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Disruptive innovation: Incumbent's response to innovation threat

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Abstract: The theory of disruptive innovation has gained significant interest from the academic and business communities, investigating the reasons for inadequate or delayed responses of incumbents to innovation which challenges their established markets. Inadequate responses were explained by the concept of innovator's dilemma, which postulates how incumbents tend to satisfy their consumers on the established markets, while overlooking opportunities with other consumers. However, the optimal incumbents' response to disruptive innovation in the normative sense has not been researched. We investigate the choice of the payoff-maximizing strategy in response to the observed market disruption as a choice between different management approaches. Our study shows the importance of the speed of innovation adoption for the choice of the optimal response to innovation. We also show that the response can be delayed in some cases, reflecting the exploitation-exploration dilemma. This new insight complements the reasons of delayed response to disruptive innovation threats.

Keywords: Innovation, Incumbent, Management strategy

JEL codes: D21, D81, L19, O33

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1. Introduction

Clayton M. Christensen and co-authors proposed the disruptive innovation theory to explain the process by which an entrant, a smaller company with limited resources, can successfully challenge established incumbent businesses (Bower & Christensen, 1996; Christensen, 1997; Christensen & Raynor, 2003; Christensen et al., 2004). The theory postulates that disruptive innovation happens as a process. Incumbents focus on satisfying their mainstream customers, while the entrants target customers at the low-end of the market with initially inferior services or products, and frequently at a lower price. No response of incumbents leaves room for the entrants to move up-market with their innovation, and the disruption happens when the mainstream customers start to adopt the innovation. There are two preconditions for market disruption: a performance overshoot on satisfying the needs of customers by the incumbent, and asymmetric incentives between the incumbent operating at higher profits and the potentially-disruptive business with lower profits. Success in not part of the definition: not every disruptive innovation has to be successful (Christensen, 2005). Disruptive innovations are often developed by new market entrants (Foster, 1986; Tushman & Anderson, 1986; Henderson & Clark, 1990; Anderson & Tushman, 1990; Bower & Christensen, 1996) while established companies often focus on sustaining innovation.

Some incumbent firms were successful in creating or commercializing disruptions (e.g., IBM's personal computers, Sony's Walkman, HP's inkjet printers, Kodak's and Kuji's digital cameras). However, many incumbent firms failed to respond. Christensen proposed "the innovator's dilemma" paradigm to explain the failure of the incumbents in responding to a disruptive thread (Christensen, 1997). Extensive research on factors enabling established companies to pursue disruptive innovation covered multiple perspectives, including human resources, organizational culture, resource allocation and organizational structure, the context and environment, the marketing and the technology perspective (Hang et al., 2011).

Since its early development, the disruptive innovation theory has undergone various enhancements, modifications and critical reviews (Adner, 2002; Danneels, 2004; Tellis, 2006; Christensen, 2006; Markides, 2006; Govindarajan & Kopalle, 2006; Schmidt & Druehl, 2008; Hang et al., 2011; Gans, 2016b; Christensen et al., 2018), and the concept of disruptive innovation has generated discussions between academics and has impacted managerial practices. Important contributions have been made to review the most effective response strategies. Early empirical evidence suggested that incumbents typically ignore disruptive innovations, yet some established companies established a separate organizational unit to develop and commercialize innovations in adjacent markets (Christensen, 1997). Another adoption strategy relies on acquiring or partnering with the entrants which challenge the business of the incumbents (Sandström et al., 2009; Christensen et al., 2011; Marx et al., 2014; Kapoor and Klueter, 2015; McDonald & Eisenhardt, 2017). Further studies suggest that accelerating innovation efforts or repositioning an existing product to niche markets can delay the potential market disruption (Utterback, 1994; Chen et al., 2010; Adner & Snow, 2010; Adner & Kapoor, 2016). Charitou & Markides (2003) have collected empirical evidence and categorized adoption tactics in response to a threat from an in-market disruption faced by the incumbent. Efforts were made in order to develop concepts which would allow managers to recognize a disruptive innovation, and distinguish it from a sustaining innovation. In this context, Schmidt and Druehl (2008) suggested a complementary framework and an alternative terminology based on the different adoption patterns. In their terminology, disruptive innovations as defined by Christensen follow the low-end encroachment (i.e., an immediate, fringe- or detached-market). The term encroachment describes the process through which the new product takes sales away from the old product. Low-end encroachment is therefore the adoption pattern happening when the innovation first displaces the existing product at the low-end, and afterwards diffuses to the mainstream market. Recently, three novel topics of research in the area of disruptive innovation have been proposed by Christensen and co-authors. The topics include: the response strategies of the incumbent, the factors shaping performance trajectories, and innovation metrics (Christensen et al., 2018).

The interest in an investment might also vary depending on the status of the market player. Investment opportunities which are attractive to entrants could be considered unattractive by incumbents when evaluated in the context of their already established business. In our study, we investigate different incumbents' strategies in responding to innovation and study the conditions under which the investment in adopting disruptive innovation is profitable. Our study shows that the interest in adopting innovation depends not only on the level of thread, but also on the speed of innovation adoption. Low thread and slow adoption can discourage the incumbent from disruptive innovation. We also show that in some cases the response can be delayed. The delay represents the dilemma between exploitation of the established Incumbent's market and exploration of the opportunities with innovation.

2. The model

We shall consider a *low-end disruptive innovation* as defined by Christensen & Raynor (2003), or an *immediate low-end encroachment* as defined by Schmidt & Druehl (2008). For the sake of simplicity, we refer to it as *innovation* which is service or product. The innovation is launched by the Entrant, the company which launches the innovation, which is not necessarily a new market entrant. The Incumbent responds to the innovation either by ignoring it or adopting it. Alternative response strategies are discussed in the next section.

