specific portfolios such as private banks and artificial intelligence stocks, as well as commodities, ETFs, foreign exchange, and cryptocurrencies. This diversity highlights the versatility of time series methods in adapting to various market environments and asset characteristics.

A notable feature of these studies is the variation in data frequency, ranging from low-frequency weekly commodity futures data to ultra-high-frequency 1-minute cryptocurrency price series. While the majority of studies rely on daily observations, the inclusion of high-frequency datasets—particularly in the post-2019 period—reflects the growing interest in exploiting intraday mean-reversion patterns, especially in cryptocurrency and futures markets. The time spans of the datasets also differ significantly, from relatively short windows of less than one year to multi-year samples exceeding a decade, suggesting that time series-based pairs trading models are applied in both short-term tactical strategies and long-term structural analyses. These patterns provide valuable context for understanding how methodological choices, data characteristics, and market conditions influence the design and performance of time series-based mean reversion strategies.

Table 14. A summary of time series methods in pair trading from year 2016 to 2023.

Articles	Publish Year	Sample	Data Frequency
2	2016	XOM and LUV, September 2011-March 2013 VALE5 and BRAP4, August 2011-April 2013	daily
23	2017	28 stocks in DJIA, 8 stocks in NYSE and NASDAQ, 2006-2014	daily
29	2017	KOSPI 100 stocks, 2005-2015	daily
36	2018	Private Banks Sector stocks, 2006-2016	daily
39	2018	Nikkei 225 stocks, 2012-2016	daily
47	2018	Nikkei 225 stocks, 2011-2016	daily
48	2018	36 stocks in DJIA, NYSE and NASDAQ, 2005-2016	daily
57	2019	AAPL, GOOGL, META, MSFT, MU, 2012-2017	daily
66	2020	EWA, EWC, IGE, 2017-2020	daily
70	2020	ETF, FX, Stocks, 2012-2019	1-min

Articles	Publish Year	Sample	Data Frequency
74	2020	Commodity Futures, 2004-2018	weekly
82	2021	AOS and DUK, 2018-2021	daily
83	2021	EWA, EWC, IGE, 2017-2020	daily
85	2021	Bitcoin, Ethereum, Litecoin and Monero, January 2019-November 2019	daily
90	2021	5 U.S. AI stocks, 2016-2019	daily
95	2021	U.S. Banks in NYSE, 2012-2019	daily
99	2022	BTC-USD, ETH-USD, 2021-2022	hourly
108	2022	CSI 300 Stocks, 2008-2019	5-min
111	2023	U.S. stocks, January 2021-December 2022	daily
120	2023	405 Cryptocurrencies, January 2022-March 2022	1-min, 5-mins, hourly

5.4.1 Use of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model

Chen et al. (2017) employ a Smooth Transition GARCH (ST-GARCH) model to enhance pairs trading performance. The ST-GARCH framework, incorporating a second-order logistic transition function, is used to model nonlinear shifts in conditional volatility. By forecasting risk measures such as Value-at-Risk (VaR), the model generates entry and exit signals and adjusts position sizing in response to changing volatility regimes. This approach enables a more adaptive risk management process compared with standard GARCH models. The authors conduct an empirical evaluation to demonstrate that the ST-GARCH-based strategy improves the timing of trades and enhances overall profitability relative to conventional threshold-based rules.

Model Selection and Specification

The paper employs a ST-GARCH model to capture conditional heteroskedasticity in financial time series. The model incorporates a second-order logistic transition function, allowing for two smooth thresholds that govern shifts between volatility regimes. This structure is particularly suited for financial data, as it can flexibly model nonlinear dynamics in the conditional variance.

The ST-GARCH specification accommodates several empirical characteristics commonly observed in asset returns. First, it captures volatility clustering, whereby periods of high (low) volatility tend to persist. Second, it accounts for asymmetric responses to positive and negative shocks, reflecting the non-symmetric nature of market reactions. Third, the model is estimated under a Student’s t-distribution for innovations, thereby accommodating fat-tailed return distributions in which extreme movements occur more frequently than under the normality assumption. While mean reversion in the spread is primarily governed by the mean equation, the ST-GARCH component allows volatility to adjust dynamically around this equilibrium, improving the robustness of risk forecasts such as VaR.

The model is specified as follows:

$$y_t = \mu_t^{(1)} + F(z_{t-d}; \gamma, c_1, c_2)\mu_t^{(2)} + a_t \tag{5.10}$$

where $y_t$denotes the demeaned return spread between the two assets in a pair, $\mu_t^{(1)}$ and $\mu_t^{(2)}$ are the conditional means in the two regimes, and a_t is the innovation term:

$$a_t = \sqrt{h_t} \cdot \varepsilon_t, \varepsilon_t \sim i.i.d.\ t^*(\nu) \tag{5.11}$$

Here, h_t is the conditional variance and ε_t follows a standardized Student's t-distribution with ν degrees of freedom. The conditional variance evolves according to:

$$h_t = h_t^{(1)} + F(z_{t-d}; \gamma, c_1, c_2)h_t^{(2)} \tag{5.12}$$

where $h_t^{(1)}$ and $h_t^{(2)}$ are the conditional variances under different regimes.

The transition function $F(z_{t-d}; \gamma, c_1, c_2)$ is specified as:

$$F(z_{t-d}; \gamma, c_1, c_2) = \frac{1}{1+\exp\{\frac{-\gamma(z_{t-d}-c_1)(z_{t-d}-c_2)}{s_z}\}} \tag{5.13}$$

where $c_1 < c_2$, c_1 and c_2 are the transition thresholds, γ controls the smoothness of regime changes, and s_z is the sample standard deviation of z_{t-d}.

Data Preparation and Normalization

To identify stock pairs with similar price dynamics, the Minimum Squared Distance (MSD) method is applied to standardized price series. Standardization removes the effects of differing price scales and volatilities, ensuring that the MSD reflects only the similarity in relative movements rather than differences in absolute price levels.

The standardized price of asset j at time t is computed as:

$$p_t^j = \frac{P_t^j - \bar{P}^j}{\sigma_j} \tag{5.14}$$

where P_t^j denotes the closing price of asset j at time t, $\bar{P}^j$ is the sample mean of the price series, and σ_j is the sample standard deviation. This transformation produces a unit-free series with zero mean and unit variance, allowing direct comparison of price trajectories across assets. The MSD is then calculated between standardized series to select pairs with the smallest average squared deviations, indicating the highest degree of co-movement.

Parameter Estimation Method

The parameters of the ST-GARCH model are estimated within a Bayesian framework using Markov Chain Monte Carlo (MCMC) simulation. This approach allows the incorporation of prior information and yields full posterior distributions for the parameters, providing a richer characterization of estimation uncertainty compared with point estimators.

Posterior sampling is conducted using algorithms from the Metropolis–Hastings family, with block sampling applied to partition parameters into groups with high intra-group correlation. This design improves sampling efficiency and accelerates convergence by reducing cross-parameter dependencies.

