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Robust optimisation in algorithmic investment strategies

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Abstract: This research develops a portfolio of four algorithmic strategies that produce Long/Short signals based on t+1 close price predictions of the underlying instrument. The main instrument used is S&P 500 index, and the data covers the period from 1990-01-01 to 2021-04-23. Each strategy is based on a different theory and aims to perform well in different market regimes. The objective is to have a set of uncorrelated investment strategies based on different logics such as trend-following, contrarian approach, statistical methods, and macro-economic news. Each strategy was individually generated following a personalized Walk-Forward optimisation, in which the model seeks to choose the most robust combination of parameters rather than the best one, in terms of risk-adjusted returns. The robustness of all strategies was tested by changing all parameters selected at the beginning of the optimisation. Additionally, the robustness of the portfolio of strategies is tested by applying it to another American index, Nasdaq Composite. Finally, the ensemble model was created based on the combination of the signals from all investment strategies for our two basis instruments. Results show that the portfolio obtains returns four (seven) times larger than the Buy & Hold strategy on S&P 500 (Nasdaq Composite) with a similar level of risk in the last 31 years.

Keywords: algorithmic trading strategies, robust optimisation criteria, overoptimisation, walk-forward optimisation, ensemble investment model

JEL codes: C4, C14, C45, C53, C58, G13

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Introduction

Forecasting returns of financial markets has been a topic of big interest for decades. Investors dedicate a large number of resources to obtain higher risk-adjusted returns than through passive investing, which is easily achieved by buying the benchmark. However, most fail to outperform the benchmark. Technology growth has evolved rapidly in the past few decades and it allows to create and analyze investment strategies in a more efficient manner than before.

There are many studies about algorithmic trading that focus on a single strategy and show its performance after the optimisation process. Many works do not explain how many attempts were needed to arrive at such an outcome and have a high risk of being overfitted to data on which they were tested. Several common mistakes are related to the data being used in each research. One is related to the common practice which is to show results from the algorithmic strategies only on historical data from previous years, when financial markets had more inefficiencies. The other one is related to not including all types of market regimes, which causes the strategy to show biased and not complete results. Another common practice is to perform the optimisation process on a single In-Sample (IS) and Out-of-Sample (OOS) window. This increases, even more, the risk of backtest overfitting. Additionally, many works do not test the robustness of the strategy by changing the default parameters of the strategy or applying them to correlated time series.

In terms of data used in this work, one main financial instrument is used, the S&P 500 index. Data range goes from 1990 to the end of April 2021. These 31 years of data allow the strategies to be tested on all combinations of market regimes in terms of the direction of trend (upper and lower), and volatility (high and low). Additionally, data for Nasdaq Composite is used to test the robustness of the portfolio of strategies on a similar stock market index in the same period.

This work tries to improve the optimisation process of investment strategies by running a Walk-Forward optimisation (WFO), in which many IS and an OOS windows are run on a rolling basis. It allows testing how the selection of parameters on different IS periods perform on OOS data. This is a way of testing whether the idea behind the strategy is stable through the years. Special attention is given in this work to the method of choosing the best combination from the IS period. Usually, the combination of parameters with the highest risk-adjusted return is considered as the best one. However, this work introduces an algorithm that aims to choose the most robust combination of parameters in terms of risk-adjusted returns, rather than the best one. This allows the strategy not to choose certain outliers that may appear in the backtesting process.

After all strategies are optimised on S&P 500 index, a sensitivity analysis is performed. This analysis consists of changing the default parameters from each WFO of each strategy to test their robustness. Additionally, the portfolio of strategies is applied to Nasdaq Composite. A similar performance on Nasdaq would allow to verify whether strategies are overfited to S&P 500.

This research tries to verify two hypotheses. The first one is based on the efficient-market hypothesis, and can be formulated as follows: *the market price reflects all public information, and therefore, it is impossible to obtain higher risk-adjusted returns than the market itself.* The second one is: *whether it is possible to obtain better risk-adjusted returns by combining signals from several investment strategies, than from each of them individually.*

The paper is organized into six chapters. The first one summarizes the literature. The second chapter describes the data used. The third one explains in detail the methodology followed in the research. The fourth chapter shows the results obtained. The fifth one shows the sensitivity analysis performed on the parameters of the chosen investment strategies. The last one describes the conclusions derived from this research. Additionally, recommendations for future work are suggested.