2.1 Diffusion process and key assumptions

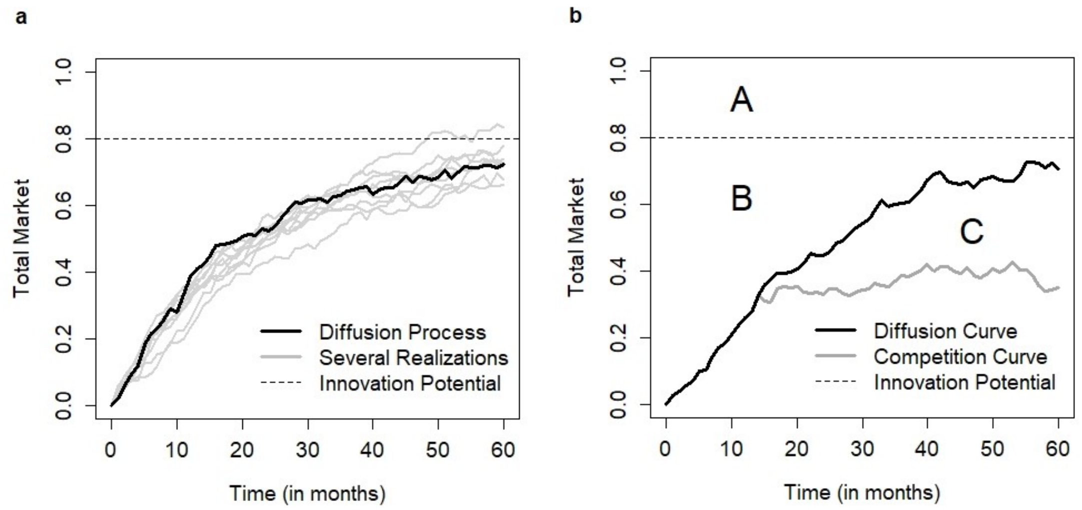
The diffusion of innovation is a process though which customers on the market adopt using the innovation. The innovation satisfies similar consumer needs as the products or services offered by the Incumbent. The market of innovation sources from the established Incumbent's business and causes a gradual encroachment of the Incumbent's market, splitting the total market into two parts along the innovation diffusion curve. Without loss of generality we shall assume the total market size equals 1.

Innovation diffusion is a stochastic process X that approaches an equilibrium state in which the innovation is eventually adopted, thereby reaching its full potential. The potential of innovation is expressed as its eventual market share μ ($0 < \mu < 1$) in the total market. The diffusion starts at 0, i.e., $X(0) = 0$ and follows the Ornstein-Uhlenbeck mean-reverting stochastic process, i.e.,

$$dX = \rho(\mu - X)dt + \sigma dW$$

where ρ ($0 < \rho < 1$) is the diffusion rate, σ is the variance (or market volatility parameter), and W is the standard Wiener process with zero mean and standard deviation equal to 1. In our model, we use uppercase letters to refer to random variables and lowercase letters to refer to values related to the realization of the process. The exogenous process parameters are denoted by Greek letters. Figure 1a shows several realizations of the mean-reverting process for some fixed parameter values. We show the process on a monthly basis, while the parameters ρ and σ are given on an annual basis. The choice of the Ornstein-Uhlenbeck process is natural in the context of a market reaching a new equilibrium state. It assumes the constant innovation diffusion rate ρ subject to stochastic fluctuations described by the parameter σ. While processes with variable diffusion rates might be considered, such considerations are beyond the scope of the current paper.

Figure 1. Innovation Diffusion and In-Market Opportunities. (a) Several realizations of the diffusion process. (b) Opportunities in different parts of the market. Parameters: μ = 0.8, ρ = 0.5, and σ = 0.05.



Source: own elaboration.

The market share of innovation X is a random variable which evolves over time and depends on the parameters of the stochastic process μ, ρ and σ. If the value of X at time t_0 is x_0 (i.e., $X(t_0) = x_0$), then the expected value of X at any future time $t > t_0$ is given by:

$$E[X(t)] = \mu + (x_0 - \mu)\, e^{-\rho(t-t_0)}$$

and the variance of $X(t)$ equals:

$$Var[X(t)] = \frac{\sigma^2}{2\rho}\,(1 - e^{-2\rho(t-t_0)})$$

Figure 1b shows the opportunities separately in each part of the market, where A denotes the sales to customers eventually unwilling to adopt the innovation, B refers to the sales to customers who have not adopted the innovation yet, and the remaining area refers to sales to customers have already switched to the innovation. This part of the market is potentially split between the Incumbent (part C) and the Entrant, provided that the Incumbent decides to compete with the Entrant by adopting the innovation. Otherwise, the part below the diffusion curve is serviced by the Entrant.

We assume that the split between C and the Entrant's market, called the competition curve, follows the trajectory of a stochastic process $Y(t)$, with $t \geq t_0$ starting at x_0 (i.e., $Y(t_0) = x_0$), where t_0 is the time when the decision of launching a competitive innovation is considered. The stochastic process Y is the Ornstein-Uhlenbeck process approaching a competitive equilibrium which is a fraction of the eventual market share of the innovation, therefore cμ ($0 < c < 1$), at the same rate ρ and the same variance σ as the innovation diffusion process X, i.e.,

$$dY = \rho(c\mu - Y)dt + \sigma dW$$

The competitive equilibrium depends on the response strategy chosen by the Incumbent and is parameterized as λ, which denotes the advantage of the leader over the follower. We set $\lambda = 2/3$, which can be derived from the Stackelberg leader-follower imperfect competition model (Stackelberg, 1934; Varian, 1992), assuming that two players with the same marginal cost compete over quantity with the leader setting the market price. This level of the leader advantage over the follower is consistent with the empirical evidence when analysing a first-mover advantage in the FMCG markets.

The alternative response strategies are considered by the Incumbent as investment opportunities. The expected payoff at time t_0 depends on the strategy s and is based on the expected Net Present Value (NPV), the standard tool for the evaluation of investment opportunities. NPV is the discounted future cash flow which depends on the expected realization of stochastic processes X and Y:

$$V(s, x_0, t_0) = E\left[\int_{t_0}^{\infty} CF(s, X, Y, t)e^{-r(t-t_0)}dt\right] - k(s)$$

where $X(t_0) = x_0$, $CF(s, X, Y, t)$ is the future cash flow associated with the response s at time t, $k(s)$ is the cost of implementing the strategy, and r is the discount factor. We set $r = 0.05$.