For each trading pair, a total of 30,000 MCMC iterations are performed. The initial 10,000 draws are discarded as a burn-in period to allow the Markov chain to converge toward its stationary distribution, thereby mitigating the influence of initial parameter values. The remaining 20,000 draws are retained for inference and model-based decision-making.

Generating Trading Signals

The paper proposes two methods for generating trading signals:

1. Threshold Method – In this approach, the upper and lower trading thresholds are estimated from the ST-GARCH model using Bayesian inference. When the observed return spread exceeds the estimated upper threshold, the strategy enters a short position in stock A and a long position in stock B. Conversely, when the spread falls below the lower threshold, a long position is taken in stock A and a short position in stock B.
2. Quantile Forecasting Method – This method relies on one-step-ahead quantile forecasts of the return spread derived from the ST-GARCH model. For example, the 20% and

80% quantiles are used as dynamic decision boundaries. A trading position is initiated when the forecasted return spread lies outside the specified quantile interval, allowing the strategy to adapt to time-varying volatility conditions.

Model Validation and Empirical Analysis

To validate the effectiveness of the MCMC sampling scheme, the paper conducts both a simulation study and an empirical analysis. In the simulation study, the accuracy and stability of parameter estimation are evaluated under various sample sizes and initial conditions, with the results confirming the robustness of the proposed MCMC approach. In the empirical analysis, two six-month out-of-sample periods in 2014, as well as the entire year, are examined. Annualized returns and profits are calculated with and without accounting for transaction costs. The findings indicate that the proposed ST-GARCH model-based methods are capable of effectively capturing market arbitrage opportunities, delivering annualized returns of at least 35.5% in the absence of transaction costs and 18.4% when transaction costs are included.

Chodchuangnirun et al. (2018) apply nonlinear autoregressive GARCH models to analyze and construct a pairs trading strategy, specifically employing the Kink-AR-GARCH, Threshold-AR-GARCH, and Markov Switching AR-GARCH (MS-AR-GARCH) models. These nonlinear specifications, incorporating GARCH effects, are designed to capture key features of financial time series such as volatility clustering, asymmetry in conditional mean and variance, and fat-tailed return distributions. By modeling the dynamic behavior of return spreads under varying market regimes, the models enhance the ability to optimize trading strategies. Model parameters are estimated using the Maximum Likelihood Estimation (MLE) method, and the optimal model is selected based on the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Trading signals are generated by defining upper and lower thresholds for return spreads predicted by the models. When the observed spread exceeds the upper threshold, a sell–buy operation is initiated, while a buy–sell operation is triggered when the spread falls below the lower threshold. Empirical results indicate that the MS-AR-GARCH model outperforms the other two in generating trading signals, as it more effectively captures structural changes and volatility regime shifts, thereby delivering superior returns compared to traditional trading rules.

Lin et al. (2021) employ the GARCH model to develop a multi-asset pairs trading strategy that optimizes trading signals through volatility forecasting. One-step-ahead volatility forecasts are generated for asset pairs, serving as the basis for identifying optimal entry and exit points according to expected movements in return spreads under different market conditions. These forecasts are combined with semi-parametric tolerance limits to refine the trading signal generation process, adjusting the spreads to define dynamic upper and lower bounds that guide trade execution. The strategy calls for selling one asset and buying the other when the adjusted spread exceeds the upper bound, and reversing the trade when it falls below the lower bound. A rolling-window framework is adopted for model training and testing, with performance assessed at the end of each testing period. Empirical findings show that this integrated approach effectively captures evolving market conditions, produces consistently positive returns across multiple periods, and offers a more precise and adaptable framework for multi-asset pairs trading.

5.4.2 Applying Ornstein-Uhlenbeck Process

Before proceeding with the analysis in this section, it is important to distinguish between the applications of the OU process in stochastic control frameworks and in time series modeling.

In stochastic control, the OU process is used to describe the dynamics of systems influenced by both deterministic trends and stochastic disturbances. The primary objective is to formulate and solve an optimization problem—typically to minimize a cost function or maximize a performance criterion—while continuously adjusting the control variables over time. This setting explicitly incorporates the randomness of the process and the impact of control actions. For instance, in portfolio optimization, the OU process can model asset prices or interest rates with mean-reverting behavior, enabling the derivation of optimal asset allocations that balance return and risk. In such cases, the OU process is embedded in a SDE framework, where the system's evolution is analyzed under active control.

By contrast, time series modeling applies the OU process primarily as a statistical tool to capture the mean-reverting property of a single stochastic variable over time. This approach is common in financial econometrics for modeling variables such as stock prices, exchange rates, or interest rate spreads that fluctuate around a long-term equilibrium. The focus is on estimating model parameters from historical data, assessing goodness-of-fit, and using the fitted model for forecasting or scenario analysis. Unlike stochastic control, there is no optimization over control variables; the goal is to understand and predict the intrinsic mean-reversion dynamics based solely on the observed time series.

The following examples illustrate these differences in application.

Stochastic Control Methods: Optimal Portfolio Management

Consider a fund manager aiming to optimize a portfolio that includes a stock whose price exhibits mean-reverting behavior. The stock price $S(t)$ is modeled using an OU process to capture its tendency to revert toward a long-term mean μ. The objective is to determine, at each time t, the optimal quantity of the stock to hold in order to maximize expected portfolio returns while controlling risk over a fixed investment horizon. This constitutes a stochastic control problem in which the holdings are continuously adjusted in response to observed prices and volatility.

The OU process for the stock price dynamics can be expressed as:

$$dS(t) = \theta(\mu - S(t))\, dt + \sigma\, dW(t) \tag{5.15}$$

where θ is the mean reversion speed, σ is the volatility, and $W(t)$ denotes a standard Wiener process.

Given this model, the optimal trading strategy $\mathrm{\pi(t)}$ is derived by solving the control problem with respect to a specified utility function, such as a mean–variance criterion or constant absolute risk aversion (CARA) utility. The strategy explicitly incorporates the mean-reverting dynamics of $S(t)$, enabling the manager to anticipate price reversion and adjust holdings dynamically to balance expected return against risk.

Time Series Modeling: Forecasting Interest Rate Movements

Consider a financial analyst aiming to forecast future interest rates for purposes such as bond pricing or interest rate risk management. Interest rates often display mean-reverting behavior, fluctuating around a long-term equilibrium level due to macroeconomic forces and monetary policy interventions. In this context, the short-term interest rate $r(t)$ can be modeled using an OU process, which in the continuous-time form is equivalent to the Vasicek model:

$$dr(t) = \alpha(\beta - r(t))\, dt + \sigma\, dW(t) \tag{5.16}$$

where $\alpha > 0$ denotes the speed of mean reversion (a higher value implies faster reversion toward the mean), β is the long-term equilibrium level of the interest rate, $\sigma > 0$ represents the instantaneous volatility, and $W(t)$ is a standard Wiener process.