1 Literature Review

Algorithmic trading disrupted the world of financial markets during the 1970s. It offers many advantages to investors related to the automation of research, investment decisions, and trade execution. Algorithmic strategies have evolved a lot since then. The first generation of algorithms were pure trading execution programs, based on simple logic. The second generation became more sophisticated and they were designed to generate their trading signals and trade them automatically. A third-generation of algorithms includes the ability to learn and adapt itself to the market conditions (Chlistalla, 2011). Newer studies focus on the impact of high-frequency trading (HFT) on liquidity of financial markets, which agrees on the fact that HFT has been improved. The main positive is that HFT can allocate trades at a lower cost. However, HFT speed could disadvantage other investors, and the resulting adverse selection could reduce market quality (Jones, 2012). The profitability of algorithmic trading has been studied for decades. Early studies had several limitations in their testing procedure and were based on technical analysis and found limited evidence of its profitability. Later studies have evolved to a more advanced and robust methodology, including a larger number of tested strategies, the control of risk, statistical tests, parameters optimisation, and out-of-sample verification. It was found that among a total of 95 modern studies, 56 studies found positive results regarding technical trading strategies, 20 studies obtain negative results, and 19 studies indicate mixed results (Park et al., 2007). Boehmer et al. (2020) studied the impact of algorithmic trading on market quality from 2001 to 2011 in 42 equity markets and concluded that AT improves liquidity and informational efficiency, but increases short-term volatility. Importantly, AT also lowers execution shortfalls for buy-side institutional investors. Their results are consistent across markets and across a wide range of AT environments.

Reducing risk has been the objective of most investors, and diversification has been proven to reduce risk. As explained by Modern Portfolio Theory an optimal asset allocation allows investors to maximize risk-adjusted returns (Markowitz, 1952). Such diversification is usually achieved by including different assets in the portfolio. Other studies aim to diversify investment decisions on a single asset class. A method which combines signals from several strategies to diversify the risk of wrong predictions by a single strategy was proposed by Kijewski and Ślepaczuk (2020). They showed that it is possible to double the compounded returns of S&P 500 index on the same level of risk. Another more complex work that proved that is possible to beat the market consecutively was achieved by building a portfolio of investment strategies on several asset classes (Ślepaczuk et. al., 2018).

The optimisation process of each strategy plays a key role in the research. Originally, researchers divided the dataset into an "In-Sample" (IS) and one "Out-of-Sample" (OOS). The IS period was used for researching and optimising investment strategies, and the OOS was used to verify whether the strategy works or not. However, this optimisation procedure has several pitfalls related to the risk of overfitting, which occurs when a model targets particular data periods rather than a general structure. This topic has been extensively explained by Bailey et al. (2013). The main problem is that it is relatively simple to overfit an investment strategy in such a way that it performs well IS and any perseverant researcher will always be able to find a backtest with the desired Sharpe ratio. They suspect that such backtest overfitting process is a large part of the reason why so many systematic hedge funds can not achieve the elevated expectations generated by their managers. Nowadays, Walk-Forward optimisation (WFO) is frequently used as the most common approach. It utilizes a rolling or expanding window as IS for parameterization and another period as OOS using the chosen parameters. Local stationarity in market conditions is assumed, which means that market conditions in IS and OOS are similar. This way, optimal parameters are chosen for the out-of-sample period without looking ahead and incurring a forward-looking bias (Peterson, 2017).