We assume that the cash flow is proportional to the market share of the Incumbent in related periods. The Incumbent enjoys the annual profitability level π_E from selling the existing service or product and the annual profitability level π_D, from selling the competitive innovation. Consistently with the condition of asymmetric incentives postulated by Christensen, we shall assume that $\pi_D < \pi_E$. For the purpose of the subsequent analysis we set $\pi_D = \pi_E/2$. The cash flow depends on the chosen response strategy s and is calculated as the sum of cash flows from all parts of the market (A, B and C). and Y:

In real-life situations, incumbents might face restrictions related to their knowledge or access to technology and the possibility of producing and offering the innovation on the market. As noted by Gans (2016a), they also face challenges in integrating the innovation in their primary business. For simplicity, in our model we disregard these challenges and assume that the Incumbent can immediately produce and market the innovation. If Incumbent decides to adopt the innovation, a flat one-off cost $k\pi_E$ is applied. The cost is expressed as a proportion of profitability level from the existing service or product where k indicates the number of annual yearly profits from the initial business of the Incumbent which have to be invested. The Incumbent can start offering the innovative service or product in the market with no delay, immediately after incurring the cost. We set $k = 1$. Note that the realization of the innovation diffusion process can fall below 0 in the initial stage of innovation adoption on the market. The negative cash flow associated with such a scenario is interpreted as an unexpected additional cost.

2.2 Response strategies

Empirical evidence provided by Charitou & Markides (2003) reveals five key responses of incumbents to disruptive threats: 1) focus on, and invest only in the traditional business, 2) ignore the innovation, 3) attack back: disrupt the disruption, 4) adopt the innovation, by playing both games at once, and 5) embrace the innovation completely and scale it up.

The first response was implemented, for instance, by Gillette in response to the threat from disposable razors. The company focused on improving the performance of their product and at a later stage launched two new products: Sensor and Mach3. The first response assumes that the innovation can gain a certain level of the market, but fails to overtake a substantial prat of it. The second response instead assumes that the disruptive service or product is divergent from the established business of the incumbent and targets different customers. In none of the two first strategies the Incumbent decides to adopt the innovation. The third response is based on offering a new service or product which disturbs the innovation diffusion process. This strategy was implemented for instance by the Swiss watch companies in response to Seiko and Timex offering watches based on quartz technology.

The fourth and the fifth strategies assume that the Incumbent adopts the innovation. The implementation of the fourth strategy typically involves establishing a new company unit such as in the case of First Direct, a subsidiary of Midland Bank in the UK, offering telephone banking. Such a strategy assumes that the Incumbent in parallel continues to service its current customers with the established product or service. The fifth strategy sees the established company become the leader, by embracing the new idea and focusing on bringing it to the market (Charitou & Markides, 2003). This strategy often assumes that the current business is de-invested or abandoned.

In our analysis, we consider three simplified strategies, based on the ones proposed by Charitou & Markides. The first is to *ignore*, which corresponds to the first and the second strategies outlined above. The second is to *react*, which corresponds to the fourth one and assumes playing both games. The third is to *embrace*, which corresponds to the fifth one and assumes converting the existing Incumbent's customers to innovation. We do not consider the third strategy, namely to attack back and disrupt the disruption. In our analysis, we assume that the diffusion process is exogenous to Incumbent.

Our *ignore* strategy (s = *I*) assumes that the Incumbent fails to adopt the innovation regardless of the underlying reasons. In this case, the Incumbent keeps on serving its customers with its existing service or product and ignores the innovation opportunities at the low end of the market. The payoff associated with this response at time t_0 equals (see Appendix):

$$V(I,\ x_0\ , t_0) = \frac{1}{r}\pi_E(1-\mu) + \frac{1}{r+\rho}\pi_E(\mu - \ x_0)$$

The first part of the sum reflects the discounted profits from the part of the market denoted by A, i.e., the sales from customers who are eventually unwilling to adopt the innovation. The second part reflects the discounted profits from part B, i.e., the sales from customers who have not adopted the innovation yet. The innovation diffusion process X makes the Incumbent's market share shrink over time, eventually approaching $1-\mu$. This equation represents the payoff at time t_0 when the Entrant's market share is x_0 = X(t_0). Ignore is the default option for the Incumbent before it decides to switch to an alternative strategy and time of consideration.

Both the *react* and the *embrace* strategies assume that the Incumbent adopts the innovation and enters the competition with the Entrant. Adopting innovation requires paying the cost $k\pi_E$. The cost is the same in case of either *react* or *embrace*, given that it is what the Incumbent has to pay to initiate the production of the innovative product irrespectively of the quantity or the market share obtained thereafter. In the case of *react*, the Incumbent adopts the innovation in addition to its existing service or product and starts competing with the Entrant in the part of the market below the diffusion curve. The competitive equilibrium is set to reflect the leadership position of the Entrant in this part of the market. The payoff from *react* strategy is given by (see Appendix):

$$V(R,\ x_0\ ,t_0) = \frac{1}{r}\pi_E(1-\mu) + \frac{1}{r+\rho}\pi_E(\mu - \ x_0) + \left(\frac{1}{r} - \frac{1}{r+\rho}\right)\pi_D\mu(1-\lambda) - \ k\pi_E$$

where the first two components are the same as for the *ignore* strategies and relate to parts A and B of the market, respectively. The third component denotes the discounted profits from offering the competitive innovation, and reflects part C of the market approaching the competitive equilibrium where Entrant is the leader. The last component is the one-off cost of adopting the innovation.

