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Abstract

This article aims to extend the evaluation of classic multifactor models of Carhart(1997) for the case of global equity indices and to expand analysis performed in Sakowski, Slepaczuk, and Wywiał (2015). Our intention is to test several modifications of these models to take into account different dynamics of equity excess returns between emerging and developed equity indices. Proposed extensions include volatility regime switching mechanism (using dummy variables and the Markov approach) and three new risk factors based on realized volatility of index returns, percentage deviation from nominal GDP trend and capitalisation relative to GDP factor. Additional modifications include introduction of common and country specific variables in order to control for global risk where fluctuations of volatility of various assets, prices of commodities, currencies and rates is really important. Moreover, instead of using data for individual stocks (which is a common approach in the literature), we evaluate the performance of these models for weekly data of 81 world investable equity indices in the period of 2000-2015. Such approach is proposed to estimate equity risk premium for a single country. Empirical evidence from the first part reveals important differences between results for classical models estimated on single stocks (either in international or US-only frame work) and models evaluated for equity indices. Additionally, we observe substantial discrepancies between results for developed countries and emerging markets. The last part of this research helps us to understand results revealed in the first part. Thanks to introduction of new risk factors, and additional common and country specific variables we were able to increase explanatory power of our factor models especially in case of emerging market indices. Finally, using weekly data for the last 15 years we illustrate importance of model risk and data overfitting effects when drawing conclusions upon results of multifactor models.

Keywords:

investment algorithms, multi-factor models, Markov switching model, asset pricing models, equity risk premia, risk factors, Markowitz model

JEL:

C15, G11, F30, G12, G13, G14, G15

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1 Introduction

The estimation of equity risk premium based on multifactor models is one of the most developed branches of financial research. Beginning with three-factor model of Fama and French (1992), who extended the classic CAPM model (Sharpe (1964), Lintner (1965) and Black, Jensen, and Scholes (1972)), the research has taken two paths. The first aims at finding a sparse set of factors, extending or replacing Fama-French factors. This includes:

- value investing strategy effects - either considered analogically to HML factor described in Fama and French (1992) (i.e. investing in stocks that have high book to market value), as included in Carhart (1997) or taking a slightly different path and employing dividends yield, earnings ratio, etc (Lakonishok, Shleifer, and Vishny (1994), Arshanapalli, Coggin, and Doukas (1998)),
- effect of low market capitalisation of single stocks, found in Fama and French (1992) and repeated e.g. in Fama and French (2012),
- momentum and reversal effect, captured first by Jegadeesh and Titman (1993) and then included in numerous factor models, most significantly in Carhart (1997) four-factor model, but also in Wu (2002), , and Asness (1995),
- effects of measured liquidity (Rahim and Noor (2006), Liu (2004)),
- other fundamental factors, including investment factor and return on equity factor (Chen, Novy-Marx, and Zhang (2011)), or profitability factor and investment factor (Fama and French (2015)), quality minus junk factor (Asness, Frazzini, and Pedersen (2013)), cash-flow-to-price factor,
- innovative factors basing on the previous research results, such as measuring CAPM beta and betting against it (Frazzini and Pedersen (2014))¹

The other direction is to implement the existing models in a different geographical and market setting, i.e. moving away from the spectrum of US equities. This includes:

- research on the three- and four-factor model in the context of global equities (Griffin (2002)), only emerging markets (Cakici, Fabozzi, and Tan (2013)) or single country (Connor and Sehgal (2001)),
- search for a different set of factors that explain variability of equities in a specific country or region, e.g. Baltic countries (Lieksnis (2010)), Eastern Europe (Foye, Mramor, and Pahor (2013)), or in a global setting (Hou, Karolyi, and Kho (2011)).

Naturally, these two paths weren't mutually exclusive and research often overlaps both of them. It is also the case of our research, initiated by Sakowski, Slepaczuk, and Wywiał (2015), where we propose a shift from single stocks to equity indices, one for each country, to reflect the point of view of a global investor allocating his portfolio between different countries. Apart from application of Carhart (1997) in this setting, we move to include extensions to the initial model, by searching for additional factors, such as realized volatility risk premium factor constructed in Sakowski, Slepaczuk, and Wywiał (2016a) or inclusion of

¹The Betting Against Beta, or shortly BAB, factor captures the phenomenon that long leveraged low beta assets and short deleveraged to 1 high-beta assets produce significant positive risk-adjusted returns.

dynamic aspects, most notably Markov regime-switching model with two volatility regimes (Sakowski, Slepaczuk, and Wywiał (2016a)). This paper continues this work, retaining dynamic extensions and focus on multifactor models on equity indices, but also extending it with search for additional factors as well as external regressors.

Our main research questions can be stated as follows:

1. Can multifactor models be used for explanation of equity risk premium for global indices?
2. Can we distinguish any common or country specific macroeconomic variables which increase explanatory power of our multi-factor models?
3. Are sensitivities to risk factors stable across countries? Do they differ during various phases of economic cycles?
4. Can we include volatility risk factor, percentage deviation from nominal GDP or realative small minus big factor to better explain variability of risk premia?
5. Does volatility regime switching mechanism enable us to explain equity risk premium for global indices in the extended models as well as in case of five-factor model presented in (Sakowski, Slepaczuk, and Wywiał (2016a))?
6. Which risk factor or macroeconomic variable may be the most important in portfolio construction based on the model?

The structure of the paper is as follows. Section 2 presents methodology of our study. Equity risk premium, functional forms of the alternative models and econometric issues are discussed in this part. Section 3 provides description of both the data and the procedure we used to build risk factors and external regressors. We also analyse dynamics of risk factors and macroeconomic variables in time here. Section 4 presents results. The last section concludes.

2 Methodology

2.1 Motivation

The methodology is based on the seminal paper of Carhart (1997), who proposed the four-factor model for analysis of mutual funds performance. One of the reason why we prefer the model of Carhart (1997) over the methodology of Fama and French (1992) (the three-factor model for stocks return analysis) are the results of Fama and French (2012) and comprehensive results obtained for emerging markets by Cakici, Fabozzi, and Tan (2013). They focused on 18 emerging markets treating each of them separately. Their results revealed significance of value and momentum everywhere except Eastern Europe and additionally showed that momentum and value factors were negatively correlated.

At this moment it is important to explain rationale for choosing equity indices instead of single stocks. The main reason behind this is that from the global investment perspective single countries may be treated as an asset class. This issue is very important from the global portfolio selection problem, where asset allocation approach seems to gain more popularity. This is confirmed by the dynamic development of ETFs and derivatives providing country exposure. This approach seems to better reveal global factors than regressions on single

stocks and enables equity risk premia for countries to be assessed separately. What is also important, the literature on this subject is currently very limited.

Taking into account that our research is intended for equity indices with special attention to emerging and developed markets, we propose several amendments to the initial methodology of Carhart (1997). Necessary modifications include:

- converting monthly to weekly data in order to reveal dynamics during shorter time intervals,
- including new risk factors that explain the diversity of returns more deeply, i.e. realized volatility as fifth factor,
- necessary conversion of well-known risk factors from the single country level to the worldwide level, including currency conversion,
- creating an adequate zero investment portfolio that fully reflects the influence of particular risk factor on equity risk premia,
- introducing volatility switching mechanism to take into account different dynamics of equity indices during high and low volatility periods.
- inclusion of common and country specific variables to n-factor regressions in order to reveal various risks affecting mainly emerging market economies.

2.2 Equity risk premium

It is also important to define the equity risk premium as the expected excess return of equities over the risk free rate. The point here is that current literature (Duarte, Rosa, et al. (2015)) proposes many alternative ways to measure it. Equity risk premium can be defined using:

- historical returns approach:

$$ERP = \sum_{t=t_0}^n R_t - Rf_t \quad (1)$$

where $R_t - Rf_t$ is excess return at time t over risk-free rate.

- earnings yield approach:

$$ERP = \frac{E}{P} - Rf \quad (2)$$

where $\frac{E}{P}$ is earnings to price ratio

- dividend yield approach:

$$ERP = \frac{D}{P} + g - Rf \quad (3)$$

where $\frac{D}{P}$ is dividend to price ratio and g is dividend growth rate.

- regression- and factor-based approach which can be characterised by point-in-time estimates instead of long-term estimates only, not dependent on e.g. tax policy, and which allows dynamic forecasts:

$$ERP_t = \alpha + \sum_{i=1}^n \beta_i X_{i,t} + \epsilon_t \quad (4)$$

where $X_{i,t}$ are the i -th risk factor at moment t and β_i is sensitivity to this factor.

- survey-based approach which is often systematically biased, negatively correlated with future returns, and positively with previous returns.

In this article, when we talk about equity risk premium we refer to the definition based on historical returns. Selection of the particular definition of equity risk premium can certainly affect final conclusions. Before we focus on this issue, we describe alternative factor models used in this research.

2.3 Multi-factor models

2.3.1 Initial model

We start with the classic Carhart four-factor model:

$$(R_i - Rf)_t = \alpha_i + \beta_{MKT,i} (Rm - Rf)_t + \beta_{HML,i} HML_t + \beta_{SMB,i} SMB_t + \beta_{WML,i} WML_t + \epsilon_{i,t} \quad (5)$$

where:

$(R_i - Rf)_t$ is weekly return of equity index in excess to weekly risk free rate,

$(Rm - Rf)_t$ is equally weighted equity index less than risk free rate,

HML_t is the weekly premium on the book-to-market factor,

SMB_t is the weekly premium on the size factor

WML_t is the weekly premium on winners-minus-losers factor.

The WML factor is calculated by subtracting the equal weighted average of the highest performing equity indices from the equal weighed average of the lowest performing equity indices (Carhart (1997)).

Next, we add to the model an additional factor based on realized volatility (VMC - volatile minus calm):

$$\begin{aligned} (R_i - Rf)_{i,t} = & \alpha + \\ & \beta_{MKT,i} (Rm - Rf)_t + \beta_{HML,i} HML_t + \beta_{SMB,i} SMB_t + \beta_{WML,i} WML_t + \\ & \beta_{VMC,i} VMC_t + \\ & \epsilon_{i,t} \end{aligned} \quad (6)$$

The VMC factor is the weekly premium on volatile minus calm (VMC) equity indices and is obtained by subtracting the equal weighted average return of the highest volatility equity indices from the equal weighted average return of the lowest volatility equity indices. The definition of high or low volatility is based on 63 days realized volatility calculated separately for each equity index.

The detailed procedure of calculating HML, SMB, VML and VMC risk factors and definitions of zero-investment portfolios based on them is summarized in Section 3.1.

Since we want to take into account impact of different market environments on the factor sensitivities, we consider adding to the model a regime switching mechanism.

The first attempt to capture a change in the volatility regime focuses on including different dynamics of equity risk premia in: 1) high and low volatility environment and 2) during upward and downward movements of the market. We add dummy variables with appropriate interactions and the functional form of the regression is then as follows:

$$\begin{aligned}
(R_i - Rf)_{i,t} = & \alpha_i + \\
& \beta_{MKT,i}(Rm - Rf)_t + \beta_{HML,i}HML_t + \beta_{SMB,i}SMB_t + \beta_{WML,i}WML_t + \\
& \beta_{VMC,i}VMC_t + \\
& \gamma_i D_t + \\
& \gamma_{MKT,i}(Rm - Rf)_t D_t + \gamma_{HML,i}HML_t D_t + \gamma_{SMB,i}SMB_t D_t + \\
& \gamma_{WML,i}WML_t D_t + \gamma_{VMC,i}VMC_t D_t + \\
& \varepsilon_{i,t}
\end{aligned} \tag{7}$$

We consider two alternative definitions of the dummy variable used in equation (6):

- $D_t = 1$ for high volatility periods, for low volatility periods, where the division is based on realized volatility calculated in USD for the market index and the periods brackets were defined *ex-ante*,
- ex-post identification of upward ($D_t = 1$) and downward ($D_t = 0$) movements of the market². Such division is especially important when we perform analysis between 2000 and 2015 since this period of equity markets was characterized by two strong bull and two strong bear markets which was not observable before within such relatively short data span.