James (1968) performed one of the first experiments using technical analysis to forecast price movements with a monthly frequency from 1926 to 1960. His article described several experiments using moving averages of different lengths and weights, intending to minimize losses. Results showed that Buy & Hold outperformed most strategies based on moving averages. Other studies involving moving averages propose using them as a trend filter for each asset in a portfolio. Such technique allows the investor to increase risk-adjusted returns with no adverse impact on return. The data used starts in 1973 and has a monthly frequency (Faber, 2007). In a similar work Gayed (2016) found that being exposed to equities with leverage in an uptrend and rotating into risk-free Treasury bills in a downtrend leads to the significant outperformance over time. For investors and traders seeking a destination with higher returns who are willing to take more risk, systematic leverage is an option to consider. Data used goes from 1928 to 2015 and results proved to be robust to various levels of leverage, moving average periods, and across multiple economic and financial market cycles. Huang and Huang (2018) arrived at a similar conclusion after analyzing the MA strategy on three major indices (the S&P 500 index, the Dow Jones Industrial Average, and the NASDAQ-100 index). MA strategies have a lower return, but better risk-adjusted performance than the buy-and-hold strategy.

Seasonality on financial markets has been studied in the past in an attempt to obtain profits or reduce losses in certain periods. It has been previously proved that the introduction of stock index futures trading reduced the presence of seasonality of mean returns (Faff and McKenzie 2002). Other studies arrived at the conclusion that it is possible to obtain significant premia within commodity and equity index universes by incorporating seasonality signals into a trend-following strategy. These seasonality signals were based on the logic of avoiding the worse performing months from the previous ten years (Baltas, 2016). Bjerring et al. (2016) also showed that seasonality in commodity instruments should be considered, and leads to a significant increase in risk-adjusted returns.

Forecasting of the time series by using statistical models started in the 1970s. In an attempt to generate investment strategies by applying them, Brock et. al. (1992) used the random walk, the AR(1), and the GARCH-M models to Dow Jones index from 1897 to 1986. They concluded that signals are not consistent among the three models. Buy signals generate high returns and returns following sell signals are negative. Furthermore, the returns following buy signals are less volatile than returns following sell signals. In another work from Devi et. al. (2013), Nifty Midcap50 companies were forecasted with ARIMA model with different parameters. The Box-Jenkins methodology is used to identify the model, and AIC BIC test criteria is applied against the data represented in the past to select the best model. This work generates investment decisions based on the minimum error percentage.

There are many studies focused on looking for macroeconomic data as leading indicators to predict the market crisis. McQueen and Roley (1990) arrived at the conclusion that macroeconomic news has little effect on stock prices. A stronger relationship is found in different stages of the business cycle are allowed in the research process. In later studies, it was shown that financial crises follow a pattern that consists of changes in asset prices, real exchange rates, investment, and employment (Zoega, 2010). In a deeper research project by Korzeń and Ślepaczuk (2019) several macroeconomic factors were analyzed to filter momentum investment strategies on the S&P 500 Index over the last 10 years. The conclusion was that the results of the momentum strategy with a macroeconomic filter were worse than the momentum strategy alone. The benchmark outperformed both strategies in terms of risk-adjusted returns.

Studied literature usually focuses on simple and usually overfitted optimisation procedures of a single investment strategy, without totally making use of all the benefits of modern quantitative tools. More attention should be paid to several aspects, such as the optimisation procedure in order to create more robust and long-lasting investment strategies. Additionally, investors should focus on the creation and combination of non-correlated investment strategies which perform well on different market regimes. Such an approach should allow investors to obtain decent risk-adjusted returns in different market conditions. Finally, studies that claim to have found profitable investment strategies should provide information about the effect of changing all strategy parameters on its performance.

