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From the Point to Chaos: The Evolution of the Concept of Equilibrium in Economic Theory

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Abstract: This article examines equilibrium as a representation of economic order. It distinguishes five properties often treated as a single feature: existence, uniqueness, stability, convergence and predictability. Twentieth century fixed-point formalisation, exemplified by Arrow-Debreu, established existence under specified assumptions but did not guarantee the other four. Their failure redirects attention to other objects of analysis: the selection of an equilibrium, persistent cycles and regimes, adjustment paths, and distributions of outcomes. Empirically, equilibrium is inferred through models and identification assumptions rather than observed directly. We therefore distinguish two ideal-typical orientations: a solution-centred approach that locates order in an equilibrium object and a process-centred approach that examines its evolution over time. The latter draws on stationary distributions, invariant measures and the analysis of ergodicity. We argue that the continued prominence of point based equilibrium reflects its operational convenience and narrative force, which do not themselves establish empirical adequacy.

Keywords: equilibrium, general equilibrium, stability, nonlinear dynamics, chaos, economic methodology

JEL codes: C62, D50, B41, B23

1 Introduction

Equilibrium is one of the most familiar, extensively studied and influential concepts in economics. Students usually encounter it in one of their first classes, as the intersection of demand and supply curves or as the solution of an optimisation problem involving familiar economic objectives: utility, cost and profit. In this familiar picture, equilibrium is a safe and stable state in which the economy, or markets, typically find themselves. Even when disturbed, they are expected to return to it. This almost tautological picture, shaped by the neoclassical tradition from Walras to Marshall and later formalised in twentieth-century general equilibrium theory, disposes us to treat equilibrium as a well-defined and methodologically neutral object. Yet what equilibrium actually is remains far from obvious. Equilibrium is a representation of order, and its meaning has shifted with developments in mathematics, economic methodology and empirical practice, as well as with changes in how time and dynamics are treated in models (Arrow & Debreu, 1954; Marshall, 1890/1920; Walras, 1874/2013). The familiar understanding of equilibrium brings together five distinct properties: existence, uniqueness, stability, convergence and predictability. It suggests that a solution exists and is unique, that the system returns to it after a disturbance, that some adjustment process actually reaches it, and that the outcome can be predicted within a model. In ordinary economic exposition, these properties are often treated as though they formed a single feature of "equilibrium". Yet the presence of one does not necessarily imply the others. That the existence of an equilibrium entails neither its uniqueness nor its stability is well known, and its consequences were described by Kirman (1989) and Ackerman (2002). Convergence was likewise a classical object of study, notably in the analysis of the approach to equilibrium through tâtonnement. Predictability, by contrast, became a distinct concern in the later literature on nonlinear dynamics. In this article we seek to systematise the concept and reconstruct its history by examining these five properties separately (Arrow & Hahn, 1971; Ingrao & Israel, 1990).

A canonical case in which all five properties co-occur is provided by Banach's contraction mapping theorem. A contraction on a complete metric space has exactly one fixed point, to which its iterations converge from any starting point. We thus obtain existence, uniqueness and convergence at once, together with stability and predictability in this ideal setting. Kakutani extended fixed-point reasoning to set-valued mappings, and Arrow and Debreu subsequently used his theorem in their canonical proof of the existence of a competitive equilibrium (Arrow & Debreu, 1954; Banach, 1922; Kakutani, 1941). Our main thesis is that the history of equilibrium can be understood as the progressive unravelling of this coherent bundle, brought about in part by the same formal tradition that had established the existence of equilibrium. Sonnenschein, Mantel and Debreu showed that aggregate excess demand can take almost any admissible form, so that existence need imply neither uniqueness nor stability. Grandmont showed that competitive equilibrium models can generate endogenous cycles, while Li and Yorke demonstrated that determinism does not guarantee predictability (Debreu, 1974; Grandmont, 1985; Li & Yorke, 1975; Mantel, 1974; Sonnenschein, 1972). Existence is therefore the only one of these properties secured by the Arrow–Debreu fixed-point argument itself.

The remaining properties can fail, but their failure does not necessarily imply the absence of economic order. This is our second, more constructive thesis. When uniqueness fails and several equilibria exist, one can analyse which of them is realised and how the outcome depends on the path the system has followed (path dependence). An unstable equilibrium may give way to ordered oscillations, whose recurring pattern can itself constitute a form of order distributed over time. Where an adjustment process fails to converge, the path of adjustment becomes an object of study in its own right. And when individual trajectories cannot be reliably predicted, a stable statistical pattern, where one exists, may still describe how frequently the system visits different regions of its state space. These possibilities suggest that the scattered "anomalies" of twentieth-century equilibrium theory are more than a collection of technical imperfections. They point towards alternative ways of representing economic order.

Once the concept of equilibrium is decomposed into its constituent properties, two different orientations become apparent. The first locates order in a solution-object, typically a point that closes the model. The second seeks it in temporal and statistical patterns: distributions, processes and recurring forms of evolution over time. Both may be described in terms of "equilibrium", although they give priority to different objects of explanation and employ different analytical tools. Distinguishing these orientations is the analytical contribution of the article. The individual elements examined below are already known, but considering them together reveals an ontological distinction[1] that is obscured when they are treated as separate qualifications to equilibrium theory. We seek to locate the point-based approach within this broader methodological space and to show that a second, less widespread orientation has developed alongside it, one that can be epistemically preferable when suited to the problem under investigation. Once time, entropy, irreversibility and non-ergodicity enter the analysis, the equilibrium point cannot be assumed to provide an adequate representation of the system on its own.

The article is organised as follows. Section 2 traces the history of equilibrium before formalisation, when it was understood through physical metaphors and tendencies rather than formal theorems. It also shows how a statistical language of order was developing in nineteenth-century physics. Section 3 examines how fixed-point theory sharpened the concept of equilibrium and established its existence without securing the remaining properties. Section 4 forms the analytical core of the article. It examines uniqueness, stability, convergence and predictability separately and considers the alternative forms of order that become relevant when these properties cannot be guaranteed. Section 5 turns to empirical practice and shows that the equilibrium

[1] *Ontology* is used here in an ideal-typical, methodological sense. The article does not claim that point-based and temporal-statistical representations occupy mathematically disjoint domains. A stationary distribution can be a fixed point of an operator, and a fixed point can be embedded in a dynamical or stochastic system. The distinction concerns the priority of explanation and the conditions under which a given representation can represent economic order.

point is inferred through a model rather than directly observed in data. Section 6 develops the temporal-statistical orientation and its analytical tools. Section 7 brings the two orientations together and draws the methodological conclusions. The question throughout is what form of order a given equilibrium object can adequately represent[2].

