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Tomasz Kopczewski
Jan Lisicki

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From the Point to Chaos: The Evolution of the Concept of Equilibrium in Economic Theory

Tomasz Kopczewski[1], *Jan Lisicki*[1*]

[1] *University of Warsaw, Faculty of Economic Sciences*
[*] *Corresponding author: jt.lisicki@uw.edu.pl*

Abstract: This article reconsiders equilibrium as a representation of economic order. It separates five properties often treated together—existence, uniqueness, stability, convergence, and predictability—and shows that fixed-point results in general equilibrium establish existence without guaranteeing the other four. When uniqueness, stability, convergence, or predictability fails, the relevant object of analysis may shift from a single point to an equilibrium-selection process, a regime, an adjustment path, or a distribution. The same distinction matters empirically because equilibrium is not directly observed but inferred from a maintained model and identification assumptions. We therefore distinguish two ideal-typical orientations: a solution-centred approach, which seeks an object that closes the model, and a temporal-statistical approach, which locates order in persistent patterns of dynamics and distributions. The article argues that the continued appeal of point-based equilibrium reflects its tractability and explanatory simplicity as well as its theoretical role.

Keywords: equilibrium, general equilibrium, stability, nonlinear dynamics, chaos, economic methodology

JEL codes: C62, D50, B41, B23

1 Introduction

Equilibrium is one of the most natural, most studied and at the same time most influential concepts in economics. Students usually encounter it in one of their first classes, as the intersection of the demand and supply curves, or as the solution of a system of equations that maximises or minimises familiar economic objectives: utility, cost, and profit. It is a safe and stable state in which the economy, or markets, typically find themselves; and even if for some reason they are momentarily out of it, they will soon return to it. This almost tautological picture, shaped by the neoclassical tradition from Walras to Marshall and later formalised in twentieth-century general equilibrium theory, disposes us to treat equilibrium as a well-defined and methodologically neutral object. Yet what equilibrium actually is remains far from obvious. Equilibrium is a representation of order, and its meaning has shifted together with developments in mathematics, in the methodology of economics, in empirical practice, and in how time and dynamics are treated in models (Arrow & Debreu, 1954; Marshall, 1890/1920; Walras, 1874/2013); it is therefore worth examining the concept closely. Equilibrium, in this fuller sense, is a state in which a solution exists and is unique, to which the system returns after a disturbance (stability), which some real adjustment process actually reaches (convergence), and whose outcome can be predicted when the concept is used in a model (predictability). In ordinary economic exposition this sounds like a single concept — "equilibrium" — yet in practice it is composed of these five properties. These distinctions matter, because the presence of one property does not necessarily imply the occurrence of the others, and these distinctions can easily become blurred. That the existence of an equilibrium entails neither its uniqueness nor its stability is well known, and its consequences were described by Kirman (1989) and Ackerman (2002). Convergence, likewise, was a classic object of study (the approach to equilibrium, tâtonnement). Predictability, by contrast, comes from the later literature on nonlinear dynamics. In this article we therefore seek to systematise the concept and to reconstruct its history by decomposing it into its constituent properties: existence, uniqueness, stability, convergence and predictability (Arrow & Hahn, 1971; Ingrao & Israel, 1990).

A canonical case in which all five features co-occur is provided by Banach's contraction mapping theorem. A contraction on a complete space has exactly one fixed point, which the iterations reach from any starting point — so we obtain existence, uniqueness and convergence at once, and in this ideal setting stability and predictability as well. Kakutani extended the fixed-point logic to set-valued mappings, and Arrow and Debreu then used it in the canonical proof of the existence of a competitive equilibrium (Arrow & Debreu, 1954; Banach, 1922; Kakutani, 1941). Our main thesis is that the history of equilibrium is the progressive negation of this coherent bundle. Interestingly, it was brought about by the same formal tradition that had earlier proved its existence. Sonnenschein, Mantel and Debreu showed that aggregate excess demand can take almost any form, so that existence need carry neither uniqueness nor stability.

Grandmont showed that a competitive equilibrium can lose stability and settle into endogenous cycles. Li and Yorke put the point most sharply: determinism does not guarantee predictability (Debreu, 1974; Grandmont, 1985; Li & Yorke, 1975; Mantel, 1974; Sonnenschein, 1972). Existence is therefore the only one of these properties secured by the Arrow–Debreu fixed-point argument itself.

The rest, one by one, can fail and does fail — both in theory, and in empirical data. However, for the theory of equilibrium, this decomposition is not purely negative, and that is our second, more constructive thesis. If a given property cannot be established for the equilibrium point, order can be discerned in other structures: a distribution, a cycle, or a path. When uniqueness fails and there are many equilibria, one can analyse which of the possible equilibria is realised — and this depends on the path the system actually followed (path dependence). The absence of stability, if the system does not come to rest, may mean that it oscillates in an ordered way, and the recurring pattern of these fluctuations can itself be a form of order — one distributed over time. When no process reaches the point (a failure of convergence), the object worth studying ceases to be the destination and becomes the adjustment path itself. And when a single trajectory cannot be predicted, order returns at the statistical level: not in what happens at the next step, but in how often, over the long run, the system visits each of its states (a distribution). Once the concept of equilibrium is decomposed into its components, the scattered "anomalies" of twentieth-century theory no longer look like a list of technical imperfections. They are evidence that economics, even if it does not say so explicitly, has arrived at two different answers to the question of what economic order is. The first locates order in a solution-object, a single point that closes the model. The second describes it in a temporal-statistical pattern: a distribution, a process, an ordered evolution over time. Both may be called "equilibrium", although they employ a different conceptual and investigative apparatus. This separation of two ways of viewing equilibrium is the analytical contribution of the article. Although the individual elements analysed below are known and have already been described, the novelty lies in bringing them together, ordering them and verifying their properties so that they reveal an ontological rift[1] rather than a loose set of caveats. We try to place the point-based approach appropriately within the methodological space and to show that a second, less widespread approach operated alongside it — one that can be epistemically preferable when well matched to the problem. Once time, entropy, irreversibility and non-ergodicity enter the analysis, the point can no longer claim by default that it is representing a complex state.

The article is organised as follows. Section 2 describes the history before formalisation, when equilibrium was a physical metaphor and a tendency rather than a sharp theorem; it also shows

[1] *Ontology* is used here in an ideal-typical, methodological sense. The article does not claim that point-based and temporal-statistical representations occupy mathematically disjoint domains. A stationary distribution can be a fixed point of an operator, and a fixed point can be embedded in a dynamical or stochastic system. The distinction concerns the priority of explanation and the conditions under which a given representation can represent economic order.

that a second, statistical language of order was already taking shape in nineteenth-century physics. Section 3 shows how fixed-point theory sharpened the concept of equilibrium and, by proving its existence, made the whole bundle of the remaining features seem natural. Section 4 is the analytical core; it discusses each feature of equilibrium separately, showing that uniqueness, stability, convergence and predictability are in no way self-evident. For each component an alternative, order-preserving variant is also presented. Section 5 turns to empirical practice and shows that, in data, the equilibrium point is constructed rather than observed. Section 6 develops in full the second, temporal-statistical conceptual axis. Section 7 sets the two paths side by side and draws the methodological conclusions. Equilibrium remains an important and useful concept, yet our article asks what form of order a given object should represent[2].

2 Before the Theorem: Metaphor, Mechanism and a Second Language

Equilibrium entered economics through a metaphor. Its source was classical mechanics — more precisely, the image of bodies at rest under opposing, mutually cancelling forces. When eighteenth- and nineteenth-century thinkers described markets and prices, the analogy suggested itself, and with it came structural commitments built into the metaphor: continuity, reversibility, and the point as the natural form of order. These entered economics as assumptions carried over from mechanics (Ingrao & Israel, 1990; Mirowski, 1989). This metaphor, although widely used, was taken over only partially — in mechanics an equilibrium can be stable, unstable or neutral. Economics borrowed the image but not the whole problem of stability: it selected only the most important element of the metaphor, the existence of a resting point, while the question of whether the system returns to it after a disturbance was set aside. The classical economists, aware of the simplifications inherent in the analogy, used it as an aid, to help readers grasp their theories — they did not formulate hard theorems or proofs. We cite a few examples to show this scholarly caution, which was only later effaced by the formalisation of economics.

In Smith the natural price is not a state that the market occupies but one to which it draws near: it is, "as it were, the central price, to which the prices of all commodities are continually gravitating", while particular circumstances may keep the market price "suspended" above or below it, with prices "constantly tending towards it" without any guarantee of arrival (A. Smith, 1776/2012). Smith already discerns here the difference between the target toward which prices continually gravitate and their actual arrival at it, which is by no means guaranteed. Hahn (1984) later named this same gap — between equilibrium as a theoretical solution and equilibrium as a state attained by the economy — one of the deepest unresolved problems of neoclassical theory.

[2] Throughout the article *equilibrium* is used in a methodological sense, and not solely in the sense of market clearing. It denotes a kind of object that the theory adopts as a representation of order: a point, a set, a distribution, a regime, or a process.