2 Data

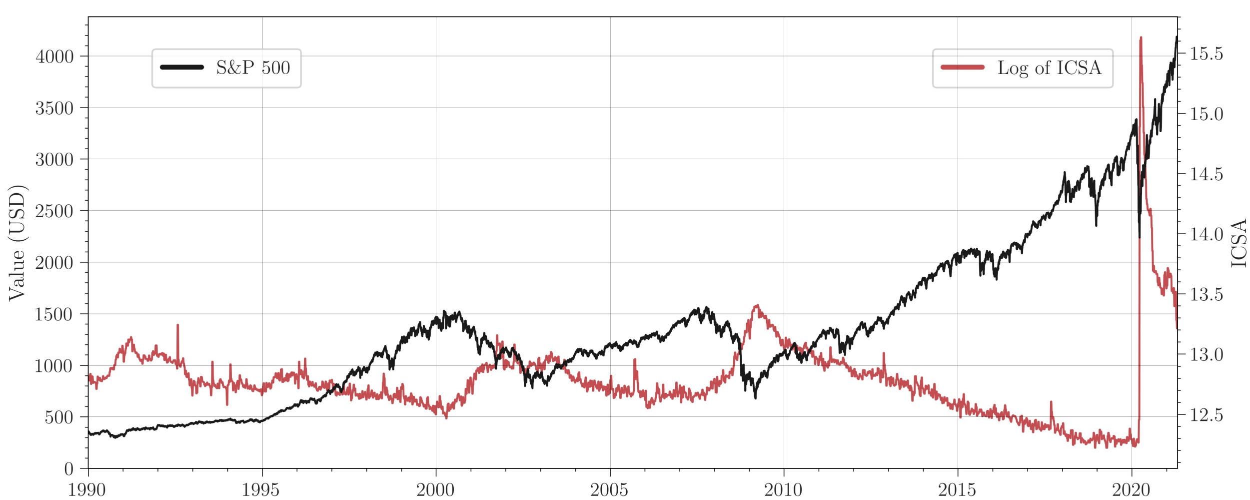
The research presented in this paper uses three time series of data. The first and most important refers to the Standard & Poor´s 500 (S&P 500) index. Daily close prices are used, and data was downloaded from Yahoo Finance. The range of data goes from 1990-01-01 to 2021-04-23, from which the first decade is used exclusively for training strategies. Using such a range allows the research to test the investment algorithms through a variety of market regimes, with different combinations of trend and volatility. It includes periods of low volatility and long uptrend, for most of the time on the 1990s and from 2010 until 2018; low volatility and long downtrends from the first years of XX century; high volatility and downside shocks, like 2008 and March of 2020; and high volatility and sharp upside trend from 2009 and in 2020 after significant turmoil in March. Figure 1 presents the fluctuations of S&P 500 index in the period our research.

Figure 1: S&P 500 index

Note: S&P 500 index fluctuations from 1990-01-01 to 2021-04-23.

Another time series used in this work is the seasonally adjusted Initial Claims (ICSA), retrieved from Federal Reserve Bank of St. Louis, FRED [1]. An initial claim is a claim filed by an unemployed individual after a separation from an employer. The claim requests a determination of basic eligibility for the Unemployment Insurance program. This data is released every Thursday by the FRED, and reports the situation of unemployement on the last Saturday. Figure 2 presents the fluctuations of ICSA and S&P 500 index.