2 Before the Theorem: Metaphor, Mechanism and a Second Language

Equilibrium entered economics through a metaphor drawn from classical mechanics: bodies at rest under opposing forces that cancel one another. When eighteenth- and nineteenth-century thinkers applied the analogy to markets and prices, they also carried over assumptions about continuity, reversibility, and the point as the natural form of order. They became part of the assumptions used in economic analysis (Ingrao & Israel, 1990; Mirowski, 1989). The borrowing was selective: in mechanics, equilibrium can be stable, unstable or neutral. Economics borrowed the image but not the whole problem of stability: it selected only the most important element of the metaphor, the existence of a resting point, while the question of whether the system returns to it after a disturbance was set aside. Aware of the analogy's limitations, classical economists used it to explain their theories without claiming to have proved that markets reach equilibrium. The following examples show the caution that later formalisation did not always preserve.

For Smith, the natural price is one towards which the market gravitates, without necessarily reaching it: it is, "as it were, the central price, to which the prices of all commodities are continually gravitating", while particular circumstances may keep the market price "suspended" above or below it, with prices "constantly tending towards it" without any guarantee of arrival (A. Smith, 1776/2012). Smith thus distinguishes the price towards which markets gravitate from their actual arrival at it. Hahn (1984) later named this same gap — between equilibrium as a theoretical solution and equilibrium as a state attained by the economy — one of the deepest unresolved problems of neoclassical theory. Mill retained a similar caution: despite his more formal language, he treated the "equation of supply and demand" as a law of tendency — a way in which the respective quantities are pushed towards equality, rather than a state they are sure to attain (Mill, 1848/1965). Walras pursued a more ambitious formalisation of market processes but also described the gap: his tâtonnement — literally groping in the dark — reaches equilibrium only under the unrealistic assumption that no transaction takes place until all markets clear. Walras reworked this adjustment device across successive editions without making it fully consistent (Walker, 1996; Walras, 1874/2013). Even Marshall, whose formal diagrams are most closely associated with entrenching point equilibrium in the pedagogical imagination, warned

[2]Throughout the article *equilibrium* is used in a methodological sense, and not solely in the sense of market clearing. It denotes a kind of object that the theory adopts as a representation of order: a point, a set, a distribution, a regime, or a process.

against reading them literally. Equilibrium, he wrote, "suggests something of statical analogy", and such "fragmentary statical hypotheses are used as temporary auxiliaries to dynamical — or rather biological — conceptions", because "the Mecca of the economist lies in economic biology rather than in economic dynamics" (Marshall, 1890/1920). Pareto, finally, showed that the existence of an equilibrium need not imply uniqueness, since the set of optima is usually a continuum (Hildenbrand, 1994; Pareto, 1906/1971). These accounts do not formally establish that markets reach equilibrium. It would be more accurate to say that, under certain conditions and over time, through an idealised process, markets may tend towards it. The metaphor left difficult questions unresolved: about existence, uniqueness, convergence, speed, and whether a single path represents the whole.

Twentieth-century formalisation gave economics a more rigorous and general language. In replacing conditional tendencies and provisional analogies with theorems and axioms, however, it did not always preserve the earlier qualifications. Jevons described exchange as "the mechanics of utility and self-interest" (Jevons, 1871/1970): utility and interest play the role of forces in this picture, and the market reaches equilibrium just as a mechanical system does. Marginal calculus, however, relies on derivatives (marginal utility is the derivative of utility) and therefore requires the relevant functions to be differentiable. Continuity and differentiability thus become formal conditions of this analysis, which cannot directly represent discrete or discontinuous choices (Mirowski, 1989). Marshall did introduce time into the theory, in the form of periods of adjustment to equilibrium — market, short and long — yet this time denoted only a measure of how long adjustment takes, not something that co-constitutes market order itself. In each of these periods the equilibrium point remained the ultimate object (Marshall, 1890/1920). The concept of equilibrium narrowed in part as conditional statements were recast as mathematical results under explicit assumptions — "May tend, under conditions, towards" became "exists, given assumptions".

Between Walras's equations and the later topological existence proofs of Arrow and Debreu, economists lacked the mathematical tools required for a general proof. Some therefore attempted a physical solution using material devices. An example is Fisher's 1891 doctoral dissertation, which contained a hydraulic device for computing equilibrium prices — before equilibrium became a theorem, it was for a time a machine (Brainard & Scarf, 2005; Fisher, 1892). Such machines did more than illustrate equations: they represented the economy as continuous flows and quantities brought smoothly into balance. They suggested that disequilibrium is a transient deviation and made the return to equilibrium appear physically natural. Later hydraulic macroeconomics sharpened this effect. Its literal embodiment was the MONIAC — a machine built by Bill Phillips in 1949 in which coloured water flowing through pipes and tanks represented the flow of money in the economy (income, spending, taxes, saving), so that macroeconomic relationships became as visible as a plumbing installation. The vividness came at a price: by presenting the economy as a system of self-balancing flows, it made the substantive and historically contingent question — whether the economy returns to a stable anchor after shocks — look like a law of nature (like water finding its level). It relegated frictions, expectations and path dependence to the role of mere disturbances around a self-correcting mechanism (Corkhill, 2012; Mirowski, 1989). Equilibrium had not yet been proved, and already it was being presented as if it were physically necessary. As this mechanical-mathematical language developed, another conception of order was taking shape in physics. In the work of Maxwell and Boltzmann, order was no longer solely a state of rest in which mutually cancelling forces act, but a statistical property of systems composed of many interacting parts, in which distribution, probability, entropy and the relation between microstates and macrostates were the central concepts (Georgescu-Roegen, 1971; von Plato, 1991)[3].