Mill likewise preserved this caution: despite a more formal language he treated the "equation of supply and demand" as a law of tendency — a way in which the respective quantities are pushed towards equality, rather than a state they are sure to attain (Mill, 1848/1965). Similarly Walras, who had the greatest ambition to formalise market processes, was at the same time the most candid in describing this gap: his tâtonnement — literally groping in the dark — reaches equilibrium only under the unrealistic assumption that no transaction takes place until all markets clear. It is worth noting that Walras reworked this adjustment device between successive editions and never made it fully consistent (Walker, 1996; Walras, 1874/2013). Even Marshall, whose formal diagrams are most closely associated with entrenching point equilibrium in the pedagogical imagination, warned against reading them literally. Equilibrium, he wrote, "suggests something of statical analogy", and such "fragmentary statical hypotheses are used as temporary auxiliaries to dynamical — or rather biological — conceptions", because "the Mecca of the economist lies in economic biology rather than in economic dynamics" (Marshall, 1890/1920). Pareto, finally, showed that the existence of an equilibrium need not imply uniqueness, since the set of optima is usually a continuum (Hildenbrand, 1994; Pareto, 1906/1971). In sum, these are not formal results establishing that markets reach equilibrium. It would be more accurate to say that, under certain conditions and over time, through an idealised process, markets may tend towards it. The caution was warranted — it showed that, despite the pictorially convenient metaphor, difficult and unresolved questions arise with it: about existence, uniqueness, convergence, speed, and whether a single path represents the whole.

Twentieth-century formalisation was, on the one hand, a great advance for economics — it allowed the discipline to be cast in a universal, rigorous form. On the other hand, from the point of view of caution and of the subtlety of the phenomena described, it had an adverse effect, because it exchanged this vocabulary of conditional tendencies and provisional analogies for a vocabulary of theorems and axioms — and the caveats did not always survive the exchange. Jevons described exchange as "the mechanics of utility and self-interest" (Jevons, 1871/1970): utility and interest play the role of forces in this picture, and the market reaches equilibrium just as a mechanical system does. His method, the marginal calculus, however, operates with derivatives (marginal utility is the derivative of utility), and these exist only for quantities that vary continuously and smoothly. Thereby continuity and differentiability cease to be a convenient simplification and become a formal condition: the theory describes only a smooth world, ruling out in advance everything discrete and discontinuous (Mirowski, 1989). Marshall did introduce time into the theory, in the form of periods of adjustment to equilibrium — market, short and long — yet this time denoted only a measure of how long adjustment takes, not something that co-constitutes market order itself. In each of these periods the equilibrium point remained the ultimate object (Marshall, 1890/1920). The narrowing of the concept of equilibrium thus took place partly through the transformation of the classical economists' hedging statements into hard mathematical proofs with appended assumptions — "May tend, under conditions, towards" became "exists, given assumptions".

It is worth noting that between Walras's equations and the later formal topological proofs of the existence of equilibrium (Arrow–Debreu) there lay an intermediate moment in which the appropriate mathematics did not yet exist. In that period some economists tried to design a physical solution, replacing the missing mathematics with material devices. An example is Fisher's 1891 doctoral dissertation, which contained a hydraulic device for computing equilibrium prices — before equilibrium became a theorem, it was for a time a machine (Brainard & Scarf, 2005; Fisher, 1892). Such machines were not merely illustrations — they quietly chose an ontology, presenting the economy as continuous flows and smoothly balancing quantities. They suggested that disequilibrium is a transient deviation and made the return to equilibrium appear physically natural. Later hydraulic macroeconomics sharpened this effect. Its literal embodiment was the MONIAC — a machine built by Bill Phillips in 1949 in which coloured water flowing through pipes and tanks represented the flow of money in the economy (income, spending, taxes, saving), so that macroeconomic relationships became as visible as a plumbing installation. This same vividness, however, came at a price: by presenting the economy as a system of self-balancing flows, it made the substantive and historically contingent question — whether the economy returns to a stable anchor after shocks — look like a law of nature (like water finding its level). It relegated frictions, expectations and path dependence to the role of mere disturbances around a self-correcting mechanism (Corkhill, 2012; Mirowski, 1989). Equilibrium had not yet been proved, and already it was being presented as if it were physically necessary. As this geometric — or rather mechanical-mathematical — language was being built, the concept of order was also evolving in physics in parallel. In the line of Maxwell and Boltzmann order was no longer a state of rest in which mutually cancelling forces act, but a statistical property of systems composed of many interacting parts, in which distribution, probability, entropy and the relation between microstates and macrostates were the central concepts (Georgescu-Roegen, 1971; von Plato, 1991)[3].

Towards the end of the nineteenth century the two languages of order began to diverge — one geometric and point-based, the other statistical and temporal. The question raised by the second — namely, when a single long trajectory of a process adequately represents the whole system — would later become one of the central questions for economics as well. Georgescu-Roegen (1971) was among the first to stress that economics had borrowed Newtonian mechanics while neglecting thermodynamics, losing time and irreversibility along the way. When Arrow and Debreu published their existence proof in 1954, roughly a hundred and fifty years lay behind them in which economics had freely used the concept without any formal proof of existence, uniqueness or stability — and in three senses at once: as an end state, a direction of motion, and a normative criterion. These meanings reinforced one another, which gave the concept its strength and concealed the questions that later mathematics would open. Is equilibrium unique?

[3] The history of the ergodic hypothesis is itself more tangled than later, simplified accounts suggest: its rigorous version was formulated only in the 1930s (Birkhoff, von Neumann), so it should not be attributed to Boltzmann in finished form (von Plato, 1991).

Is it stable? Does the system converge to it? Does a single trajectory represent the whole? (Hahn, 1984; Ingrao & Israel, 1990). These questions lead us to the next section.

3 Formalisation: Equilibrium as a Fixed Point

By the middle of the twentieth century economic theory found itself in a difficult position. For over a century it had spoken of equilibrium, depicted it through supply and demand curves, and described it as being attained through auctioneers and processes of mutual adjustment, and yet it still had no general proof that the central object of all this theory existed at all. The decisive move came when equilibrium ceased to be the expected point at which economic forces settle and became the fixed point of a mathematical mapping — a state that the mapping leaves unchanged ($f(x) = x$). This changed the sense of what it means for an economy to have reached equilibrium: it was no longer a matter of the end of an adjustment process. Equilibrium became an abstract object whose existence could be guaranteed by an appropriate mathematical structure. What earlier economists had called a tendency was reformulated as a theorem (Arrow & Debreu, 1954; Ingrao & Israel, 1990). However, this would not have been possible without the use of new mathematical tools developed shortly beforehand, particularly in topology. The point of departure for the formalisation of equilibrium in economics was Banach's fixed-point theorem, which stated that if we have a mapping that shortens the distance between two points, then after its repeated application everything concentrates at exactly one place. More formally, Banach's theorem proved that every contraction mapping on a complete space has exactly one fixed point, to which the iterations converge from any starting point (Banach, 1922). This tool provided proofs of both existence and uniqueness, together with a constructive route by which the iterates converge to that point. In this picture the problem of equilibrium would be solved in its strongest form: without multiplicity, without ambiguity of choice among states, without tension between statics and dynamics, and without a gap between the solution and the process that reaches it. The possibility of reproducing equilibrium under the conditions required for Banach's theorem to hold would justify treating its five components as combined into a single object.

We draw attention to this because that picture, though never attained, persisted in the common understanding as a regulative ideal[4] — a template for what a "well-behaved" equilibrium should look like. Banach's assumptions, however, proved too tight for the general theory of competitive equilibrium. In general equilibrium theory, the relevant mappings are often set-valued rather than single-valued, so tools suited to correspondences were needed. Such a tool was provided by Kakutani's theorem, which was based not on the idea of contraction but on convexity. It generalised Brouwer's earlier theorem to set-valued mappings — those assigning a whole set of values to a single point — provided that they are sufficiently regular and take non-empty, compact, convex values (Kakutani, 1941). Arrow and Debreu used it in 1954 to prove the existence of an

[4]The *regulative ideal* is used here in the Kantian sense: not as a state actually attained, but as an ordering template that sets a direction and a norm for the theory, even when it remains unattainable.

equilibrium for a competitive economy (Arrow & Debreu, 1954). Lionel McKenzie published a related result in the same year. The model labelled "Arrow–Debreu" thus made it possible to order this long and difficult history into an elegant, clean story formalised through mathematical equations (Düppe & Weintraub, 2014; McKenzie, 1954).

What matters for us is that the new tool, while it allowed a breakthrough in the formalisation of equilibrium theory, also narrowed the range of what could actually be proved. The new theory established only the existence of equilibrium. It said nothing about its uniqueness, nor did it offer any realistic, decentralised process of convergence and it did not touch the old Walrasian problem: how the economy actually reaches equilibrium. The only thing established was that equilibrium belongs to a mathematically admissible class of structures. This was enough to found postwar mainstream theory on that discovery, and at the same time it gave far less than many would expect of economic equilibrium. From that moment equilibrium was no longer a dynamic balance of the forces of demand and supply; it became a mathematical object whose existence was guaranteed by topology — but only within a world constrained by continuity, convexity and compactness. The cornerstone of this theory — the existence proof — was thus not merely a mathematical achievement; it also carried ontological consequences: it made a tacit assumption about what the economy is like. The paradox that matters here is that this foundation became central precisely through its invisibility. Economists tell the story of the mathematisation of their discipline starting from Walras, through marginalism and optimisation. Yet it was the fixed-point reasoning, which gave equilibrium its modern form, that was the turning point in the process of mathematisation, even though it is far less visible. Students absorb the point ontology through comparative statics, welfare theorems and game-theoretic notions of equilibrium, and rarely learn where it comes from. The cornerstone became at the same time the background of the whole architecture of the discipline (Düppe, 2010). Owing to this formal naturalisation of equilibrium, dynamic difficulties — instability, multiplicity, divergent adjustment, path dependence — came to look like secondary complications rather than fundamental issues whose resolution is no less important. At this stage the point-based approach to equilibrium was the standard, even though its difficulties had not been resolved — it simply became the default image of what order should look like (Hands, 2016).