In this setting, the analyst's primary task is to estimate α, β, and σ from historical interest rate data. Estimation methods may include MLE, generalized method of moments (GMM), or ordinary least squares (OLS) applied to a discretized version of the process. Once calibrated, the fitted model can be used to produce probabilistic forecasts of future interest rates, evaluate portfolio VaR, or assess potential outcomes under different macroeconomic scenarios. The OU process framework captures the tendency of interest rates to revert toward a central value while allowing for random fluctuations around it.

Thus, unlike the stochastic control example in the previous section, where the OU process is embedded within an optimization problem involving continuous portfolio rebalancing decisions, time series modeling focuses solely on characterizing and predicting the intrinsic mean-reverting behavior of the observed variable. The former integrates control actions to influence the system's evolution in real time, whereas the latter emphasizes statistical inference and predictive accuracy based on past observations.

Lee and Leung (2020) employ the OU process to construct and optimize a pairs trading strategy by modeling the mean-reverting behavior of asset pairs' return spreads. These spreads typically revert to a long-term average after short-term fluctuations. In their framework, the value of each asset pair portfolio is fitted to an OU process, enabling the capture of the mean-reversion characteristics commonly observed in financial markets. The model parameters—including the speed of mean reversion, long-term mean, and volatility—are estimated via MLE using historical price data for each asset pair. This parameterization allows the OU model to accurately describe the dynamics of portfolio values over time.

Within this OU-based framework, the authors develop an optimized exit rule. Specifically, they analyze the effect of liquidating a position when the portfolio value deviates from its mean by a multiple of the standard deviation, and derive an analytical expression for the optimal exit point that maximizes expected profitability. This optimized rule is then compared to a conventional mean-reversion exit strategy. Empirical tests using eight asset pairs—including stocks, ETFs, currencies, and futures—demonstrate that the optimized rule substantially increases annualized returns while reducing trade frequency, thereby lowering transaction costs. As the OU process is a standard model in time series analysis, this work primarily falls within the time series methods category, relying on historical data to estimate model parameters and design an optimal exit strategy.

Xiang et al. (2023) extend the classic OU process by adopting the fractional Ornstein–Uhlenbeck (fOU) process to capture the long-range dependence and anti-persistence observed in asset price spreads. The fOU process incorporates fractional Brownian motion, offering a more flexible representation of spread dynamics compared to the traditional OU model. Parameters such as the mean-reversion speed and the fractional order (Hurst exponent) are estimated from historical data, and these estimates are used to adaptively determine optimal trading thresholds. This adaptive mechanism dynamically adjusts trade entry and exit points, triggering trades when the spread deviates from its mean beyond the estimated threshold, thereby improving the responsiveness and profitability of the strategy.

The performance of the fOU-based strategy is evaluated through both simulation and empirical analysis. Simulation results show that the fOU model outperforms the traditional OU model in capturing spread dynamics, delivering higher returns and lower risk across different market scenarios. Empirical tests on real market data further confirm its robustness in generating trading signals and enhancing strategy performance. Although the fOU process builds on concepts from stochastic processes and fractional Brownian motion, its application here is firmly rooted in time series analysis, where historical data are used to model and forecast the mean-reverting behavior of asset price spreads.

5.5 Other Methods

Table 15 summarizes studies that adopt alternative approaches to pairs trading between 2016 and 2023, covering a wide spectrum of asset classes, markets, and data frequencies. The reviewed literature spans traditional equity markets—including U.S. stocks (NYSE, NASDAQ, AMEX), major indices such as the S&P 500, S&P 100, DJIA, and FTSE 100, as well as sector-specific portfolios like energy sector equities—to non-equity assets such as foreign exchange (EUR/USD, EUR/SGD), cryptocurrencies (Bitcoin and altcoins), and electricity spot prices (EPEX SPOT). The temporal coverage varies considerably, with datasets ranging from short-term intraday samples (e.g., 1-minute, 5-minute, and 15-minute intervals in Forex and cryptocurrency markets) to long-term daily series spanning several decades (e.g., 1962–2014 for U.S. stocks).

The methods employed in these studies are diverse, reflecting the exploratory nature of this category. While some research applies statistical or econometric techniques—such as regime-switching models, distributional fitting, and spread-based filters—others integrate more recent computational tools, including clustering algorithms, graph-theoretic approaches, and machine learning-based signal generation. High-frequency studies tend to focus on markets where microstructure effects and rapid price adjustments are prominent, such as Forex and cryptocurrency markets, whereas daily-frequency analyses dominate in equities and other traditional assets. Overall, the studies in Table 15 illustrate the expanding scope of pairs trading beyond classical cointegration or time series frameworks, highlighting both methodological innovation and diversification in targeted asset classes.

Table 15. A summary of other methods in pair trading from year 2016 to 2023.

Articles	Publish Year	Sample	Data Frequency
9	2016	NYSE, AMEX, and NASDAQ stocks, 2003-2012	daily
13	2016	U.S. stocks, 1962-2014	daily
16	2017	-	-
17	2017	U.S. stocks, 1987-2011 UPM and Stora Enso, 1987-2003	daily
19	2017	DJIA stocks, 2000-2015	daily
21	2017	S&P 100 stocks, 1990-2014	daily
38	2018	EUR/USD and EUR/SGD, June 2017-November 2017	1-min
42	2018	-	-
58	2019	U.S. Nasdaq energy sector stocks, 2012-2014	daily
71	2020	FTS 100 stocks, 2010-2019	daily
79	2020	Nasdaq 100 stocks, 1999-2003 2007-2012	daily
80	2021	Nasdaq stocks, 2000-2021	daily
87	2021	38 Forex, September 2017-July 2018	15-min
98	2021	Toronto Stock Exchange, January 2017-June 2020	daily
101	2022	103 NASDAQ 100 stocks, 2000-2021	daily
110	2022	NYSE:LUV, NASDAQ:AAPL, 2015-2020	daily
112	2023	S&P 500 stocks, 1990-2015	daily
113	2023	64 PSX stocks, 2017-2019	daily

Articles	Publish Year	Sample	Data Frequency
116	2023	EPEX SPOT, 2020-2022	15-min
120	2023	405 Cryptocurrencies, January 2022-March 2022	1-min, 5-mins, hourly

5.5.1 Copula Approach

Within the category of alternative methods, the Copula approach offers a flexible and robust framework for modeling the dependence structure between asset returns without relying on linear correlation assumptions. Unlike traditional cointegration or time series techniques, which primarily capture linear relationships, Copula models can describe complex, nonlinear, and tail-dependent relationships that are often observed in financial markets.

Xie et al. (2016) employs the Copula method to enhance pairs trading strategies by more accurately capturing the joint distribution and dependency structure between stock pairs. The following subsection provides a detailed description of the process used in their study.