By the late nineteenth century, these two languages of order had begun to diverge: one geometric and point-based, the other statistical and temporal. The latter raised a question that would become important for economics: when can a single long trajectory adequately represent the behaviour of an entire system? Georgescu-Roegen (1971) later argued that economics had borrowed Newtonian mechanics while neglecting thermodynamics and, with it, time and irreversibility. When Arrow and Debreu published their existence proof in 1954, economists had used equilibrium for roughly a century and a half without a general proof of its existence, uniqueness or stability. It had served at once as an end state, a direction of movement and a normative criterion. The overlap gave the concept much of its appeal while leaving unresolved whether equilibrium is unique or stable, whether a system converges to it and whether one trajectory represents the whole (Hahn, 1984; Ingrao & Israel, 1990). Formalisation would make these questions more precise.

3 Formalisation: Equilibrium as a Fixed Point

By the middle of the twentieth century, economic theory faced a difficulty. For over a century, economists had described equilibrium through supply and demand curves, auctioneers and processes of mutual adjustment, yet lacked a general proof that their central object existed. The decisive move came when equilibrium ceased to be the expected point at which economic forces settle and became the fixed point of a mathematical mapping — a state that the mapping leaves unchanged ($f(x) = x$). This changed the meaning of equilibrium: its existence no longer depended on describing the end of an adjustment process. Equilibrium became an abstract object whose existence could be guaranteed by an appropriate mathematical structure. What earlier economists had called a tendency was reformulated as a theorem (Arrow & Debreu, 1954; Ingrao & Israel, 1990). The change relied on mathematical tools developed in topology. Banach's fixed-point theorem supplied an especially strong template: a mapping that contracts distances

[3]The history of the ergodic hypothesis is itself more tangled than later, simplified accounts suggest: its rigorous version was formulated only in the 1930s (Birkhoff, von Neumann), so it should not be attributed to Boltzmann in finished form (von Plato, 1991).

has a unique fixed point, towards which repeated iterations converge. More formally, Banach's theorem proved that every contraction mapping on a complete space has exactly one fixed point, to which the iterations converge from any starting point (Banach, 1922). The theorem thus gives existence and uniqueness together with convergence of the iterates. In this picture the problem of equilibrium would be solved in its strongest form: without multiplicity, without ambiguity of choice among states, without tension between statics and dynamics, and without a gap between the solution and the process that reaches it. The possibility of reproducing equilibrium under the conditions required for Banach's theorem to hold would justify treating its five components as combined into a single object.

Although this picture was not attained in general equilibrium theory, it persisted as a regulative ideal[4] — a template for what a "well-behaved" equilibrium should look like. Banach's assumptions were too restrictive for the general theory of competitive equilibrium. In general equilibrium theory, the relevant mappings are often set-valued rather than single-valued, so tools suited to correspondences were needed. Kakutani's theorem provided such a tool, replacing contraction with conditions involving convexity. It generalised Brouwer's earlier theorem to set-valued mappings — those assigning a whole set of values to a single point — provided that they are sufficiently regular and take non-empty, compact, convex values (Kakutani, 1941). Arrow and Debreu used it in 1954 to prove the existence of an equilibrium for a competitive economy (Arrow & Debreu, 1954). Lionel McKenzie published a related result in the same year. These developments gave the long effort to formalise equilibrium a precise mathematical form (Düppe & Weintraub, 2014; McKenzie, 1954).

The new mathematical tool established a narrower result than the broader picture of equilibrium might suggest. The new theory established only the existence of equilibrium. It did not establish uniqueness or offer a realistic decentralised convergence process, leaving open the Walrasian question of how an economy reaches equilibrium. The proof established existence under the mathematical assumptions of the model. This was enough to found postwar mainstream theory on that discovery, and at the same time it gave far less than many would expect of economic equilibrium. From that moment equilibrium was no longer a dynamic balance of the forces of demand and supply; it became a mathematical object whose existence was guaranteed by topology — but only within a world constrained by continuity, convexity and compactness. The cornerstone of this theory — the existence proof — was thus not merely a mathematical achievement; it also carried ontological consequences: it made a tacit assumption about what the economy is like. This foundation became central even as its assumptions receded from view. Economists tell the story of the mathematisation of their discipline starting from Walras, through marginalism and optimisation. Yet it was the fixed-point reasoning, which gave equilibrium its modern form, that was the turning point in the process of mathematisation, even though

[4]The *regulative ideal* is used here in the Kantian sense: not as a state actually attained, but as an ordering template that sets a direction and a norm for the theory, even when it remains unattainable.

it is far less visible. Students absorb the point ontology through comparative statics, welfare theorems and game-theoretic notions of equilibrium, and rarely learn where it comes from. The existence proof became part of the background of the discipline (Düppe, 2010). Owing to this formal naturalisation of equilibrium, dynamic difficulties — instability, multiplicity, divergent adjustment, path dependence — came to look like secondary complications rather than fundamental issues whose resolution is no less important. At this stage the point-based approach to equilibrium was the standard, even though its difficulties had not been resolved — it simply became the default image of what order should look like (Hands, 2016).

Historians and methodologists have examined different aspects of this transformation. Mirowski (1989) argued that neoclassical economics borrowed from physics not only loose metaphors but an entire formal apparatus. In this light the turn to the fixed point can be read less as a neutral refinement of Walras's theory and more as part of a broad refashioning of the scientific order in economics. Ingrao and Israel (1990) come closest to the reading developed in this article: their history traces the long road equilibrium travelled from an intuitive, partly physical metaphor to a formal mathematical object. They treat the Arrow–Debreu proof as the culmination of a century-long effort to give economic order the standing that mathematical physics had earlier attained. Weintraub (1991) showed that postwar stability analysis, which investigated whether and how the economy returns to equilibrium, did not so much apply ready-made tools to an existing problem as construct the "relevant dynamical problem" itself — the analysis of the process of arrival through price adjustment. Framing the problem in this way was a consequence of applying a particular mathematical apparatus. McCloskey (1994) examined how the rhetoric of formalism and the prestige of mathematical theorems increase the persuasive force of arguments. She explains how theorems and theories based on the fixed point could become central to economics despite significant ontological gaps: what operated here was disciplinary authority more than substantive superiority. Mäki (1992, 2002) probed the underlying question of realism: once equilibrium is turned into an axiomatic object, is the result valid only in an artificial formal world, or does it also have real consequences? Similarly, Düppe (2010) showed how Debreu's axiomatic method separated mathematical form from economic content: the mathematics became self-contained, valid as a pure proof regardless of what it means for a real economy. This enhanced the standing of mathematical economics while leaving unresolved how its theorems relate to economic phenomena. Hands (2016) drew attention to two phases in the history of postwar research on stability. Early results, obtained under sharp assumptions, strengthened confidence in the Walrasian model: it seemed that price adjustment converges to equilibrium. But later, more general theorems could not be proved — counterexamples appeared in which prices do not converge. The same line of research, in seeking a proof of stability, thus produced results that undermined confidence in the model.