In case of *embrace*, the Incumbent convers the customers to innovation and takes the leadership of bringing the innovation to the market. It is assumed that the conversion happens with no delay and sales from part B of the market are serviced by innovation. Implementing the innovation is less profitable than the traditional business, but allows the Incumbent to get long-term leadership advantage over the Entrant. It is assumed part A of the market is serviced with the established service or product as the Incumbent is unable and unwilling to convert customers in this part of the market to innovation. The payoff is given by (see Appendix):

$$V(E,\ x_0\ ,t_0) = \frac{1}{r}\pi_E(1-\mu) + \frac{1}{r+\rho}\pi_D(\mu - \ x_0) + \left(\frac{1}{r} - \frac{1}{r+\rho}\right)\pi_D\mu\lambda - k\pi_E$$

where the first component is the same as in the case of *ignore* and *react* tactics and relates to part A of the market. The second component reflects part B of the market where, from now on, the Incumbent sells its innovation at a lower profitability level π_D. Part C is almost the same as in the *react* response, except that now the competition curve approaches the equilibrium state where the Incumbent is the leader.

2.3 The response of the incumbent

In this section we investigate the choice of the optimal response which maximizes the expected payoff of the Incumbent. The choice of the strategy depends on the values of parameters μ and ρ (when $\rho > r$), while other parameters are kept constant. The analysis assumes that the Incumbent knows their values. This assumption is not necessarily realistic, but allows us to consider the optimal strategy in absence of knowledge constraints. Market volatility plays no role of choice of the strategy, as payoffs based on the expected net present value do not depend on σ.

The following equations define the iso-profit curves at time t_0 ($x_0 = X(t_0)$), i.e., the parameters' values for which alternative strategies (I and R, I and E, and R and E) offer equal payoffs:

$$\mu\rho(1-\lambda) = r(r+\rho)k\frac{\pi_E}{\pi_D}$$

$$\mu\rho\lambda = r(\mu - x_0)\left(\frac{\pi_E}{\pi_D} - 1\right) + r(r+\rho)k\frac{\pi_E}{\pi_D}$$

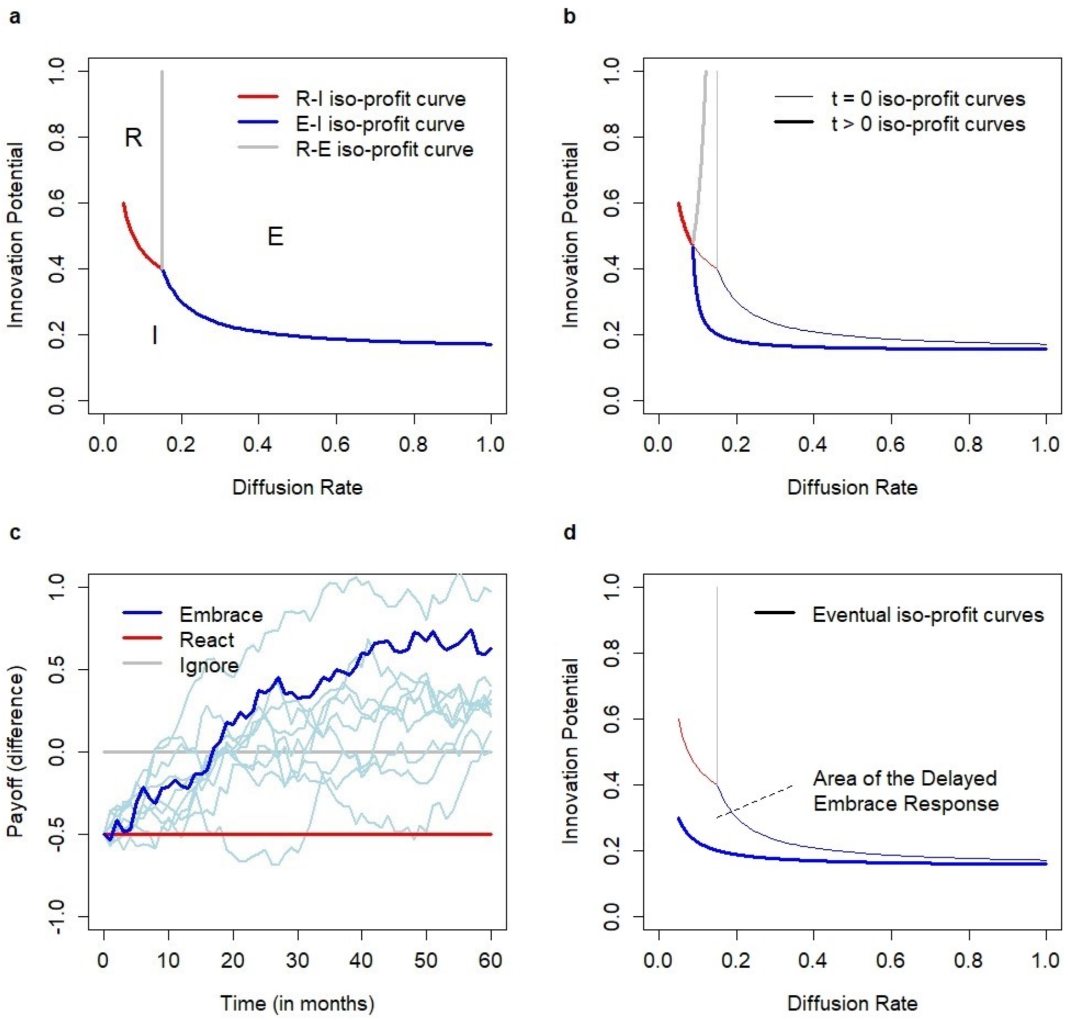
$$\mu\rho(2\lambda - 1) = r\left(\frac{\pi_E}{\pi_D} - 1\right)(\mu - x_0)$$

It is important to note that all equations depend only on the ratio π_E/π_D, but not on the particular values of π_E and π_D. When $x_0 = 0$, the last equation does not depend on μ and hence, the choice between *embrace* and *react* depends only on the rate of innovation diffusion. In order to determine the optimal response as a function of μ and ρ, we plot the iso-profit curves on the parameter space (Fig. 2a). The values of other parameters are fixed, namely $\pi_E/\pi_D = 2$, $k = 1$, $\lambda = 2/3$, and $r = 0.05$. Note that when $x_0 = 0$, for $\rho \leq (r/\lambda)\cdot(\pi_E/\pi_D - 1)$ the *ignore* strategy is better than the *embrace* strategy regardless of μ.