Motivation for Using Copula

The study begins by noting that traditional pairs trading strategies frequently adopt the so-called "distance method," which measures the normalized price distance between two stocks to detect potential mispricing. This method is effectively analogous to relying on linear correlation to assess the relationship between the assets, which implicitly presumes that their returns follow a joint normal distribution. However, empirical evidence suggests that stock returns rarely conform to joint normality and may exhibit nonlinear dependence structures, including tail dependence. Consequently, conventional approaches risk overlooking critical aspects of the dependence structure, potentially leading to suboptimal or misleading trading signals.

Modeling Joint Distribution with Copula

To more precisely characterize the joint distribution between stock returns, the study adopts a Copula-based approach. According to Sklar's theorem, if two random variables X and Y have marginal distribution functions $F_X(x)$ and $F_Y(y)$, respectively, their joint cumulative distribution function $H(x, y)$ can be expressed as:

$$H(x, y) = C(F_X(x), F_Y(y)) \tag{5.17}$$

where C denotes a Copula function that fully captures the dependence structure between the variables. This framework allows the modeling of the joint distribution independently of the marginal distributions, thereby accommodating non-normal and potentially heavy-tailed or skewed marginals frequently observed in financial returns.

Constructing Mispricing Measures

To quantify the relative valuation between two stocks, X and Y, the paper defines two mispricing indices, $|MI_t^{X|Y}|$ and $|MI_t^{Y|X}|$, which measure the degree of mispricing between the two assets at time t. Let R_t^X and R_t^Y denote the daily returns of stocks X and Y, with marginal distribution functions F_X and F_Y, respectively. According to Sklar's theorem, their joint distribution function H can be expressed as:

$$H(r_t^X, r_t^Y) = C(F_X(r_t^X), F_Y(r_t^Y)) \tag{5.18}$$

where C denotes the copula function that captures the dependency structure between R_t^X and R_t^Y.

The mispricing indexes are defined in terms of conditional probabilities:

$$MI_t^{X|Y} = P(R_t^X < r_t^X | R_t^Y = r_t^Y), MI_t^{Y|X} = P(R_t^Y < r_t^Y | R_t^X = r_t^X) \tag{5.19}$$

By the properties of copulas, these conditional probabilities can be expressed using the partial derivatives of C:

$$MI_t^{X|Y} = \frac{\partial C(u,v)}{\partial v}, MI_t^{Y|X} = \frac{\partial C(u,v)}{\partial u} \tag{5.20}$$

where $u = F_X(r_t^X)$ and $v = F_Y(r_t^Y)$.

The indices $|MI_t^{X|Y}|$ and $|MI_t^{Y|X}|$ take values in the range $[0,1]$. A value close to 0.5 indicates that the two assets are relatively fairly valued, given their historical joint distribution. Values above 0.5 suggest that the asset in the numerator is relatively overvalued with respect to the other asset, while values below 0.5 indicate relative undervaluation.

Strategy Construction and Trading Signal Generation

The trading strategy consists of two phases: the formation period and the trading period. During the formation period, the daily return series of the candidate stocks are calculated, and their marginal distributions are estimated. Different types of Copulas (e.g., Gumbel, Frank, Clayton, normal, and Student's t Copulas) are fitted to the joint distribution, with the Copula that has the highest likelihood being chosen as the final model.

In the trading period, the Copula obtained from the formation period is used to compute the daily mispricing indexes $|MI_t^{X|Y}|$ and $|MI_t^{Y|X}|$. Trading rules are based on two key parameters: the deviation threshold D (set to 0.6) and the stop-loss parameter S (set to 2). The deviation threshold D determines the minimum level of cumulative relative mispricing needed to trigger a trade, while the stop-loss parameter S acts as a risk control mechanism, forcing the closure of positions when the accumulated deviation moves unfavorably beyond a certain magnitude. Economically, S limits downside risk by ensuring that adverse price movements do not escalate into large losses, particularly during abnormal market conditions where the mean-reversion assumption may temporarily fail.

Two auxiliary variables, FlagX and FlagY, are introduced to accumulate the relative mispricing indexes of the two stocks over time, thereby capturing persistent deviations rather than reacting to transient noise. Specifically, FlagX tracks the cumulative deviation of stock X, defined as $|MI_t^{X|Y} - 0.5|$, while FlagY tracks the cumulative deviation of stock Y, defined as $|MI_t^{Y|X} - 0.5|$. At the beginning of the trading period, both flags are initialized to zero. Each day, the value of $|MI_t^{X|Y} - 0.5|$ is added to FlagX, and the value of $|MI_t^{Y|X} - 0.5|$ is added to FlagY.

A trading signal is triggered when either flag reaches D or falls below $-D$. If FlagX reaches D, stock X is deemed overvalued relative to stock Y, prompting a short-sell of X and a purchase of Y; if FlagX reaches $-D$, stock X is considered undervalued, leading to a purchase of X and a short-sell of Y. Similarly, if FlagY reaches D, stock Y is considered overvalued relative to stock X, triggering a short-sell of Y and a purchase of X; if FlagY falls to $-D$, stock Y is considered undervalued, prompting a purchase of Y and a short-sell of X.

Finally, when the absolute value of either FlagX or FlagY reaches S, the position is forcibly closed to limit potential losses. This symmetrical decision framework ensures that both overvaluation and undervaluation scenarios are treated consistently, while the combined use of D and S balances return-seeking behavior with risk management.

Empirical Analysis and Return Data

The effectiveness of the Copula-based method is evaluated through an empirical analysis involving different stock pairs, such as Brookdale Senior Living Inc. and Emeritus Corporation, as well as a large sample of utility sector stocks. The results demonstrate that the Copula strategy substantially outperforms the traditional distance method, delivering higher excess returns and reducing the frequency of negative returns.

When the Copula method is applied without the "one-day waiting" strategy—that is, trades are executed immediately on the day of price divergence—an initial investment of USD 10,000 generates a profit of USD 847. In contrast, the same setting under the traditional distance method results in a loss of USD 592. When incorporating the "one-day waiting" rule, the performance gap widens further: the Copula method yields a profit of USD 1,060, whereas the distance method produces a loss of USD 1,526.

A broader sample analysis, conducted on utility sector stocks, reinforces these findings. Across different subsets of stock pairs—including the top 5, top 20, and those ranked 101–120—the Copula-based strategy consistently achieves higher annualized excess returns than the distance method. This performance advantage is particularly pronounced in the top-ranked pairs, suggesting that the Copula approach is more effective in capturing persistent mispricing patterns.

These empirical results highlight the Copula strategy's ability to identify a greater number of profitable trading opportunities while simultaneously reducing transaction costs, thus offering a robust enhancement over traditional correlation-based approaches in pairs trading.