Together, these accounts show that formalisation did more than fill a gap in mathematical theory. This process changed the meaning of equilibrium, transforming the classical economists' cautious, conditional statements about tendencies into a single, concrete, theorem-based object, and at the same time narrowing other aspects that ought to be taken into account when trying to describe economic order. The analytical burden then shifted from existence to uniqueness, stability, convergence and the adequacy of the equilibrium point itself.

4 Existence Is Not Enough

The Arrow–Debreu proof showed that a competitive economy satisfying appropriate assumptions has at least one equilibrium. It did not establish uniqueness, stability or convergence through a realistic process, nor did it determine the full structure of equilibrium beyond market clearing. Already in General Competitive Analysis, Arrow and Hahn (1971) devoted much space to the question of whether and how equilibrium is attained — precisely because the existence proof itself does not settle it. This section discusses the remaining features of equilibrium. For each of them we present examples showing that the property cannot be guaranteed within the point-based scheme. We also consider how order can be described without requiring each property. Each limitation brings a different object of analysis into view.

4.1 Uniqueness: From the Point to a Choice

The uniqueness problem begins with the demand functions of consumers maximising utility subject to budget constraints. These functions satisfy a few basic properties: they are continuous, homogeneous of degree zero, and satisfy Walras's law; but, thanks to consumers' rationality, they also satisfy the weak axiom of revealed preference and have a negative semi-definite substitution matrix. Sonnenschein (1972) asked which of these properties carry over to the demand of the whole market when we sum the demands of all consumers. The result was restrictive: only the three most basic ones survived — continuity, homogeneity of degree zero, and Walras's law. Aggregate excess demand — the difference between what the market wants to buy and what it already holds — can take practically any continuous shape, provided it satisfies these three conditions. Mantel (1974) generalised this result and later showed that it holds even under homothetic preferences — that is, even when consumers behave regularly (Mantel, 1976). Debreu (1974) gave the strongest version: one can choose a set of utility-maximising consumers whose aggregate demand reproduces exactly — at least locally — any continuous function satisfying homogeneity and Walras's law. Thus individually rational consumers can generate almost any admissible shape of aggregate excess demand. The textbook of Mas-Colell, Whinston and Green (1995) sums this up with the slogan "anything goes", which captures well the methodological weight of the result. Similar conclusions were drawn by Bliss (1993) and by Arrow (1986), who conceded that the assumptions of rationality have very weak implications at the aggregate level (Debreu, 1974; Mantel, 1974; Sonnenschein, 1972). Aggregate excess demand can therefore admit multiple equilibria — its graph may cross zero at many points. Individual rationality alone does not determine which equilibrium is selected (Kirman, 1992; Rizvi, 1994).

A common response is to introduce a representative agent. It is assumed that the whole economy behaves like a single consumer with a single utility function, so that aggregate demand has a more regular structure. Uniqueness, however, requires additional assumptions. This simplifies the aggregation problem by imposing a representative structure. Kirman (1992) showed that this convenience can be misleading — a representative agent can respond to changes in the direction opposite to that of the actual individuals in the economy, so that welfare conclusions drawn from the representative-agent model may differ from those implied by the underlying heterogeneous economy.

The existence proof still holds under its assumptions, but the equilibrium may be one of several admissible solutions. The theory does not even say how many there are, or which one will be realised. The object used to represent economic order can therefore be reconsidered. Since the point is not unique, the key question is no longer "where does this equilibrium lie?" but "which of the many equilibria will be realised, and how is it reached?" The possible multiplicity makes selection (which equilibrium is chosen) and path dependence (the dependence of the outcome on the path taken) into real objects of study. The analysis focuses on the process that selects one of the many states. From the perspective of the process, multiplicity places a different demand on explanation: the explanatory burden shifts to the dynamics of selection and to the historical path.

4.2 Stability: From the Point to the Regime

Even if we identify a particular equilibrium point, it may be unstable. The classical literature on stability already examined this possibility. Scarf's simple examples from 1960 already showed that Walrasian tâtonnement can be globally unstable: prices circle around the equilibrium instead of converging to it. Smale's later work showed, in turn, that the static conditions for the existence of equilibrium permit no conclusions about its dynamics — these questions must be considered entirely separately (Gintis, 2007; Scarf, 1960; Smale, 1976). Systems of this kind are studied by the mathematical theory of bifurcation, which treats instability as a standard object of study rather than a disappointing exception. It examines how stability can change as the parameters of a system change, so that a state described as a fixed point may even exist and yet lose its role as the dominant long-run attractor. Small parameter changes can alter the dynamics and produce periodic motions, coexisting regimes, or quasi-periodic paths ("Local Bifurcation Theory Applied to OLG Models," 2005). An example is Grandmont's model in On Endogenous Competitive Business Cycles. The model assumes a competitive economy, markets clear in every period, and the participants have perfect foresight. Even so, persistent, deterministic cycles emerge within the model, driven by the tension between the wealth effect and the intertemporal substitution effect across generations. Cyclical motion can therefore arise from the internal logic of the equilibrium dynamics itself, and the tools of Hopf bifurcation, used for physical and biological systems, can be carried over to the study of a competitive economy (Grandmont, 1985). Instability poses a further problem: the sought-after fixed point ceases to be the only relevant object. The relevant dynamical object may instead be a cycle, a saddle, or a chaotic attractor, each of which behaves quite differently under a shock. A chaotic attractor can attract nearby trajectories even when motion along it is difficult to predict in detail (Dore & Rosser, 2007; Kaldor, 2020).

Failure to stabilise at a point need not mean an absence of structure. Adjustment may be cyclical, quasi-periodic or organised around distinct regimes. A bifurcation diagram can then reveal aspects of long-run behaviour that an intersection of demand and supply curves cannot show. The questions concern which motions are possible and how parameter changes alter them (Grandmont, 1985; "Local Bifurcation Theory Applied to OLG Models," 2005). This perspective also informs research on critical transitions. In certain nonlinear systems, approaching a threshold between regimes can produce warning signals, including critical slowing down, increased variance and greater autocorrelation. Their interpretation depends on the system and the assumptions of the model; critical slowing down, in particular, concerns the rate of recovery from small disturbances (Scheffer et al., 2009). Research on systemic crises uses related ideas to study changes in regimes through indicators of the system's dynamics rather than solely through deviations from a market-clearing price (Battiston et al., 2016; May et al., 2008).

