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CAN WE INVEST BASED ON EQUITY RISK PREMIA AND RISK FACTORS FROM MULTI-FACTOR MODELS?

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Can We Invest Based on Equity Risk Premia and Risk Factors from Multi-Factor Models?

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Abstract

We find that detailed analysis of multi-factor models makes it possible to propose investment strategies based on equity risk premium disequilibrium. We examine two investment algorithms built on weekly data of world equity indices for emerging and developed countries in the period of 2000-2015. We create seven risk factors using additional data about market capitalisation, book value, country GDP and betas of equity indices. The first strategy utilises theoretical value of equity risk premium from seven-factor Markov-switching model with variables common for all countries and variables specific to developed/emerging countries. We compare theoretical with realised equity risk premium for a given index to undertake the buy/sell decisions. The second algorithm works only on eight risk factors and applies them as input variables to Markowitz models with alternative optimisation criteria (target risk, target return, maximum Sharpe ratio, minimum variance and equally weighted assets). Finally, we notice that the impact of risk factors on final results of investment strategy is much more important than the selection of a particular econometric model in order to correctly evaluate equity risk premium.

Keywords:

investment algorithms, multi-factor models, Markov switching model, asset pricing models, equity risk premia, risk factors, Markowitz model

JEL:

C15, G11, F30, G12, G13, G14, G15

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1 Introduction

The idea of this article comes from analysis of various multi-factor models with and without regime switching mechanism. After detailed analysis of many approaches focusing on proper evaluation of equity risk premium (Sakowski, Slepaczuk, and Wywiał (2016)) we decided to turn from theoretical modeling to a practical one. Our main objective is to construct investment algorithms based on the information coming from multi-factor models and factors themselves.

We decided to accomplish this aim in two separate steps. The first one will focus on direct extraction of equity risk premia. The investment decision will be based on simple comparison of actual and theoretical equity risk premium for a given index. In the second step we propose an entirely new approach. We incorporate Markowitz (1952) methodology to risk factors from multifactor models, which are treated as single investment assets and then we allocate funds into these risk factors.

The literature describing the first approach in a direct way is rather limited. We hardly find any studies that focus on the second idea. However, as far as the first part is concerned, we can find several examples where authors, having introduced a new multi-factor model, try to turn to the practical world and show how their new methodology could work in real financial markets. Examples of such attempts can be summarized as follows:

- creating new factors for multi-factor models: Asness, Frazzini, and Pedersen (2013), Arshanapalli, Coggin, and Doukas (1998), Fama and French (2012), Jegadeesh and Titman (1993), Rahim and Noor (2006), Asness (1995), Chen, Novy-Marx, and Zhang (2011), Hou, Karolyi, and Kho (2011), Foye, Mramor, and Pahor (2013), Griffin (2002),
- introducing new model functional forms: Tan (2013), Ammann and Verhofen (2006), Angelidis and Tessaromatis (2014), Hammerschmid and Lohre (2014),
- new approaches to extract information from risk factors (Fama and French (2012)),
- application of an existing model in a different geographical setting: Connor and Sehgal (2001); Griffin (2002), Cakici, Fabozzi, and Tan (2013),
- developing a model tailored to specific markets and countries: Hou, Karolyi, and Kho (2011), Lieksnis (2010) and many others,
- application of the multi-factor models to economic forecasting: Liew and Vassalou (2000); Vassalou (2000),
- using model-derived forecasts in capital budgeting: Hu (2003).

It is also important to mention Mulvey (2010) who describes investment approach similar to some extent to our first strategy. Constructing long/short US equity portfolios, the authors employ a set of risk factors, including market index, treasury yields, implied volatility changes, US dollar index, and credit spread. The authors augment it with Markov regime switching model. However, unlike our approach, instead of predicting equity risk premium for a single asset, they jointly estimate the portfolio weights and portfolio covariance matrix. They find that the selected portfolio produces significantly higher risk-adjusted returns with lower risk, comparing to popular US equity indices.

Regarding our second strategy, employing the multi-factor models and risk factors themselves is not a completely new idea. However, setting them in Markowitz framework adds value. The interest in investing directly in the market portfolio, instead of selecting individual stocks, has surged since the publication of Sharpe (1964) paper. In 1971, Wells Fargo/Nikko first “market” index fund appeared, and five years later first mutual fund targeting market portfolio has been set up by Vanguard. Seminal papers of Fama and French (1992) and Carhart (1997) sparked interest in what was later called “smart beta”. Investing directly in the risk factors themselves became feasible not only for the largest financial institutions, but also for the wider spectrum of indices, when first “fundamentally-weighted” Exchange-Traded Funds appeared. Since then, risk factors have been used either directly as investable asset classes or as a proxy to evaluate exposure of the portfolio consisting of individual equities as documented in Ang (2014). The robustness of treating factors as investable indices, presented as a reasonable alternative to cap-weighted indices has been evaluated by e.g. Amenc, Goltz, Sivasubramanian, and Lodh (2015).

Cazalet, Grison, and Roncalli (2013) evaluated construction of “smart beta” indices based on risk factors. They find that factor-based indices and portfolios of these indices exhibit generally higher returns and lower volatility than capitalisation-weighted indices, such as market portfolio. However, the authors point out that the source of this supposed outperformance may lie in greater difficulty of passive execution of the strategy, namely higher tracking error. Other attempts to incorporate dynamic factor portfolio selection have been made by Komatsu and Makimoto (2011), where authors search for an optimal algorithm for choosing factor portfolios. However, the article contained no empirical analysis and a numerical simulation was applied as a sole means to experimentally verify the model. Ma and Maclean (2011) evaluated a dynamic portfolio selection algorithm based on regime switches, applied to 9 ETFs, with the equally-weighted portfolio as a benchmark. They found that applying dynamic selection to generalized asset classes embodied by ETFs outperforms the naive equally-weighted portfolio.

To sum up, what is novel in our approach, is incorporating the risk factors into the Markowitz portfolio selection framework. We choose a simpler but hopefully more robust approach to selecting factor portfolios.

The structure of this paper is as follows. After introduction we carefully describe the methodology of investment strategies used in this paper i.e. equity risk premium approach and Markowitz model based on risk factors. Data are described in the third section. Fourth section contains our results and shortly addresses sensitivity analysis. Last section explains our results and concludes.

2 Methodology

Methodological section of this paper is divided into two subsections. First, we define strategy directly based on equity risk premium inequilibrium with detailed references to theoretical multi-factor model used in this approach. In the second part we use the Markowitz model with some amendments focused on optimisation criteria, input variables and technical issues.