It is worth looking a little more closely at the work of the historians and methodologists who described this phenomenon — a debate into which the present work also enters. Mirowski (1989) argued that neoclassical economics borrowed from physics not only loose metaphors but an entire formal apparatus. In this light the turn to the fixed point can be read less as a neutral refinement of Walras's theory and more as part of a broad refashioning of the scientific order in economics. Ingrao and Israel (1990) come closest to the reading developed in this article: their history traces the long road equilibrium travelled from an intuitive, partly physical metaphor to a formal mathematical object. They treat the Arrow–Debreu proof as the culmination of a century-long effort to give economic order the standing that mathematical physics had earlier attained. Weintraub (1991) showed that postwar stability analysis, which investigated whether and how the economy returns to equilibrium, did not so much apply ready-made tools to an existing problem as construct the "relevant dynamical problem" itself — the analysis of the process of arrival through price adjustment. Framing the problem in this way was a consequence of applying a particular mathematical apparatus. McCloskey (1994) noticed something else — the rhetoric of formalism and the prestige attached to mathematical theorems substantially increase the persuasive force of arguments. She explains how theorems and theories based on the fixed point could become central to economics despite significant ontological gaps: what operated here was disciplinary authority more than substantive superiority. Mäki (1992, 2002) probed the underlying question of realism: once equilibrium is turned into an axiomatic object, is the result valid only in an artificial formal world, or does it also have real consequences? Similarly, Düppe (2010) showed how Debreu's axiomatic method separated mathematical form from economic content: the mathematics became self-contained, valid as a pure proof regardless of what it means for a real economy. This simultaneously raised the prestige of mathematical economics, making it resemble pure mathematics, but also deepened doubts as to whether its theorems speak of economic phenomena at all. Hands (2016) drew attention to two phases in the history of postwar research on stability. Early results, obtained under sharp assumptions, strengthened confidence in the Walrasian model: it seemed that price adjustment converges to equilibrium. But later, more general theorems could not be proved — counterexamples appeared in which prices do not converge. The same line of research, in seeking a proof of stability, thus produced results that undermined confidence in the model.

All these authors drew attention to different issues arising from the formalisation of equilibrium, which together add up to the conclusion that the mathematisation was not merely an innocent description of a theoretical gap. This process changed the meaning of equilibrium, transforming the classical economists' cautious, conditional statements about tendencies into a single, concrete, theorem-based object, and at the same time narrowing other aspects that ought to be taken into account when trying to describe economic order. From then on the burden of the problem shifted from existence to uniqueness, stability, convergence and the adequacy of the equilibrium point itself.

4 Existence Is Not Enough

The Arrow–Debreu proof showed that a competitive economy satisfying appropriate assumptions has at least one equilibrium. But it said nothing about whether that equilibrium is unique, whether it is stable, whether any realistic process converges to it, or whether it has any determinate structure beyond clearing the market. Already in General Competitive Analysis, Arrow and Hahn (1971) devoted much space to the question of whether and how equilibrium is attained — precisely because the existence proof itself does not settle it. This section discusses the remaining features of equilibrium. For each of them we present examples showing that the property cannot be guaranteed within the point-based scheme. We then set out the positive consequences — that order can still be described even when a given property is not required. The violation of each of these four features shows which objects can be studied, and how, despite a far more complex toolbox, they can allow economic order to be described.

4.1 Uniqueness: From the Point to a Choice

Here the problem started from the analysis of the demand functions of individual consumers maximising utility within their budgets. These functions satisfy a few basic properties: they are continuous, homogeneous of degree zero, and satisfy Walras's law; but, thanks to consumers' rationality, they also satisfy the weak axiom of revealed preference and have a negative semi-definite substitution matrix. Sonnenschein (1972) asked which of these properties carry over to the demand of the whole market when we sum the demands of all consumers. The answer proved disappointing: only the three most basic ones survived — continuity, homogeneity of degree zero, and Walras's law. Aggregate excess demand — the difference between what the market wants to buy and what it already holds — can take practically any continuous shape, provided it satisfies these three conditions. Mantel (1974) generalised this result and later showed that it holds even under homothetic preferences — that is, even when consumers behave regularly (Mantel, 1976). Debreu (1974) gave the strongest version: one can choose a set of utility-maximising consumers whose aggregate demand reproduces exactly — at least locally — any continuous function satisfying homogeneity and Walras's law. This means that almost any admissible shape of aggregate demand can be obtained from the behaviour of fully rational individuals. The textbook of Mas-Colell et al. (1995) sums this up with the slogan "anything goes", which captures well the methodological weight of the result. Similar conclusions were drawn by Bliss (1993) and by Arrow (1986), who conceded that the assumptions of rationality have very weak implications at the aggregate level (Debreu, 1974; Mantel, 1974; Sonnenschein, 1972). Consequently, since aggregate excess demand can take almost any admissible shape, nothing guarantees a unique equilibrium — its graph may cross zero at many points. Moreover, individual consumers' rationality in no way affects which equilibrium is selected (Kirman, 1992; Rizvi, 1994).

A popular way of circumventing this problem is to use the representative agent — the device the point-based approach uses to hold a coherent whole together. It is assumed that the whole economy behaves like a single consumer with a single utility function, so that aggregate demand is well-behaved and, consequently, there is a single equilibrium. This, however, bends reality: it solves the aggregation problem simply by assuming it away. Kirman (1992) showed that this convenience can be misleading — a representative agent can respond to changes in the direction opposite to that of the actual individuals in the economy, so that welfare conclusions drawn from the representative-agent model may differ from those implied by the underlying heterogeneous economy.

In short, equilibrium still exists — the Arrow–Debreu proof holds — but it ceases to be a single, sharply determined outcome of optimisation and becomes one of many admissible solutions. The theory does not even say how many there are, or which one will be realised. Rather than bending reality to a difficult result, one can instead change the object sought as a representation of economic order. Since the point is not unique, the key question is no longer "where does this equilibrium lie?" but "which of the many equilibria will be realised, and how is it reached?" The possible multiplicity makes selection (which equilibrium is chosen) and path dependence (the dependence of the outcome on the path taken) into real objects of study. The analysis focuses on the process that selects one of the many states. What the point ontology registers as a failure (the absence of a unique solution) is, from the perspective of the process, a positive fact: the explanatory burden shifts to the dynamics of selection and to the historical path.

4.2 Stability: From the Point to the Regime

Even if we identify a particular equilibrium point, it may be unstable. This possibility was described in the classical literature on stability. Scarf's simple examples from 1960 already showed that Walrasian tâtonnement can be globally unstable: prices circle around the equilibrium instead of converging to it. Smale's later work showed, in turn, that the static conditions for the existence of equilibrium permit no conclusions about its dynamics — these questions must be considered entirely separately (Gintis, 2007; Scarf, 1960; Smale, 1976). Systems of this kind are studied by the mathematical theory of bifurcation, which treats instability as a standard object of study rather than a disappointing exception. It examines how stability can change as the parameters of a system change, so that a state described as a fixed point may even exist and yet lose its role as the dominant long-run attractor. Even small changes in parameters can produce atypical behaviour of the equilibrium process, such as periodic motions, coexisting regimes, or quasi-periodic paths ("Local Bifurcation Theory Applied to OLG Models," 2005). An example is Grandmont's model in On Endogenous Competitive Business Cycles. The model assumes a competitive economy, markets clear in every period, and the participants have perfect foresight. Even so, persistent, deterministic cycles emerge within the model, driven by the tension between the wealth effect and the intertemporal substitution effect across generations. Cyclical motion can therefore arise from the internal logic of the equilibrium dynamics itself, and the tools of Hopf bifurcation, used for physical and biological systems, can be carried over to the study of a competitive economy (Grandmont, 1985). Thus, alongside existence and the failure of uniqueness, a third feature is instability: the sought-after fixed point ceases to be the only relevant object. The relevant dynamical object may instead be a cycle, a saddle, or a chaotic attractor, each of which behaves quite differently under a shock. Even a chaotic attractor is stable in the sense that trajectories are drawn to it, though their exact motion along it cannot be predicted (Dore & Rosser, 2007; Kaldor, 2020).

Yet the absence of a guarantee of stabilisation does not immediately mean disorder. The adjustment process can still take structured forms: cyclical, quasi-periodic, or regime-dependent. In this vein the appropriate image of the dynamics is a bifurcation diagram rather than the textbook intersection of demand and supply curves. The guiding question shifts from "where is the equilibrium?" to "what long-run motions does this structure allow, and when does one regime give way to another?" (Grandmont, 1985; "Local Bifurcation Theory Applied to OLG Models," 2005). The regime approach also has an important practical application. An extensive literature on critical transitions shows that, as a nonlinear system approaches a threshold at which one regime is replaced by another, it typically exhibits certain early-warning signals, such as critical slowing down, rising variance and increasing autocorrelation. All three measure how quickly the system returns to equilibrium after small disturbances — the slower it is, the greater the risk of instability (Scheffer et al., 2009). On this reading, systemic crises are regime shifts rather than deviations from an equilibrium point, and their onset is sought by examining certain statistical indicators of the dynamics rather than the distance from a market-clearing price (Battiston et al., 2016; May et al., 2008).