4.3 Convergence: From the Point to the Process

Stability describes how a system responds to disturbances around equilibrium; convergence concerns whether a specified adjustment process approaches it from given initial conditions. Walras's auctioneer, who announces prices until markets clear, is an idealisation that Walras revised across editions (Walker, 1996). The Sonnenschein–Mantel–Debreu results also limit the restrictions on aggregate excess demand that can be inferred from individual rationality. The positive global stability result of Arrow, Block and Hurwicz (1959) depended on the strong condition of gross substitutability. Without additional restrictions beyond continuity and Walras's law, convergence of tâtonnement cannot be established in general (Ingrao & Israel, 1990). Even the usual price-adjustment rule, under which prices rise when excess demand is positive, must be specified as part of the dynamics rather than derived from an existence theorem (Hahn, 1984; Rizvi, 1994). Gintis (2007) therefore questioned whether Walrasian dynamics supplies a credible decentralised process reliably leading to market-clearing prices.

Even where competitive equilibrium is proved to exist, the proof alone does not guarantee that a particular market procedure or algorithm can locate it (Velupillai, 2000). An empirical counterpart to the study of convergence is the phenomenon of hysteresis, which shows that after a large shock the economy need not return to its earlier configuration. A typical illustration of this problem is persistently high unemployment after deep recessions. Since existence does not guarantee convergence, the adjustment path itself becomes an object of study: how long adjustment takes, which route it follows, whether it ends, how it responds to a disturbance. Such studies analyse the learning process, and convergence time, basins of attraction and path dependence are the key indicators.

4.4 Predictability: From the Point to the Distribution

The final, less formal expectation associated with equilibrium is that a fully specified model yields a predictable outcome. Li and Yorke (1975) challenged the broader identification of determinism with predictable dynamics. For a continuous map of an interval into itself, they showed that an orbit of period three entails periodic points of every period and an uncountable set of non-periodic points. Their article's title, "Period three implies chaos", captures the possibility of irregular motion within a simple deterministic system. Economic models provide related examples. Benhabib and Day (1981) showed that rational choice in a stationary environment can generate irregular behaviour when preferences depend on experience. Day (1982) obtained irregular growth cycles in a neoclassical setting, while Benhabib and Day (1982) found bounded, non-periodic trajectories in a deterministic overlapping-generations model. Baumol and Benhabib (1989) surveyed the implications of chaos for economics. As Boldrin and Woodford (1990) and Benhabib (1992) emphasised, nonlinear dynamics can arise within equilibrium models themselves. Internal consistency and determinism do not guarantee that a system's long-run trajectory can be forecast in detail.

Two qualifications matter. Chaos is invoked here as a possibility established within theoretical models, not as a claim that actual economies follow low-dimensional chaotic dynamics. Empirical attempts to distinguish deterministic chaos from stochastic irregularity in economic and financial data have been inconclusive (Hsieh, 1991). Economic forecast errors also have many possible causes, including misspecified models, shocks, structural change and feedback as agents respond to forecasts and models of their behaviour (the Lucas critique). Where a system admits a suitable invariant measure, however, its long-run statistical properties may be studied even when individual trajectories cannot be forecast reliably. Such a measure can describe the relative frequency with which trajectories visit regions of the state space under the appropriate dynamical assumptions. The possibility of statistical regularity therefore provides an additional object of analysis, without guaranteeing that every unpredictable system has a stable long-run distribution.

4.5 One Point, Four Dynamics

An existence proof does not establish uniqueness, stability, convergence or predictability; these properties require separate analysis. Each time one of the features is called into question, the object of analysis changes from a single point to an object distributed in time or in the space of states: to a choice among equilibria, to a regime, to an adjustment path, or to a distribution. The next section turns from these theoretical results to the empirical problem of identifying equilibrium.

5 The Point in Empirical Research

The properties of market equilibrium also pose problems for empirical and experimental research. In standard theory, determining the price is simple: one need only read it off the point where the demand and supply curves intersect on a diagram. Empirical identification is more difficult: the prices and quantities of goods are jointly determined, demand and supply schedules are not directly observed, and the analyst often sees aggregated outcomes, produced moreover within a specific institutional setting and under particular market conditions. Equilibrium cannot be read directly from the data. It must be inferred through a model and an identification strategy linking observations to the unobserved structure of demand and supply.

Haavelmo (1944) introduced the probabilistic approach to econometrics, in which economic data should be treated as generated by a random process rather than as an exact law with a small measurement error. An economic relationship — for example, how demand depends on price — must therefore first be written down within a statistical model, with a random component and assumptions about how the observations came about. Only within such a model can the data say anything about the true, hidden relationship. In the same spirit, the work of Berry, Levinsohn and Pakes reconstructs demand, supply, mark-ups and equilibrium prices from a mixture of theory, instruments, distributional assumptions and estimation (Berry et al., 1995). Such analyses show that equilibrium can be derived under certain substantive assumptions, and not simply read off the data or computed from the point where two curves intersect. There is an analogy with the SMD theorem. SMD says that, in theory, rationality alone does not suffice to determine a well-behaved aggregate demand — assumptions must be added. The problem of identifying an empirical equilibrium is similar: the data alone do not suffice to recover the demand and supply curves — here too assumptions must be added. In both cases, additional assumptions are required (Hildenbrand, 1994; Mas-Colell et al., 1995). Identifying demand and supply curves is only one difficulty in estimating equilibrium. Markets consist of finitely many agents making discrete decisions — often with minimum lot sizes, price ticks and indivisible goods. The demand and supply schedules reconstructed from observed data are usually step-like, and their jumps reflect the discreteness of real actions (Varian, 2014). Such curves may not meet at a single point at all — they may overlap over an interval, jump over each other, or generate several sets of prices at which the market is almost cleared. In such a situation the natural object of analysis becomes an interval of prices or an admissible set of goods. The theoretical literature that has studied indivisibility confirms that competitive equilibrium in such markets requires additional special conditions (Quinzii, 1984; Yokote, 2020). When market behaviour is discrete, treating the economy as continuous curves intersecting at a single point is, in effect, a strong assumption. In some settings, the data and assumptions support only an approximate estimate or a range of values. An approximate equilibrium can nonetheless be informative, as Budish (2011) showed by analysing an approximate competitive equilibrium from equal incomes, using the example of course selection for MBA students. Order is then a bounded region within which imperfections are tolerated to some degree, and the allocations of goods are approximately efficient. Similarly, Yokote (2020) shows that in heterogeneous, discrete markets one can identify goods that are over-demanded or under-supplied and read off the directions of price adjustment even where no point equilibrium exists.