Our analysis shows that if the innovation potential μ is sufficiently low, then the *ignore* strategy provides the Incumbent with the highest payoff (in the graph, this represents the area below the red and blue curves). This stays true regardless of the fact that the market for the established product shrinks. If instead μ is high, the choice between *react* and *embrace* is determined by the rate of innovation adoption ρ. If its value is sufficiently low, the *react* tactic is the optimal strategy. Alternatively, if the rate of innovation adoption is fast enough, then *embrace* is the optimal response of the Incumbent.

However, over time the choice of the optimal strategy changes (Fig. 2b). In some parameters' ranges, it becomes more profitable to implement the *embrace* strategy instead of *ignore* or *react*. This is because the opportunities in part B of the market shrink over time and it becomes more profitable to abandon the increased profitability associated with the established product in favor of assuming the leadership in bringing the innovation into market. This dynamic assumes that the Incumbent could assume the leadership despite delayed response in applying the *embrace* strategy. This assumption is not necessarily realistic in the real marketplace.

Figure 2. Optimal Response. (a) Optimal strategies at time t = 0 based on the iso-profit curves for alternative strategies. (b) Optimal strategies at time t > 0 when x(t) = 0.2. (c) Differences in payoffs associated with alternative strategies as a function of time (μ = 0.3, ρ = 0.15, and σ = 0.05). Light blue lines represent different realizations of the innovation diffusion process. (d) Long-run optimal strategies and the area of delayed embrace response. Time of the delayed response can very significantly, as shown in (c).



Source: own elaboration.

The timing for such a response is determined by reaching a specific level of innovation adoption and could significantly vary depending on the realization of the diffusion process (Fig. 2c). In our example, the incentive to change from the *ignore* strategy to the *embrace* strategy occurred after 18 months (μ = 0.3, ρ = 0.15, and σ = 0.05). For the considered parameters, such an incentive happens when the market share of innovation exceeds 10%.

Our analysis suggests the existence of the optimal strategy in the long run which might be different than the optimal strategy at time t = 0. Long-term optimal strategies are shown in Figure 2d. The delayed incentive to implement the embrace strategy represents the conflict between mid-term profitability perspective which suggests exploitation of the existing opportunities with the established product and long-term profitability perspective which suggests maximizing profits in the long run by assuming the leadership in the changing market.

Another interesting observation is that the *react* strategy is hardly optimal at t = 0 except for the limited range of parameters' values. The react strategy can only be optimal if both the innovation potential is high and the rate of innovation adoption is very low, close to the interest rate. The *react* strategy is not optimal in the long-run. Long term, the *embrace* strategy dominates the *react* strategy for all parameters' values. Our analysis stands in contrast with the previous considerations and the common business practice. Indeed, Christensen (2005) asked whether there was a universally effective response strategy to a disruptive threat and suggested that a separate division, one which explores the new disruptive model, could be the solution. This strategy calls for the exploitation of the existing opportunities with the established product which could potentially undermine the opportunity of assuming the leadership in the growing innovation market.

In the analysis, we assumed the absence of knowledge constraints and postulated that the choice of the optimal strategy is based on knowing the exact values of μ and ρ, the parameters of the innovation diffusion process. In reality, the eventual innovation potential and the rate of adoption might not be known to the Incumbent. Instead, the Incumbent can only rely on the expectation or belief of their true values. Mismatch between Incumbent's expectations and the true values of the diffusion process can produce all sorts of non-optimal response: ignoring the innovation, if the Incumbent underestimates its potential or adoption rate, or inadequate choice between the react and embrace strategies. Understanding how the lack of prior knowledge about the innovation potential and adoption rate is beyond the scope of the current paper.

Our analysis was conducted for fixed values of the other model parameters. Higher values of k result in the *ignore* strategy becoming relatively more attractive to Incumbent by increasing the area where *ignore* is the optimal response. Higher values of π_D in relation to π_E make the *react* and the *embrace* strategies relatively more attractive to the Incumbent by decreasing the area where *ignore* is the optimal strategy. The choice between the *embrace* and the *react* tactics is driven by the value of π_E/π_D and the value of the discount factor. The lower values of the discount factor make future profits more attractive to the Incumbent, and hence promote choosing the *embrace* response over *react*. We have set the value of $\lambda = 2/3$, which is the advantage of the leader over the follower. If the parameter approaches $1/2$, by which there is no advantage of the leader, then the threshold value of ρ moves to the right and the *react* response becomes more attractive in relation to the *embrace* response. In particular for $\lambda = 1/2$, the *embrace* response is always worse than the *react* response. Indeed, if there is no leadership advantage, the Incumbent has no incentive to convert its customers to an innovation which is assumed to be less profitable than the established business. paper.

It should be stressed that the assumption about asymmetric incentives ($\pi_E > \pi_D$) is essential in all of our analyses. Otherwise, adopting the innovation is always profitable provided that the one-off cost does not exceed the future profits from launching the innovation itself. Apart from the asymmetric incentives, Christensen (2005) postulates the second condition for market disruption to happen, that is the overshot in satisfying customers' needs by the traditional business. Note that we have not included any explicit assumption about the performance of the traditional product or the inferior performance of the innovation. However, the innovation performance implicitly affects the Incumbent's expectations towards innovation potential μ and the diffusion dynamics. If the performance of innovation is inferior, the Incumbent could expect the market potential of the innovation μ to be low, by which ignoring the innovation seems reasonable. Thus, the condition about performance overshot of the traditional business makes the Incumbent vulnerable to an inferior service or product and might lead to creating the gap between Incumbent's expectations and the actual innovation potential.