Figure 2: S&P 500 index and the logarithm of initial jobless claims

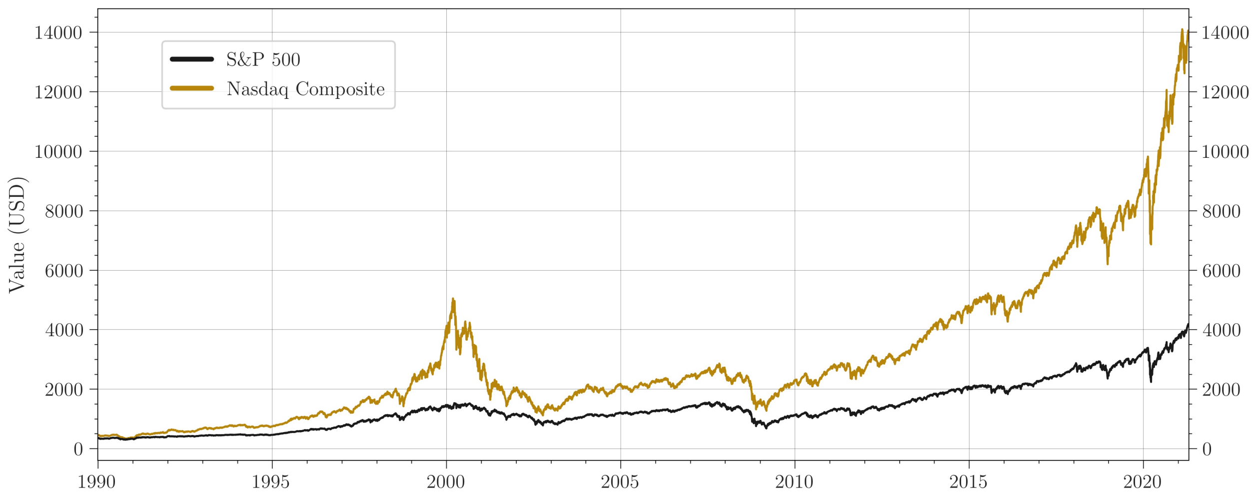


Note: S&P 500 index and the logarithm of ICSA weekly data with from 1990-01-01 to 2021-04-23.

[1]Data obtained from: https://fred.stlouisfed.org/series/ICSA

Due to the fact that Nasdaq Composite index is used in the sensitivity analysis we present the fluctuations in Figure 3. Like in the case of S&P 500, it was downloaded from Yahoo Finance, and it goes from 1980-01-01 to 2021-04-23. It is one of the most popular market indices in the United States, and companies from the information technology sector have a big weight in Nasdaq Composite. This time series is used in the sensitivity analysis of this work, with the purpose of testing the robustness of the algorithmic strategies on a different, but highly correlated time series. Both market indices and their performance metrics are shown in Figure 3 and Table 1.

Figure 3: S&P 500 vs Nasdaq Composite



Note: S&P 500 and Nasdaq Composite indices from 1990-01-01 to 2021-04-23.

Table 1: Performance metrics of S&P 500 and Nasdaq Composite indices

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All Risk	ARC MD	ARC AMD	Num. trades	Neutral
S&P 500	8.15	0.45	18.19	56.78	14.04	7.17	0.06	10.40	0.14	0.58	1	0
Nasdaq	11.54	0.50	23.18	77.93	19.22	15.11	0.07	52.48	0.15	0.60	1	0

Note: Performance metrics of S&P 500 and Nasdaq Composite indices.

3 Methodology

This work focuses on creating signals from several invesment strategies to later combine them into an ensemble model to produce a complex single signal on S&P 500 futures. This can also be understood as a portfolio of algorithmic strategies on a single asset. The objective is to have a set of uncorrelated investment strategies based on different logics such as trend-following, contrarian approach, statistical methods, and macro-economic news. Thus, the chosen strategies are moving-average crossover, *sell in May and go away*, ARIMA, and macro-economic factor-based.

Each of the strategies gives a signal of -1 (short), 0 (market neutral), or 1 (buy). After obtaining the signal from each of the strategies, they are combined to produce one signal between -1 and +1 every day to trade on S&P 500 index. This work supposes that it is possible to buy and sell any fraction of S&P 500 futures contracts, including several decimals. Additionally, a commission of 0.02 % is added to every transaction on S&P 500 index futures.

Another assumption is that the strategy uses daily close prices in order to generate the forecast for the next day and makes the trade at the end of the same day. In practice it requires the trade execution just several seconds before each day close time. It is equally possible in real-trading and requires only that the investor will take the price a few minutes before the close of the trading day and change the position accordingly to the investment signal before the market closes.

3.1 Performance metrics

This work uses many metrics to evaluate return and risk from all strategies. They are defined and explained by Kijewski and Ślepaczuk (2020), and by Korzeń and Ślepaczuk (2019). The chosen metrics are:

- Annualized return compounded (ARC):

$$ARC = \prod_{i=1}^{n} (r_i + 1)^{252/n} - 1 \tag{1}$$

  where:
  $r_i - is\ the\ daily\ percentage\ return\ at\ time\ i$

  $n - is\ the\ number\ of\ trading\ days$

- Annualized standard deviation (ASD):

$$aSD = \sqrt{252} * \frac{1}{n-1} * \sum_{i=1}^{n} (r_i - \bar{r})^2 \tag{2}$$

where:

$\bar{r} - is\ the\ average\ daily\ percentage\ return$

- Information ratio* (IR*):

$$IR^{*} = \frac{ARC}{aSD} \tag{3}$$

- Maximum drawdown (MD):

$$MD = sup_{x,y\ \epsilon\ \{[t_1,t_2]^2\ :\ x \leq y\}} \frac{P_x - P_y}{P_x} \tag{4}$$

where:

$P_t - is\ the\ equity\ line\ level\ at\ time\ t$

- Average maximum drawdown (AMD):

$$AMD = \frac{\sum_{i=1}^{n} MD_i^{yearly}}{n} \tag{5}$$

where:

$MD_i^{yearly} - is\ the\ maximum\ drawdown\ in\ trading\ year\ number\ i$

$n - is\ the\ number\ of\ trading\ years$

- Maximum Loss Duration (MLD): longest time needed to surpass a maximum value of the strategy returns. Measured in years.

- Information ratio** (IR**):

$$IR^{**} = \frac{ARC * ARC * sign(ARC)}{aSD * MD} \tag{6}$$

- All risk:

$$All\ Risk = aSD * MD * AMD * MLD \tag{7}$$

- Annualized return compounded / Maximum drawdown (ARC MD):

$$ARC\ MD = \frac{ARC}{MD} \tag{8}$$

- Annualized return compounded / Average maximum drawdown (ARC AMD):

$$ARC\ AMD = \frac{ARC}{AMD} \tag{9}$$

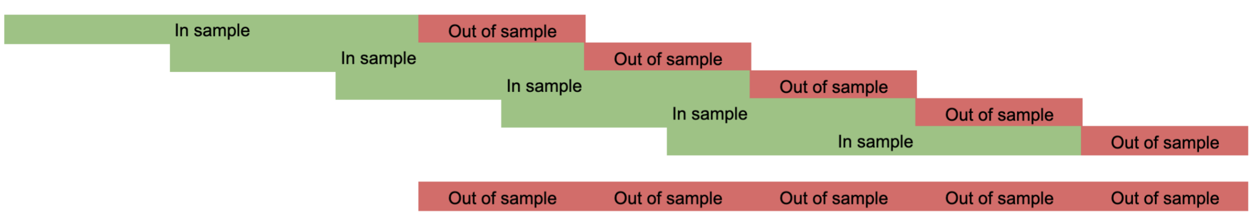
- Num. trades: Sum of all changes in position on S&P 500 index.

- No signal: Number of days with a neutral position on S&P 500 index.

3.2 *Walk-Forward optimisation*

Each strategy is optimised by using a Walk-Forward optimisation (WFO) with a special algorithm to choose the most robust combinations of parameters. WFO has several advantages over the traditional optimisation method with a single in-sample (IS) and out-of-sample (OOS). WFO uses rolling IS and an OOS windows. The most robust combination of parameters in terms of risk-adjusted returns is chosen from each IS window and then used to trade on the OOS window. This decreases the chances of parameter overfitting, as all combinations are tested on different rolling windows. This method also allows the strategy to adapt itself to different market regimes, allowing each strategy to choose different parameters for different periods. Another advantage is that it produces a longer overall OOS, as at the end of the optimisation, all OOS windows are combined creating one large OOS sample. Each strategy uses different lengths for IS and OOS periods.

Figure 4: Walk-Forward optimisation



Note: Example of In-Sample and Out-of-Sample windows in a Walk-Forward optimisation. At the end we are able to create an equity line for the tested investment strategy on the sum of all OOS periods.

The logic of WF optimisation is the following:

- Split dataset in many IS and OOS rolling windows.
- For each IS and OOS window:

- Run all combinations of parameters in the IS window in order to optimise robust optimisation criterion.
- Choose the most robust combination of parameters from the IS window.
- Use that combination of parameters to produce signals on the OOS window.

- Combine all OOS periods in order to create one equity line for the investment strategy and calculate all performance metrics based on it.