The very shape of such price regions — the exact set of prices at which the market is approximately in equilibrium — becomes an object of study. Partial identification follows a related logic: it reports the set of values consistent with the maintained assumptions instead of imposing stronger restrictions solely to obtain one number (Budish, 2011; Manski, 2003). Such a set expresses what can be inferred from the data under the adopted assumptions. The inferred equilibrium point also depends on the model. Two analysts studying the same market but under different assumptions — different instruments used to separate demand and supply, a different assumed form of the curves, or a different treatment of unobserved differences between agents — will usually obtain different equilibrium prices and different elasticities. Financial markets offer another illustration, despite their detailed price and order data. Yet a listed asset does not have, at any given moment, a single market-clearing price. It has a bid-ask spread — the difference between the highest price offered by buyers and the lowest price asked by sellers — and an order book, that is, a list of all buy and sell orders at various price levels and of various volumes. Even the most liquid markets therefore display, at every moment, a set of prices rather than a single point. The price quoted is only a convention chosen by the analyst: the midpoint between bid and ask, the price of the last transaction, or the volume-weighted average. What appears as a single intersection in a textbook diagram may therefore correspond to a sequence of transaction prices summarised by one number. Independently of econometric research, the way market equilibrium comes about has also been studied in experimental economics.

In experiments on controlled double-auction markets conducted by Vernon Smith, transaction prices converge towards the competitive equilibrium price (V. L. Smith, 1962). The process takes time: the range of transaction prices gradually narrows over successive rounds. The trajectory of transaction prices is therefore part of the experimental result, alongside the end state. Such a distribution of prices changing over time is worth treating as the primary object, not as an approximation of some hidden, true point — and a statistical account of equilibrium in this spirit is developed in Section 6.

Empirical analysis adds another dimension to the theoretical argument. The SMD theorem showed that the rationality of individuals does not suffice to preserve the structure of demand at the aggregate level. The theories of bifurcation and chaos showed that equilibrium objects may be structurally ill-suited to static analysis. The problem of identifying demand and supply, in turn, shows that even if a precise empirical equilibrium point is attainable, it is a product of both methodological and statistical assumptions. The assumptions and constraints supporting a point estimate or an identified set should therefore be made explicit. Equilibrium may be exact or approximate, point-like or set-valued; the analysis should therefore ask which kind of order its data actually support: point-like, region-based, statistical or temporal.

6 A Second Language: Time, Entropy, Ergodicity and Stylized Facts

6.1 Two Orientations of Order

The preceding sections identified questions left open by point equilibrium. Here we develop the temporal-statistical orientation, in which order can be described through the properties of a process over time. In much of the equilibrium tradition, time, in so far as it is considered at all, is regarded as a less important element than the end point itself — the destination matters more than the journey. The solution-centred orientation usually treats equilibrium as an object — a fixed point, or, less often, a mapping (for example, a decision function), or a set of admissible solutions. The orientation described here asks what kind of order the system displays over time. On this understanding, order can take several forms: it may be a distribution of outcomes (for example, how often, over the long run, the system visits particular states), it may have a stationary regime (a persistent, recurring statistical pattern), or it may be an ergodic process, in which outcomes are considered in relation to the trajectory. The temporal-statistical orientation is concerned with persistence, stationarity, the reproducibility of averages, and the relation between different realisations of a process. The following subsections examine its history, rooted in statistical physics; its treatment of time and of ergodicity; and the various methodological currents in which it can be found — econophysics, the Austrian tradition, mainstream dynamic economics, and complexity economics. At the end we propose a coherent taxonomy of the objects used in this approach.

6.2 From Statistical Physics to Statistical Equilibrium in Economics

A statistical language of order developed in nineteenth-century physics. Where mechanics saw order as a resting point of balancing forces, statistical physics discerned it, in a system of billions of particles, as a statistical property — a stable distribution of microstates, defined by probability and entropy rather than by a single configuration (von Plato, 1991). The line of development runs from Maxwell and Boltzmann, through Gibbs's ensembles, to the ergodic theorems of Birkhoff and von Neumann. Ergodic theory made precise a further question: when does the long-run behaviour of a single trajectory coincide with the average over the whole distribution? Prigogine went further and showed that in systems held far from equilibrium an irreversible process can itself generate structure — "order out of chaos" (Prigogine & Stengers, 1984). These approaches show how order can be described through the temporal and statistical organisation of a process. This way of thinking passed into economics. Georgescu-Roegen (1971) argued that the economic process is irreducibly entropic — it turns low-entropy resources into high-entropy products — and hence irreversible, so that time is indispensable for understanding it.

An account of the economy based exclusively on reversible mechanical analogies risks leaving this irreversibility outside the model. Foley offered a formulation of statistical equilibrium in which an entropy-maximising distribution of transactions replaces the single intersection of Walrasian curves as the object of analysis (Foley, 1994). Scharfenaker and Foley (2017) developed the maximum-entropy approach to infer aggregate distributions from limited information. These contributions illustrate how equilibrium can be formulated statistically, although the role of time depends on the particular model and should not be assumed from the use of a distribution alone.