The last form of the multi-factor model tested in the first part of this paper is a simple form of Markov switching model with two states of the world. For this purpose we used five-factor model in the form presented in equation (6). The rationale for the selection of the last model was based on several studies which showed among other things that various types of regime switching models can be useful in explaining equity risk premia Tan (2013). Ammann and Verhofen (2006) revealed that value investing seems to be a rational strategy in the High-Variance Regime, while momentum investing in the Low-Variance Regime. They additionally presented an empirical out-of-sample backtest indicating that this switching strategy can be profitable. Moreover, Angelidis and Tassaromatis (2014) show that there are significant costs to investors who fail to take into account the existence of regimes in portfolio construction and asset allocation. Hammerschmid and Lohre (2014) showed regime shifts is preserved in the presence of fundamental variables known to predict equity risk premia.

Hence, the complete list of models used in the first part of this study is presented below:

1. Carhart.4F.localcurrency (four-factor model, factors based on local currency),
2. Carhart.4F.USD (four-factor model, factors based on USD),
3. Carhart+VMC.5F.USD (five-factor model, VMC added, factors based on USD),

²In the course of our analysis, periods of high/low volatility occurred to coexist with periods of upward and downward market movements.

4. Carhart+VMC.5F.dummyRVinteract.USD (five-factor model, VMC and volatility dummies added, factors based on USD),
5. Carhart+VMC.5F.dummyUp&Downinteract.USD (five-factor model, VMC and market trend dummies added, factors based on USD),
6. Carhart+VMC.5F.MarkovSwitch2Reg.USD (five-factor model, VMC and added, factors based on USD)

2.3.2 Extensions - additional factors and

Based on the initial results we decided to develop this research and perform additional analysis in order to increase explanatory power of our models. For this purpose we defined two additional factors (QMS-positive minus negative, SMBrel-relative smal minus big) and additionally we added common and country specific variables which then will be used in our multifactor model. After this step our general functional form of multifactor model has the following shape:

$$\begin{aligned}
 (R_i - Rf)_{i,t} = & \alpha + \beta_{CV,i}(CV)_t + \beta_{CspV,i}(CspV)_t + \\
 & \beta_{MKT,i}(Rm - Rf)_t + \beta_{HML,i}HML_t + \beta_{SMB,i}SMB_t + \beta_{WML,i}WML_t + \\
 & \beta_{VMC,i}VMC_t + \beta_{QMS,i}QMS_t + \beta_{SMBrel,i}SMBrel_t + \\
 & \varepsilon_{i,t}
 \end{aligned} \tag{8}$$

The QMS factor is the weekly premium on positive minus negative (QMS) equity indices and is obtained by subtracting the equal weighted average return of equity indices characterized by the highest positive percentage deviation of nominal GDP from its 5y moving average and the equal weighted average return of the equity indices characterized by the lowest positive percentage deviation of nominal GDP from its 5y moving average.

The SMBrel factor is similar to SMB factor with one important exception. The sorting of equity indices to each decile group is based on the ratio of index capitalization to GDP of the given country instead of capitalization only. Such approach enable us to refer in this factor to the level of capital market development and not only to absolute value of capitalisation which in reality is quite hard to compare between countries.

The detailed procedure of calculating QMS and SMBrel risk factors and definitions of zero-investment portfolios based on them is summarized in Section 3.1.

Common variables are the same for all equity indices under investigation. The list of common variables which will be added to our regression is as follows:

1. GSCI - S&P Goldman Sachs Commodity Index. This variable should represents main trends on aggregated commodity prices. We will use weekly differences of this index. The rationale for this variable comes from substantial dependence between commodity dependent countries (producers and importers) and the prices of commodities.
2. SKEW - skew of S&P500 index options. We will use weekly differences of 4 weeks moving average of this index. This variable can be very important in signalling market crashes.
3. OVXGVZ - the average of annualized monthly RV of crude oil prices and GVZ volatility indices. We will use weekly differences of average of these indices. These volatility indices should be a good proxy on sentiment of oil and gold investors.

4. VX - the average of VIX, VXEEM, VXEWS and VVIX volatility indices. We will use weekly differences of average of these indices. These volatility indices should be a good proxy on sentiment of investors focused on US market, emerging markets, Brazil and China.
5. DXY - USD index. We will use weekly differences of this index. The rationale for this variable is its high negative correlation with commodity prices which are mainly priced in USD.
6. CLF_dummy - dummy variable which has value of 1 when Crude oil price is lower than 65 USD and 0 otherwise.
7. VVIX - volatility of VIX index, i.e. volatility of volatility of SPX index. We will use weekly differences of this index. The rationale for this variable is its very high correlation with equity market crashes.

Country specific variables are aggregated in two groups, separately for developed and emerging countries. They are built based on rates and currency fluctuations:

1. STdev - short-term rates for developed countries. This variable will be built based on single short-term rates for developed countries. It is moving average of weekly returns from ST rates.
2. STdev - short-term rates for emerging countries. This variable will be built based on single short-term rates for emerging countries. It is moving average of weekly returns from ST rates.
3. LTdev - long-term rates for developed countries. This variable will be built based on single long-term rates for developed countries. It is moving average of weekly returns from LT rates.
4. LTem - long-term rates for emerging countries. This variable will be built based on single long-term rates for emerging countries. It is moving average of weekly returns from LT rates.
5. Cdev - currencies for developed countries. This variable will be built based on single currency rate fluctuations for developed countries. It is moving average of weekly returns from currency rates.
6. Cem- currencies for emerging countries. This variable will be built based on single currency rate fluctuations for emerging countries. It is moving average of weekly returns from currency rates.

Hence, the complete list of models used in the second part of this study is presented below:

1. Carhart+VMC.5F.USD (five-factor model, VMC, factors based on USD),
2. Carhart+VMC+QMS+SMBrel.7F.USD (five-factor model, VMC, QMS and SMBrel added, factors based on USD),
3. Carhart+VMC+CspV.5F.USD (five-factor model, VMC and CspV added, factors based on USD),

4. Carhart+VMC+CV.5F.USD (five-factor model, VMC and CV added, factors based on USD),
5. Carhart+VMC+CV+CspV.5F.USD (five-factor model, VMC, CV and CspV added, factors based on USD),
6. Carhart+VMC+QMS+SMBrel+CV+CspV.7F.USD (seven-factor model, VMC, QMS, SMBrel, CV and CspV added, factors based on USD).

2.3.3 Third part

Based on the results presented in Sakowski, Slepaczuk, and Wywiał (2016a) and preliminary ones from first and second part we decided to test Markov switching model with two states of the world on the same set of variables as presented in Formula (8). Our intuition was that adding new set of variables to Markov switching models could help us to explain such big differences in explanatory power of our models between emerging and developed markets.

Hence, the complete list of models used in the third part of this study is presented below:

1. Carhart+VMC+QMS+SMBrel+CV+CspV.7F.MSM.USD (seven-factor Markov switching model, VMC, QMS, SMBrel, CV and CspV added, factors based on USD),
2. Carhart+VMC+QMS+SMBrel+CV.7F.MSM.USD (seven-factor Markov switching model, VMC, QMS, SMBrel and CV added, factors based on USD),
3. Carhart+VMC+QMS+SMBrel+CspV.7F.MSM.USD (seven-factor Markov switching model, VMC, QMS, SMBrel and CspV added, factors based on USD).
4. Carhart+VMC+QMS+SMBrel+CV+CspV.5F.MSM.USD (five-factor Markov switching model, VMC, QMS, SMBrel, CV and CspV added, factors based on USD),
5. Carhart+VMC+QMS+SMBrel+CV.5F.MSM.USD (five-factor Markov switching model, VMC, QMS, SMBrel and CV added, factors based on USD),
6. Carhart+VMC+QMS+SMBrel+CspV.5F.MSM.USD (five-factor Markov switching model, VMC, QMS, SMBrel and CspV added, factors based on USD).
7. Carhart+VMC.5F.MSM.USD (five-factor Markov switching model, Carhart+VMC, factors based on USD).

2.4 Methodological and diagnostic issues

In the process of estimation of multi-factor models using time-series data, we can potentially suffer from several econometric problems or issues which should be solved in the process of estimation (possible ARCH effect, autocorrelation, heteroscedasticity of the error term or differences between various methods of estimation of our models). Surprisingly, this issue is barely ever discussed in the literature of multi-factor models.

An attempt to estimate our regressions correctly with full econometric diagnostics takes us to the point where we should go one of the below paths:

1. To estimate models with the same functional forms and compare their results across all markets, ignoring any diagnostic issues, as it is presented in financial literature for years.

2. To perform all diagnostics concerning time-series issues and correct the first estimations, which will most probably result in different model functional forms across investigated markets and hence make it difficult to compare results for them.

Taking into account that we do intend to compare single alpha, beta coefficients and R squared coefficient among equity indices, we decided to choose the first approach. We believe that this allows to analyze explaining power of models estimated for different markets. All in all, the issue of performing model diagnostics seems to be important and we decided to devote to this issue separate paper Sakowski, Slepaczuk, and Wywiał (2016b).

3 Data

We gathered the data for the most comprehensive set of investable equity indices³ covering the period between 1990 and 2015. However the study was intentionally limited to 2000-2015 because of unavailability of longer time series for some of risk factors, especially for emerging markets countries.

The analysis was performed on weekly data for 81 most representative and investable equity indices, covering all continents. We included data for 27 developed and 54 emerging markets indices. The detailed list of all equity indices, risk factors and common and country specific variables and their descriptive statistics can be obtained upon request.

The reason behind selection of weekly instead of monthly data was the intention to evaluate theoretical value of excess returns for the given equity index more frequently. All returns and risk factors, with exception of the four-factor model of Carhart with factors based on local currency (Carhart.4F.localcurrency), were calculated after converting local prices to USD. Surprisingly however, results did not differ significantly between the same model calculated in local currency and in USD.

3.1 Description of risk factors

Detailed analysis of dynamics of standard four factors from the Carhart model helped us to define the final specification of the five factor model. Below we present the detailed description of procedure of calculating HML, SMB, VML, VMC, QMS and SMBrel risk factors, definitions of zero-investment portfolios based on them and then our observation concerning these factors dynamics.

The $(R_m - R_f)$ factor represents weekly excess return of the market portfolio over the risk-free rate. The market portfolio consists of 81 equity indices which are equally weighted.

The HML is a zero-investment portfolio that is long on the highest decile group of book-to-market (B/M) equity indices and short on the lowest decile group. The difference of returns of these extreme decile groups is calculated in each weekly interval, which finally constitutes HML factor. Based on these returns we created cumulative returns for HML and then LMH zero investment portfolio.

The SMB is a zero-investment portfolio that is long on the highest decile group of small capitalization (cap) equity indices and short on the lowest decile group. The difference of returns of these extreme decile groups is calculated in weekly interval as well. Similarly, based on these returns we created cumulative returns for SMB and then BMS zero investment portfolio.

³For practical purposes we used only these indices which can be easily invested through options, futures or ETFs.

The WML is a zero-investment portfolio that is long on the highest decile group of previous 1-year return winner equity indices and short on its lowest decile group (loser equity indices). The difference of returns of these extreme decile groups is calculated again for each weekly interval and based on that we create cumulative returns for WML and then LMH zero investment portfolio.

The VMC is a zero-investment portfolio that is long on the highest decile group of high volatility equity indices and short on its lowest decile group (low volatility equity indices). The difference of returns of these extreme decile groups is calculated again for each weekly interval and based on that we create cumulative returns for VMC and then CMV zero investment portfolio.

The QMS factor is a zero-investment portfolio that is long on the highest decile group equity indices characterized by the highest positive percentage deviation of nominal GDP from its 5y moving average and short on its lowest decile group (the highest negative percentage deviation of nominal GDP from its trend). The difference of returns of these extreme decile groups is calculated again for each weekly interval and based on that we create cumulative returns for QMS and then SMQ zero investment portfolio.