3.2.1 Robust optimisation criterion

An important decision is how to choose the best performing combination of parameters on each IS period. This work does it based on risk-adjusted returns - information ratio (IR). Additionally, in order to increase the robustness of optimisation in each IS window, a *robust IR* is computed. The main motivation for this change is to make the algorithm choose the best zones of parameters, rather than the best single combination. The exact formula to calculate the *robust IR* is:

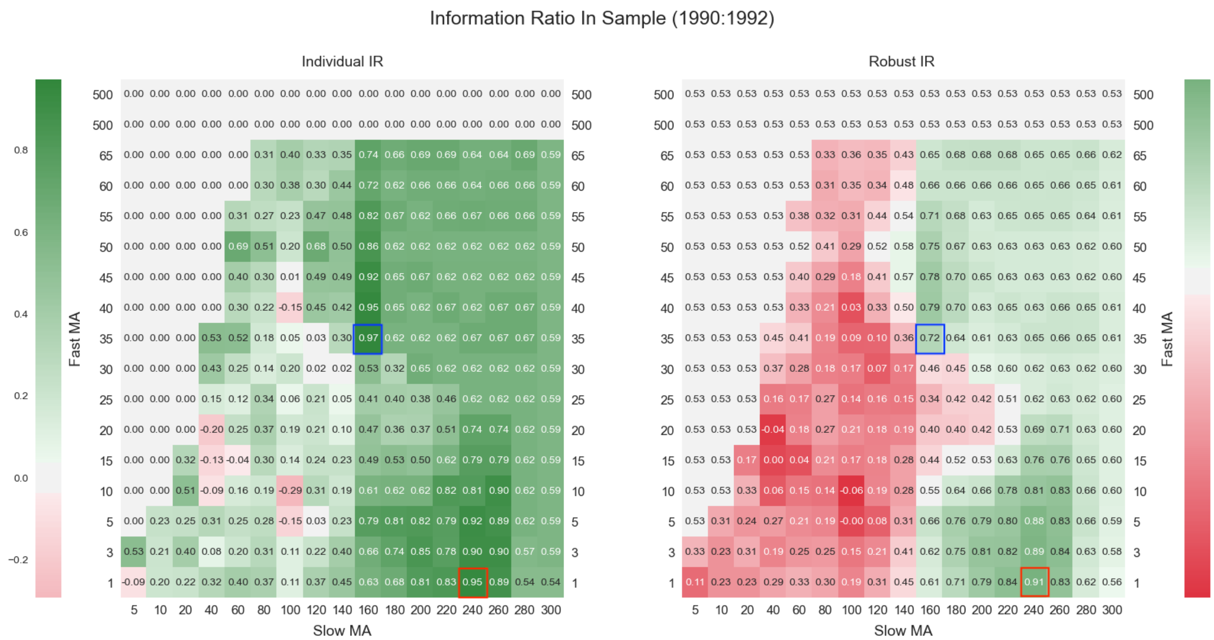
$$Robust\ IR = \frac{IR + average(IR_{neighbors})}{2} \tag{10}$$

where:

$IR_{neighbors} - is\ the\ IR\ of\ each\ of\ the\ neighboring\ combination\ in\ the\ optimisation\ matrix$

Figure 5 presents the optimisation matrix for individual IR and *robust IR* in order to better explain the logic used in this study:

Figure 5: Information ratio vs Robust information ratio



Note: Example of a calculation of the Robust Information Ratio used to choose the best combination of parameters from each IS window in the WFO. Best combination in terms of IR is marked with a blue square. Best combination in terms of Robust IR is marked with a red square. Combinations of parameters that were not run are filled with 0.00 on the left plot, and with the IR from Buy & Hold strategy on the right plot.

3.2.2 Long memory Walk-Forward optimisation procedure

Furthermore, this work studies an algorithm that aims to choose the combination that has a more robust performance over time. To choose the best combination of parameters on each IS this work introduces the concept of assigning weights to each parameter combination based on its *robust IR*. After this, weights from several IS periods are added. The logic for choosing the most robust parameter combination is the following:

1. Calculate IR from all combinations of parameters.
2. Compute the *robust IR*.
3. Calculate weights assigned to each combination of parameters. This weight is proportional to the *robust IR* of each combination from the actual IS window, and the weight is limited to 10 times the inverse function of the total number of tested combinations (Max. weight = 10/(total number of tested combinations). The exact formula is defined as by:

$$weight_x = \begin{cases} \min(\frac{Robust\ IR_x}{\sum_{i=1}^{n} \max(0, Robust\ IR_i)}, \frac{10}{n}) & \text{if } Robust\ IR_x > 0 \\ 0 & \text{if } Robust\ IR_x \leq 0 \end{cases} \tag{11}$$

where:

$Robust\ IR_x - is\ the\ robust\ IR\ of\ each\ combination\ with\ positive\ IR$

$n - is\ the\ total\ number\ of\ tested\ combinations$

4. Add weights from the most recent m IS windows.
5. Choose the combination of parameters with the highest accumulated weight.