2.1 Equity risk premia approach

2.1.1 Model overview

In this study we use seven-factor Markov-switching model with additional variables common for all countries (CV) and variables country-specific for developed/emerging markets (CspV)¹. These variables were previously presented and described in detail in Sakowski, Slepaczuk, and Wywiał (2016). The model has the following functional form:

$$(R_i - Rf)_{i,t} = \alpha + \beta_{CV,i}(CV)_t + \beta_{CspV,i}(CspV)_t + \beta_{MKT,i}(Rm - Rf)_t + \beta_{HML,i}HML_t + \beta_{SMB,i}SMB_t + \beta_{WML,i}WML_t + \beta_{VMC,i}VMC_t + \beta_{QMS,i}QMS_t + \beta_{SMBrel,i}SMBrel_t + \varepsilon_{i,t} \quad (1)$$

where:

- $(R_i - Rf)_t$ is weekly return of equity index in excess to weekly risk free rate,
- $(Rm - Rf)_t$ is equally weighted equity index less than risk free rate,
- HML_t is the weekly premium on the book-to-market factor (high minus low),
- SMB_t is the weekly premium on the size factor (small minus big),
- WML_t is the weekly premium on winners-minus-losers factor (winners minus losers),
- VMC_t is the weekly premium on volatile minus calm (VMC) equity indices (volatile minus calm),
- QMS_t is the weekly premium on equity indices characterised by positive minus negative percentage deviation of nominal GDP from its 5y moving average (quick minus slow),
- $SMBrel_t$ is similar to SMB factor with one important exception, i.e. the sorting of equity indices to each decile group is based on the ratio of index capitalization to GDP of the given country instead of capitalization only (small minus big relative)
- CV denotes variables common for all countries, among them S&P Goldman Sachs Commodity Index (GSCI), CBOE SKEW index (SKEW), average of annualized monthly realized volatility of crude oil prices and CBOE GVZ volatility index (OVXGVZ), average of CBOE VIX, VXEEM, VXEWS and VVIX volatility indices (VX), USD index (DXY), dummy variable which has value of 1 when crude oil price is lower than 65 USD and 0 otherwise (CLF_dummy), CBOE VVIX index (VVIX)
- $CspV$ denotes country specific variables, among them short-term interest rates, long-term interest rates and currencies for emerging and developed countries. CV and $CspV$ are described in detail in Sakowski, Slepaczuk, and Wywiał (2016).

¹We refer to this model as 7F+Markov+CV+CspV.

The set of common variables used has been limited to VX, OVXGVZ, DXY and CLF_dummy variable in order to minimize collinearity of the model. In addition to the linear form outlined in an equation above, the model is augmented with Markov-switching model with two states. The model is estimated using a standard Expectation Maximization procedure, separately for each of 81 indices on the period 2000-2015. The output of the estimation procedure contains two sets of parameters, one for high-volatility regime and one for low-volatility regime. Based on the estimated probability of each state in a given week, we select the parameter set for a regime with greater probability. Using these parameters, theoretical risk premia are calculated and used in the investment algorithm outlined below.

2.1.2 Investment algorithm for ERP approach

Based on this model we obtain weekly theoretical equity risk premia for all equity indices which then will be compared with its practical weekly realisation. The investment algorithm works as follows:

$$\text{ERP approach buy/sell signal}_t = \begin{cases} \text{long} & \text{if } ERP_{theoretical,t} > ERP_{realised,t}, \\ \text{short} & \text{if } ERP_{theoretical,t} < ERP_{realised,t}. \end{cases} \quad (2)$$

where:

$ERP_{theoretical,t}$ denotes theoretical equity risk premium in week t calculated from seven-factor markow-switching model with additional CV and CspV.

$ERP_{realised,t}$ denotes practical realisation of equity risk premium in week t .

2.2 Markowitz model

2.2.1 Investment algorithm for Markowitz model

The second algorithm does not use any specific multi-factor model but it focuses only on risk factors. Seven of them, RmRf, HML, SMB, WML, VMC, QMS, and SMBrel, have been calculated and described in details in Sakowski, Slepaczuk, and Wywiał (2016). Additionally, we use a modified *betting-against-beta* factor (BAB) calculated for worldwide equity indices based on the methodology presented in Frazzini and Pedersen (2014). Our modifications focus on more aggressive leveraging of low-beta indices and applying lower leverage to high-beta indices. These modifications' purpose is to limit the volatility of the factor and increase its stability, in particular during extreme market moves.

We use these eight risk factors as input data in the Markowitz portfolio optimisation framework.

The risk factors mentioned above are treated as investable indices with the initial value of 100. This value then changes in time according to returns obtained by specific risk factor in each week. For example, the dynamics of the WML factor is described by:

$$\text{WML}_{\text{investable index}, T} = 100 * \prod_{t=1}^T (1 + \text{WML}_{\text{return}, t}) \quad (3)$$

where:

- $\text{WML}_{\text{investable index}, T}$ denotes value of investable index based on WML risk factor for seven-factor Markow-switching model with CV and CspV at the moment T .
- $\text{WML}_{\text{return}, t}$ denotes single week returns from WML risk factor at the moment t .

Table 1: Profitability statistics for investable indices based on risk factors which are inputs for Markowitz model.

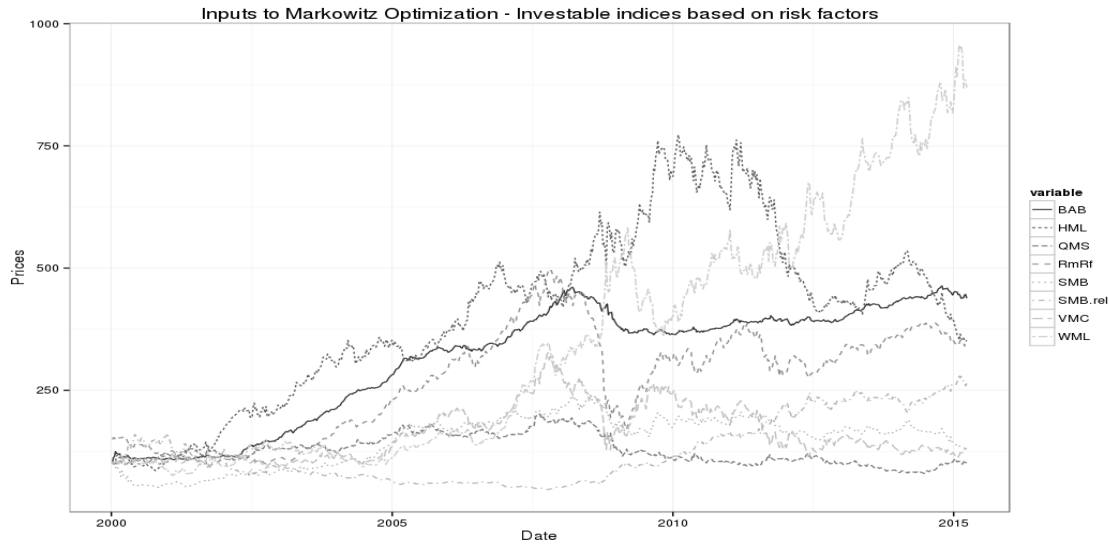
	RmRf	HML	SMB	WML	VMC	QMS	SMB.rel	BAB
Cagr	5.59	7.8	3.95	15.16	1.07	0.28	5.59	9.25
Sharpe	0.42	0.5	0.32	0.79	0.17	0.09	0.39	1.52
Volatility	15.81	18.03	16.22	20.53	25.8	15.13	17.72	5.94
MaxDD	-65.69	-54.73	-46.68	-38.22	-65.4	-60.49	-71.29	-21.05
VaR	-3.26	-3.52	-3.43	-4.56	-5.23	-3.21	-3.69	-1.17
CVaR	-5.71	-5.01	-5.11	-6.44	-7.58	-4.55	-5.19	-1.73