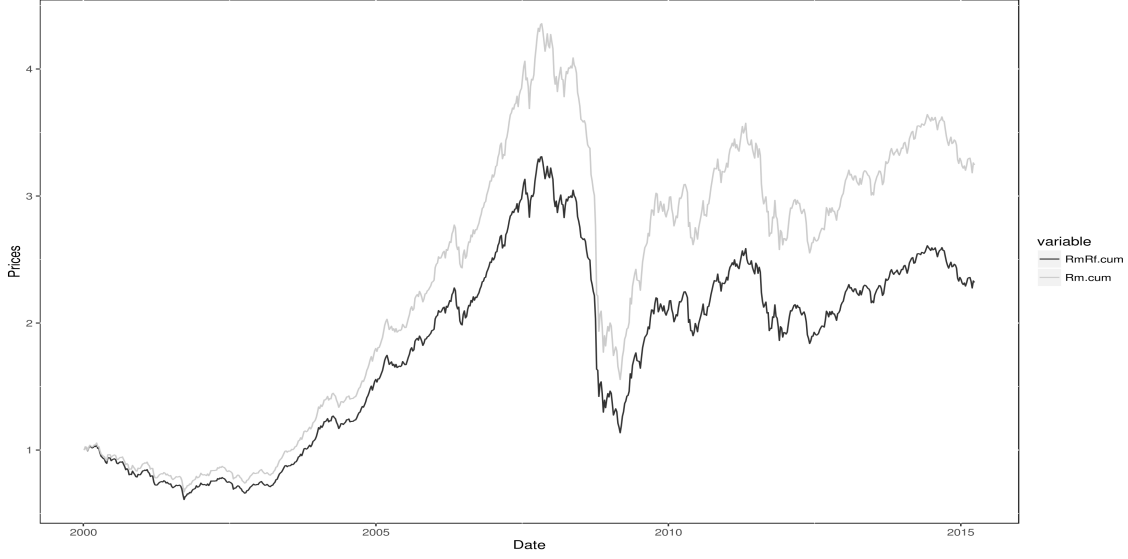
Finally, the SMBrel is a zero-investment portfolio that is long on the highest decile group of equity indices characterized by the lowest ratio of cap to nominal GDP and short on its low highest decile group. The difference of returns of these extreme decile groups is calculated again for each weekly interval and based on that we create cumulative returns for SMBrel and then BMSrel zero investment portfolio.

3.2 Analysis of risk factors dynamics, common variables and country specific variables

3.2.1 Risk Factors

Figure 1 shows the dynamics of the market index factor ($Rm - Rf$) and market index returns (Rm). We cannot observe any substantial differences between them. This actually informs us that we analyzed the period of exceptionally low rates, and that interest rates had only marginal impact on the value of this factor.

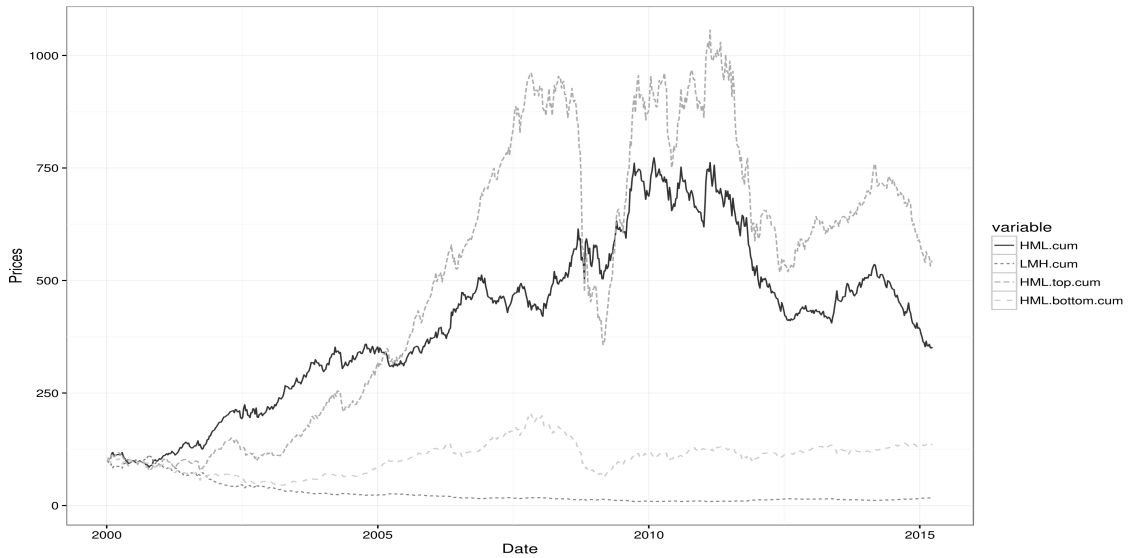
Figure 1: Dynamics of cumulative Rm-Rf factor and separately for market index (Rm).



Rm-Rf factor was calculated on weekly data in USD between 2000-2015. Rm represents equally weighted market index based in USD. Lines present cumulative returns for Rm-Rf and Rm factors.

Figure 2 presents fluctuations of the second factor (HML_t). It reveals two distinct periods. The first one (2000-2011) shows a strong HML effect showing much better performance of equity indices with high book-to-market characteristics. Similar phenomenon has been heavily presented in the literature for stock returns. However, in the second period, starting from 2012, the HML effect disappeared and has been entirely transformed into the LMH effect which is quite surprising and requires additional research.

Figure 2: Cumulative returns of HML factor with top/bottom 10% percentiles

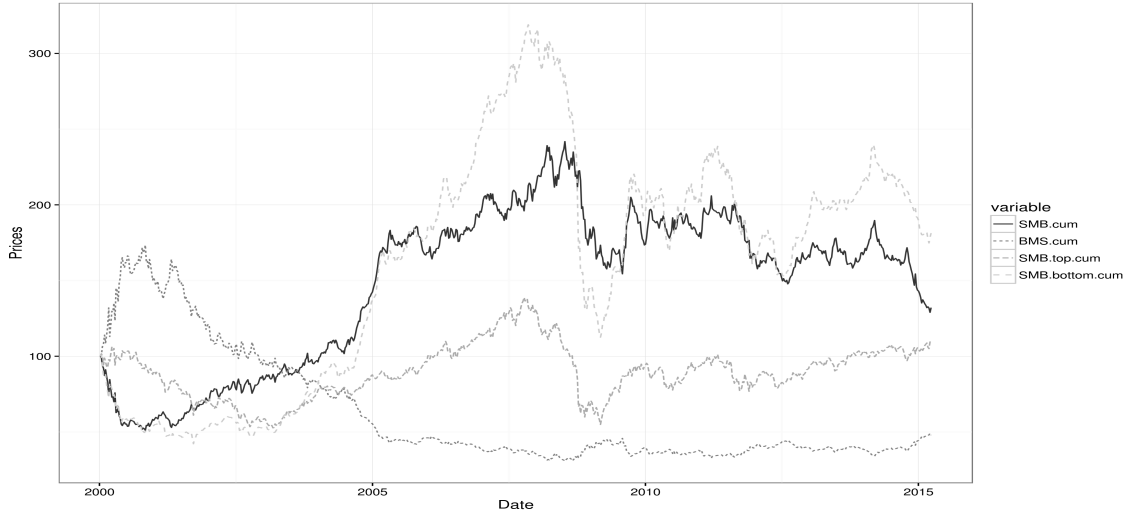


HML factor was calculated on weekly data between 2000-2015. Lines present cumulative returns for, respectively, HML, LMH, top and bottom book-to-market values decile portfolios.

Fluctuations of the third risk factor (SMB_t) are presented in the Figure 3. Again,

it can be divided into two periods. The first one, between 2000-2008, is characterised by outperformance of small capitalisation equity indices what was revealed in the literature for single stocks. In the second period (2008-2015), this effect is totally reversed and we can observe outperformance of big capitalisation equity indices.

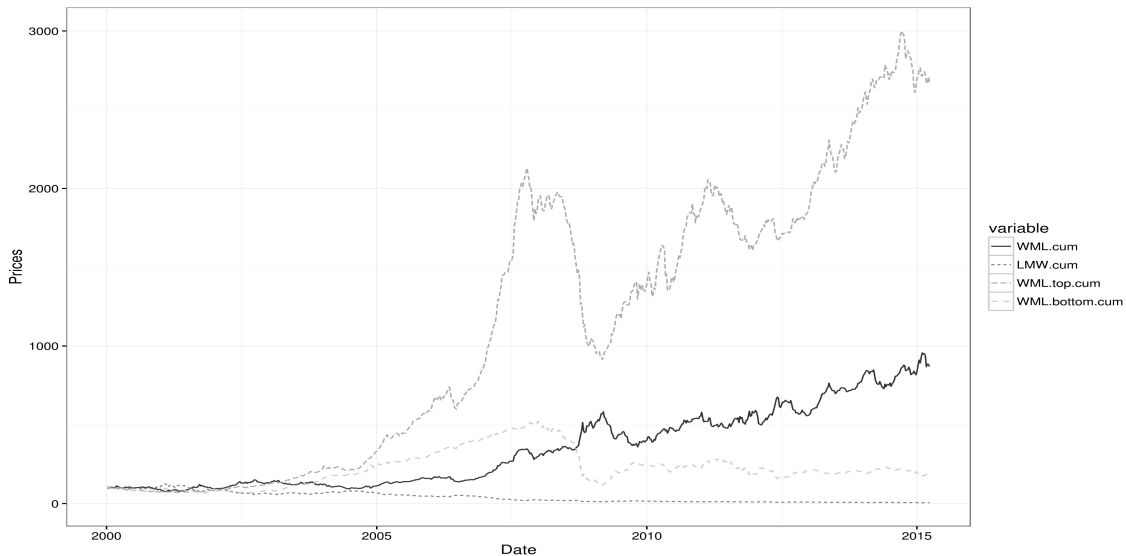
Figure 3: Cumulative returns of SMB factor with top/bottom 10% percentiles.



SMB factor was calculated on weekly data in USD between 2000-2015. Lines present cumulative returns for, respectively, SMB, BMS, top and bottom capitalization decile portfolios.

The fourth risk factor (WML_t) shows that WML effect is the strongest one (Figure 4) and that it is pretty stable during the whole period and exactly confirms the short-term momentum effect observed in financial literature.

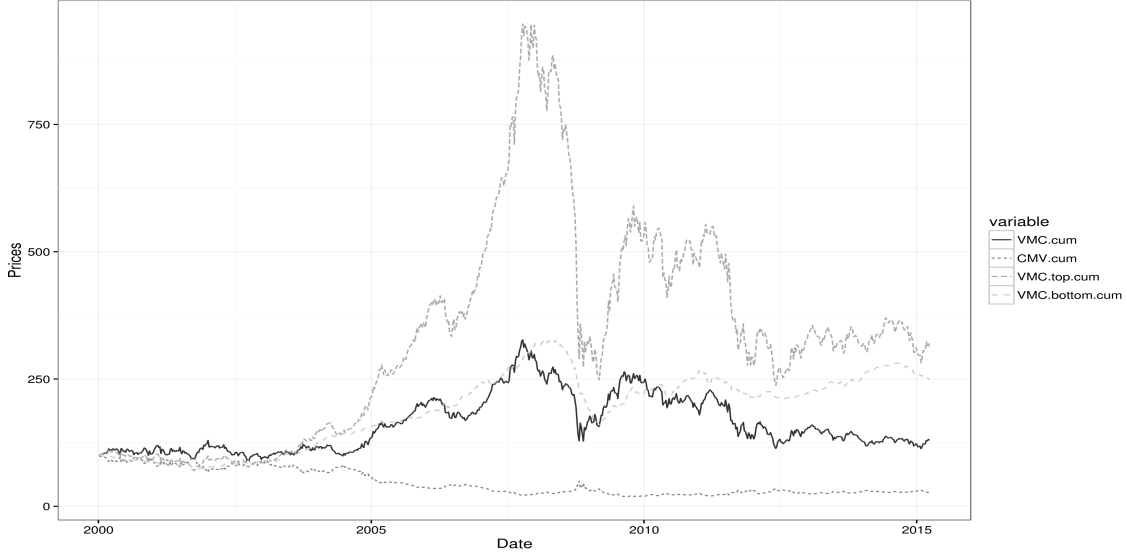
Figure 4: Cumulative returns of WML factor with top/bottom 10% percentiles.