4.3 Convergence: From the Point to the Process

Stability concerns the behaviour of the system near an equilibrium point, provided it is already there. Convergence asks whether any real process reaches it at all. The auctioneer who calls out prices until all markets clear is a fiction, one that Walras himself altered between successive editions (Walker, 1996). Here too the cause is the SMD theorem. The classical positive result confirming convergence — the global stability theorem of Arrow et al. (1959) — required a very strong assumption: gross substitutability. Yet without structural restrictions on aggregate excess demand beyond continuity and Walras's law, there is no possibility that tâtonnement will converge in general (Ingrao & Israel, 1990). The direction of adjustment itself — a rise in prices when demand exceeds supply — is intuitively plausible, but it is not a theorem, only an assumption imposed from outside (Hahn, 1984; Rizvi, 1994). Gintis (2007) described this result in Walrasian dynamics as the absence of a credible, decentralised process shown to lead reliably to market-clearing prices.

So even where there is a proof of the existence of a competitive equilibrium, it need not be computable — still less constructively attainable — that is, nothing guarantees that any real procedure, market or algorithm, could locate it (Velupillai, 2000). An empirical counterpart to the study of convergence is the phenomenon of hysteresis, which shows that after a large shock the economy need not return to its earlier configuration. A typical illustration of this problem is persistently high unemployment after deep recessions. Thus, since existence does not guarantee that any process reaches the point, the adjustment path itself becomes worth studying: how long adjustment takes, which route it follows, whether it ends, how it responds to a disturbance. Such studies analyse the learning process, and convergence time, basins of attraction and path dependence are the key indicators.

4.4 Predictability: From the Point to the Distribution

The last, informal property of classical equilibrium suggests that a fully specified model yields a predictable outcome. This intuition was put to the test by the theorem of Li and Yorke (1975), who showed that for a continuous map of an interval into itself, an orbit of period three entails the existence of periodic points of every period and of an uncountable set of non-periodic points — chaos. The title of their article, "Period three implies chaos", shows how highly irregular behaviour can arise in very simple deterministic systems. This result had a significant impact on economics as well. Building on it, Benhabib and Day (1981) showed that rational choice in a stationary environment can produce erratic behaviour when preferences depend on experience. Day (1982) found irregular growth cycles within the familiar logic of the neoclassical model, and Benhabib and Day (1982) obtained bounded, non-periodic trajectories in a standard, deterministic overlapping-generations model. The survey by Baumol and Benhabib (1989) presented chaos as an increasingly important problem: fully deterministic systems generate behaviour that looks random. Chaos can therefore arise in economic models as a consequence of rational behaviour under nonlinear conditions. As Boldrin and Woodford (1990) and Benhabib (1992) stress, this is an internal property of equilibrium models. A model can be fully deterministic, internally consistent and fully specified, and yet fail to yield a long-run path that can be forecast.

This result, however, requires two qualifications. First, chaos is used here as a theoretical possibility within equilibrium models, not as an empirical description of real economies. Attempts to detect deterministic, low-dimensional chaos in economic and financial data have been largely inconclusive, because irregular data are hard to distinguish from stochastic processes (Hsieh, 1991). Second, the empirical argument against predictability rests less on chaos and more on the historical record of failed economic forecasts, which can have many different causes. They reflect model misspecification, ineradicable shocks, structural change, and feedback in which agents respond to the models and forecasts made about them (the Lucas critique). Thus, since one cannot predict the specific trajectory of a system, one can instead analyse the distribution of its trajectories by means of an invariant measure, which makes it possible to say how often, over the long run, the system visits a given region of its space. That is, the very feature that limits point forecasting — deterministic irregularity — allows order to be analysed in a new, statistical form.

4.5 One Point, Four Dynamics

The existence of equilibrium has been formally proved, but the remaining four properties — uniqueness, stability, convergence and predictability — do not follow from it and can fail independently of one another. Each time one of the features is called into question, the object of analysis changes from a single point to an object distributed in time or in the space of states: to a choice among equilibria, to a regime, to an adjustment path, or to a distribution. The results above are largely theoretical; the problem of the properties of equilibrium and of the manner of reaching it has, however, also been examined in empirical research, as described in the next section.

5 The Point in Empirical Research

The analysis of the features characterising market equilibrium is not merely a theoretical problem — this issue has also been examined many times through the observation of empirical data and through the tools of experimental economics. In standard theory, determining the price is simple: one need only read it off the point where the demand and supply curves intersect on a diagram. Empirical verification of this phenomenon is more difficult: the prices and quantities of goods are jointly determined, demand and supply schedules are not directly observed, and the analyst often sees aggregated outcomes, produced moreover within a specific institutional setting and under particular market conditions. Moreover, equilibrium cannot be read directly from the data — it has to be determined on the basis of an assumed model and an identification strategy, that is, assumptions linking observations to the hidden structure of demand and supply.

Haavelmo (1944) introduced the probabilistic approach to econometrics, in which economic data should be treated as generated by a random process rather than as an exact law with a small measurement error. An economic relationship — for example, how demand depends on price — must therefore first be written down within a statistical model, with a random component and assumptions about how the observations came about. Only within such a model can the data say anything about the true, hidden relationship. In the same spirit, the work of Berry, Levinsohn and Pakes reconstructs demand, supply, mark-ups and equilibrium prices from a mixture of theory, instruments, distributional assumptions and estimation (Berry et al., 1995). Such analyses show that equilibrium can be derived under certain substantive assumptions, and not simply read off the data or computed from the point where two curves intersect. It is worth noting an analogy with the SMD theorem. SMD says that, in theory, rationality alone does not suffice to determine a well-behaved aggregate demand — assumptions must be added. The problem of identifying an empirical equilibrium is similar: the data alone do not suffice to recover the demand and supply curves — here too assumptions must be added. In both cases a well-behaved equilibrium is not obtained for free — we must add constraining assumptions (Hildenbrand, 1994; Mas-Colell et al., 1995). Determining the curves from data is only one of the difficulties in trying to estimate the equilibrium point. Markets consist of finitely many agents making discrete decisions — often with minimum lot sizes, price ticks and indivisible goods. The demand and supply schedules reconstructed from observed data are usually step-like, and their jumps reflect the discreteness of real actions (Varian, 2014). Such curves may not meet at a single point at all — they may overlap over an interval, jump over each other, or generate several sets of prices at which the market is almost cleared. In such a situation the natural object of analysis becomes an interval of prices or an admissible set of goods. The theoretical literature that has studied indivisibility confirms that competitive equilibrium in such markets requires additional special conditions (Quinzii, 1984; Yokote, 2020). When market behaviour is discrete, treating the economy as continuous curves intersecting at a single point is, in effect, a strong assumption. The empirical literature responds to these challenges by stating openly that sometimes only an approximate estimate is possible, and only over a certain interval of values. Such an approximate equilibrium can nonetheless have great explanatory value and is worth studying, as Budish (2011) showed by analysing an approximate competitive equilibrium from equal incomes, using the example of course selection for MBA students. Order is then a bounded region within which imperfections are tolerated to some degree, and the allocations of goods are approximately efficient. Similarly, Yokote (2020) shows that in heterogeneous, discrete markets one can identify goods that are over-demanded or under-supplied and read off the directions of price adjustment even where no point equilibrium exists.

The very shape of such price regions — the exact set of prices at which the market is approximately in equilibrium — becomes an object of study. The same conclusion is reached by the approach of partial identification in econometrics, which, rather than adding strong assumptions in order to obtain a single number, reports the set of all values consistent with better-justified assumptions (Budish, 2011; Manski, 2003). Such a set should not be treated as a failed attempt to identify a point we could not pin down exactly — it is the strongest honest conclusion that the data and the adopted assumptions allow. This is seen still more clearly once we notice that the equilibrium point itself depends on the assumed model. Two analysts studying the same market but under different assumptions — different instruments used to separate demand and supply, a different assumed form of the curves, or a different treatment of unobserved differences between agents — will usually obtain different equilibrium prices and different elasticities. This is seen still more clearly in financial markets, which ostensibly provide very precise data on both demand and supply. Yet a listed asset does not have, at any given moment, a single market-clearing price. It has a bid-ask spread — the difference between the highest price offered by buyers and the lowest price asked by sellers — and an order book, that is, a list of all buy and sell orders at various price levels and of various volumes. Even the most liquid markets therefore display, at every moment, a set of prices rather than a single point. The price quoted is only a convention chosen by the analyst: the midpoint between bid and ask, the price of the last transaction, or the volume-weighted average. It is worth noting that what the textbook depicts as a single intersection of two curves is, in the data, a whole distribution of transaction prices compressed into a single number. Independently of econometric research, the way market equilibrium comes about has also been studied in experimental economics.