Krauss and Stübinger (2017) apply the Copula method to pairs trading with the aim of more accurately capturing the non-linear dependence structure between stocks. The dataset comprises S&P 100 index constituents over the period 1990–2014. The strategy employs a 60-month rolling formation window, divided into a 12-month estimation period followed by a 1-month pseudo-trading period, during which all possible stock pairs are fitted using a Student's t-Copula model. The choice of the t-Copula is motivated by its ability to capture tail dependencies often observed in financial return series. In the estimation phase, a semi-parametric procedure is adopted: empirical marginal distribution functions are first computed for each stock's log returns, which are then transformed into uniform variables and used to fit the t-Copula via MLE. The fitted Copula model allows for the computation of conditional distribution functions for each pair, from which trading signals are derived based on deviations from the equilibrium relationship. When the conditional probability indicates overvaluation or undervaluation of one stock relative to the other, the model generates the corresponding buy or sell signals.

The approach is tested in an out-of-sample trading period, where each selected pair is traded according to individualized rules, including profit-taking and stop-loss thresholds determined from the cumulative return series in the estimation period. Furthermore, the study distinguishes between mean-reverting and momentum pairs: while the former tends to revert to equilibrium after a Copula signal, the latter exhibit further divergence, prompting a reversal of the trading rules to capture the momentum effect. Empirical results indicate that the Copula-based pairs trading strategy effectively captures complex dependencies between stocks, producing statistically significant annualized returns and elevated Sharpe ratios during the out-of-sample period, even under challenging market conditions.

Nadaf et al. (2022) also adopt a Copula-based approach to pairs trading, enhancing its effectiveness by integrating the Laplace marginal distribution. The study examines two stocks—Apple (AAPL) and Southwest Airlines (LUV)—using in-sample data from January 1, 2015, to December 31, 2019, and out-of-sample data from January 4, 2020, to November 20, 2020. Daily closing prices are converted to log returns, which are then modeled with the Laplace distribution to account for the pronounced leptokurtic (fat-tailed) nature of financial return data, a feature less accurately captured by the normal distribution. Parameters for the Laplace distribution are estimated via MLE, and goodness-of-fit tests confirm its adequacy. The marginal Laplace distributions are then combined through a Gaussian Copula, also estimated via MLE, to model the joint dependence structure between the two assets.

Trading signals are generated from the joint probabilities implied by the Copula model: when the conditional probability of one stock relative to the other reaches extreme values, indicating potential overvaluation or undervaluation, buy or sell orders are initiated. The strategy assumes negligible transaction costs and operates with a one-day holding period, opening positions at the market open and closing them at the close based on the generated signals. Out-of-sample testing demonstrates that this Copula-based framework successfully captures the non-linear dependencies between the assets, resulting in consistent and significant profitability.

5.5.2 Hurst Exponent Approach

Ramos-Requena et al. (2017) introduce a novel pairs trading framework that integrates the Hurst exponent as a key criterion for identifying asset pairs with strong mean-reverting behavior. The Hurst exponent, denoted as H, measures the long-term memory or persistence of a time series. Values of $H < 0.5$indicate anti-persistence, meaning the series exhibits mean-reverting tendencies; $H = 0.5$ corresponds to a random walk with no memory; and $H > 0.5$ suggests persistence or trend-following behavior. This property makes the Hurst exponent a valuable tool for distinguishing between pairs whose price spreads are likely to revert to a long-term mean and those that are not.

Traditional pairs trading methods, such as those based solely on distance measures or correlation coefficients, typically capture only short-term linear relationships and may fail to reflect the long-range dependence structure inherent in financial time series, particularly under volatile or non-stationary market conditions. By incorporating the Hurst exponent into the pair selection process, the proposed approach aims to identify spreads with stronger mean-reverting dynamics, thereby improving the robustness and profitability of the trading strategy.

Understanding the Hurst Exponent

The H is a statistical measure used to assess whether a time series tends to revert to its mean, persist in a given trend, or behave like a random walk. It is estimated by analyzing the scaling behavior of the rescaled range (R/S) statistic over different time windows, following the method introduced by Hurst (1951). The relationship can be expressed as:

$$\frac{R(n)}{S(n)} \propto n^H \tag{5.21}$$

where $R(n)$ is the range of cumulative deviations from the mean within a time window of size n (i.e., the maximum value minus the minimum value of the cumulative sum of deviations), $S(n)$ is the standard deviation of the time series over the same window, and H is the Hurst exponent.

The value of H provides insight into the underlying behavior of the time series:

- $H = 0.5$: The series behaves like a random walk, with no significant correlation between increments.

- $H < 0.5$: The series exhibits anti-persistent or mean-reverting behavior, indicating negative autocorrelation.
- $H > 0.5$: The series displays persistence, meaning that positive (negative) changes tend to be followed by further positive (negative) changes, reflecting long-term positive autocorrelation.

Using the Generalized Hurst Exponent (GHE)

To provide a more accurate estimation, the study employs the GHE, which extends the concept of the Hurst exponent to different moments of the distribution, offering a more flexible tool for capturing mean reversion over various time scales. The GHE is calculated using:

$$E[|X(t+\tau) - X(t)|^q] \propto \tau^{qH(q)} \tag{5.22}$$

where $X(t)$ is the log-price of a stock at time t, τ represents the time lag, q is the order of the moment (typically set to 1 or 2 for mean-reversion analysis), and $H(q)$ is the generalized Hurst exponent.

Unlike the classical Hurst exponent, which is restricted to analyzing the second moment ($q = 2$) and thus provides only a single measure of persistence or mean reversion, the GHE evaluates the scaling behavior for different moments q. This allows the method to capture a richer set of dependencies in financial time series, including asymmetries between small and large fluctuations.

If $H(q)$ remains constant across different q values, the process is considered monofractal, meaning its scaling properties are uniform. In contrast, variations in $H(q)$ across q indicate multifractality, a property often observed in financial markets, where different magnitudes of price changes may follow different scaling laws.

In the context of pairs trading, using the GHE offers two main advantages. First, it enhances the detection of mean reversion by analyzing $H(q)$ for values below 0.5 across multiple scales, making it easier to identify stock pairs whose spread dynamics consistently revert to the mean. Second, it improves robustness to non-Gaussianity, as the GHE does not assume normally distributed returns and accounts for higher-order moments, enabling it to capture the heavy tails and volatility clustering present in financial data.

Pair Selection Process

The study begins by collecting historical price data for a given universe of stocks. In the empirical example, data from constituents of the Dow Jones Index are used over the designated sample period. For each possible stock pair, the log-price differences are computed over time, generating a time series that reflects the relative price movements between the two assets.

The mean-reversion characteristics of each pair are then assessed by computing the GHE, as discussed above, across a range of time lags τ. This approach allows the analysis to capture persistence or anti-persistence in the price spread over multiple time scales, providing a more nuanced measure than the traditional Hurst exponent.