This table presents profitability statistics for investable indices based on risk factors from 2000 to 2015. Cagr means annualized rate of return. Sharpe denotes average yearly rate of return divided by volatility. Volatility means annualized standard deviation of returns. MAXDD is maximum drawdown. VaR means value at risk while CVaR denotes conditional VaR.

2.2.2 Visualisation of all risk factors

Visualisation of all risk factors in comparison with market index can be found on Figure 1. The market index (RmRf) is built as an equally weighted average of all equity indices, with the same assumptions as in case of risk factors - its initial value is scaled to 100. Dynamics of risk factors presented on Figure 1 shows that their correlations with broad market index returns are time-varying. What is more important, these correlations could be positive as well as negative, depending on the time frame we consider.

Figure 1: Investable indices based on risk factors from seven-factor model + BAB risk factor which are inputs to Markowitz model versus market index.



This chart presents risk factors dynamics in the period between 2000-2015 for seven-factor Markov-switching model with CV and CspV + BAB risk factor compared with market index (Rm). Investable indices for risk factors were calculated starting from value of 100 as a cumulative returns of each risk factors separately. Market index was obtained starting from value of 100 as a cumulative returns of equilly weighted returns of all equity indices under investigation.

Table 1 shows profitability statistics for investable indices based on risk factors which are inputs to the Markowitz model. We can see that their risk return ratio (Sharpe ratio) ranges between 0.09 and 1.52 indicating that a few of them can be quite good investment itself.

3 Data

The analysis was performed on weekly data from 2000 to 2015 for 81 most representative and investable equity indices, covering all continents. We include data for 27 developed and 54 emerging markets indices. The detailed list of all equity indices and their descriptive statistics can be obtained upon request.

Apart from price data for equity indices we required the following weekly data in order to properly calculate investable indices for seven risk factors: risk free interest rate (3mLiborUSD), book value for equity indices, currency rates, market capitalisation for equity indices recalculated from local currency into USD and quaterly GDP data for all countries.

Additionally, to calculate all necessary variables for 7F-Markov+CV+CspV we used following data: skew for S&P500 index, GSCI index, VIX, VVIX, OVX and GVZ volatility indices, DXY index, crude oil prices, short-term and long-term interest rates and currencies for emerging and developed countries.

It is important to underline that although our analysis was performed in the period between 2000-2015, the results for Markowitz model starts in 2001 while the results for ERP algorithm starts in 2003. The reason for different start point is connected with lookback period for Markowitz model (52 weeks) and minimum number of data which was left in order to estimate our model (3 years).

Additionally, it is important to underline that the series were appropriately lagged, in order to ensure that no strategy suffered from look ahead bias. Also, we made efforts avoid other flaws often encountered in investment research like sample selection bias or overoptimisation effect. This is additionally confirmed by sensitivity analysis in section 4.3.

4 Results

This section is divided into three subsections. Two of them describe two investments strategies while Subsection 4.3 addresses detailed sensitivity analysis of results.

4.1 Equity risk premia approach results

Equity risk premia investment strategy is based on following assumptions:

1. ERP calculations are based on 7F-Markov+CV+CspV model,
2. Degree of financial leverage (DFL) for long and for short postions are set to 100%,
3. Percentage of stocks we go long and short in the strategy equal 20%.

Buy/sell signals were generated according to the rules presented in Subsection 2.1.2.

Profitability statistics for ERP approach are reporten in Table 2, while Figure 2 presents equity lines.

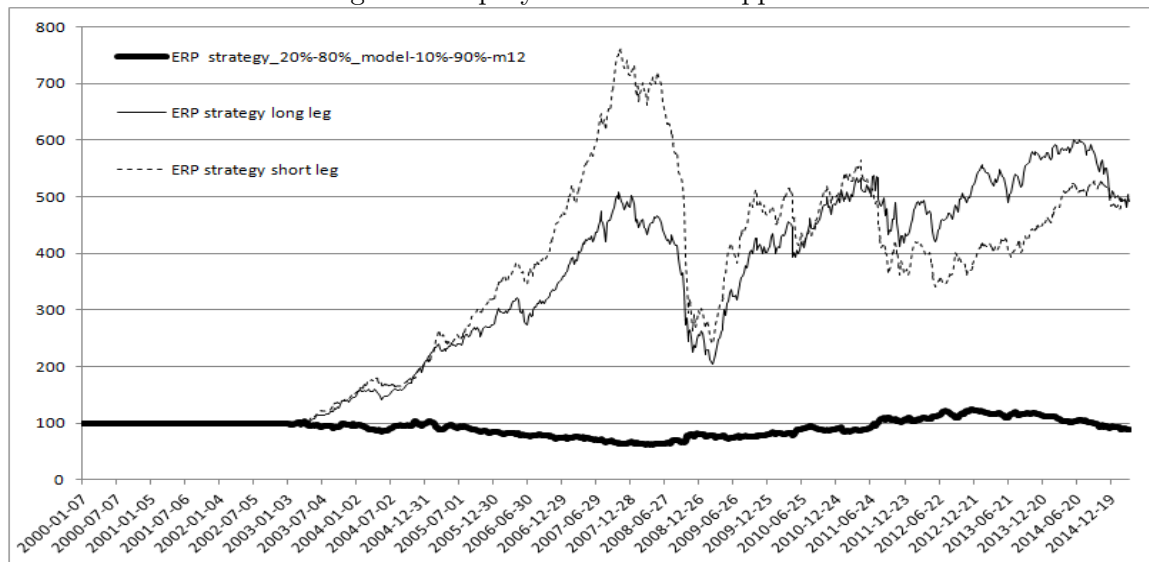
We can see that main version of our investment strategy does not produce any cumulative returns across the whole investment horizon. The difference between long and short leg equity lines is negligible what results in final equity lines oscillating around 0% returns almost for the whole period. Unfortunately, it can mean that investment algorithm proposed for extraction of information hidden in ERP estimated by our model is not able to produce abnormal returns. This is also in contrast with the results of a similar research found in Mulvey (2010), who achieved Sharpe ratio as high as 2.07. We try to analyse and explain this issue more deeply in Section 4.3.