WML factor was calculated on weekly data in USD between 2000-2015, with returns based on last 1 year. Lines present cumulative returns for, respectively, WML, LMW, top and bottom momentum decile portfolios.

The fifth risk factor (VMC_t) reveals similar dynamics to HML and SMB effects (Fig-

Figure 5: Cumulative returns of VMC factor with top/bottom 10% percentiles.

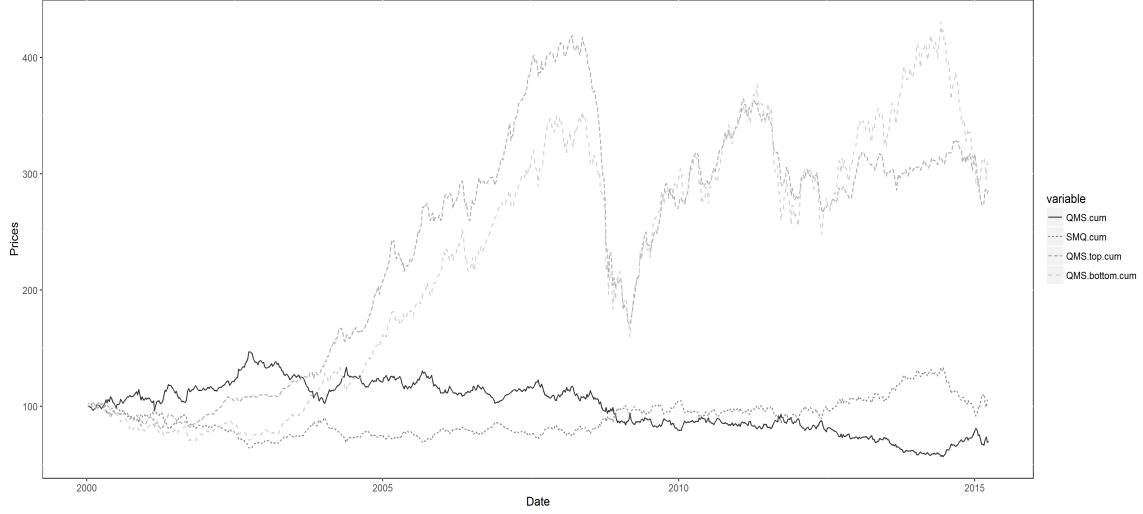


VMC factor was calculated on weekly data in USD between 2000-2015. Lines present cumulative returns for, respectively, VMC, CMV, top and bottom 63 days realized volatility decile portfolios.

ure 5) dividing the period into two different subperiods. The first one ends exactly before bear market in 2008 and is characterised by outperformance of high volatility equity indices. In the second period (2008-2015) this effect is exactly reversed and we can observe outperformance of low volatility equity indices.

The sixth risk factor (QMS_t) built based on percentage deviation from nominal GDP reveals once again similar dynamics to HML and SMB effects (Figure 6) and divides the period into two subperiods. The first one showing better behaviour of QMS portfolios lasted from 2000 to 2003, while the second one characterized by the better behaviour of SMQ portfolios was much longer and lasted from 2003 until 2015.

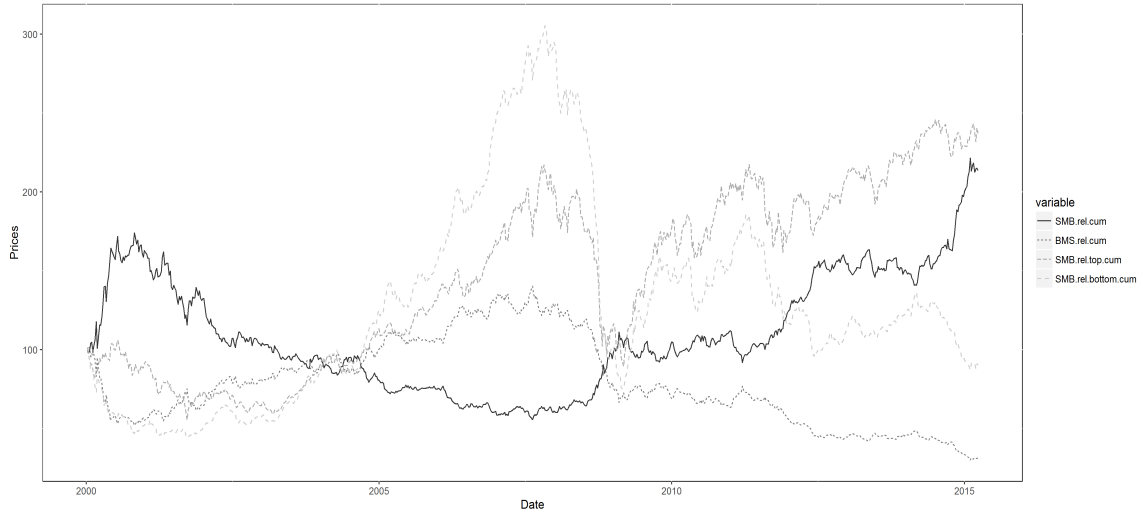
Figure 6: Cumulative returns of QMS factor with top/bottom 10% percentiles.



QMS factor was calculated on weekly data in USD between 2000-2015. Lines present cumulative returns for, respectively, QMS, SMQ, top and bottom decile portfolios.

The seventh risk factor ($SMBrel_t$) reveals completely different dynamics in comparison to quite similar SMB factor (Figure 7). Once again we can distinguish two distinctive periods but currently BMSrel behaves much better in the first period between 2000 and 2008, while SMBrel is significantly stronger in the second period ending in 2015.

Figure 7: Cumulative returns of SMBrel factor with top/bottom 10% percentiles.



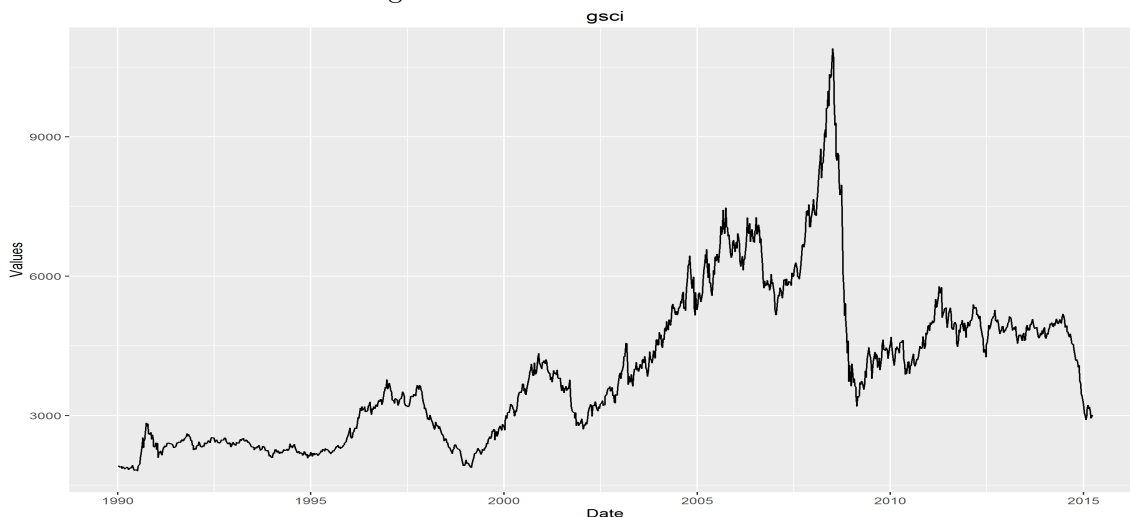
SMBrel factor was calculated on weekly data in USD between 2000-2015. Lines present cumulative returns for, respectively, SMBrel, BMSrel, top and bottom decile portfolios.

Presented dynamics of seven factors suggest that their explanatory power with respect to excess returns can be rather limited with exception of the first factor. What is important is that the analysis of fluctuations of portfolios based on our risk factors in USD in comparison to their dynamics in local currency (Sakowski, Slepaczuk, and Wywiał (2015)) reveals very similar dynamics. This informs us that currency effect is not the main driver which can be used in order to explain these effects.

3.2.2 Common Variables

Before we go to results section it is important to analyse the fluctuations of our new set of variables, named common variables, which are the same for all equity indices under investigation. First one is S&PGSCI (Goldman Sachs Commodity Index) which serves as benchmark for investment in the commodity markets and as a measure of commodity performance over time. The performance of this index in the period 1990-2015 can be found on Figure 8.

Figure 8: S&PGSCI fluctuations



SPGSCI was calculated on weekly data in USD between 1990-2015.

We can distinguish very strong bull market which lasts from the beginning of the eighties until 2008 and then very strong bear market which is present on commodity markets until the end of our data set. The inclusion of this variable in our model was dictated by its strong correlation with emerging markets.

The next variable is CBOE SKEW Index. The CBOE SKEW Index is an index⁴ derived from the price of S&P 500 tail risk. Similar to VIX, the price of S&P 500 tail risk is calculated from the prices of S&P 500 OTM options. SKEW typically ranges from 100 to 150. A SKEW value of 100 means that the perceived distribution of S&P 500 log-returns is normal, and the probability of outlier returns is therefore negligible. As SKEW rises above 100, the left tail of the S&P 500 distribution acquires more weight, and the probabilities of outlier returns become more significant. The fluctuations of SKEW can be found on Figure 9.

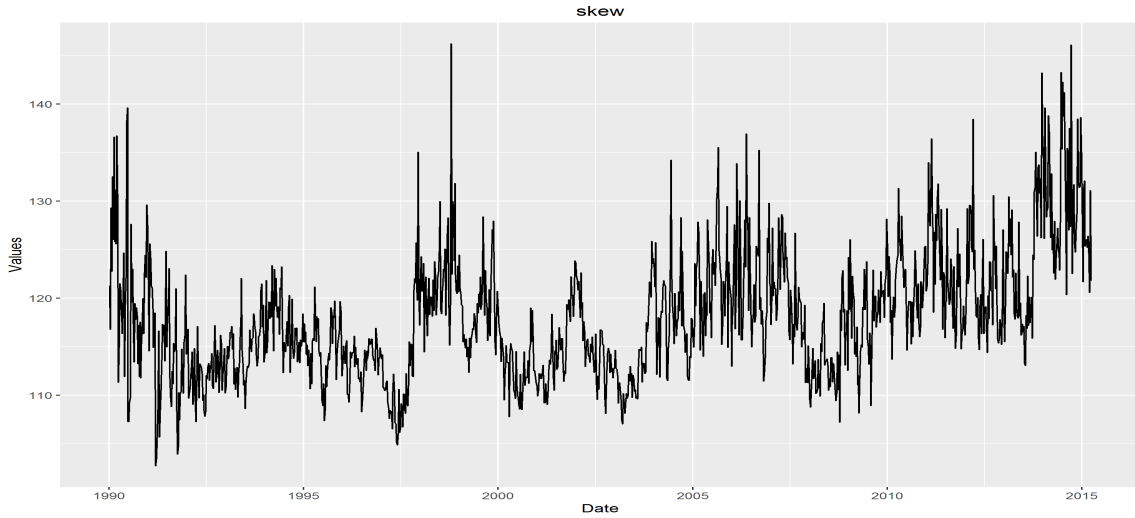
We can see that this measure most often ranges between 110 and 130 confirming that distribution of returns of S&P500 index rather departs from normal one. We included this variable in our models in order to be able to refer to high stress periods on capital markets.