In experiments on controlled double-auction markets conducted by Vernon Smith, transaction prices converge towards the competitive equilibrium price (V. L. Smith, 1962). Yet this is still a process that takes time — the price interval selected by the participants gradually narrows over successive rounds of the experiment. The whole outcome should therefore be considered from the standpoint of the process and of a broad analysis of the approach to order, rather than of a point-like end state. Such a distribution of prices changing over time is worth treating as the primary object, not as an approximation of some hidden, true point — and a statistical account of equilibrium in this spirit is developed in Section 6.

This section thus adds a different but complementary point of view to the earlier theoretical argument. The SMD theorem showed that the rationality of individuals does not suffice to preserve the structure of demand at the aggregate level. The theories of bifurcation and chaos showed that equilibrium objects may be structurally ill-suited to static analysis. The problem of identifying demand and supply, in turn, shows that even if a precise empirical equilibrium point is attainable, it is a product of both methodological and statistical assumptions. In such analyses, more important than retaining a point outcome may be a transparent presentation of the assumptions and constraints that can lead to an outcome in the form of a set of points. Equilibrium may be exact or approximate, point-like or set-valued; the analysis should therefore ask which kind of order its data actually support: point-like, region-based, statistical or temporal.

6 A Second Language: Time, Entropy, Ergodicity and Stylized Facts

6.1 Two Orientations of Order

Earlier we focused on analysing the point-based view of equilibrium and its features, pointing to possible alternative approaches. Here we develop those alternatives — describing the temporal-statistical orientation, in which order can be described as a changing property of a process over time. In much of the equilibrium tradition, time, in so far as it is considered at all, is regarded as a less important element than the end point itself — the destination matters more than the journey. The solution-centred orientation usually treats equilibrium as an object — a fixed point, or, less often, a mapping (for example, a decision function), or a set of admissible solutions. The orientation described here approaches this differently: it asks not where the point is, but what kind of order the system displays over time. On this understanding, order can take several forms: it may be a distribution of outcomes (for example, how often, over the long run, the system visits particular states), it may have a stationary regime (a persistent, recurring statistical pattern), or it may be an ergodic process, in which outcomes are considered in relation to the trajectory. The temporal-statistical orientation is concerned with persistence, stationarity, the reproducibility of averages, and the relation between different realisations of a process. The remainder of this section analyses various aspects of the temporal-statistical orientation: its history, rooted in statistical physics; its treatment of time and of ergodicity; and the various methodological currents in which it can be found — econophysics, the Austrian tradition, mainstream dynamic economics, and complexity economics. At the end we propose a coherent taxonomy of the objects used in this approach.

6.2 From Statistical Physics to Statistical Equilibrium in Economics

The second language of order was born in nineteenth-century statistical physics. Where mechanics saw order as a resting point of balancing forces, statistical physics discerned it, in a system of billions of particles, as a statistical property — a stable distribution of microstates, defined by probability and entropy rather than by a single configuration (von Plato, 1991). The line of development runs from Maxwell and Boltzmann, through Gibbs's ensembles, to the ergodic theorems of Birkhoff and von Neumann. Ergodic theory added, however, a crucial question: when does the long-run behaviour of a single trajectory coincide with the average over the whole distribution? Prigogine went further and showed that in systems held far from equilibrium an irreversible process can itself generate structure — "order out of chaos" (Prigogine & Stengers, 1984). The common core of these works is that order can be located in the temporal and statistical organisation of a process rather than in a geometric point. This way of thinking passed into economics. Georgescu-Roegen (1971) argued that the economic process is irreducibly entropic — it turns low-entropy resources into high-entropy products — and hence irreversible, so that time is indispensable for understanding it.

An economics built exclusively on reversible, mechanical images of equilibrium loses this irreversibility as a key characteristic. Foley gave this language a precise equilibrium formulation: statistical equilibrium replaces the Walrasian intersection of curves with a distribution of transactions that maximises entropy, so that equilibrium simply is a distribution (Foley, 1994). Scharfenaker and Foley (2017) developed this into a general maximum-entropy method that predicts aggregate distributions from limited information. The second language is thus a different way of describing equilibrium — one in which order is statistical and, from the very outset, depends substantially on time.

6.3 The Role of Time and Ergodicity

In the earlier part we showed how order can be described as a distribution; this, however, poses a further question. Does such a distribution, understood as an average over many runs of a process, faithfully capture what a single system experiences over a longer time? This is the question of ergodicity, and on this issue the temporal-statistical orientation substantially changes the standard view. In this approach we can define two averages: the time average, computed from a single trajectory of the system over the long run, and the ensemble average (the expected value), computed over the whole distribution of states at a single moment. A process is ergodic when the two coincide, and the classical theorems of Birkhoff and von Neumann showed that this condition is not self-evident and is not satisfied by every process (Birkhoff, 1931; von Neumann, 1932). Let us illustrate this with a simple example. Take a multiplicative game, repeated over time, with a fixed proportional gain or loss at each step. The averaged outcomes of a large ensemble of parallel players will yield growing wealth, because a few extreme winners pull the average of the outcomes upward. Yet the typical single trajectory, considered over time, will often show a result close to zero. The ensemble average and the time average are different, and only the latter describes what a typical player actually experiences. This is of direct significance for economics, because many models — especially those based on expected utility — freely substitute the ensemble average for the time average. When a process is ergodic, this substitution is harmless. When it is not — and under multiplicative, path-dependent dynamics it usually is not — it means that we define the wrong objective for the modelled agent (Peters, 2019). Davidson (1988) put it this way: in a non-ergodic world the future cannot be described by a stable distribution constructed from historical data. This may have deep consequences for money, contracts and expectations. The claim is disputed, however — Doctor et al. (2020) acknowledge that the ergodicity problem is real but reject Peters's conclusion that it replaces expected-utility theory, pointing out that maximising logarithmic utility yields the same result. Peters (2020) replies that agreement of results does not necessarily settle the interpretation. This very dispute is evidence of the importance of analysing the ergodicity of processes: it bears directly on whether the ensemble approach can replace the individual trajectory of events — which could translate into a revision of many classical economic theories.

6.4 Many Schools and Traditions

This same temporal-statistical orientation is present in various schools of economics, whose common element is the rejection of treating the point as the key reference. The most empirical of them is econophysics, which arose in the 1990s from the statistical analysis of financial time series (Mantegna & Stanley, 1999). Its main objects of study are stylized facts, the recurring statistical regularities: fat tails of distributions, scaling laws, long-range dependence and volatility clustering. The market does not present itself here as a system visibly converging to a market-clearing point, but as a stream of varied data, and order can emerge from the study of the distribution — its shape and its correlation structure. The conclusions are therefore purely statistical-empirical (Gallegati et al., 2006; Kutner et al., 2019; McCauley, 2006).

By a completely different method, the Austrian tradition reaches a similar conclusion; it bases its arguments largely on qualitative tools, and derives the system dynamics from a praxeological approach rather than through opposing forces balancing at a point. Hayek argued that formal equilibrium is close to a tautology until it is filled with an account of how dispersed knowledge is acquired and communicated, and that the market is an ongoing process of coordination, a "discovery procedure", rather than a solved allocation (Hayek, 1937, 1945). Kirzner (1973) framed this as the entrepreneurial market process, Lachmann (1976) described a "kaleidic" economy in which divergent expectations continually reshape the pattern, and von Mises (1949) treated the "evenly rotating economy" openly as a fiction — a theoretical thought experiment. It is worth stressing that a statistical element is present in this current too — Hayek's pattern prediction shows how, in complex phenomena, science can predict only a recurring pattern rather than a point outcome (Hayek, 1964).

Another strand is represented by recursive methods (Stokey et al., 1989), which describe models by means of a precise definition of the current state of the economy and rules for its change — a "law of motion". Incomplete-markets models (Aiyagari, 1994) and industry-dynamics models (Hopenhayn, 1992) showed that the equilibrium object can be a stationary distribution, and Krusell and Smith (1998) treat the distribution as an equilibrium at the level of the aggregate economy. Thus, in this current, the closure and stability of a model depended on determining the distribution and correctly specifying the law of motion. A point-like price vector was not enough to describe the outcomes fully, or was simply unattainable.

From the 1990s these various strands began to coalesce into a coherent research programme with defined tools, assumptions and methodology — complexity economics. This research programme developed primarily around the Santa Fe Institute — a centre for the study of complex systems founded in 1984, which brought together physicists, biologists and economists around the question of how complex order emerges from the interaction of many elements. Its leading figure in economics is W. Brian Arthur, known for work on increasing returns, path dependence and "lock-in", who developed and popularised the very notion of complexity economics (Arthur, 1999, 2015). On this view the economy is a set of patterns and structures that form and dissolve through the interaction of adapting agents, rather than a system resting in a static and ultimately solved allocation.

After the 2008 crisis Farmer and Foley (2009) argued that economics needs agent-based modelling, and Kirman (2010) reframed rationality so that it rests on interactions and emergence rather than on a representative optimising agent. These strands differ in their methods — statistical, praxeological, recursive and simulation-based — yet agree on one thing: order can be a property of a process rather than of a single equilibrium point.