Once the GHE values are obtained for all candidate pairs, the pairs are ranked in ascending order. Those with the lowest GHE values—indicating the strongest mean-reverting behavior—are selected as candidates for trading. A low GHE suggests a higher probability that the spread will return to its historical mean, which is a desirable characteristic for pairs trading strategies.

The selected pairs are then carried forward into the trading phase, where entry and exit signals are generated based on subsequent deviations from the estimated equilibrium relationship. This ensures that the pair selection process is directly linked to the profitability potential of the trading strategy.

Trading Strategy Based on Hurst Exponent

Opening positions occur when the log-price spread of a selected pair deviates from its rolling mean by a predefined threshold, such as one standard deviation. A sell signal is generated when the spread exceeds the upper threshold, indicating that the spread is unusually wide and likely to contract, while a buy signal is generated when the spread falls below the lower threshold, suggesting that the spread is unusually narrow and likely to widen. The use of pairs with low Hurst exponents ensures that these signals are more reliable, as low H values (below 0.5) indicate strong mean-reverting tendencies, meaning the spread is statistically more likely to return to its equilibrium level.

Closing positions take place when the spread reverts to its mean or crosses another predefined boundary, thereby capturing the profit from the reversion.

The strategy also requires setting specific parameters for entry and exit points, with thresholds typically defined as multiples of the standard deviation of the spread. Both the rolling mean and standard deviation are calculated over a chosen rolling window, such as 60 days, with the window length selected to balance responsiveness and stability—shorter windows react faster but may generate more false signals, whereas longer windows provide smoother signals but respond more slowly to market changes.

Trades are executed according to the generated signals. When a pair's spread moves above or below the threshold, the corresponding long or short positions are taken and held until the spread reverts to the mean or another threshold triggers an exit.

Empirical Validation

The paper validates the effectiveness of the Hurst exponent-based strategy using historical data from the Dow Jones Index. In the backtesting, the strategy achieved an annualized return of 17.14% with a Sharpe ratio of 1.34, compared to 12.56% and a Sharpe ratio of 0.97 for the traditional distance method. The proportion of profitable trades was 64%, which is higher than the 55% observed for the distance-based approach. These results indicate that the Hurst exponent method not only delivers higher returns but also improves risk-adjusted performance. The method performed particularly well during volatile market periods, showing greater robustness and adaptability to changing conditions by consistently selecting stock pairs with stronger mean-reverting properties.

Fernández-Pérez et al. (2020) enhance pairs trading strategies by incorporating the Hurst exponent to identify stock pairs with pronounced mean-reverting behavior. Originally developed in hydrology, the Hurst exponent has been adapted to financial time series to detect long-term memory and autocorrelation, thereby capturing the extent to which asset prices revert to their historical equilibrium levels. In their approach, the GHE is employed to evaluate mean-reversion characteristics by analyzing the scaling behavior of the time series. Stock pairs with a Hurst exponent value below 0.5—indicating statistically significant mean-reverting properties—are selected for trading. The strategy is implemented in two stages: a 250-day formation period, during which the pairs with the lowest Hurst exponent values are identified, followed by a trading phase where positions are opened when the price spread between the selected stocks deviates from its moving average by a predefined threshold. Empirical tests on FTSE 100 constituents from 2010 to 2019 show that the optimal threshold depends on portfolio size: a standard deviation threshold of 1 yields the best results for smaller portfolios, whereas 1.5 is optimal for larger portfolios. Performance metrics such as the Sharpe ratio and Maximum Drawdown reveal that the strategy delivers superior risk-adjusted returns, particularly during periods of heightened market volatility such as the 2016 Brexit referendum.

Bui and Ślepaczuk (2022) also apply the Hurst exponent to pairs trading, focusing on stocks in the NASDAQ 100 index to identify pairs with strong mean-reverting tendencies. Using daily price data from January 1, 2000, to December 31, 2018, and an out-of-sample period extending to July 1, 2021, the study computes the Hurst exponent for the log-price ratio of each stock pair via the GHE method. Every six months, the ten pairs with the lowest Hurst exponent values are selected. Trading signals are generated when the spread between two stocks exceeds an upper threshold (sell signal) or falls below a lower threshold (buy signal), with positions closed when the spread reverts to its mean or crosses the opposite threshold. The parameters for moving averages, rolling standard deviations, and thresholds are optimized using historical data to minimize bias. Empirical findings suggest that while the Hurst exponent serves as a viable alternative to correlation-based selection methods, its performance in terms of risk-adjusted returns is sensitive to factors such as the number of pairs selected, rebalancing frequency, and the application of leverage.

5.5.3 Entropic Approach

Amer and Islam (2023) apply an entropic approach to pairs trading, aiming to mitigate model uncertainty and determine optimal trading thresholds. The method addresses issues such as non-converging pairs and model misspecifications that may cause substantial losses, by incorporating entropy as a penalty term in the optimization process. This penalization framework not only enhances robustness but also reduces downside risk. To empirically test the approach, the authors use daily data from 64 companies listed on the Pakistan Stock Exchange (PSX) between 2017 and 2019, spanning sectors including cement, chemicals, automobiles, food, oil, gas, and power. Firms are selected based on their price-to-earnings ratio (PER) within each sector, with those having a PER below the sectoral median considered undervalued and included in the sample.

The Johansen cointegration test is employed to identify stock pairs that exhibit a long-term equilibrium relationship, resulting in 79 unique cointegrated pairs suitable for trading. The entropic approach builds upon the OU process to model the mean-reverting behavior of these pairs, expressed as:

$$dX_t = -\mu(X_t - \alpha)d_t + \sigma dB_t, X_0 = \alpha \tag{5.23}$$

where μ is the speed of mean reversion, α is the mean-reversion level, σ is the volatility, and B_t represents Brownian motion. Parameters μ, α, and σ are estimated using maximum likelihood methods. The selection of these parameters, along with the discount rate ρ and confidence level λ, follows the empirical settings and theoretical guidance from prior studies, ensuring consistency with established literature.

The optimization of boundary points is framed as an optimal stopping problem, aiming to maximize profit while minimizing relative entropy (model risk). The optimal boundaries are calculated using the solution:

$$v_0(t, x) = sup_{\tau \in \mathfrak{I}} E_x^S[e^{-\rho(\tau - t)} X_\tau] \tag{5.24}$$

where $\mathfrak{I}$ is the set of all stopping times, ρ is the discount rate, and S represents the stock pair. The trading strategy involves shorting the stock pair when it reaches its highest value and liquidating it when it reverts to the mean, or taking a long position when it reaches the mean and liquidating it at the boundary.

The study explores different values of λ (0.001, 0.01, 0.1, and ∞) to compute optimal boundary points and returns, with lower λ values corresponding to higher confidence levels. The empirical results indicate that lower λ values yield better returns, highlighting a higher confidence level in the reference measure. Moreover, paired t-tests confirm that the performance differences between the entropic approach and benchmark strategies are statistically significant at the 5% level, further validating the robustness of the results.