The subsequent variable used in our new seven-factor model is OVXGVZ, which is the average of annualized monthly RV of crude oil prices and GVZ volatility index published by CBOE based on the same methodology like for VIX⁵. Our intention was to connect volatility of two main commodities (crude oil and gold) in one index in order to reflect their

⁴The definition was found on CBOE website: <http://www.cboe.com/micro/skew/introduction.aspx>

⁵The definition of GVZ index can be found on the following CBOE website: <http://www.cboe.com/micro/gvz/introduction.aspx>

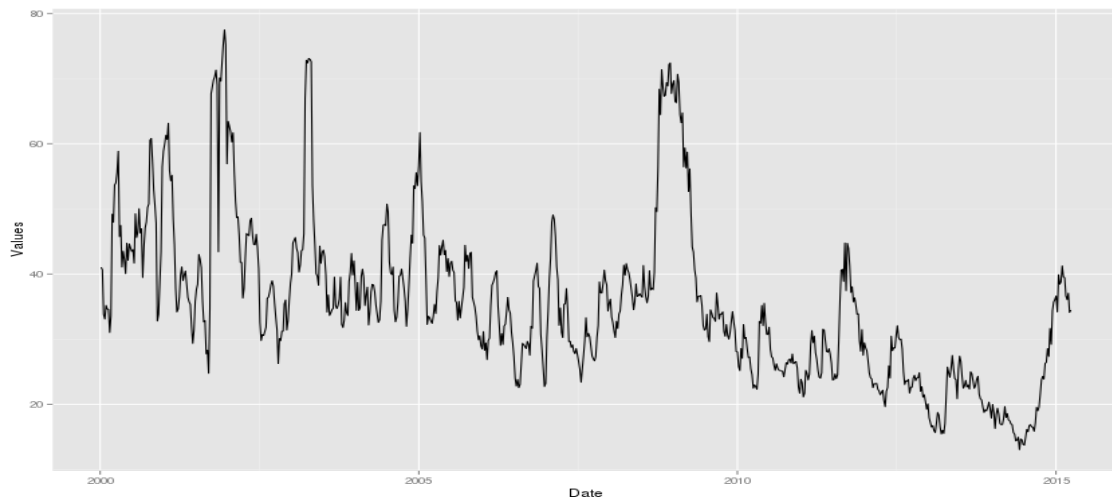
Figure 9: Skew



Skew is presented on weekly data between 1990-2015.

connection with emerging market economies. The behaviour of OVXGVZ index is presented on Figure 10.

Figure 10: OVXGVZ index fluctuations



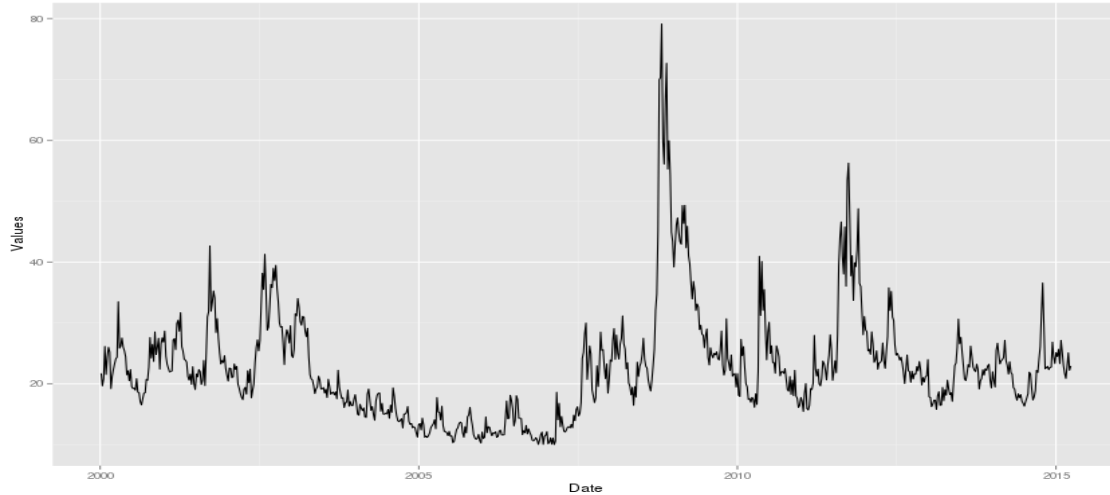
OVXGVZ was calculated on weekly data between 2008-2015. OVXGVZ is calculated as the average of annualized monthly RV of crude oil prices and GVZ (volatility index of gold).

Figure 10 shows that the average level of volatility presented by OVXGVZ index went gradually lower during presented 15years period but at the same time volatility jumes were much severe, partly because much lower starting point.

Next variable (VX) once again refers to volatility and is the average of VIX (volatility index of S&P500 index), VXEEM (volatility index of emerging markets EEM ETF, the iShares MSCI Emerging Markets Index), VXEWS (volatility index of Brazil markets EWZ ETF) and VXFBI (volatility index of China markets FXI ETF) volatility indices published

by CBOE⁶. The fluctuations of VX variable are shown on Figure 11.

Figure 11: VX index



VX index was calculated on weekly data between 2011-2015. VX is calculated as a average between values of VXEEM (volatility of emerging equity indices) and VXEWS index (volatility of Brazil equity index) and VXFBI (volatility of China equity index).

We can see that 90% of fluctuations of our index ranges between 15 and 25 but it explodes even to 80 in the time of stress on equity markets. therefore we presume that it should be correlated with markets turmoils on equity markets.

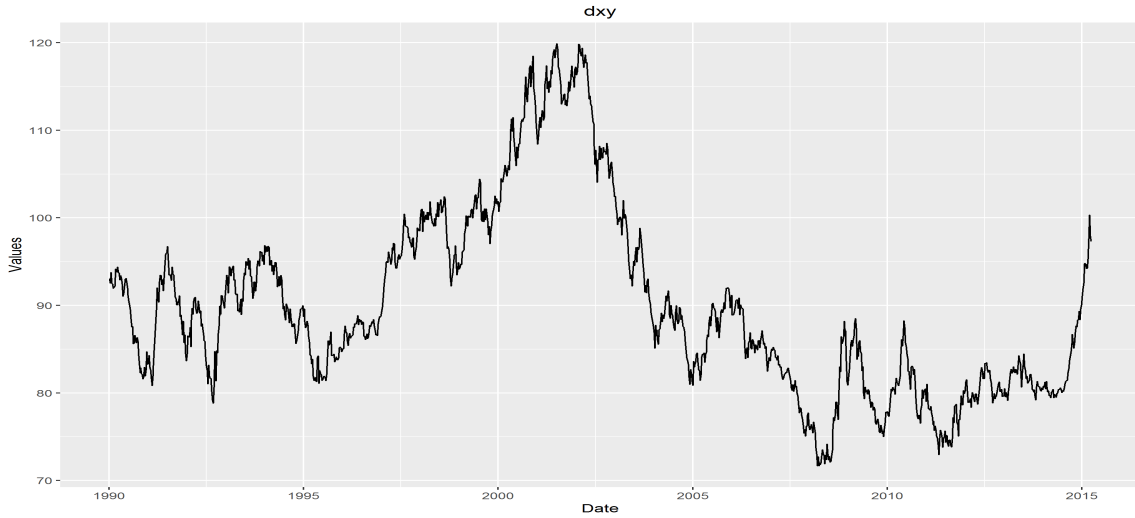
Next variable is US Dollar Index (DXY) which is an index (or measure) of the value of the United States dollar relative to a basket of foreign currencies, often referred to as a basket of US trade partners' currencies. Once again, our main intention to add this variable to our model was to indicate the time of stress on capital markets which usually characterize by "flight to hard currencies", mainly USD. The behaviour of DXY is presented on Figure 12.

Figure 12 shows two strong upward trends on DXY index which lasted between 1995 and 2002 (culminating at the bottom of bear market after internet bubble) and current one which started in 2011.

The subsequent variable (CLF_dummy) was dummy variable which had value of 1 when Crude oil price is lower than 65 USD and 0 otherwise. This variable was intentionally created as a dummy one only in order to indicate period of high and low crude oil prices which are pretty important for many emerging markets economies. Figure 13 shows the fluctuations of CLF_dummy.

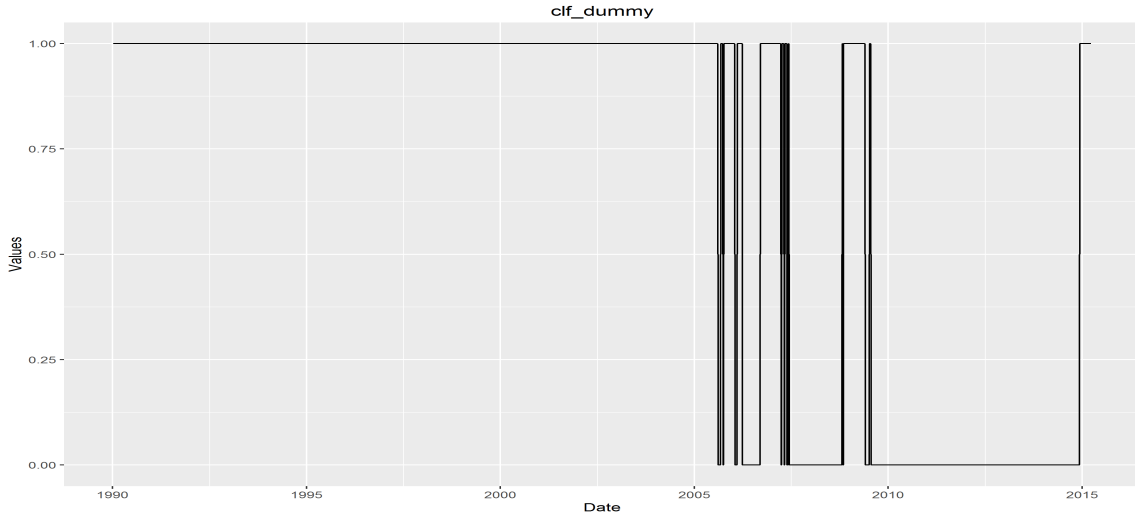
⁶The definitions of these volatility indices can be found on the following CBOE websites: <http://www.cboe.com/micro/vix-options-and-futures.aspx>, <http://www.cboe.com/micro/vixetf/vxeem/>, <http://www.cboe.com/micro/vixetf/vxewz/>, <http://www.cboe.com/micro/vixetf/vxfbi/>

Figure 12: DXY index



DXY was calculated on weekly data between 1990-2015.

Figure 13: CLF_dummy



SPGSCI was calculated on weekly data in USD between 1990-2015.

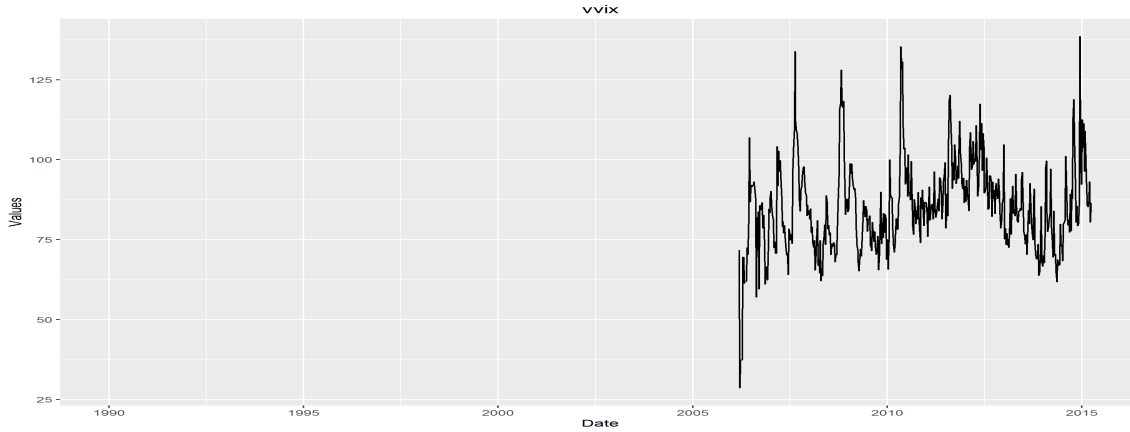
Last variable among common variables is VVIX, once again calculated by CBOE, as volatility of volatility of S&P500 index⁷. The rationale for this variable is its very high correlation with equity market crashes. We decided to use weekly differences of this index in our model. The behaviour of this index can be found on Figure 14.