6.3 The Role of Time and Ergodicity

Describing order through a distribution raises another question. Does such a distribution, understood as an average over many runs of a process, faithfully capture what a single system experiences over a longer time? This is the question of ergodicity. In this approach we can define two averages: the time average, computed from a single trajectory of the system over the long run, and the ensemble average (the expected value), computed over the whole distribution of states at a single moment. Under the appropriate assumptions, ergodicity implies that long-run time averages of suitable observables coincide with their ensemble averages, as clarified by the theorems of Birkhoff and von Neumann (Birkhoff, 1931; von Neumann, 1932). Let us illustrate this with a simple example. Take a multiplicative game, repeated over time, with a fixed proportional gain or loss at each step. The averaged outcomes of a large ensemble of parallel players will yield growing wealth, because a few extreme winners pull the average of the outcomes upward. Yet the typical single trajectory, considered over time, will often show a result close to zero. The ensemble average and the time average are different, and only the latter describes what a typical player actually experiences. The distinction matters when interpreting expectations in economic models, including models based on expected utility. When a process is ergodic, this substitution is harmless. For some multiplicative, path-dependent dynamics, Peters argues that the substitution can lead to an inappropriate objective for the modelled agent (Peters, 2019). Davidson (1988) put it this way: in a non-ergodic world the future cannot be described by a stable distribution constructed from historical data. This may have deep consequences for money, contracts and expectations. The claim is disputed, however — Doctor, Wakker and Wang (2020) acknowledge that the ergodicity problem is real but reject Peters's conclusion that it replaces expected-utility theory, pointing out that maximising logarithmic utility yields the same result. Peters (2020) replies that agreement of results does not necessarily settle the interpretation. The dispute illustrates why the ergodicity of the process must be specified: it bears directly on whether the ensemble approach can replace the individual trajectory of events — which could translate into a revision of many classical economic theories.

6.4 Many Schools and Traditions

Related approaches appear in several schools of economics, although they differ in method and do not all reject point equilibrium. Econophysics, which arose in the 1990s from the statistical analysis of financial time series (Mantegna & Stanley, 1999). Its main objects of study are stylized facts, the recurring statistical regularities: fat tails of distributions, scaling laws, long-range dependence and volatility clustering. The market does not present itself here as a system visibly converging to a market-clearing point, but as a stream of varied data, and order can emerge from the study of the distribution — its shape and its correlation structure. The conclusions in this setting are principally statistical and empirical (Gallegati et al., 2006; Kutner et al., 2019; McCauley, 2006).

The Austrian tradition approaches market processes through a different, largely qualitative and praxeological method. Hayek argued that formal equilibrium requires an account of how dispersed knowledge is acquired and communicated. He described the market as a continuing process of coordination and a “discovery procedure” rather than a completed allocation (Hayek, 1937, 1945). Kirzner (1973) developed the entrepreneurial market-process account, while Lachmann (1976) described a “kaleidic” economy in which changing expectations repeatedly alter the pattern of activity. Mises (1949) explicitly presented the “evenly rotating economy” as a theoretical fiction. Hayek’s account of pattern prediction also addresses the limits of precise forecasts for complex phenomena: in some cases, science can identify recurring patterns without predicting a unique outcome (Hayek, 1964). This is a point of contact with statistical approaches, despite the traditions’ different methods.

Recursive methods provide a further route to analysing economic dynamics. They specify the economy’s current state together with a rule for its evolution, or “law of motion” (Stokey et al., 1989). Incomplete-markets models (Aiyagari, 1994) and industry-dynamics models (Hopenhayn, 1992) can take a stationary distribution as part of their equilibrium object; Krusell and Smith (1998) likewise use a distribution to describe the aggregate economy. Closure of the model then requires determining the distribution and specifying its law of motion. A price vector alone does not describe the full state of such an economy.

From the 1990s, complexity economics developed as a research programme with its own tools and questions. This research programme developed primarily around the Santa Fe Institute — a centre for the study of complex systems founded in 1984, which brought together physicists, biologists and economists around the question of how complex order emerges from the interaction of many elements. W. Brian Arthur, known for work on increasing returns, path dependence and “lock-in”, helped develop and popularise complexity economics (Arthur, 1999, 2015). On this view the economy is a set of patterns and structures that form and dissolve through the interaction of adapting agents, rather than a system resting in a static and ultimately solved allocation.

After the 2008 crisis, Farmer and Foley (2009) called for greater use of agent-based models, while Kirman (2010) examined rationality in relation to interaction and emergent outcomes rather than through a single representative optimising agent. These approaches use different methods, from statistical analysis and recursive modelling to qualitative argument and simulation. Their point of contact is the possibility of studying economic order through the behaviour of processes over time.

6.5 A Constructive Catalogue of Order-Objects

The temporal-statistical orientation uses several kinds of analytical object. Section 4 examined four properties that cannot be inferred from an existence proof. The first concerned uniqueness. Where solutions are multiple, one can study the admissible set and, given a probabilistic model, the likelihood of different selections. As a rule, the elements that the geometric tradition described as a single admissible state, the temporal-statistical orientation presents as a generalised family of objects, which can nonetheless describe a certain stable pattern. This approach can be related to models of social learning, in which the participants do not always reach agreement, and yet one can delimit a concrete region within which all the possible outcomes must fall — and this very constraint is a form of order (Balbus et al., 2025). The next issue was stability near an equilibrium point. This is not always possible, but instead we can analyse whether there is any recurring pattern of motion in the vicinity of the solution. The system then follows a fixed cycle or remains within a stable regime — a behaviour that persists until some parameter crosses a threshold. Such a cycle can still be fully ordered. Convergence raises the question of whether the process approaches the point at all. Here, in line with the temporal-statistical orientation, the journey matters more than the destination. Convergence time and the evolution of state variables along a trajectory can provide information about an individual agent's experience. In this approach, too, one can analyse the form of order, but it is shifted from the end state to the patterns that emerge over the course of the system's evolution. The final issue was predictability. Where point forecasts are unreliable, one can instead analyse distributions over possible outcomes. This is not about the confidence intervals around a baseline path already present in many classical models, but about considering and quantifying entirely alternative scenarios. Where suitable long-run regularities exist, simulations may reveal how frequently the system visits particular states. If a stochastic system admits a stationary distribution, that distribution is preserved by its dynamics. If instead the dynamics are deterministic but chaotic, trajectories may be difficult to predict even when suitable invariant measures describe their long-run frequencies. Even in financial data one can find persistent patterns, described by certain stylized facts — fat tails and volatility clustering.

Each object answers a different analytical question and may be useful where a point solution leaves important features unexplained. It is also worth stressing that each of them can be measured, though doing so often requires the additional assumption of certain thresholds whose crossing marks a transition between a stable and an unstable state. Distributions can be estimated from simulation data; assessing whether time averages correspond to ensemble averages requires explicit assumptions and appropriate evidence. One can also measure how long it takes to reach the pattern and on what initial conditions this depends, and detect a regime change directly from the data series.