Table 2: Profitability statistics for ERP approach

	ERP strategy_20%-80%_model-10%-90%-m12
Cagr	-0,85%
Volatility	11,41%
MD	40,32%
Sharpe	-0,07
Sharpe/MD	-0,18
Sharpe*Cagr/MD	0,00
DFL long	100%
DFL short	100%

This table presents profitability statistics for ARP approach. DFLlong and DFLshort means that we invest only 100% of asset in long and short leg of the strategy and that we do not use any additional leverage.

Figure 2: Equity lines for ERP approach



This chart presents equity lines for ERP approach. Black thick line shows results for long-short strategy. Black thin line shows results only for long leg of the strategy. Black thin dotted line shows results only for short leg of the strategy.

Table 3: Profitability statistics for Markowitz model with various optimisation criteria.

	TRISK.8	TRET.12	MS	MV	EW
Cagr	9.09	8.17	12.48	7.72	8.15
Sharpe	1.93	1.99	1.98	1.92	1.65
Volatility	4.54	3.98	6.03	3.91	4.81
MaxDD	-17.31	-12.71	-10	-16.78	-12.5
VaR	-0.91	-0.74	-1.07	-0.74	-0.91
CVaR	-1.32	-1.15	-1.77	-1.13	-1.25

This table presents profitability statistics for Markowitz model with five optimisation criteria: target risk 8% (TRISK.8), target return 12% (TRET.12), max Sharpe ratio (MS), min variance (MV) and equally weighted (EW). Cagr means annualized rate of return. Sharpe denotes average yearly rate of return divided by volatility. Volatility means standard deviation of returns. MAXDD is maximum drawdown. VaR means value at risk. Lookback period for variance-covariance matrix is set to 52 weeks. Rebalancing is done every quarter. Long/short restriction is set to long only positions.

4.2 Markowitz model results

The second investment approach shows entirely different results. We decided to take the following assumptions with respect to the Markowitz model:

- optimisation criteria: target risk 8% (TRISK.8), target return 12% (TRET.12), max Sharpe ratio (MS), min variance (MV) and equally weighted (EW),
- long/short restriction: long only,
- lookback for variance-covariance matrix: 1 year,
- rebalancing: quarterly,
- weights restrictions: from 0 to 100% for single asset,
- weekly data,
- DFL for the strategy equals 100%,

Table 3 presents profitability statistics for Markowitz model with five different optimisation criteria.

Table 3 shows that our second investment algorithm produces very promising results with relatively high return to risk ratio. What is more important, our results are quite stable across different optimisation criteria. We can see that Sharpe ratio oscillates around 1.9, which is relatively a high value for strategies working on frequencies lower than daily. Additionally, risk statistics like MaxDD, VaR or CVaR are on the level 3 to 4 times lower than the same for buy&hold strategies on equity indices for the same period. Most notably, Markowitz models consequently outperform in terms of Sharpe ratio both each individual factor and equally-weighted portfolio. Presented investment approach enabled us to practically insulate our investment against all main financial crashes or bear markets encountered in our research period (2000-2002, 2008-2009, August 2011).

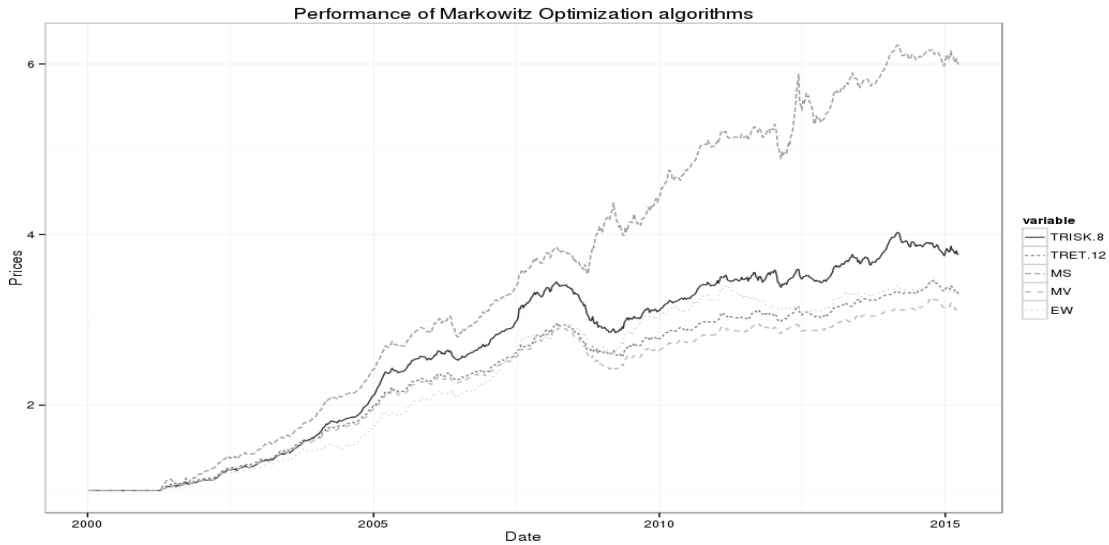
Figure 3 presents equity lines for Markowitz model for five various optimisation criteria with assumptions summarized at the beginning of this section.

4.3 Sensitivity analysis

4.3.1 Equity risk premia approach

In order to check the robustness of the results we decided to perform sensitivity analysis of our results focusing on the main important assumptions. We focused on:

Figure 3: equity lines for Markowitz model with five various optimisation criteria



This chart presents equity lines for Markowitz model. Equity lines present results for 5 different optimisation criteria. TRISK.8 means total risk with 8% threshold. TRET.12 denotes total return with 12% threshold. MS means maximum Sharpe ratio. MV denotes minimum variance. EW means equally weighed weights which does not change for the whole period. Lookback period for vaiance-covariance matrix is set to 52 weeks. Rebalancing is done every quarter. Long/short restriction is set to long only positions.

- DFL for long and short positions. We check the following version: $DFL_{long}=100\%$ and $DFL_{short}=25\%$
- The number of stock we decided to go long or short based on the highest departure of practical ERP from theoretical ERP. We check two variants: 10% and 50%.

Table 4 presents profitability statistics for ERP approach when we change the assumptions concerning equal DFL for both legs of the strategy (long and short one). The rationale for such parameter change can be twofold. The first one is to just check sensitivity of our results to one of the parameters. The second one is that we wanted to check to what extent the assumptions of the strategy should take into account the fact that the average broad market was in upward trend in examined investment horizon.

Table 4: Profitability statistics for ERP approach with various DFL

	ERP_strategy_model-10%-90%-m12
Cagr	8,23%
Volatility	13,74%
MD	46,24%
Sharpe	0,60
Sharpe/MD	1,29
Sharpe*Cagr/MD	0,11
DFL long	100%
DFL short	25%
ranks	20%

This table presents profitability statistics for ERP approach. DFL_{long} and DFL_{short} means that we invest only 100% of asset in long and 25 in short leg of the strategy and that we do not use any additional leverage. Ranks means that we took only 20% of indices with highest difference between realised and theoretical ERP.