We can see that for the majority of time VVIX fluctuates between 70 and 110 and it spikes even to 200 in the time of equity market turmoils.

Generally, we can summarize that we have chosen two set of common variables: widely connected with the behaviour of emergin market economies (S&PGSCI, OVXGVZ or CLF_dummy) or the ones indicating equity or commodity markets turmoils (OVXGVX, VX, SKEW or VVIX).

⁷The definition of this index can be found here: <http://www.cboe.com/micro/vvix/>

Figure 14: VVIX fluctuations



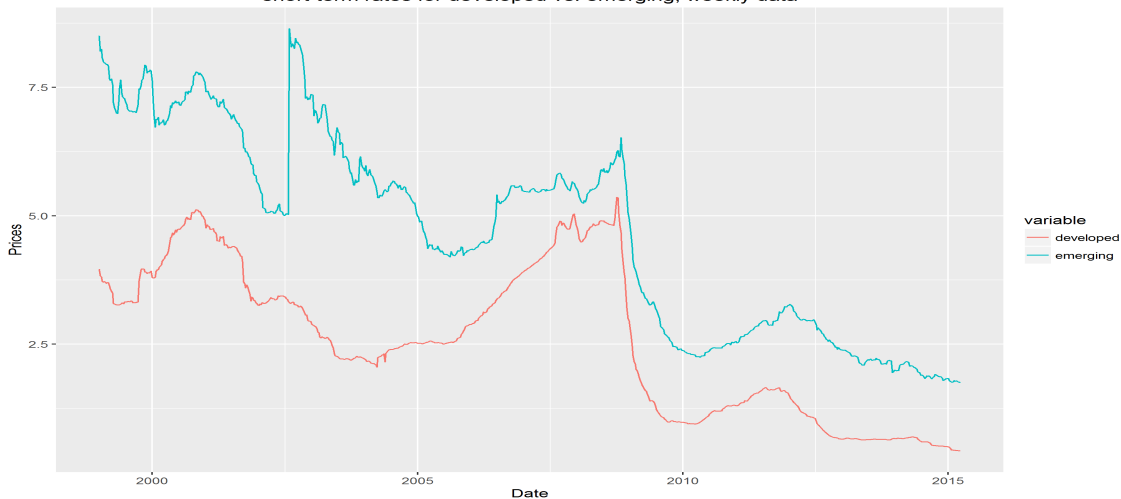
VVIX was calculated on weekly data in USD between 2007-2015.

3.2.3 Country Specific Variables

The last set of variables which were introduced to our model were country specific variables, i.e. variables that were aggregated in two groups, separately for developed and emerging countries. They were built based on rates and currency rates fluctuations.

Figure 15 presents fluctuations of short-term interest rates separately for developed and emerging markets countries. We can see that through the whole period under investigation “the average rate” was in downward trend with one substantial spike before financial crisis in 2008. The average rate of short-term rates decreased in this period from 8% for emerging markets economies (4% in case of developed ones) to 2.5% (and around 1% for developed ones).

Figure 15: Short-term interest rates for developed vs. emerging market countries, short-term rates for developed vs. emerging, weekly data

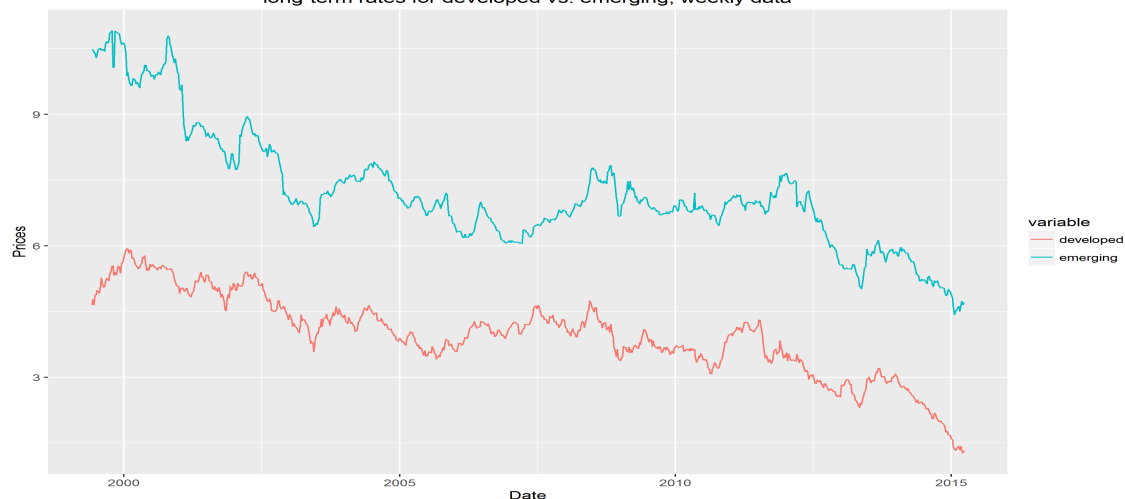


Short-term interest rates were calculated on weekly data in USD between 2000-2015. Lines present average values of short-term interest rates separately for emerging (upper line) and developed countries (lower line).

The similar situation is presented on Figure 16 where we observe long-term interest rates separately for developed and emerging markets. The fluctuations are quite similar to

short-term rates but their path downward is even more stable in case of long-term rates.

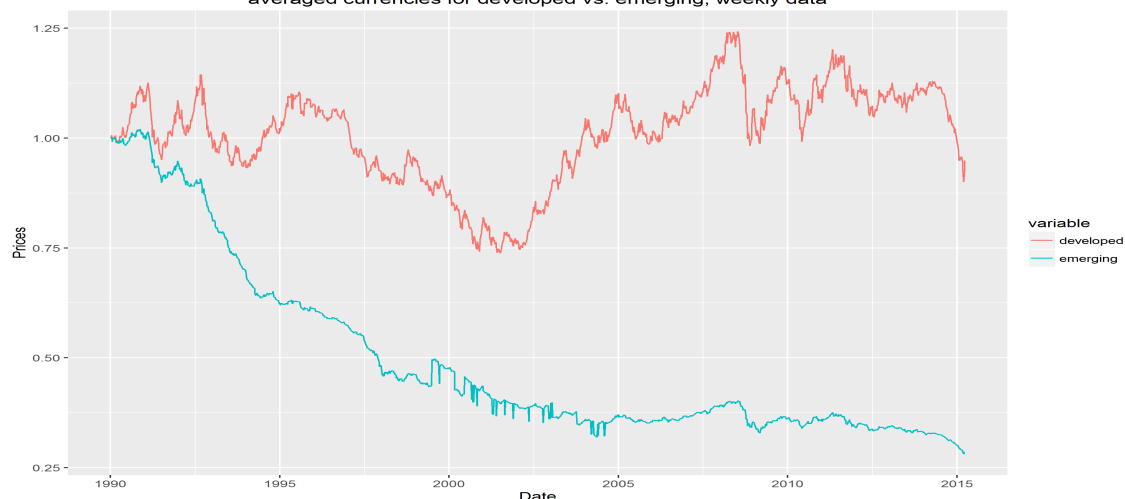
Figure 16: Long-term interest rates for developed vs. emerging,
long-term rates for developed vs. emerging, weekly data



Long-term interest rates were calculated on weekly data in USD between 2000-2015. Lines present average values of long-term interest rates separately for emerging (upper line) and developed countries (lower line).

The last set of variables in the group of country specific ones are currency rates once again separately aggregated for developed and emerging markets. Figure 17 shows their fluctuations. We can see depreciation of emerging market currencies against USD through all the period between 1990 and 2015. Quite different situation was observed in case of developed markets currencies which depreciated against USD in two period: 1990-2002 and 2009-2015 and appreciated between this two periods.

Figure 17: Currency indices for developed and emerging,
averaged currencies for developed vs. emerging, weekly data



Currency indices were calculated on weekly data in USD between 2000-2015. Lines present average values of currency index calculated separately for emerging (lower line) and developed countries (upper line). Currency index is calculated based on avarege currency returns calculated separately for emerging and developed countries.

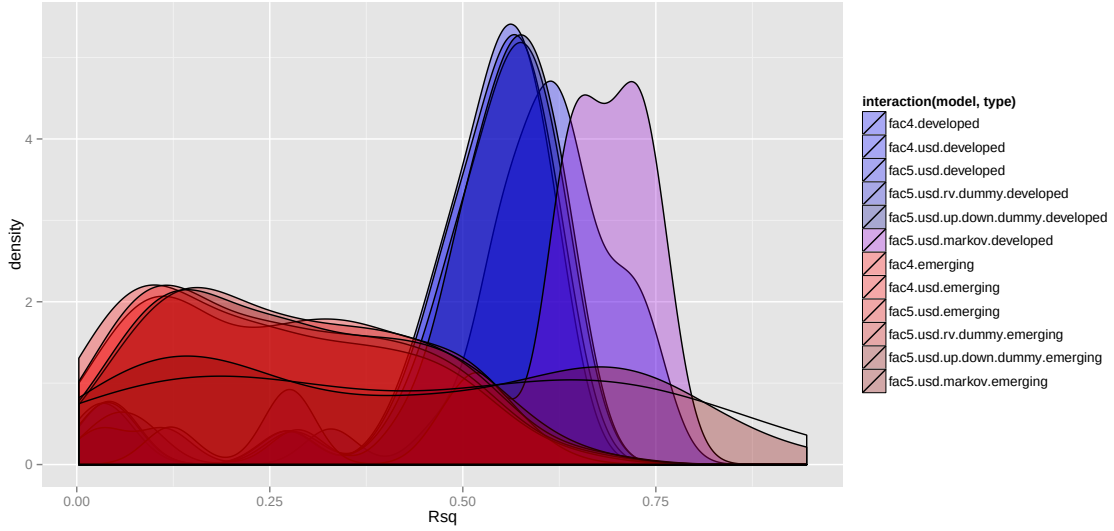
4 Results

4.1 Comparison of various models' results

4.1.1 First part of research. Variations of five factor models.

After estimation of six multi-factor models from the first step of our research (six first regressions described in chapter 2.3) for 81 different countries it is really hard to present results in a form that is understandable to the reader. For clarity purposes, we start with comparison of densities of RSquared coefficients for all models separately for developed and emerging markets (Figure 18).

Figure 18: Density of RSquared coefficients for six models from the first part of research, separately for developed and emerging countries.



Densities of RSquared coefficients for emerging markets are marked with red colors palette, while densities of RSquared for developed countries are marked with blue colors palette. We can see that densities of RSquared for developed countries are centered around much higher values in comparison to densities for emerging ones.

Such comparison enable us to formulate our first conclusion that the results of six multi-factor models (including regime switching mechanism as well) do not differ significantly for all countries. This conclusion does not change when we analyse results separately for emerging and developed countries. However, focusing on the results between two groups of emerging and developed countries we come to two crucial observations.

First, the highest explanatory power of the five-factor model can be observed for developed equity indices (blue palette densities in comparison to red palette ones). In this group almost all RSquared values are higher than 50%. On the other hand, for emerging markets they get much lower values and are characterized with higher dispersion. This conclusion does not differ when we analyse the results of different models tested.