3. Conclusions

Christensen (1997) proposed "the innovator's dilemma" to explain how incumbents fail to respond to a disruptive threat by focusing on sustaining innovations which improve their current products and services for the up-market and mainstream market customers and overlooking opportunities at the bottom of the market and with non-consumption. In this context, managerial and organizational constraints have been extensively studied. Our model shows that in some cases the delayed adoption of innovation could be linked to the exploitation-exploration dilemma in the absence of managerial and organizational constraints. Thus, our study provides a new perspective of the potential casual mechanism of incumbents' failure in response to a disruptive threat. The existing empirical evidence (Charitou & Markides, 2003) shows a range of response strategies used in the past. Our analysis confirms that different response strategies should indeed be considered. For a low potential innovation, ignoring it could be the optimal strategy maximizing incumbents' payoff despite the shrinkage of their established markets. The opportunity of assuming leadership in driving the disruptive innovation should be considered for innovations with medium to high potential, if their speed of adoption is high. In such a scenario abandoning the opportunities in exploitation of the established market is profitable. Conversely, if the speed of adoption is low, the exploitation of the established market might be profitable for the incumbent which suggests implementing the strategy of playing both games at once (Charitou & Markides, 2003). Playing both games by establishing a separate division exploring the opportunities with innovation has been previously considered as the effective incumbents' response to market disruption (Christensen, 2005). However, our analysis shows that playing both games at once is not an optimal strategy in the long-run if it undermines the opportunity of taking leadership in the disruptive innovation market and regardless of the speed of innovation adoption. Assessing which market strategies could be sound in responding to disruptions was considered one of the potential directions of future research (Christensen et al., 2018). Our analysis provides a partial response in this area.

Previous research investigated the organizational constrains in adopting disruptive innovation. Gans (2016b) outlined the essential challenges of disrupted incumbents in transferring the disruption into their primary business. Such constraints were not assumed in our analysis. They should be considered when considering the optimal response in the marketplace, as they influence the range of incumbents' response choices and timing.

Schmidt & Druehl (2008) provide a compelling argument that high-end encroachment, in contrast to immediate low-end encroachment analysed here, is unlikely to lead to in-market disruption. Our model makes no distinction between these two cases, yet we can briefly discuss our predictions for high-end encroachments. First, the assumption about asymmetric incentive might be violated. Second, high-end customers are responsible for a disproportionally high level of market share in comparison to low-end customers. Therefore, the introduction of the innovation operating on the high end of the Incumbent market leads to an effectively faster diffusion of the innovation and a potentially higher eventual market share. This means that incumbents are more likely to respond vigorously to the threat, consistently with the analysis of Schmidt & Druehl (2008).

Disruptive innovations often expand the existing markets (Govindarajan & Kopalle, 2006; Markides, 2006) where the innovations (*new-market disruptions)* compete against non-consumption. Schmidt & Druehl (2008) classify such innovations as *fringe-market low-end* *encroachments* and *detached-market low-end encroachments*. We analyzed the possibility of market expansion and showed that market expansion leads to increased attractiveness of the *embrace* and *react* strategies as compared to the *ignore* strategy. Additionally, market expansion makes the *embrace* strategy relatively more attractive than the *react* strategy. Otherwise, the market expansion does not qualitatively change the landscape of optimal response strategies compared to the baseline scenario with no market expansion. Thus, our general conclusions are similar regardless of whether market expansion is or is not assumed

To optimize choices over time, many existing studies employ the Bellman equation (Bellman, 1952). In such an approach, beliefs are updated according to the changes in the observed conditions. The most recent applications of the Bellman equation include Dangl & Wirl (2004), Doraszelski (2004), Cao & Wan (2009), Walsh et al. (2014), Dang & Forsyth (2016), O'Donoghue et al. (2018). In the paper of Farzin et al. (1998), the authors investigated the optimal timing of adoption of innovative technologies, when decisions are irreversible and the actual evolution of the technologies is a random process. The study compares the net present value approach with the dynamic programming approach, represented by the use of the Bellman equation, and shows how the two methods differ in the conclusions about what is the optimal timing of adoption of an innovative technology. Net present value approach does not account for the option value of waiting, as argued also by Doraszelski (2001). Applying the Bellman equation instead of the net present value considered here could be an interesting extension of our modeling approach.

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Appendix – Payoffs derivation

In this section, we provide the derivation of payoffs for the Incumbent in each part of the market. The payoff is calculated at t_0, which is the moment when the Incumbent considers a decision to adopt the innovation. We assume $X(t_0) = x_0$, by which is the share of innovation in the initial Incumbent's market which is also the market share of the Entrant.

Part A

In part A, the Incumbent serves the customers who are eventually unwilling to switch to the innovation. This part is deterministic, and does not depend on the realization of the processes X and Y). The cash flow is constant over time and equals:

$$CF_A(s, X, Y, t) = \pi(1 - \mu) \quad \text{(A1)}$$

where $\pi = \pi_E$.

Part B

In part B, the Incumbent serves the customers who will eventually switch to the innovation, but have not done it yet till the moment of deciding by the Incumbent. Cash flow in this area depends on the realization of the innovation diffusion stochastic process X and equals:

$$CF_B(s, X, Y, t) = \pi(\mu - X(t)) \quad \text{(A2)}$$

In the *ignore* and *react* strategies this part of the market is served by the existing service or product, hence $\pi = \pi_E$. In case of the *embrace* response, we assume that the Incumbent can convert its customers to innovation at the moment of making the decision, hence going forward is serving them with the disruptive innovation and $\pi = \pi_D$. Note that when the innovation diffusion process approaches the eventual market share μ, then the process can exceed μ and the calculated cash flow in this region can become negative. This is not a concern since the cash flow from this region is analyzed jointly with the cash flow from part A which is positive and covers time intervals when X(t) exceeds μ, and $\pi_E \geq \pi_D$.