When compared with buy-and-hold, distance-based, and machine learning methods, the entropic approach consistently delivers higher returns, demonstrating its effectiveness in managing model uncertainty and enhancing profitability in the volatile Pakistani market.

Yoshikawa (2017) develops a robust control framework for pairs trading that explicitly accounts for model uncertainty by adopting an entropic approach. Conventional pairs trading strategies often rely on a single reference model—typically assuming mean-reversion dynamics—which can lead to substantial losses when the model is misspecified or when the assumed mean-reversion fails to hold. To mitigate these risks, the study formulates the optimal stopping problem under uncertainty using relative entropy (Kullback–Leibler divergence) as a penalty term. This approach seeks the trading policy that maximizes performance in the worst-case scenario over all alternative probability measures within a prescribed entropy bound from the reference measure.

The reference model assumes that the spread between two stocks follows an OU process, characterized by a mean-reversion speed, a long-term equilibrium level, and volatility. These parameters are estimated via MLE to reflect the observed spread dynamics in the Tokyo Stock Exchange data.

Incorporating the entropic penalty modifies the optimal entry and exit thresholds relative to the standard, uncertainty-free case. As the level of model uncertainty increases—corresponding to a larger entropy bound—the optimal boundaries shift inward, resulting in more conservative entry and exit decisions. This contraction reduces exposure to adverse price movements when the true dynamics deviate from the reference model.

A numerical example demonstrates the approach by calculating optimal trading boundaries under various uncertainty levels. The results show that higher uncertainty leads to systematically narrower trading bands, thereby improving robustness against model misspecification and enhancing the resilience of the pairs trading strategy.

6. Conclusion and Future Research Direction

We have conducted a thorough review of the literature related to the broad concept of pairs trading. Organized by the different pairs trading approaches, our findings can be summarized along with recommendations for future research as follows.

6.1 Distance Methods

The review of distance-based approaches in pairs trading underscores their robustness and adaptability across diverse market environments and asset classes. Originating from the seminal work of GGR, the distance method has been widely adopted and refined to improve profitability and mitigate risk. As a model-free approach, it selects pairs based solely on historical price relationships—measured via Euclidean distance—without imposing the structural constraints of economic or cointegration models. This simplicity, combined with the implicit assumption of mean-reverting price differentials, has enabled the method to consistently identify relative mispricing opportunities in financial markets.

Subsequent extensions have demonstrated the method's versatility. For instance, Bowen and Hutchinson (2016) applied the GGR framework to the UK equity market, showing that its profitability persisted, albeit with increased volatility, during the global financial crisis. Other studies have extended the method across multiple geographies and asset classes, including commodities, cryptocurrencies, and high-frequency data environments. Overall, empirical evidence suggests that the distance method retains profitability under varied market conditions, though performance can be sensitive to factors such as market liquidity, transaction costs, and the statistical properties of the selected asset pairs.

Nonetheless, the distance method is subject to certain limitations. High transaction costs can erode returns, particularly in illiquid markets or those with significant trading frictions. Its reliance on historical price data may also reduce its effectiveness when market dynamics undergo structural shifts—such as changes in market microstructure, regulatory frameworks, or the competitive landscape. Some studies (e.g., Chen et al., 2019) report a decline in profitability in more recent years, underscoring the need for adaptive methodologies.

Future research could focus on integrating the traditional distance metric with advanced statistical or machine learning techniques to improve both pair selection and trade execution. Dynamic modeling frameworks that capture evolving market states, or the incorporation of alternative data sources—such as news sentiment, order book information, or macroeconomic indicators—may enhance predictive accuracy and robustness. Additionally, examining the effects of regulatory changes, market microstructure evolution, and the increasing prevalence of algorithmic trading could yield valuable insights for optimizing strategy design.

In summary, the distance method remains a cornerstone of pairs trading research and practice. Continued innovation, particularly through the integration of adaptive and data-driven approaches, holds promise for enhancing its effectiveness in increasingly complex and competitive financial markets.

6.2 Cointegration Methods

The review of cointegration-based approaches to pairs trading underscores their theoretical rigor and broad applicability across asset classes and market conditions. Cointegration strategies exploit long-term equilibrium relationships between asset prices, enabling the identification of mean-reverting opportunities even when individual price series are non-stationary. From the foundational work of Engle and Granger (1987), who introduced the concept of cointegration and the error correction model (ECM) via a two-step estimation procedure, to Vidyamurthy's (2004) application of these concepts to pairs trading, the method has evolved into a cornerstone of statistical arbitrage. Subsequent developments maximum likelihood approach, have expanded the framework to multivariate settings.

Recent advances have shifted from static testing to dynamic and adaptive cointegration frameworks, enhancing robustness to changing market conditions. Techniques such as rolling-window estimation, Kalman filter–based cointegration, VECM, and Bayesian inference approaches allow continuous recalibration of model parameters, maintaining the validity of cointegration relationships across different regimes. Applications have extended beyond equities to include derivatives, CDSs, and cryptocurrencies, illustrating the method's adaptability across fundamentally different market structures.

Empirical evidence generally supports the profitability of cointegration-based pairs trading, even after accounting for transaction costs and market frictions, with notable success during periods of market stress or structural breaks. However, performance can be sensitive to pre-selection metrics, estimation windows, trading frequency, and the stability of the identified relationships. In illiquid markets or under high transaction costs, profitability can be substantially diminished.

Future research directions include integrating cointegration frameworks with advanced machine learning and artificial intelligence methods for pair selection and trade execution, as well as exploring nonlinear cointegration models to capture more complex dependencies. The incorporation of alternative data sources—such as news sentiment, order book dynamics, and macroeconomic variables—may further enhance predictive accuracy. Additionally, examining the role of ESG-related signals and extending the methodology to emerging asset classes could open new avenues for application.

Overall, cointegration methods provide a theoretically sound and empirically validated foundation for pairs trading. Continued methodological innovation, particularly through adaptive modeling and the incorporation of alternative datasets, holds promise for improving strategy resilience in increasingly complex financial markets.

6.3 Stochastic Control Methods

Stochastic control approaches to pairs trading provide a rigorous and adaptable framework for managing the dynamic and often volatile nature of financial markets. These methods formulate the trading problem as a continuous-time optimization task, enabling the systematic adjustment of trading positions in response to evolving market conditions. A central component in many stochastic control models is the OU process, which characterizes the mean-reverting behavior of asset spreads.

Foundational work, such as Jurek and Yang (2007), applies the OU process to model spread dynamics and explicitly addresses key risks in pairs trading, notably horizon risk—the risk that the spread has not converged by the end of the investment horizon—and divergence risk—the risk of adverse price movements prior to convergence. By deriving optimal dynamic strategies that adjust capital allocation based on both the spread level and the time remaining, stochastic control methods represent a significant advance over static, threshold-based trading rules.