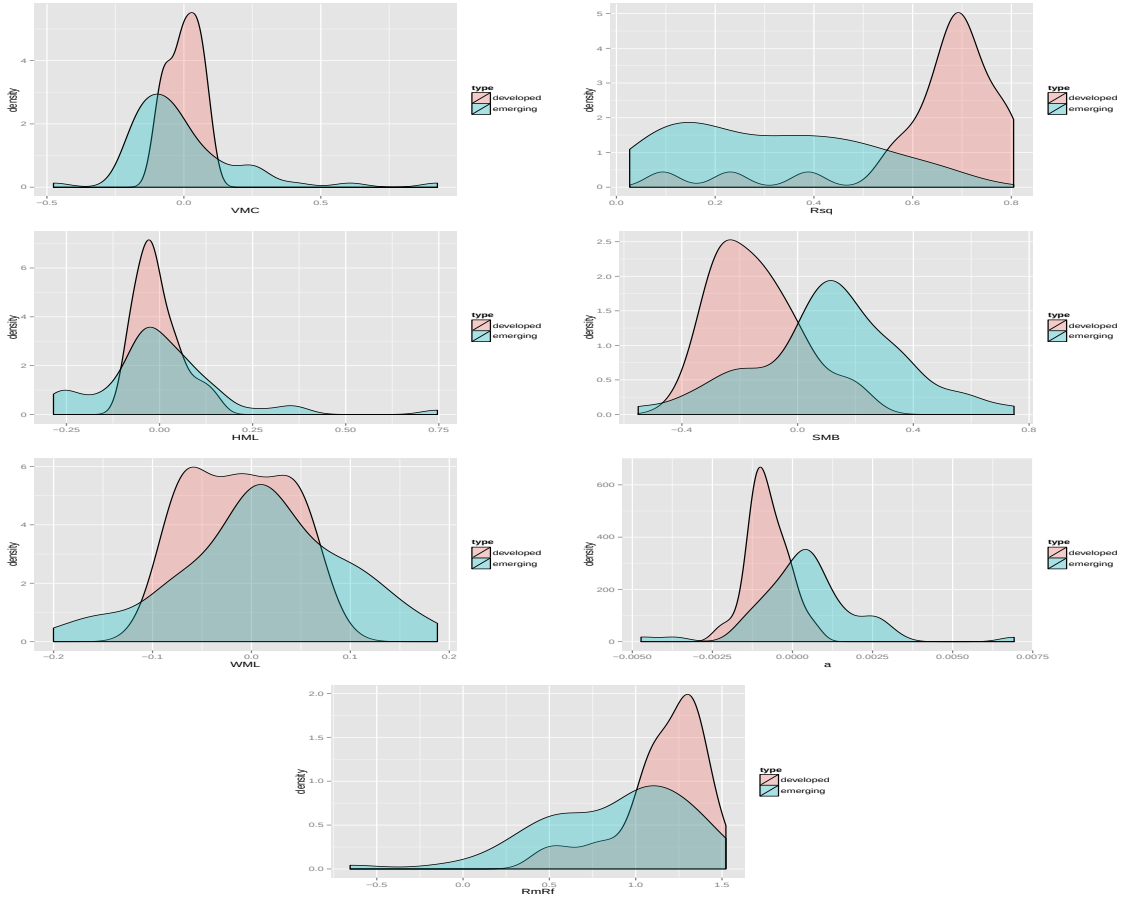
The second issue noticed here, partly connected with the first one, is that multifactor models have correct functional form for developed countries while they could be misspecified for the emerging markets subgroup. The reason for this difference could be that majority of all models proposed during the last 30 years were prepared for developed countries on the basis of thorough empirical investigations of developed market data while emerging market

data were practically unavailable. In the final part of results section we try to present rationale for this phenomenon.

Taking into account the fact that one of our main outcomes from this part is that results do not differ significantly between tested models we decided to focus on interpretation of the five-factor model (Carhart+VMC.5F.USD) which then will be the benchmark model for the second and third part of the research section. Our results for equity indices are in many ways quite similar to well know studies for stock returns (Lieksnis (2010), Davis, Fama, and French (2000)), however they do not reveal so strong effects connected with our risk factors as was presented in the literature before.

In order to draw more conclusions with regard to different results for developed and emerging markets, we analysed the densities of parameters estimates and Rsquared coefficients separately for these two types of equity indices (Figure 19).

Figure 19: Density of parameters estimates and Rsquared separately for developed and emerging equity indices.



The data cover the period between 2000-2015. Five-factor model. Factors based on USD.

Our additional conclusions can be summarized as follows:

1. The results of regressions for developed countries with highest Rsquared coefficients have negative (but close to zero) alpha coefficients (significant in 50% of cases) which informs us that there was no any excess returns which were not explained by our five-factor model. On the other hand, on average alpha coefficients for emerging equity

indices are positive but still rather insignificant. It could mean that our model for emerging markets require further investigations.

2. Beta for $(R_m - R_f)$ factor is on average higher for developed countries and additionally less diversified across countries in comparison to emerging markets.
3. The sensitivity to HML factor is similar for developed and emerging markets, however again it is much more diversified for emerging equity indices.
4. The average values of SMB beta are negative for developed countries and lower in comparison to emerging markets, however their diversity is much higher for emerging markets once again.
5. Characteristics of WML beta estimates is very similar between emerging and developed markets.
6. The only important observation concerning VMC beta is that they are close to zero and one more time dispersion is much higher among emerging markets,
7. Separate densities for Rsquared for developed and emerging markets confirmed previous observations based on figure 18 that regression for developed markets have significantly higher explanatory power than these for emerging markets.

These observations suggest that five-factor model can be a quite robust approach for developed markets and then it is characterised with high explanatory power. However, it should be amended and enhanced with additional risk factors and probably some state variables for emerging markets. We decided to do this in the second part of this research.

4.1.2 Second part of research. Seven factor models

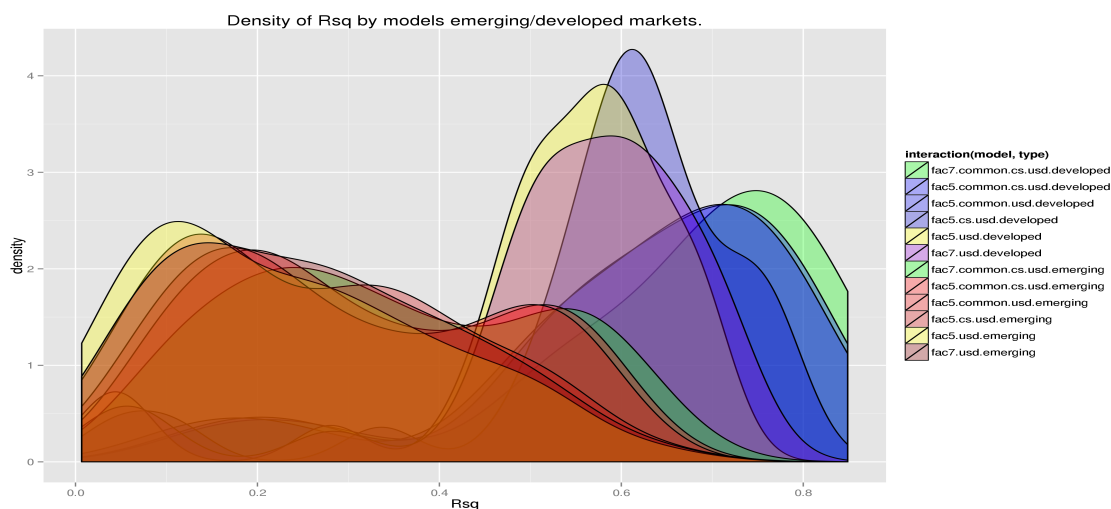
In the second part of the research we decided to introduce several amendments to our models (new risk factors, common variables and country specific variables described in details in section 2.3). This step resulted in new estimations of our 5-factor and 7-factor models. Densities of RSquared coefficients for these new models can be seen on Figure 20.

The most important observation based on Figure 20 is that densities for all new models are moved to the right side what means that on average they had higher explanatory power in comparison to the set of models from the first part. This feature can be seen both for developed countries and emerging ones. Moreover, the main conclusions from the first part stating that our models for developed countries have higher explanatory power in comparison to emerging ones is still valid.

We can observe that new set of variables (CV and CspV) had significant influence on our models (densities of RSquared for these models are the farthest on the right). We can notice that the best model occurred to be Carhart+VMC+QMS+SMBrel+CV+CspV.7F.USD, i.e. the model with 2 additional risk factors, common variables and country specific variables. What is more important is that based on new amendments we can distinguish two separate groups of equity indices among emerging markets countries. It can be seen based on two-modal shape of RSquared densities for emerging countries moved the farthest to the right.

The most convincing explanation of the above phenomenon is that currently we can observe three types of countries when we consider the level of development instead of only two ones (emerging and developed). This third group consist of countries which according to the most popular classifications are still at emerging markets group (like Poland or South

Figure 20: Density of RSquare coefficients for six models from the second part of research, separately for developed and emerging countries.

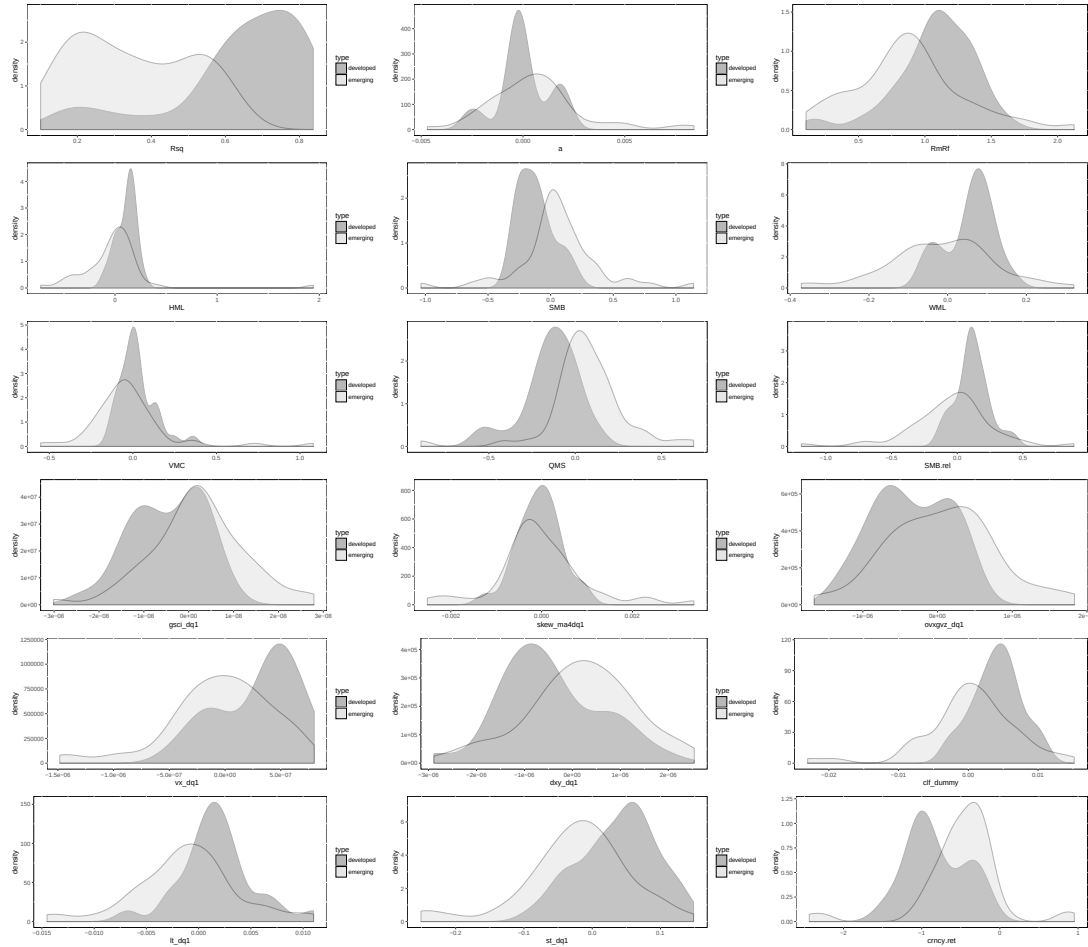


Densities of RSquared coefficients for emerging markets are marked with red colours palette, while densities of RSquared for developed countries are marked with blue colors palette. We can see that densities of RSquared for developed countries are centered around much higher values in comparison to densities for emerging ones and what is more important densities for new models are moved to the right hand side in comparison to benchmark model from the first part (intentionally indicated with yellow colour - Carhart+VMC.5F.USD). The best model is indicated with green colour (Carhart+VMC+QMS+SMBrel+CV+CspV.7F.USD).