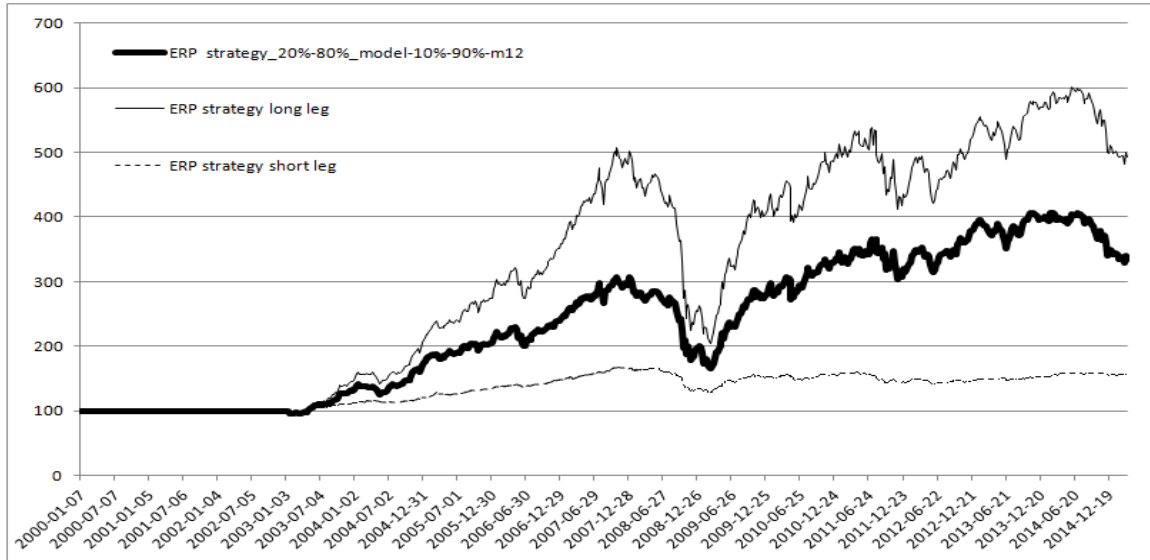
Table 5: Profitability statistics for ERP approach with various Ranks (10% and 50%).

	ERP strategy_50%_model-10%-90%-m12	ERP strategy_10%_model-10%-90%-m12
Cagr	-1,77%	0,32%
Volatility	5,99%	16,53%
MD	33,60%	44,36%
Sharpe	-0,29	0,02
Sharpe/MD	-0,88	0,04
Sharpe*Cagr/MD	0,02	0,00
DFL long	100%	100%
DFL short	100%	100%
ranks	50%	10%

This table presents profitability statistics for ERP approach. DFLlong and DFLshort means that we invest only 100% of asset in long and short leg of the strategy and that we do not use any additional leverage. Ranks means that we took only 10% (or 50%) of indices with highest difference between realised and theoretical ERP.

Figure 4 presents equity lines for ERP approach. We can observe much better results in comparison to the main version presented in Section 4.1 The difference between long and short leg equity lines is quite substantial and it could be compared for example by Sharpe ratio (0.6 versus -0.07) presented in Table 4. The problem is that observed improvement in results should be mainly attached to larger weight of long position (100% versus 25% for short leg) in upward trend instead of much better efficiency of the investment algorithm used.

Figure 4: Equity lines for ERP approach. DFLlong=100%. DFLshort=25%.

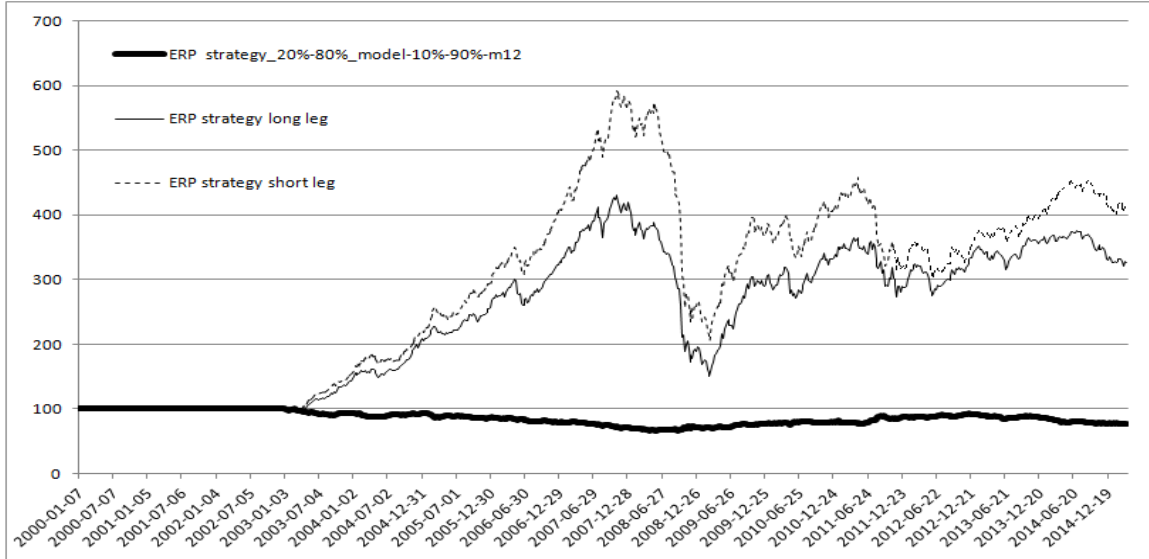


This chart presents equity lines for ERP approach. Black thick line shows results for long-short strategy. Black thin line shows results only for long leg of the strategy. Black thin dotted line shows results only for short leg of the strategy.

The second change with respect to sensitivity analysis for ERP approach focused on the number of indices used in our analysis. Table 5 presents results of this analysis. We can see that these change did not affect results in comparison to the main version. Sharpe ratio is still very close to zero or even smaller (for Ranks=50%). We can observe that generally better results can be obtained when we focus only on indices with highest differences between realised and theoretical ERP (left column from Table 5).