6.5 A Constructive Catalogue of Order-Objects

In this part we focus on describing the methodological apparatus of the temporal-statistical orientation. In Section 4 we set out four features of the classical understanding of equilibrium for which the tools of the point-based approach did not yield unambiguous answers. The first of the properties analysed was the problem of the uniqueness of the solution. In such a situation it is possible to find a set of solutions and to analyse which of them are more probable. As a rule, the elements that the geometric tradition described as a single admissible state, the temporal-statistical orientation presents as a generalised family of objects, which can nonetheless describe a certain stable pattern. This approach can be related to models of social learning, in which the participants do not always reach agreement, and yet one can delimit a concrete region within which all the possible outcomes must fall — and this very constraint is a form of order (Balbus et al., 2025). The next issue examined was the stability of the system and its capacity to keep the system at a single determinate point. This is not always possible, but instead we can analyse whether there is any recurring pattern of motion in the vicinity of the solution. The system then follows a fixed cycle or remains within a stable regime — a behaviour that persists until some parameter crosses a threshold. Such a cycle can still be fully ordered. Next, convergence was analysed, and the question of whether the process reaches the point at all. Here, in line with the temporal-statistical orientation, the journey matters more than the destination. What matters is the convergence time and the observation of how the state variables evolve along a given trajectory — this allows genuine conclusions about the situation of an individual agent. In this approach, too, one can analyse the form of order, but it is shifted from the end state to the patterns that emerge over the course of the system's evolution. The last element considered was the predictive capacity of models based on the equilibrium approach. Here, in place of sharp predictions that nonetheless have a low probability of being realised, the alternative approach offers distributions over possible outcomes. This is not about the confidence intervals around a baseline path already present in many classical models, but about considering and quantifying entirely alternative scenarios. Even without knowing what will happen next, through the analysis of simulations we can say how often, over the long run, the system visits particular states — and this pattern of frequencies can be treated as order. If the system is random, it can be described by a certain distribution that reproduces itself period after period. If instead it is deterministic but chaotic, the trajectory may wander unpredictably, yet the system may nonetheless spend a fixed fraction of time in each region. Even in financial data one can find persistent patterns, described by certain stylized facts — fat tails and volatility clustering.

Each of these four objects should be treated as an equally valid element of the analysis, especially where the point-based approach, owing to its structural limitations, does not provide an adequate explanation. It is also worth stressing that each of them can be measured, though doing so often requires the additional assumption of certain thresholds whose crossing marks a transition between a stable and an unstable state. Distributions can be estimated from simulation data, and whether a single trajectory actually reproduces them (ergodicity) can be tested on the data. One can also measure how long it takes to reach the pattern and on what initial conditions this depends, and detect a regime change directly from the data series.

The strands described in this section — the historical roots of treating distribution and time as key properties of complex systems, the approaches present in various economic schools and currents, and the conceptual-methodological apparatus — define the second orientation, the temporal-statistical one, an alternative to the point-based view. Its central claim is simple: the solution of a model need not be a good description of order, which should rather be understood as a complex though recurring process over time. In the next section we sum up the considerations discussed in this article.

7 Summary: Two Orientations of Order

7.1 Two Orientations

Contemporary economics uses the concept of equilibrium in several ways; in this article we have presented two orientations — the point-based and the temporal-statistical. The first is solution-centred: its aim is to find a stable outcome that will allow conclusions to be drawn. It asks whether the model contains a specific point at which opposing forces balance, at which no party finds it worthwhile to change its decision, and to which, even if the system is for some reason displaced from it, it will return — this point is the solution that closes the whole system. The second is centred on the adjustment process itself, and its conclusions may be formulated conditionally or in the form of a distribution. It checks whether the system reproduces a certain order over time — persistent regimes, stable distributions, ergodic processes — and even the absence of such a pattern is likewise treated as a result. In this orientation the aim is harder to define, and the specific measures and definitions may be more diffuse and tailored to the particular problem, but it also has a certain advantage: its language allows the representation of far more complex systems. The two approaches overlap in part as regards their mathematical tools, but differ fundamentally in their principal object of study — in what they treat as equilibrium (Aumann, 1974; Davidson, 1988; Peters, 2019). This can mean a precise point, a certain approximate clearing region, or even a whole distribution of outcomes. The proper question is therefore: in which situations should one use the tools and methodology of each of the two orientations?

7.2 What Happened with the Geometric–Point Approach?

In Section 4 we described the specific challenges that the geometric tradition faced in the twentieth century — from the loss of uniqueness to the absence of predictability. Despite them it held its place in the mainstream, but it changed what it counts as a solution, although the aim of the search remained the same. It still seeks a solution that closes the model, but this can now be an ever wider class of objects: regions instead of points, distributions, attractors, and even states emerging from interaction. Some of them are the same objects that the temporal-statistical orientation employs — the difference lies not in the types themselves, but in the fact that here a closed solution is still being sought. The following examples show how this sought-after object shifted. When goods are indivisible, an exact market-clearing price may not exist, so one accepts an approximate equilibrium — a whole interval of prices at which the market is almost cleared (Budish, 2011) — and the equilibrium object then becomes a region. In game theory a Nash equilibrium is a single profile of mutually best responses, but the correlated equilibrium introduced by Aumann (1974) allows players to coordinate their actions on a shared random signal — and then the solution becomes a probability distribution over combinations of actions. Economists now routinely solve dynamic models in which the equilibrium object is a stationary distribution and a rule for its change (a law of motion) rather than a price vector. Even methods based on the fixed point have broadened: Balbus et al. (2025) obtain a region of admissible outcomes instead of a point outcome, applying this to models of social learning in which consensus may be unattainable. In all these cases the equilibrium object is still treated as the solution of the model — only its shape changes: from a point to a region, a distribution, or a constraint. The geometric tradition thus survived by giving up its oldest premise — the claim that order must be a single, exact, unique and stable point.

The temporal-statistical orientation works differently — it analyses when the solution-object is a justified object of study at all. What is crucial in this consideration is the ergodicity of the process: if the process in question is not reproducible, then time cannot be removed from the model without changing its conclusions (Davidson, 1988; Peters, 2019). The fixed point is still an attainable outcome, but it is an important special case rather than an end in itself (Mantegna & Stanley, 1999). This shows that the two orientations interpenetrate in many places, though they place their emphases on different issues. The point-based tradition can use stationary distributions and sets of outcomes, but even so its priority remains the solution of the model. The temporal-statistical orientation can use fixed points and invariant measures, yet seek the solution in the behaviour of the process over time. Even the same words — stability, convergence, equilibrium — can mean different things in each of the views, and the boundary between them cuts across the customary division into orthodox and heterodox currents. Contemporary economics has broadened the ontology of equilibrium and now operates with a family of order-objects: points, sets, regions, distributions, trajectories, attractors, constraints and regimes.

7.3 The Narrative Force of the Point

There is one more reason why it is hard to depart from the scheme of the fixed point. A formalised theory that has been proved mathematically has far greater persuasive force than cautious conditional statements, which are often the outcome of research in the temporal-statistical strand. Cautious statements of the kind "under certain conditions, over time, the market may tend towards balance" are more honest than mathematical theorems resting on unrealistic assumptions, but they have far less influence. They cannot be drawn on a chart, one cannot compute a specific outcome, they do not allow simple comparisons, and often not predictions either — whereas in science measurable results are received more favourably than caution. Theorems are portable, easy to teach and hard to challenge — they have been proved; only the assumptions can be questioned, which still does not make them untrue within their closed world. This asymmetry of influence is documented in several fields of research. Simple, fluently processed statements are judged more likely to be true (Alter & Oppenheimer, 2009; Reber & Schwarz, 1999). In economics the prestige of mathematical proof functions as a source of persuasion and authority more than as a neutral demonstration of truth (McCloskey, 1994). People who build their message around a single, confident idea prove more influential and better rewarded than those who are accurate but cautious. This is the finding of Tetlock (2005): the "hedgehog", who holds to one big idea, beats the versatile "fox" in influence, though it loses to it in the accuracy of its forecasts. This is also why the later negative results we described — from Sonnenschein–Mantel–Debreu to chaos — are often absorbed as footnotes to the theorem rather than replacing it. This is the pattern that Kuhn (1962) described as normal science accommodating anomalies and that "Falsification and the Methodology of Scientific Research Programmes" (1970) described as a protective belt around the hard core.

It is precisely these robust mathematical theories that make teaching, modelling and the comparison of results possible, and bracketing their troubles is easier than demolishing the construction built upon them. This has a direct bearing on the strategy for strengthening the influence of the temporal-statistical current. Since so powerful a formalisation cannot be displaced merely by showing that its assumptions are unrealistic, the aim should be to place the point appropriately as a special case and to endow the temporal-statistical current with objects of comparable operational strength. Such an object could be a concrete benchmark of the temporal-statistical approach — based on a stationary distribution, an invariant measure, or a convergence time. A caveat alone will not defeat a theorem — an alternative should be equally well equipped with tools.

7.4 Conclusions

In this article we have described the road that economics travelled in describing, and later formally defining, the concept of equilibrium. We drew attention to the fundamental differences between these views — from the standpoint of methodology and of modelling. Initially, the point-based approach tried to establish several properties of equilibrium — existence, uniqueness, stability, convergence and predictability. Yet the failure to establish them in general showed that order can also be non-unique, cyclical, dynamic, statistical or history-dependent. This alternative, temporal-statistical current, which rests on defining equilibrium as a process, offers broader analytical possibilities for certain problems and may allow a better explanation of complex, unstable economic processes. Although this point of view has gained prominence in recent years, its development is not uniform — there are many alternative currents united by a process-based approach to economic mechanisms. What is still lacking are objective, standardised and generally accepted measures of the quality of dynamic models; only such tools would allow the full potential of the process-based approach to be shown.