Korea) but they behave rather like developed country and many of their economic indicators and measures are much closer to these for developed ones.

Above observations enticed us to thoroughly analyse densisties of parameters for our seven factor model with additonal variables (Carhart+VMC+QMS+SMBrel+CV+CspV.7F.USD) what can be seen on Figure 21.

Figure 21: Density of parameters estimates and Rsquared for seven-factor model separately for developed and emerging equity indices.



The data cover the period between 2000-2015. Seven-factor model with Cv and CspV. Factors based on USD.

The observations based on densities of parameters estimates of seven factor model can be compared with these for 5-factor model:

1. Densities of Rsquared for developed and emerging markets confirmed previous observations based on figure 20 that regressions for developed markets have higher explanatory power than these for emerging markets.
2. Density of alpha parameter did not change significantly.
3. Beta for $(R_m - R_f)$ factor for developed countries is on average higher than 1 and higher than average for emerging market countries and higher than beta for $(R_m - R_f)$ for five-factor model.
4. The sensitivity to HML factor for developed markets, is higher than for five-factor model while we do not observe any significant differences in comparison to emerging markets.
5. The average values of SMB beta did not change at all.

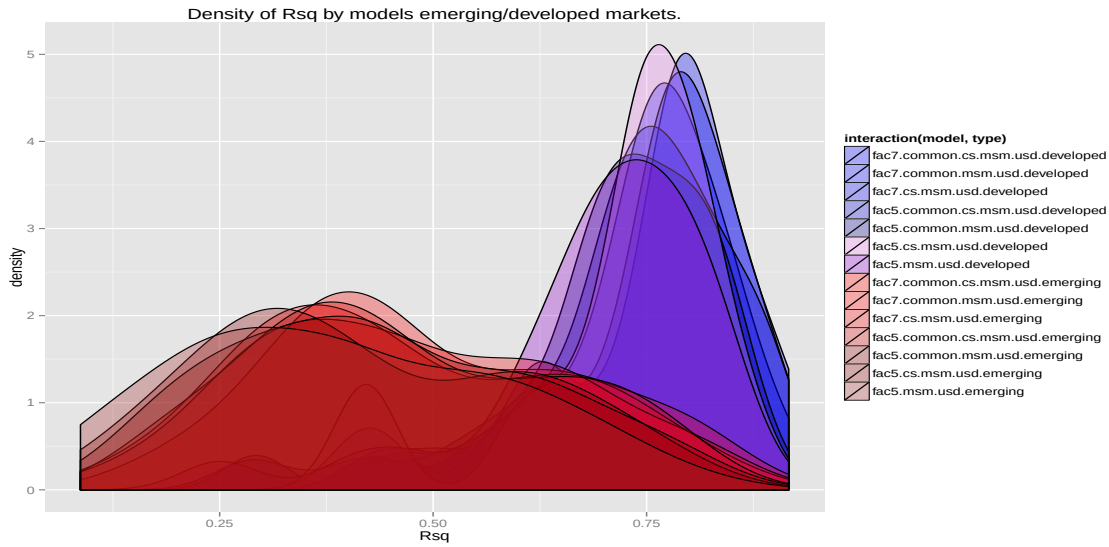
6. Characteristics of WML beta estimates for developed countries is on average higher than WML beta for emerging ones.
7. The average values of VMC beta did not change at all.
8. The average values of QMS beta did not differ significantly between developed and emerging markets countries.
9. Characteristics of SMBrel beta estimates for developed countries is on average higher than SMBrel beta for emerging markets and additionally it is less dispersed.
10. Analysing the sensitivity to common and country specific variables we can say that they were very similar between emerging and developed countries.

Summarizing the results from the second step we can say that new variables and risk factors add value but main conclusions from the first step are still valid. In the third section we will analyze results of Markov switching models described in section 2.3.3.

4.1.3 Third part of research. Seven factor Markov switching models

The third part of results is devoted to Markov switching models based on the same set of variables as used in the second part. Figure 22 presents densities of RSquare coefficients for six Markov switching models analysed in this section.

Figure 22: SMDensity of RSquare coefficients for six Markov switching models from the third part of research, separately for developed and emerging countries.



Densities of RSquared coefficients for emerging markets are marked with red colors palette, while densities of RSquared for developed countries are marked with blue colors palette. We can see that densities of RSquared for developed countries are centered around much higher values in comparison to densities for emerging ones.

What is important here is that the results for Markov switching models are much less dispersed inside groups of emerging and developed countries although we still observed much higher average RSquared values for developed ones. The second important issue in case of the results of the third part is that after adding new variables and risk factors to our model the previous advantage of Markov switching models observed in the first section is not so

obvious currently. We can even say that the difference between RSquared presented on Figure 22 is really insignificant.

In the next section we will try to understand why our results are so different in comparison to these observed for multifactor models single stocks for the last several decades.

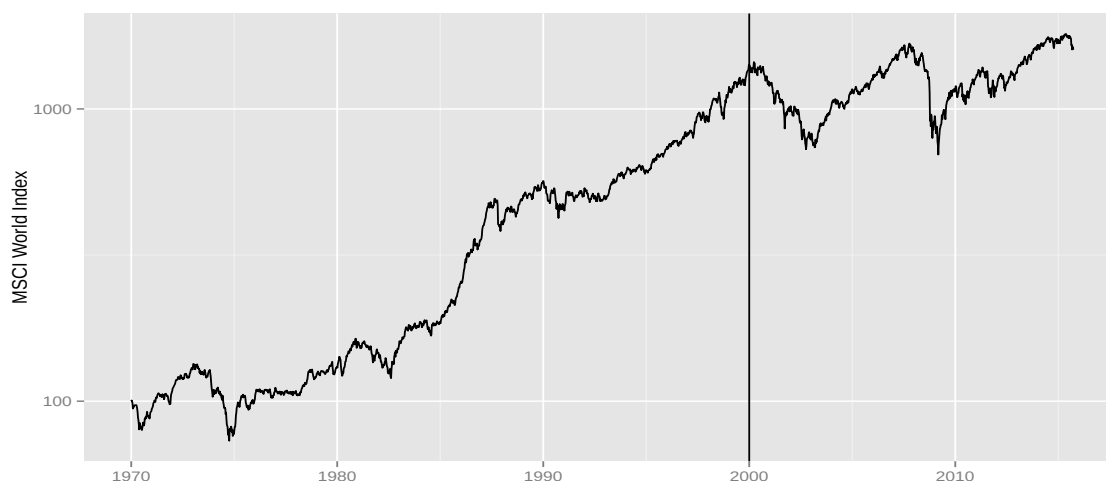
4.2 Explanation of results

Having analysed our results we can ask a natural question why they are different than those presented in the literature for single stocks. We would like to investigate why our multi-factor models are not able to fully explain variability of excess returns for emerging markets.

We see four possible explanations for such results.

The first one is the different time span. In most of previous studies the data covered more than 80% of bull markets - from late 1960 until the beginning of 2000s (Figure 23). Contrary to that, in our research we had two distinct bull markets (2003-2007 and 2009-2015) and two distinct bear markets (2000-2003 and 2007-2009) what resulted in rather horizontal long-term trend during the last 15 years. These could be the reason for substantially lower R-squared in our research when compared with results of studies for equity stock returns.

Figure 23: Dynamics of MSCI World index 1969-2015



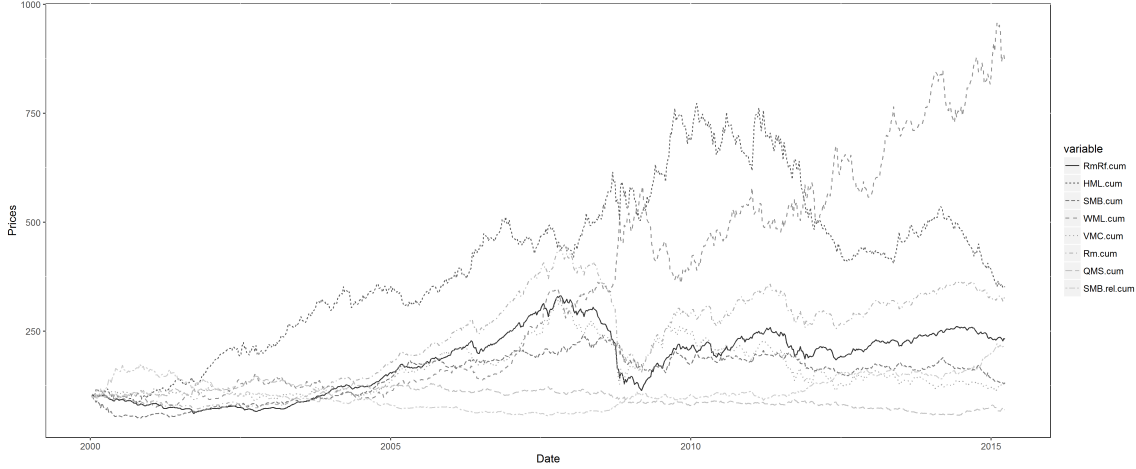
This chart presents weekly data for MSCI index in the period between 1969-2015. MSCI index is cap weighted index for global equity indices.

The second reason could be different explanatory power of risk factors in a strong upward trend vs up and down movements in horizontal trend. This is well illustrated on Figure 24, where behaviour of zero-cost portfolios built based on our risk factors are presented with comparison to fluctuations of broad index (RM.cum).

The third explanation is that most of previous studies used mainly the data for developed markets. Moreover, various modifications of multifactor models (i.e. additional factors, functional form) were introduced based on the analysis of such data, what in our opinion could illustrate strong overfitting bias and model risk.

Lastly, the reason could be also different time frequency. We used weekly data instead of monthly data as we want to explain variability of excess returns on more frequent basis.

Figure 24: Risk factors from the seven-factor model versus market index returns (R_m).



This chart presents risk factors dynamics in the period between 2000-2015 for our seven factor model.

The interpretation presented above is only a possible explanation of different results in our research. However, if it turns out to be the correct one, then it is a very convincing example of data overfitting problem and model risk what was repeated for quite long period of time.

5 Summary

It is important to underline that these results are only the first part of rather larger attempt to fully understand reasons behind the variability of world equity indices excess returns, with special attention to various behaviour of developed markets in comparison to emerging ones.

The most surprising issue concerning our results is that the differences among various multi-factor models are not very significant but last attempt to add new risk factors and additional variables showed that there is some field for models improvements.

Our second observation is that, we observe substantial differences between model explanatory power for developed and emerging markets. In this group almost all R-squared values are higher than 50%. On the other hand, for emerging markets they get much lower values. This conclusion does not differ when we analyse the results of different models tested.

These two points lead us to our major hypothesis which is at least partially confirmed: multi-factor models do have a correct functional form for developed countries, while they could be misspecified for the emerging markets subgroup. Therefore, we claim that results for emerging market equity indices require further investigation and future research should be focused mainly on two issues.

The first is the continuous search for additional factors. There are numerous ideas present in the single stocks literature that are not yet researched in our current context. This includes for example liquidity risk (Rahim and Noor 2006, Liu 2004), return-on-equity effects, earning surprises or macro surprises, systemic risk, liquidity risk or betting-against-beta effects (Frazzini, Pedersen 2014)). Of course, the obtained model should be sparse and robust. Secondly, researchers should concentrate on the novel model implementations concerning its functional form and introducing state variables. Our research confirms that

both of these paths lead to some improvement in the quality of estimates as well as increases in explanatory power. Two additional factors - percentage deviation from nominal GDP and relative capitalisation factor - help to better explain variability of risk premia, in particular when combined with additional macroeconomic variables. It seems however that the most important effect, especially in the case of developed markets, is brought by addition of Markov regime switches.

To conclude, we believe that further research should additionally address the questions whether:

- sensitivities to risk factors are stable during various phases of economic cycles,
- correlations among international equity markets differ between high and low volatility periods,
- we can build a zero investment portfolio with positive alpha based on analysed risk factors.

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