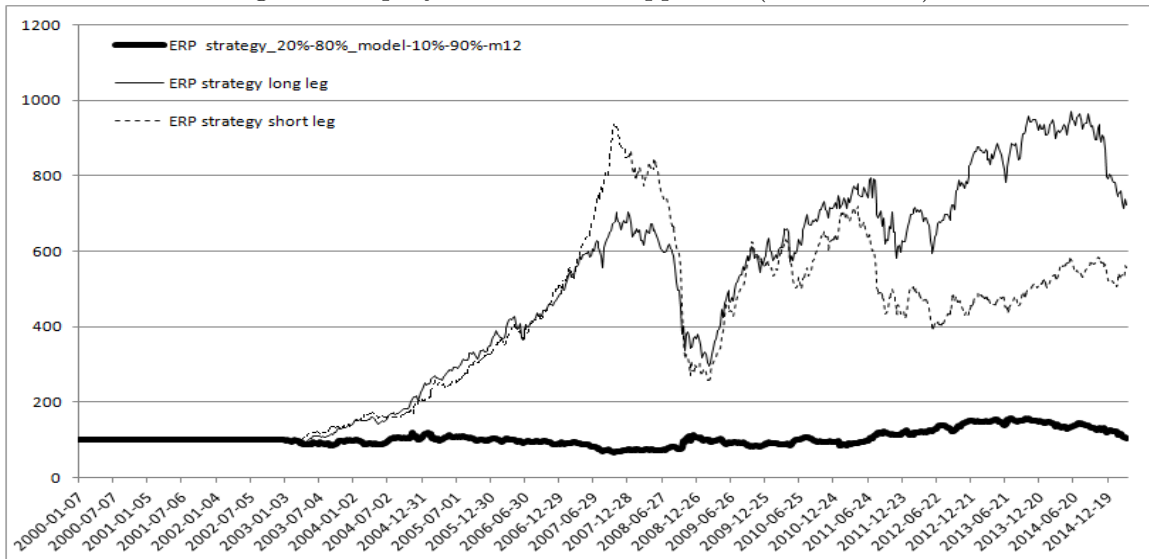
These observations are confirmed on Figures 5 and 6 where are shown equity lines for two versions of ERP strategy presented in Table 5.

Figure 5: Equity lines for ERP approach (Ranks=50%)



This chart presents equity lines for ERP approach. Black thick line shows results for long-short strategy. Black thin line shows results only for long leg of the strategy. Black thin dotted line shows results only for short leg of the strategy. DFLlong and DFLshort equals 100%. Ranks parameter equals 50%.

Figure 6: Equity lines for ERP approach (Ranks=10%)



This chart presents equity lines for ERP approach. Black thick line shows results for long-short strategy. Black thin line shows results only for long leg of the strategy. Black thin dotted line shows results only for short leg of the strategy. DFLlong and DFLshort equals 100%. Ranks parameter equals 10%.

To summarize results for the first tested algorithm (its main version and alternative modifications examined sensitivity analysis) it is necessary to admit that better results were expected. We notice that the only promising result in this approach was obtained when we set DFLlong on much higher level than DFLshort. As a result, we change the profile of the strategy from market neutral to heavy long biased. As a result, it cannot account for the strength of the model (not mentioning the possible overfitting or data-snooping bias). Taking into account all results mentioned above, we acknowledge that first algorithm although based on relatively complex multifactor model is not able to produce returns characterized by high return to risk ratio.

4.3.2 Markowitz model

We decided to check the robustness of our results to changes in the following assumptions:

- lookback for variance-covariance matrix: 0.5 year and 2 years (instead of 1 year),
- rebalancing: monthly and yearly (instead of quarterly),

Table 6 presents profitability statistics for Markowitz model with lookback period changed to 26 weeks while in Table 7 we can see results for much longer lookback period set to 2 years.

Table 6: Profitability statistics for Markowitz model with various optimisation criteria and lookback period set to 26 weeks (with quarterly rebalancing).

	TRISK.8	TRET.12	MS	MV	EW
Cagr	9.61	9.22	13.02	8.14	8.07
Sharpe	1.79	2.02	1.84	1.82	1.59
Volatility	5.17	4.39	6.75	4.33	4.94
MaxDD	-17.03	-9.35	-10.8	-16.85	-12.5
VaR	-0.98	-0.82	-1.38	-0.85	-0.93
CVaR	-1.48	-1.24	-1.95	-1.26	-1.28

This table presents profitability statistics for Markowitz model with five optimisation criteria: target risk 8% (TRISK.8), target return 12% (TRET.12), max Sharpe ratio (MS), min variance (MV) and equally weighted (EW). Cagr means annualized rate of return. Sharpe denotes average yearly rate of return divided by volatility. Volatility means standard deviation of returns. MaxDD is maximum drawdown. VaR means value at risk. Lookback period for variance-covariance matrix is set to 26 weeks. Rebalancing is done every quarter. Long/short restriction is set to long only positions.

We observe that generally results presented in Tables 6 and 7 do not differ in comparison to the main version presented in Table 3. They show that the outcome of our second algorithm are rather stable with respect to lookback period (although for longer lookback period they are slightly worse) and that this algorithm enables us to produce high risk adjusted returns when we compare Sharpe ratios between above mentioned tables. Small differences can be found when we put attention to the best and worst results in each table taking into account optimisation criteria. However, what is more important, for every lookback period we can see that EW results are much worst in comparison to four other optimisation criteria. The last observation regards maximum returns. In each table maximum Cagr is obtained for maximum Sharpe ratio optimisation criterion.

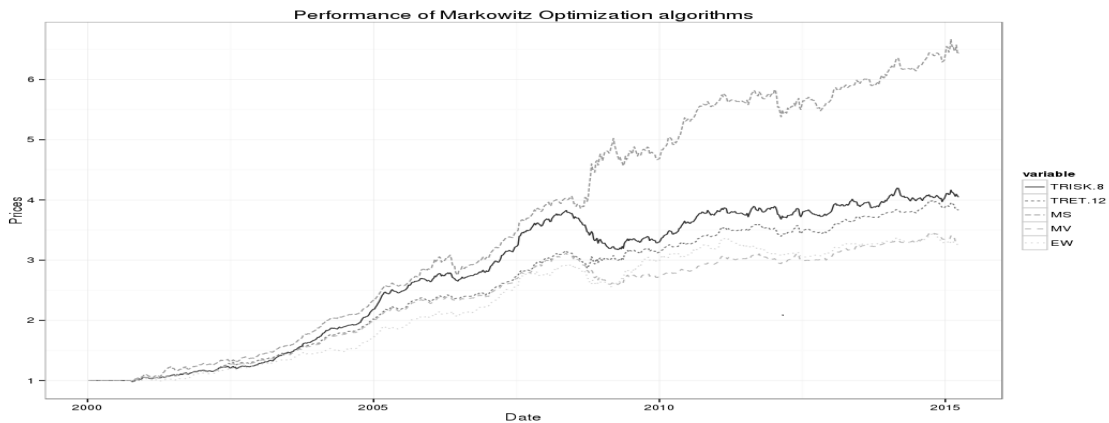
Figure 7 and Figure 8 additionally confirm our observations made based on above tables. We can see very smooth upward fluctuations of equity lines no matter which optimisation criterion we focus on.

Table 7: Profitability statistics for Markowitz model with various optimisation criteria and lookback period set to 104 weeks (with quarterly rebalancing).