Subsequent research has extended the classical OU framework to incorporate jump-diffusion processes, regime-switching dynamics, and stochastic volatility, thereby capturing non-normality, state dependence, and time-varying volatility in spread behavior. These extensions have been applied across diverse contexts, including high-frequency data environments, commodity futures, cryptocurrencies, and energy markets. Empirical studies frequently report improvements in Sharpe ratios and annualized returns relative to simpler strategies, particularly in complex or rapidly changing markets.

Theoretical innovations—such as Markov-modulated OU processes and the incorporation of explicit risk constraints into the control problem—have deepened understanding of optimal trading behavior under uncertainty and risk aversion. Many of these models require solving Hamilton–Jacobi–Bellman equations or employing simulation-based numerical methods, reflecting the computational complexity of advanced stochastic control approaches.

Despite these advancements, practical challenges remain. Many models assume frictionless trading, whereas real-world markets impose transaction costs and liquidity constraints that can materially affect performance. Although some recent studies have begun to incorporate such frictions into the control framework, further work is needed to develop tractable yet realistic formulations.

Future research could explore extending stochastic control methods to alternative investments and emerging markets, as well as integrating them with machine learning and artificial intelligence techniques to enhance adaptability and predictive capability. AI-driven approaches may uncover nonlinear dependencies and structural patterns beyond the reach of traditional models, potentially improving trade timing and profitability. Additionally, greater attention to the role of market frictions and the development of robust cost-aware control strategies will be critical for enhancing the practical applicability of these methods.

In summary, stochastic control methods offer a robust, theoretically grounded, and highly flexible framework for pairs trading. Continued methodological innovation—particularly in integrating advanced modeling techniques with realistic trading constraints—holds promise for improving the resilience and profitability of such strategies across diverse market environments.

6.4 Time Series Methods

Time series approaches have proven effective in modeling the mean-reverting behavior of asset spreads and in providing robust frameworks for exploiting short-term pricing inefficiencies. Compared with cointegration-based methods, which emphasize long-term equilibrium relationships, time series techniques such as the GARCH family and OU processes focus more on short-term dynamics, volatility clustering, and higher-frequency market fluctuations, offering greater flexibility and responsiveness in pairs trading strategies.

GARCH-type models—including ST-GARCH and nonlinear autoregressive GARCH (NAR-GARCH) variants—have been employed to capture complex volatility structures in spread returns. These models are capable of modeling features commonly observed in financial markets, such as volatility clustering, asymmetry, and heavy tails. By forecasting conditional variances and adjusting trading thresholds based on volatility predictions, GARCH-based frameworks can generate risk-adjusted entry and exit signals, potentially improving profitability while mitigating downside risk. Empirical studies have reported that incorporating GARCH volatility forecasts into pairs trading can enhance performance, even after accounting for transaction costs.

The OU process has also been widely used to model mean-reversion in asset spreads. Extensions such as the fOU process allow for the modeling of long-range dependencies and anti-persistence, offering a richer representation of spread dynamics. Although primarily examined in academic research, fOU-based strategies—when combined with adaptive trading thresholds—have been shown to outperform traditional OU implementations in certain market settings, delivering improved returns and risk metrics.

Future research directions include the integration of time series models with machine learning algorithms, such as neural networks and reinforcement learning, to enhance adaptability and predictive accuracy. Hybrid frameworks that combine time series methods with other quantitative approaches—such as copula-based dependence modeling or stochastic control techniques—may yield greater robustness and profitability. Incorporating transaction cost analysis, slippage, and liquidity constraints into time series models would also improve the realism and practical applicability of these strategies, as real-world trading frictions can significantly influence net performance.

Expanding the application of time series methods to emerging markets, alternative asset classes, and high-frequency data environments presents further opportunities for research. As financial markets continue to evolve, the adaptability and precision of time series modeling will remain crucial for identifying and exploiting arbitrage opportunities in increasingly complex and volatile environments.

In summary, time series methods offer a versatile and theoretically grounded toolkit for pairs trading, capable of capturing intricate short-term dynamics and volatility features. Continued methodological refinement, coupled with attention to practical trading constraints, holds significant potential for advancing both the robustness and profitability of such strategies.

6.5 Other Methods

Beyond the widely studied approaches to pairs trading, several alternative methodologies—most notably the Copula approach, the Hurst exponent approach, and the entropic approach—have been developed to address specific limitations of traditional techniques. These methods aim to capture non-linear dependencies, account for long-memory effects, and incorporate model uncertainty, thereby enhancing the robustness and adaptability of pairs trading strategies across diverse market environments.

The Copula approach provides a flexible framework for modeling the joint distribution of asset returns, enabling the capture of non-linear and tail dependencies that cannot be represented by simple linear correlation. By selecting appropriate Copula families—such as t-Copulas or Archimedean Copulas (e.g., Clayton, Gumbel)—researchers can model dependence structures more accurately, particularly under extreme market conditions. Empirical studies have reported that Copula-based pairs trading strategies may outperform conventional distance or correlation methods, especially for volatile or weakly correlated asset classes. Future research could focus on exploring a wider range of Copula families, improving parameter estimation under time-varying dependence, and incorporating real-time analytics to increase responsiveness to changing market conditions.

The Hurst exponent approach measures the degree of long-term memory or persistence in time series, offering a means to identify pairs with stronger mean-reverting characteristics. By applying the GHE, this method evaluates persistence or anti-persistence across multiple time scales, providing a more comprehensive assessment of mean-reversion potential than single-horizon metrics. Empirical evidence suggests that Hurst-based pair selection can, in certain settings, improve profitability relative to standard approaches, particularly during volatile periods. Promising future directions include developing dynamic models that adjust the Hurst exponent estimates in real time and integrating machine learning algorithms to forecast changes in persistence, thereby enabling more adaptive trading strategies.

The entropic approach, exemplified by Yoshikawa (2017), addresses model uncertainty by incorporating relative entropy (Kullback–Leibler divergence) as a penalty function in the optimization of entry and exit boundaries. This robust control framework accounts for deviations between a reference model and alternative probability measures within a specified entropy bound, mitigating the risk of significant losses from model misspecification or non-converging spreads. Empirical studies indicate that entropy-based strategies can enhance robustness and, in some contexts, improve returns, even in challenging market conditions. Future research could extend this method to multi-asset-class contexts, investigate alternative entropy measures, and evaluate its performance in high-frequency trading environments where rapid adjustments are essential.

In summary, Copula, Hurst exponent, and entropic approaches offer valuable extensions to traditional pairs trading techniques by capturing complex dependence structures, incorporating long-term memory, and managing model uncertainty. Integrating these methods with modern data analytics, machine learning, and artificial intelligence could further advance the sophistication and adaptability of pairs trading, enabling strategies to remain effective across a wide range of asset classes, market conditions, and trading horizons.

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