References

Ackerman, F. (2002). Still dead after all these years: Interpreting the failure of general equilibrium theory. *Journal of Economic Methodology*, *9*(2), 119–139. https://doi.org/10.1080/13501780210137083

Aiyagari, S. R. (1994). Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics*, *109*(3), 659–684. https://doi.org/10.2307/2118417

Alter, A. L., & Oppenheimer, D. M. (2009). Uniting the Tribes of Fluency to Form a Metacognitive Nation. *Personality and Social Psychology Review*, *13*(3), 219–235. https://doi.org/10.1177/1088868309341564

Arrow, K. J. (1986). Rationality of Self and Others in an Economic System. *The Journal of Business*, *59*(S4), S385. https://doi.org/10.1086/296376

Arrow, K. J., Block, H. D., & Hurwicz, L. (1959). On the Stability of the Competitive Equilibrium. *Econometrica*, *27*(1), 82. https://doi.org/10.2307/1907779

Arrow, K. J., & Debreu, G. (1954). Existence of an Equilibrium for a Competitive Economy. *Econometrica*, *22*(3), 265. https://doi.org/10.2307/1907353

Arrow, K. J., & Hahn, F. (1971). *General competitive analysis* (1. publ). Holden-Day [u.a.]

Arthur, W. B. (1999). Complexity and the Economy. *Science*, *284*(5411), 107–109. https://doi.org/10.1126/science.284.5411.107

Arthur, W. B. (2015). *Complexity and the economy*. Oxford Univ. Press.

Aumann, R. J. (1974). Subjectivity and correlation in randomized strategies. *Journal of Mathematical Economics*, *1*(1), 67–96. https://doi.org/10.1016/0304-4068(74)90037-8

Balbus, Ł., Olszewski, W., Reffett, K., & Woźny, Ł. (2025). A Tarski–Kantorovich theorem for correspondences. *Journal of Mathematical Economics*, *118*, 103106. https://doi.org/10.1016/j.jmateco.2025.103106

Banach, S. (1922). Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fundamenta Mathematicae*, *3*, 133–181. https://doi.org/10.4064/fm-3-1-133-181

Battiston, S., Farmer, J. D., Flache, A., Garlaschelli, D., Haldane, A. G., Heesterbeek, H., Hommes, C., Jaeger, C., May, R., & Scheffer, M. (2016). Complexity theory and financial regulation. *Science*, *351*(6275), 818–819. https://doi.org/10.1126/science.aad0299

Baumol, W. J., & Benhabib, J. (1989). Chaos: Significance, Mechanism, and Economic Applications. *Journal of Economic Perspectives*, *3*(1), 77–105. https://doi.org/10.1257/jep.3.1.77

Benhabib, J. (Ed.). (1992). *Cycles and chaos in economic equilibrium*. Princeton University Press.

Benhabib, J., & Day, R. H. (1981). Rational Choice and Erratic Behaviour. *The Review of Economic Studies*, *48*(3), 459. https://doi.org/10.2307/2297158

Benhabib, J., & Day, R. H. (1982). A characterization of erratic dynamics in the overlapping generations model. *Journal of Economic Dynamics and Control*, *4*, 37–55. https://doi.org/10.1016/0165-1889(82)90002-1

Berry, S., Levinsohn, J., & Pakes, A. (1995). Automobile Prices in Market Equilibrium. *Econometrica*, *63*(4), 841. https://doi.org/10.2307/2171802

Birkhoff, G. D. (1931). Proof of the Ergodic Theorem. *Proceedings of the National Academy of Sciences*, *17*(12), 656–660. https://doi.org/10.1073/pnas.17.2.656

Bliss, C. (1993). *General equilibrium theory*. Edward Elgar.

Boldrin, M., & Woodford, M. (1990). Equilibrium models displaying endogenous fluctuations and chaos. *Journal of Monetary Economics*, *25*(2), 189–222. https://doi.org/10.1016/0304-3932(90)90013-T

Brainard, W. C., & Scarf, H. E. (2005). How to Compute Equilibrium Prices in 1891. *The American Journal of Economics and Sociology*, *64*(1), 57–83. https://doi.org/10.1111/j.1536-7150.2005.00349.x

Budish, E. (2011). The Combinatorial Assignment Problem: Approximate Competitive Equilibrium from Equal Incomes. *Journal of Political Economy*, *119*(6), 1061–1103. https://doi.org/10.1086/664613

Corkhill, A. (2012). A Superb Explanatory Device': The MONIAC, an Early Hydraulic Analog Computer. *University of Melbourne Collections*, *10*, 24–28.

Davidson, P. (1988). A technical definition of uncertainty and the long-run non-neutrality of money. *Cambridge Journal of Economics*, *12*(3), 329–337. https://doi.org/10.1093/oxfordjournals.cje.a035063

Day, R. H. (1982). Irregular Growth Cycles. *American Economic Review*, *72*(3), 406–414.

Debreu, G. (1974). Excess demand functions. *Journal of Mathematical Economics*, *1*(1), 15–21. https://doi.org/10.1016/0304-4068(74)90032-9

Doctor, J. N., Wakker, P. P., & Wang, T. V. (2020). Economists' views on the ergodicity problem. *Nature Physics*, *16*(12), 1168–1168. https://doi.org/10.1038/s41567-020-01106-x

Dore, M. H., & Rosser, J. B., Jr. (2007). Do nonlinear dynamics in economics amount to a Kuhnian paradigm shift? *Nonlinear Dynamics, Psychology, and Life Sciences*, *11*(1), 119–147.

Düppe, T. (2010). Debreu's apologies for mathematical economics after 1983. *Erasmus Journal for Philosophy and Economics*, *3*(1), 1. https://doi.org/10.23941/ejpe.v3i1.37

Düppe, T., & Weintraub, E. R. (2014). *Finding Equilibrium: Arrow, Debreu, McKenzie and the Problem of Scientific Credit*. Princeton University Press. https://doi.org/10.1515/9781400850129

Falsification and the methodology of scientific research programmes. (1970). In I. Lakatos (Ed.), *Criticism and the Growth of Knowledge: Proceedings of the International Colloquium in the Philosophy of Science, London, 1965* (1st ed.). Cambridge University Press. https://doi.org/10.1017/CBO9781139171434

Farmer, J. D., & Foley, D. (2009). The economy needs agent-based modelling. *Nature*, *460*(7256), 685–686. https://doi.org/10.1038/460685a

Fisher, I. (1892). Mathematical investigations in the theory of value and prices. *Transactions of the Connecticut Academy of Arts and Sciences*, *9*, 1–124.

Foley, D. K. (1994). A Statistical Equilibrium Theory of Markets. *Journal of Economic Theory*, *62*(2), 321–345. https://doi.org/10.1006/jeth.1994.1018

Gallegati, M., Keen, S., Lux, T., & Ormerod, P. (2006). Worrying trends in econophysics. *Physica A: Statistical Mechanics and its Applications*, *370*(1), 1–6. https://doi.org/10.1016/j.physa.2006.04.029

Georgescu-Roegen, N. (1971). *The Entropy Law and the Economic Process:* Harvard University Press. https://doi.org/10.4159/harvard.9780674281653

Gintis, H. (2007). The Dynamics of General Equilibrium. *The Economic Journal*, *117*(523), 1280–1309. https://doi.org/10.1111/j.1468-0297.2007.02083.x

Grandmont, J.-M. (1985). On Endogenous Competitive Business Cycles. *Econometrica*, *53*(5), 995. https://doi.org/10.2307/1911010

Haavelmo, T. (1944). The Probability Approach in Econometrics. *Econometrica*, *12*, iii. https://doi.org/10.2307/1906935

Hahn, F. H. (1984). *Equilibrium and macroeconomics*. Blackwell.

Hands, D. W. (2016). Derivational robustness, credible substitute systems and mathematical economic models: The case of stability analysis in Walrasian general equilibrium theory. *European Journal for Philosophy of Science*, *6*(1), 31–53. https://doi.org/10.1007/s13194-015-0130-0

Hayek, F. A. (1937). Economics and Knowledge. *Economica*, *4*(13), 33. https://doi.org/10.2307/2548786

Hayek, F. A. (1945). The Use of Knowledge in Society. *The American Economic Review*, *35*(4), 519–530.

Hayek, F. A. (1964). The Theory of Complex Phenomena. In M. Bunge (Ed.), *The critical approach to science and philosophy* (pp. 332–349). Free Press.

Hildenbrand, W. (1994). *Market Demand: Theory and Empirical Evidence*. Princeton University Press. https://doi.org/10.1515/9781400863716

Hopenhayn, H. A. (1992). Entry, Exit, and firm Dynamics in Long Run Equilibrium. *Econometrica*, *60*(5), 1127. https://doi.org/10.2307/2951541

Hsieh, D. A. (1991). Chaos and Nonlinear Dynamics: Application to Financial Markets. *The Journal of Finance*, *46*(5), 1839–1877. https://doi.org/10.1111/j.1540-6261.1991.tb04646.x

Ingrao, B., & Israel, G. (1990). *The invisible hand: Economic equilibrium in the history of science*. MIT Press.