Part C

In part C, the Incumbent serves the customers with the innovation in case it decides to compete with the Entrant, which is in case of *react* and *embrace* responses only. The competitive equilibrium is set at the level cμ that is proportional to the eventual market share of the innovation, where the coefficient c depends on the response chosen (which means that $c = \lambda$ for the *react* strategy, and $c = 1 - \lambda$ for the *embrace* strategy.

$$CF_C(s,X,Y,t) = \alpha\pi(X(t) - Y(t)) \tag{A3}$$

Regardless of the strategy, this part of the market is served with the innovation, hence $\pi = \pi_D$. We also introduce here the market expansion constant α, which expands the cash flow from this part of the market only. Note that if the stochastic fluctuations of the processes X and Y are uncorrelated, the process Y can exceed X. This is not a concern, since the cash flow is considered jointly from parts B and C, and $\pi_E \geq \pi_D$.

Derivation of payoffs

If F(s, X, Y, t) denotes the discounted cash flow from applying the strategy s at the time t_0, then:

$$F(s,X,Y,t) = (CF_A(s,X,Y,t) + CF_B(s,X,Y,t) + CF_C(s,X,Y,t))\, e^{-r(t-t_0)} \tag{A4}$$

Regardless of the strategy s, the function is a linear function of X and Y and can be written as:

$$F(s,X,Y,t) = \big(a(s)X + b(s)Y + c(s)\big)\, e^{-r(t-t_0)} \tag{A5}$$

where:

$$a(I) = -\pi_E\,, b(I) = 0, c(I) = \pi_E \tag{A6}$$

$$a(R) = -\pi_E + \alpha\pi_D, b(R) = -\alpha\pi_D, c(R) = \pi_E \tag{A7}$$

$$a(E) = (\alpha-1)\pi_D\,, b(E) = -\alpha\pi_D, c(E) = \pi_E(1-\mu) + \pi_D\mu \tag{A8}$$

We now apply the two-dimensional Ito lemma to F:

$$dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial x}dX + \frac{\partial F}{\partial y}dY + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}(dX)^2 + \frac{1}{2}\frac{\partial^2 F}{\partial y^2}(dY)^2 + \frac{\partial^2 F}{\partial x \partial y}dXdY \tag{A9}$$

It should be noted that, since F is linear with regard to X and Y and do not contain terms dependent on XY, it is verified that:

$$\frac{\partial^2 F}{\partial x^2} = \frac{\partial^2 F}{\partial y^2} = \frac{\partial^2 F}{\partial x \partial y} = 0 \tag{A10}$$

Hence, in our case:

$$dF = -r\big(a(s)X + b(s)Y + c(s)\big)\, e^{-r(t-t_0)}dt + a(s)\, e^{-r(t-t_0)}dX + b(s)\, e^{-r(t-t_0)}dY \tag{A11}$$

And thus:

$$dF = -r\big(a(s)X + b(s)Y + c(s)\big)\, e^{-r(t-t_0)}dt + \rho(\mu - X)a(s)\, e^{-r(t-t_0)}dt + \rho(c\mu - Y)b(s)\, e^{-r(t-t_0)}dt + \sigma(dW_X + dW_Y) \tag{A12}$$

where $W_X + W_Y$ is the Wiener process with zero mean (i.e., $E[W_X + W_Y] = 0$) and its variance depends on the possible correlation between the Wiener processes W_X and W_Y. Thus:

$$dF = [\rho\mu a(s) + \rho c\mu b(s) + \rho c(s) - Xra(s) - X\rho a(s) - Yrb(s) - Y\rho b(s) - rc(s) - \rho c(s)]\, e^{-r(t-t_0)} + \sigma' dW \quad \text{(A13)}$$

where σ' is some constant. Therefore:

$$dF = [\rho\mu a(s) + \rho c\mu b(s) + \rho c(s)]\, e^{-r(t-t_0)} - (r+\rho)F + \sigma' dW \quad \text{(A14)}$$

This stochastic process is analytically solvable:

$$F(t) = \big(\mu a(s) + c\mu b(s) + c(s)\big)\, e^{-r(t-t_0)} - f_0(s) e^{-(r+\rho)(t-t_0)} + e^{-(r+\rho)(t-t_0)} \int_{t_0}^{t} \sigma'\, e^{(r+\rho)(t-t_0)}\, dW \quad \text{(A15)}$$

where $f_0(s)$ is a constant such that $F(t_0)$ is the cash flow at t_0 associated with the chosen strategy, namely $F(t_0) = \pi_E(1 - x_0)$ for the *ignore* and *react* strategies, and $F(t_0) = \pi_E(1 - \mu) + \pi_D(\mu - x_0)$ for the *embrace* strategy. $f_0(s)$ is calculated as follows:

$$f_0(I\ or\ R) = \mu a(s) + c\mu b(s) + c(s) - \pi_E(1 - x_0) \quad \text{(A16)}$$

$$f_0(E) = \mu a(s) + c\mu b(s) + c(s) - \pi_E(1 - \mu) + \pi_D(\mu - x_0) \quad \text{(A17)}$$

And hence, the expected value of F is given by the following formula:

$$E[F(t)] = \big(\mu a(s) + c\mu b(s) + c(s)\big)\, e^{-r(t-t_0)} - f_0(s)\, e^{-(r+\rho)(t-t_0)} \quad \text{(A18)}$$

Now, we can calculate the expected $NPV(s, x_0, t_0)$:

$$NPV(s, x_0, t_0) = E\left[\int_{t_0}^{\infty} CF(s, X, Y, t)\, e^{-r(t-t_0)} dt\right] = E\left[\int_{t_0}^{\infty} F(t) dt\right] = \int_{t_0}^{\infty} \big(\mu a(s) + c\mu b(s) + c(s)\big) e^{-r(t-t_0)} - f_0(s)\, e^{-(r+\rho)(t-t_0)}\,)\, dt = \frac{1}{r}\big(\mu a(s) + c\mu b(s) + c(s)\big) - \frac{1}{r+\rho} f_0(s) \quad \text{(A19)}$$