The historical roots, economic traditions and analytical objects discussed here help define the temporal-statistical orientation. Its central claim is simple: the solution of a model need not be a good description of order, which should rather be understood as a complex though recurring process over time. The next section draws together the methodological implications.

7 Summary: Two Orientations of Order

7.1 Two Orientations

Contemporary economics uses equilibrium in several ways. This article has distinguished a solution-centred orientation from a temporal-statistical one. The first seeks an object that closes the model and permits further analysis. It asks whether the model contains a specific point at which opposing forces balance, at which no party finds it worthwhile to change its decision, and to which, even if the system is for some reason displaced from it, it will return — this point is the solution that closes the whole system. The second examines adjustment processes and may express its conclusions conditionally or through distributions. It checks whether the system reproduces a certain order over time — persistent regimes, stable distributions, ergodic processes — and even the absence of such a pattern is likewise treated as a result. Its measures depend on the problem being studied and can represent features of systems that a single solution leaves unspecified. The two approaches overlap in part as regards their mathematical tools, but differ fundamentally in their principal object of study — in what they treat as equilibrium (Aumann, 1974; Davidson, 1988; Peters, 2019). This can mean a precise point, a certain approximate clearing region, or even a whole distribution of outcomes. The proper question is therefore: in which situations should one use the tools and methodology of each of the two orientations?

7.2 What Happened with the Geometric–Point Approach?

The geometric tradition faced challenges to uniqueness, stability, convergence and predictability. It remained influential while broadening what could count as a solution. It still seeks a solution that closes the model, but this can now be an ever wider class of objects: regions instead of points, distributions, attractors, and even states emerging from interaction. Some of them are the same objects that the temporal-statistical orientation employs — the difference lies not in the types themselves, but in the fact that here a closed solution is still being sought. The shift appears in several familiar models. When goods are indivisible, an exact market-clearing price may not exist, so one accepts an approximate equilibrium — a whole interval of prices at which the market is almost cleared (Budish, 2011) — and the equilibrium object then becomes a region. In game theory a Nash equilibrium is a single profile of mutually best responses, but Aumann's correlated equilibrium (1974) allows players to coordinate their actions on a shared random signal — and then the solution becomes a probability distribution over combinations of actions. Economists now routinely solve dynamic models in which the equilibrium object is a stationary distribution and a rule for its change (a law of motion) rather than a price vector. Even methods based on the fixed point have broadened: Balbus, Olszewski, Reffett and Woźny (2025) obtain a region of admissible outcomes instead of a point outcome, applying this to models of social learning in which consensus may be unattainable. In all these cases the equilibrium object is still treated as the solution of the model — only its shape changes: from a point to a region, a distribution, or a constraint. A solution-centred approach thus need not assume that order is represented by a single, exact, unique and stable point.

The temporal-statistical orientation works differently — it analyses when the solution-object is a justified object of study at all. Ergodicity matters when a model substitutes averages across realisations for averages along a trajectory: without the required correspondence, suppressing time can change its conclusions (Davidson, 1988; Peters, 2019). The fixed point is still an attainable outcome, but it is an important special case rather than an end in itself (Mantegna & Stanley, 1999). The two orientations therefore overlap in their tools but differ in explanatory emphasis. The point-based tradition can use stationary distributions and sets of outcomes, but even so its priority remains the solution of the model. The temporal-statistical orientation can use fixed points and invariant measures, yet seek the solution in the behaviour of the process over time. Even the same words — stability, convergence, equilibrium — can mean different things in each of the views, and the boundary between them cuts across the customary division into orthodox and heterodox currents. Contemporary economics has broadened the ontology of equilibrium and now operates with a family of order-objects: points, sets, regions, distributions, trajectories, attractors, constraints and regimes.

7.3 The Narrative Force of the Point

The persistence of point-based representations also has a rhetorical and practical dimension. A formalised theory that has been proved mathematically has far greater persuasive force than cautious conditional statements, which are often the outcome of research in the temporal-statistical strand. Conditional statements such as "under certain conditions, over time, the market may tend towards balance" can be harder to communicate than a theorem with an explicit conclusion. An explicit solution is often easier to display, compute and compare than an account of adjustment that depends on initial conditions and time. Theorems are portable and teachable, and their conclusions are valid under their stated assumptions, even when those assumptions are contested as descriptions of real economies. This asymmetry of influence is documented in several fields of research. Simple, fluently processed statements are judged more likely to be true (Alter & Oppenheimer, 2009; Reber & Schwarz, 1999). In economics the prestige of mathematical proof functions as a source of persuasion and authority more than as a neutral demonstration of truth (McCloskey, 1994). A related contrast appears in research on judgement and forecasting. This is Tetlock's (2005) finding: the "hedgehog", who holds to one big idea, beats the versatile "fox" in influence, though it loses to it in the accuracy of its forecasts. This is also why the later negative results we described — from Sonnenschein–Mantel–Debreu to chaos — are often absorbed as footnotes to the theorem rather than replacing it. This is the pattern that Kuhn (1962) described as normal science accommodating anomalies and that Lakatos (1970) described as a protective belt around the hard core.

It is precisely these robust mathematical theories that make teaching, modelling and the comparison of results possible, and bracketing their troubles is easier than demolishing the construction built upon them. This also has implications for how temporal-statistical results are presented. Since so powerful a formalisation cannot be displaced merely by showing that its assumptions are unrealistic, the aim should be to place the point appropriately as a special case and to endow the temporal-statistical current with objects of comparable operational strength. Such an object could be a concrete benchmark of the temporal-statistical approach — based on a stationary distribution, an invariant measure, or a convergence time. Methodological objections are more useful when accompanied by clearly defined analytical alternatives.

7.4 Conclusions

This article has traced the development of equilibrium from a descriptive concept to a formal mathematical object. The distinction has consequences for both methodology and modelling. Initially, the point-based approach tried to establish several properties of equilibrium — existence, uniqueness, stability, convergence and predictability. Yet the failure to establish them in general showed that order can also be non-unique, cyclical, dynamic, statistical or history-dependent. This alternative, temporal-statistical current, which rests on defining equilibrium as a process, offers broader analytical possibilities for certain problems and may allow a better explanation of complex, unstable economic processes. Although this point of view has gained prominence in recent years, its development is not uniform — there are many alternative currents united by a process-based approach to economic mechanisms. The comparison of dynamic models would benefit from clearly defined and transparent measures suited to their explanatory purposes.

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