	TRISK.8	TRET.12	MS	MV	EW
Cagr	7.6	7.3	10.25	7.23	7.19
Sharpe	1.75	1.84	1.89	1.83	1.52
Volatility	4.22	3.86	5.23	3.85	4.62
MaxDD	-18.73	-18.61	-9.98	-18.59	-12.5
VaR	-0.76	-0.7	-1.01	-0.71	-0.87
CVaR	-1.26	-1.11	-1.51	-1.11	-1.24

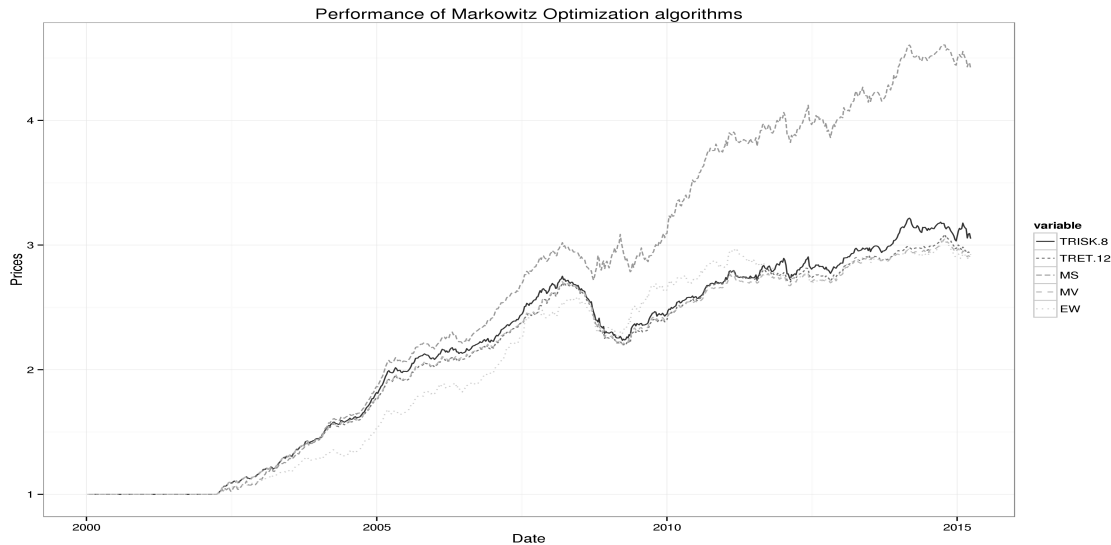
This table presents profitability statistics for Markowitz model with five optimisation criteria: target risk 8% (TRISK.8), target return 12% (TRET.12), max Sharpe ratio (MS), min variance (MV) and equally weighted (EW). Cagr means annualized rate of return. Sharpe denotes average yearly rate of return divided by volatility. Volatility means standard deviation of returns. MaxDD is maximum drawdown. VaR means value at risk. Lookback period for variance-covariance matrix is set to 104 weeks. Rebalancing is done every quarter. Long/short restriction is set to long only positions.

Figure 7: Equity lines for Markowitz model with five various optimisation criteria and lookback period set to 26 weeks (with quarterly rebalancing).



This chart presents equity lines for Markowitz model. Equity lines present results for 5 different optimisation criteria. TRISK.8 means total risk with 8% threshold. TRET.12 denotes total return with 12% threshold. MS means maximum Sharpe ratio. MV denotes minimum variance. EW means equally weighed weights which does not change for the whole period. Lookback period for variance-covariance matrix is set to 26 weeks. Rebalancing is done every quarter. Long/short restriction is set to long only positions.

Figure 8: Equity lines for Markowitz model with five various optimisation criteria and lookback period set to 104 weeks (with quarterly rebalancing).



This chart presents equity lines for Markowitz model. Equity lines present results for 5 different optimisation criteria. TRISK.8 means total risk with 8% threshold. TRET.12 denotes total return with 12% threshold. MS means maximum Sharpe ratio. MV denotes minimum variance. EW means equally weighed weights which does not change for the whole period. Lookback period for vaiance-covariance matrix is set to 104 weeks. Rebalancing is done every quarter. Long/short restriction is set to long only positions.

The next important issue we wanted to check within sensitivity analysis was the frequency of rebalacing. Instead of quarterly rebalancing (Section 4.2), we repeat our analysis with monthly rebalancing (Table 8) and with yearly rebalancing (Table 9).

Again we can see that the differences between various values of rebalancing are rather irrelevant although they are slightly better for shorter frequency of rebalancing. Additionally, we can see that MaxDD slightly increases when we rebalance more frequently. Figure 9 and Figure 10 confirm these conclusions.

Generally, sensitivity analysis confirm preliminary results for our second investment algorithm. We can see that the results are quite robust to initial assumptions about lookback window and rebalancing period. We can observe that they can be even slightly enhanced when we shorten rebalancing period and lookback window at the same time.

Table 8: Profitability statistics for Markowitz model with various optimisation cirteria and lookback period set to 52 weeks (with monthly rebalancing).

	TRISK.8	TRET.12	MS	MV	EW
Cagr	9.46	8.15	12.74	7.68	7.75
Sharpe	2.02	2.01	1.95	1.96	1.56
Volatility	4.5	3.93	6.22	3.8	4.84
MaxDD	-17.27	-13.8	-8.69	-17.18	-13.57
VaR	-0.81	-0.77	-1.16	-0.76	-0.88
CVaR	-1.28	-1.12	-1.82	-1.08	-1.27

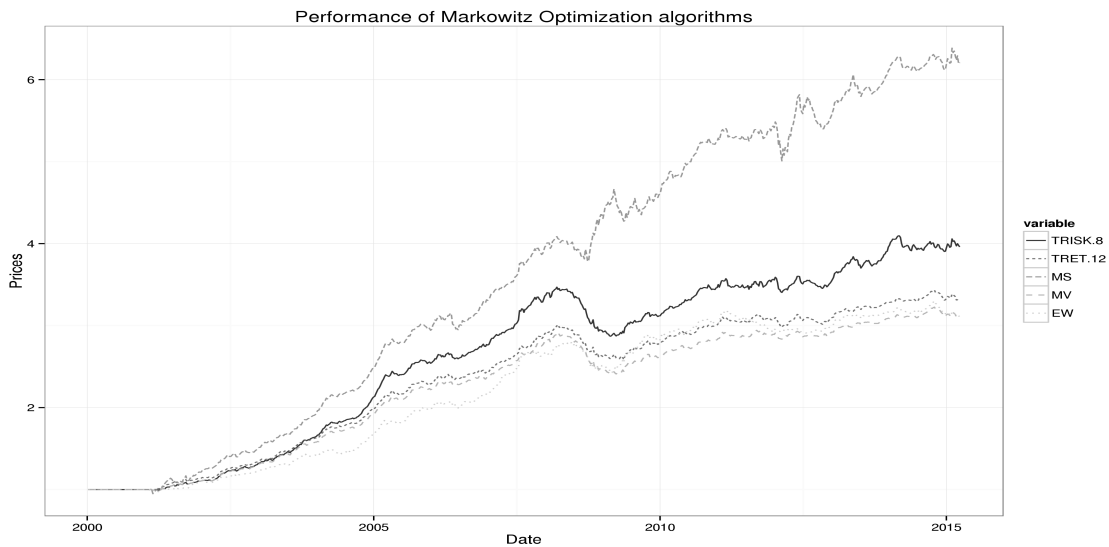
This table presents profitability statistics for Markowitz model with five. Cagr means annualized rate of return. Sharpe denotes average yearly rate of return divided by volatility. Volatility means standard deviation of returns. MaxDD is maximum drawdown. VaR means value at risk. Lookback period for vaiance-covariance matrix is set to 52 weeks. Rebalancing is done every month. Long/short restriction is set to long only positions.