Finally, we can plug in the values of the constants $a(s)$, $b(s)$, $c(s)$, $f_0(s)$ and the cost $k(s)$ for each response strategy, as well as the constant $c = 1 - \lambda$ for the *react* strategy and $c = \lambda$ for the *embrace* strategy:

$$V(I, x_0, t_0) = \frac{1}{r}\pi_E(1 - \mu) + \frac{1}{r+\rho}\pi_E(\mu - x_0) \quad \text{(A20)}$$

$$V(R, x_0, t_0) = \frac{1}{r}\pi_E(1 - \mu) + \frac{1}{r+\rho}\pi_E(\mu - x_0) + \alpha\left(\frac{1}{r} - \frac{1}{r+\rho}\right)\pi_D \mu(1 - \lambda) - k\pi_E \quad \text{(A21)}$$

$$V(E, x_0, t_0) = \frac{1}{r}\pi_E(1 - \mu) + \frac{1}{r+\rho}\pi_D(\mu - x_0) + \alpha\left(\frac{1}{r} - \frac{1}{r+\rho}\right)\pi_D \mu\lambda - k\pi_E \quad \text{(A22)}$$

It can be noted that each value function is a sum of two or three terms and the cost. The terms in the sums actually relate to the expected NPV from the separate parts of the market. The payoff associated with part A is given by:

$$\frac{1}{r}\pi_E(1-\mu) \tag{A23}$$

While the payoff from part B equals:

$$\frac{1}{r+\rho}\pi(\mu - x_0) \tag{A24}$$

where the value of π differs between *embrace* and the other strategies. The payoff from part C is given by:

$$\alpha\left(\frac{1}{r}-\frac{1}{r+\rho}\right)\pi_D\mu c \tag{A25}$$

where α is the market expansion factor ($\alpha = 1$, if there is no expansion) and the value of c differs between react and embrace strategies.

Iso-profit curves

The iso-profit curve between ignore and react is given by the following equation:

$$\frac{1}{r}\pi_E(1-\mu) + \frac{1}{r+\rho}\pi_E(\mu - x_0) = \frac{1}{r}\pi_E(1-\mu) + \frac{1}{r+\rho}\pi_E(\mu - x_0) + \alpha\left(\frac{1}{r}-\frac{1}{r+\rho}\right)\pi_D\mu(1-\lambda) - k\pi_E \tag{A26}$$

As the first two terms are the same on both sides, the equation is equivalent to:

$$0 = \alpha\left(\frac{1}{r}-\frac{1}{r+\rho}\right)\pi_D\mu(1-\lambda) - k\pi_E \tag{A27}$$

which can be written as:

$$\alpha\mu\rho(1-\lambda) = r(r+\rho)k\frac{\pi_E}{\pi_D} \tag{A28}$$

where α is the market expansion constant, λ is the first mover advantage, k is the cost coefficient, r is the discount factor, and π_E and π_D are the profit coefficients for the established business and the innovation, respectively. Note that the iso-profit curve between ignore and react does not depend on the market share of the innovation of the Entrant, and stays the same over time. The iso-profit curve between ignore and embrace is given by the equation:

$$\frac{1}{r}\pi_E(1-\mu) + \frac{1}{r+\rho}\pi_E(\mu - x_0) = \frac{1}{r}\pi_E(1-\mu) + \frac{1}{r+\rho}\pi_D(\mu - x_0) + \alpha\left(\frac{1}{r}-\frac{1}{r+\rho}\right)\pi_D\mu\lambda - k\pi_E \tag{A29}$$

The first term cancels and the equation can be rearranged to:

$$\frac{1}{r+\rho}(\frac{\pi_E}{\pi_D}-1)(\mu- x_0) = \alpha\left(\frac{1}{r}-\frac{1}{r+\rho}\right)\mu\lambda - k\frac{\pi_E}{\pi_D} \quad (A30)$$

which can be written as:

$$\alpha\mu\rho\lambda = r(\mu- x_0)\left(\frac{\pi_E}{\pi_D}-1\right)+r(r+\rho)k\frac{\pi_E}{\pi_D} \quad (A31)$$

where α is the market expansion constant, λ is the first mover advantage, k is the cost coefficient, r is the discount factor, π_E and π_D are profit coefficients, and x_0 is the share of the Entrant in the initial market of the Incumbent. The iso-profit curve between react and embrace is given by the equation:

$$\frac{1}{r}\pi_E(1-\mu)+\frac{1}{r+\rho}\pi_E(\mu- x_0)+ \alpha\left(\frac{1}{r}-\frac{1}{r+\rho}\right)\pi_D\mu(1-\lambda) - k\pi_E = \frac{1}{r}\pi_E(1-\mu)+ \frac{1}{r+\rho}\pi_D(\mu- x_0)+ \alpha\left(\frac{1}{r}-\frac{1}{r+\rho}\right)\pi_D\mu\lambda - k\pi_E \quad (A32)$$

This can be rearranged to:

$$\frac{1}{r+\rho}\pi_E(\mu- x_0)+ \alpha\left(\frac{1}{r}-\frac{1}{r+\rho}\right)\pi_D\mu(1-\lambda) = \frac{1}{r+\rho}\pi_D(\mu- x_0)+ \alpha\left(\frac{1}{r}-\frac{1}{r+\rho}\right)\pi_D\mu\lambda \quad (A33)$$

which is equivalent to:

$$\frac{1}{r+\rho}\left(\frac{\pi_E}{\pi_D}-1\right)(\mu- x_0) = \alpha\left(\frac{1}{r}-\frac{1}{r+\rho}\right)\mu(2\lambda-1) \quad (A34)$$

And finally to:

$$\alpha\rho\mu(2\lambda-1) = r\left(\frac{\pi_E}{\pi_D}-1\right)(\mu- x_0) \quad (A35)$$

where α is the market expansion constant, λ is the first mover advantage, r is the discount factor, π_E and π_D are profit coefficients, and x_0 is the share of Entrant in the initial market of the Incumbent.

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