Jevons, W. S. (1970). *The theory of political economy* (4. ed., repr.). Penguin Books. (Original work published 1871)

Kakutani, S. (1941). A generalization of Brouwer's fixed point theorem. *Duke Mathematical Journal*, *8*(3). https://doi.org/10.1215/S0012-7094-41-00838-4

Kaldor, N. (2020). *Economics without Equilibrium* (0th ed.). Routledge. https://doi.org/10.4324/9781003069713

Kirman, A. (1989). The Intrinsic Limits of Modern Economic Theory: The Emperor has No Clothes. *The Economic Journal*, *99*(395), 126–139. https://doi.org/10.2307/2234075

Kirman, A. (1992). Whom or What Does the Representative Individual Represent? *Journal of Economic Perspectives*, *6*(2), 117–136. https://doi.org/10.1257/jep.6.2.117

Kirman, A. (2010). *Complex Economics*. Routledge. https://doi.org/10.4324/9780203847497

Kirzner, I. M. (1973). *Competition and Entrepreneurship*. University of Chicago Press. https://doi.org/10.7208/chicago/9780226375489.001.0001

Krusell, P., & Smith, A. A., Jr. (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, *106*(5), 867–896. https://doi.org/10.1086/250034

Kuhn, T. S. (1962). *The Structure of Scientific Revolutions*. University of Chicago Press. https://doi.org/10.7208/chicago/9780226458106.001.0001

Kutner, R., Ausloos, M., Grech, D., Di Matteo, T., Schinckus, C., & Eugene Stanley, H. (2019). Econophysics and sociophysics: Their milestones & challenges. *Physica A: Statistical Mechanics and its Applications*, *516*, 240–253. https://doi.org/10.1016/j.physa.2018.10.019

Lachmann, L. M. (1976). From Mises to Shackle: An Essay on Austrian Economics and the Kaleidic Society. *Journal of Economic Literature*, *14*(1), 54–62.

Li, T.-Y., & Yorke, J. A. (1975). Period Three Implies Chaos. *The American Mathematical Monthly*, *82*(10), 985–992. https://doi.org/10.1080/00029890.1975.11994008

Local bifurcation theory applied to OLG models. (2005). In M. Lines (Ed.), *Nonlinear Dynamical Systems in Economics* (Vol. 476). Springer Vienna. https://doi.org/10.1007/3-211-38043-4

Mäki, U. (1992). On the method of isolation in economics. *Poznan Studies in the Philosophy of the Sciences and the Humanities*, *26*, 19–54.

Mäki, U. (Ed.). (2002). *Fact and Fiction in Economics: Models, Realism and Social Construction* (1st ed.). Cambridge University Press. https://doi.org/10.1017/CBO9780511493317

Manski, C. F. (2003). *Partial Identification of Probability Distributions*. Springer-Verlag. https://doi.org/10.1007/b97478

Mantegna, R. N., & Stanley, H. E. (1999). *Introduction to Econophysics: Correlations and Complexity in Finance* (1st ed.). Cambridge University Press. https://doi.org/10.1017/CBO9780511755767

Mantel, R. R. (1974). On the characterization of aggregate excess demand. *Journal of Economic Theory*, *7*(3), 348–353. https://doi.org/10.1016/0022-0531(74)90100-8

Mantel, R. R. (1976). Homothetic preferences and community excess demand functions. *Journal of Economic Theory*, *12*(2), 197–201. https://doi.org/10.1016/0022-0531(76)90073-9

Marshall, A. (1920). *Principles of Economics*. Palgrave Macmillan UK. https://doi.org/10.1057/9781137375261 (Original work published 1890)

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic theory* (Oxford student ed., [Nachdr.]). Oxford Univ. Press.

May, R. M., Levin, S. A., & Sugihara, G. (2008). Ecology for bankers. *Nature*, *451*(7181), 893–894. https://doi.org/10.1038/451893a

McCauley, J. L. (2006). Response to "Worrying Trends in Econophysics". *Physica A: Statistical Mechanics and its Applications*, *371*(2), 601–609. https://doi.org/10.1016/j.physa.2006.05.043

McCloskey, D. N. (1994). *Knowledge and Persuasion in Economics* (1st ed.). Cambridge University Press. https://doi.org/10.1017/CBO9780511599347

McKenzie, L. (1954). On Equilibrium in Graham's Model of World Trade and Other Competitive Systems. *Econometrica*, *22*(2), 147. https://doi.org/10.2307/1907539

Mill, J. S. (1965). *Principles of political economy* (W. J. Ashley, Ed.). Augustus M. Kelley. (Original work published 1848)

Mirowski, P. (1989). *More Heat than Light: Economics as Social Physics, Physics as Nature's Economics* (1st ed.). Cambridge University Press. https://doi.org/10.1017/CBO9780511559990

Pareto, V. (1971). *Manual of political economy* (A. N. Page, Ed.; A. S. Schwier, Trans.). A.M. Kelley. (Original work published 1906)

Peters, O. (2019). The ergodicity problem in economics. *Nature Physics*, *15*(12), 1216–1221. https://doi.org/10.1038/s41567-019-0732-0

Peters, O. (2020). Reply to: Economists' views on the ergodicity problem. *Nature Physics*, *16*(12), 1169–1169. https://doi.org/10.1038/s41567-020-01108-9

Prigogine, I., & Stengers, I. (1984). *Order out of chaos: Man's new dialogue with nature* (5. [print.]). Bantam Books.

Quinzii, M. (1984). Core and competitive equilibria with indivisibilities. *International Journal of Game Theory*, *13*(1), 41–60. https://doi.org/10.1007/BF01769864

Reber, R., & Schwarz, N. (1999). Effects of Perceptual Fluency on Judgments of Truth. *Consciousness and Cognition*, *8*(3), 338–342. https://doi.org/10.1006/ccog.1999.0386

Rizvi, S. A. T. (1994). The microfoundations project in general equilibrium theory. *Cambridge Journal of Economics*, *18*(4), 357–377. https://doi.org/10.1093/oxfordjournals.cje.a035280

Scarf, H. (1960). Some Examples of Global Instability of the Competitive Equilibrium. *International Economic Review*, *1*(3), 157. https://doi.org/10.2307/2556215

Scharfenaker, E., & Foley, D. (2017). Quantal Response Statistical Equilibrium in Economic Interactions: Theory and Estimation. *Entropy*, *19*(9), 444. https://doi.org/10.3390/e19090444

Scheffer, M., Bascompte, J., Brock, W. A., Brovkin, V., Carpenter, S. R., Dakos, V., Held, H., Van Nes, E. H., Rietkerk, M., & Sugihara, G. (2009). Early-warning signals for critical transitions. *Nature*, *461*(7260), 53–59. https://doi.org/10.1038/nature08227

Smale, S. (1976). Dynamics in general equilibrium theory. *American Economic Review*, *66*(2), 288–294.

Smith, A. (2012). *An inquiry into the nature and causes of the wealth of nations*. Wordsworth. (Original work published 1776)

Smith, V. L. (1962). An Experimental Study of Competitive Market Behavior. *Journal of Political Economy*, *70*(2), 111–137. https://doi.org/10.1086/258609

Sonnenschein, H. (1972). Market Excess Demand Functions. *Econometrica*, *40*(3), 549. https://doi.org/10.2307/1913184

Stokey, N. L., Lucas, R. E., & Prescott, E. C. (1989). *Recursive Methods in Economic Dynamics*. Harvard University Press. https://doi.org/10.2307/j.ctvjnrt76

Tetlock, P. E. (2005). *Expert Political Judgment: How Good Is It? How Can We Know?* Princeton University Press. https://doi.org/10.1515/9781400830312

Varian, H. R. (2014). *Intermediate microeconomics: A modern approach* (Ninth edition). W.W. Norton & Company.

Velupillai, K. (2000). *Computable Economics: The Arne Ryde Memorial Lectures* (1st ed.). Oxford University PressOxford. https://doi.org/10.1093/0198295278.001.0001

von Neumann, J. (1932). Proof of the Quasi-Ergodic Hypothesis. *Proceedings of the National Academy of Sciences*, *18*(1), 70–82. https://doi.org/10.1073/pnas.18.1.70

von Plato, J. (1991). Boltzmann's ergodic hypothesis. *Archive for History of Exact Sciences*, *42*(1), 71–89. https://doi.org/10.1007/BF00384333

von Mises, L. (1949). *Human action: A treatise on economics*. Yale University Press.

Walker, D. A. (1996). *Walras's Market Models* (1st ed.). Cambridge University Press. https://doi.org/10.1017/CBO9780511664502

Walras, L. (2013). *Elements of Pure Economics*. Routledge. https://doi.org/10.4324/9781315888958 (Original work published 1874)

Weintraub, E. R. (1991). *Stabilizing Dynamics: Constructing Economic Knowledge* (1st ed.). Cambridge University Press. https://doi.org/10.1017/CBO9780511571831

Yokote, K. (2020). The discrete separation theorem and price adjustment directions in markets with heterogeneous commodities. *Discrete Applied Mathematics*, *275*, 134–143. https://doi.org/10.1016/j.dam.2019.08.022

University of Warsaw
Faculty of Economic Sciences
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00-241 Warsaw
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