Table 9: Profitability statistics for Markowitz model with various optimisation criteria and lookback period set to 52 weeks (with yearly rebalancing).

	TRISK.8	TRET.12	MS	MV	EW
Cagr	7.97	7.1	8.94	7.04	7.86
Sharpe	1.7	1.72	1.54	1.73	1.62
Volatility	4.55	4.01	5.64	3.96	4.73
MaxDD	-21.08	-18.46	-20.52	-18.94	-9.87
VaR	-0.83	-0.74	-1.08	-0.72	-0.91
CVaR	-1.36	-1.16	-1.82	-1.14	-1.25

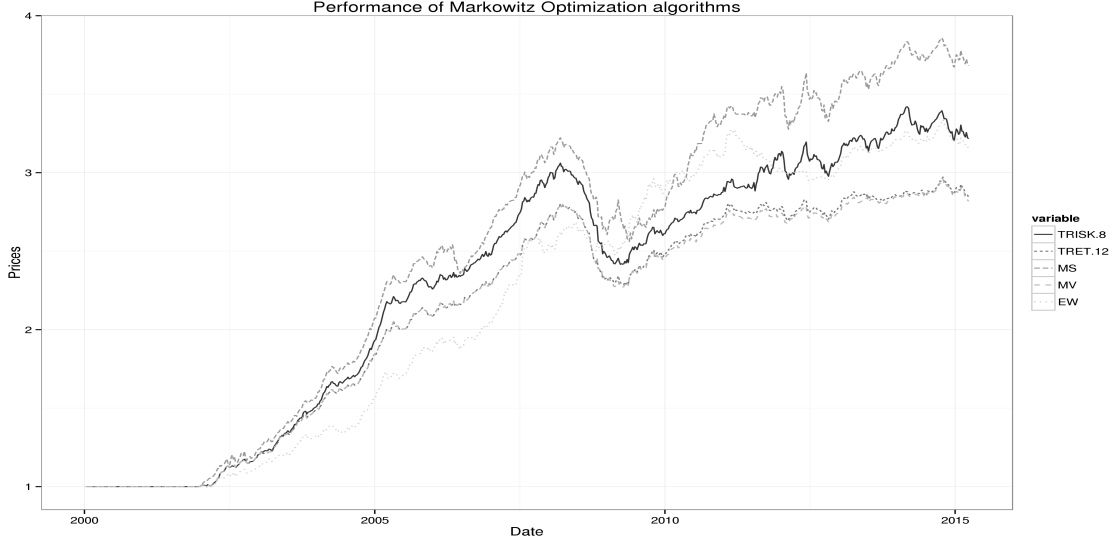
This table presents profitability statistics for Markowitz model with five. Cagr means annualized rate of return. Sharpe denotes average yearly rate of return divided by volatility. Volatility means standard deviation of returns. MaxDD is maximum drawdown. VaR means value at risk. Lookback period for variance-covariance matrix is set to 52 weeks. Rebalancing is done every year. Long/short restriction is set to long only positions.

Figure 9: Equity lines for Markowitz model with five various optimisation criteria and lookback period set to 52 weeks (with monthly rebalancing).



This chart presents equity lines for Markowitz model. Equity lines present results for 5 different optimisation criteria. TRISK.8 means total risk with 8% threshold. TRET.12 denotes total return with 12% threshold. MS means maximum Sharpe ratio. MV denotes minimum variance. EW means equally weighed weights which does not change for the whole period. Lookback period for variance-covariance matrix is set to 52 weeks. Rebalancing is done every month. Long/short restriction is set to long only positions.

Figure 10: Equity lines for Markowitz model with five various optimisation criteria and lookback period set to 52 weeks (with yearly rebalancing).



This chart presents equity lines for Markowitz model. Equity lines present results for 5 different optimisation criteria. TRISK.8 means total risk with 8% threshold. TRET.12 denotes total return with 12% threshold. MS means maximum Sharpe ratio. MV denotes minimum variance. EW means equally weighed weights which does not change for the whole period. Lookback period for vaiance-covariance matrix is set to 52 weeks. Rebalancing is done every year. Long/short restriction is set to long only positions.

5 Summary

The main important result of this study is that based on the information hidden in multi-factor models we can built profitable investment algorithms. The point is that extraction of this information is not straightforward. We proposed two types of investment strategies.

The first one was based on comparison of realised and theoretical ERP estimated with seven-factor Markov-switching model with additional variables common for all countries (CS) and variables specific to emergin and developed markets (CspV). The second strategy was based on utilising risk factor as investable indices in Markowitz framework.

Based on detailed analyses it has occurred that these two approches lead us to two different results. Results of the first approach show that proposed way of information's extraction does not lead us to profitable results when we consider risk adjusted return, perhaps counterintuitively and contrary to what has been suggested by literature so far. Entirely different results can be observed in case of the second investment algorithm. We can see that simple Markowitz framework applied to risk factors from multi-factor models can lead to abnormal risk-adjusted returns (Sharpe ratio close to 2).

After performing sensitivity analysis we can confirm that our preliminary results do not differ when we change our main assumptions. In fact, we noticed that they can be even improved when we shorten lookback period and rebalancing frequency. This is consistent with current financial practice of factors investing and search for smart beta. However, what is most remarkable, the simple but time-tested and robust approach of portfolio selection can greatly enhance the investment results beyond investing in a single factor or equally-weighted factor portfolio.

In final summary, we can state that the answer to the main research question is positive. It is possible to extract information from the multi-factor models in order to build investment

algorithms that produce returns that are significantly exceeding those of the benchmarks. However, it is important to notice that such extraction is not simple and rather requires precise and thorough analysis to do this properly. In particular, employing risk factors as separate asset classes in the Markowitz model allows to consistently achieve abnormally high returns, while direct application of the multi-factor equity risk premium model to generate trading signals does not.

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