After the investigation of this cumulative weighting algorithm, it was found that it has a similar result to increasing the size of the IS window. Therefore, the authors decided that this added complexity does not improve results in this study and that it is not worth it compared to choosing the best parameter combination in terms of *robust IR* from each IS window.

3.3 Individual strategies

The following investment strategies were optimised individually before assembling them to generate a position signal on S&P 500.

3.3.1 Moving average crossover

This strategy is very well-known and is based on the rolling average of the price. A rolling moving average (MA) is calculated from the previous n periods of the price. This creates a smoother time series that follows the price.

This is a trend-following strategy and its objective is to forecast trends. Since upper trends are usually longer in time and have lower volatility, which allows this strategy to detect the beginning of such trends early on time, and is its main advantage. On the other hand, the beginning of downward trends usually occurs in periods of high volatility, this strategy is not very accurate in detecting such trend changes because the moving averages are slower than the price. For this reason, this work does not short the index S&P 500. MA crossover produces a buy signal when the fast MA is higher than the slow MA. This means that the price has been going up, and the strategy predicts that it will keep going up. Oppositely, if the fast MA is lower than the slow MA, the strategy creates a 0 signal and exits the market.

Usually, several combinations between fast and slow moving averages are used, but they usually have too short and limited ranges. This causes the strategy not to be able to adapt itself to different market regimes, and results are not as good as they could be. The chosen parameter combinations from this work are:

- Fast MA: 1, 3, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65.
- Slow MA: 5, 10, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300.

A fast MA of one period is equal to the price, allowing many combinations for a crossover between price and a MA. Additionally, if Buy & Hold performs better than all other parameter combinations, Buy & Hold is chosen for the following OOS period (equivalent to a combination in which there are not any MA crossovers in the whole period).

One constraint for the MA crossover is that the fast MA has to be faster than the slow one, so not all combinations are run. In total, this optimisation runs a total of 212 parameter combinations on each IS period. The size of each IS window is three years, and each OOS window is one year. All parameters are changed in the sensitivity analysis to test their robustness.

3.3.2 Sell in May and go away

This strategy is very well known and it is based on the old idea that equities fall during the summer season. It consists of keeping a long position for most of the year and being neutral during several months that have delivered the worse risk-adjusted returns in the past years.

This is a seasonal strategy and its purpose is to avoid certain months of the year. If exiting a long position on a specific month and being out of the market during several months has proven to improve risk-adjusted returns from Buy & Hold, this strategy will take advantage of such behavior. *Sell in May and go away* strategy will keep a long position for most of the year and have a neutral position during a few months. The list of parameters for this strategy is:

- Selling month: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
- Selling duration: 1, 2, 3, 4, 5, 6.

Selling month refers to the number of the month in the year in which we exit a long position. Such position is taken on the first day of such month. Selling duration refers to the number of the month for which we keep the neutral position. The default position of the strategy is long on S&P 500. In total, this optimisation runs a total of 72 combinations of parameters.

The length of each IS window is ten years, and each OOS window is one year. All parameters are changed in the sensitivity analysis to test their robustness.

3.3.3 ARIMA

An autoregressive integrated moving average (ARIMA) model is a generalization of an ARMA process and its objective is to forecast data based on past values of data. This type of model is specially precise when data is non-stationary.

ARIMA(p, d, q) combines AR and MA models. The AR part forecasts the next data point based on a linear combination of the previous p values. The I (integration) part corresponds to the number of differentiations (d) needed to transform the original time series into a stationary one. The differentiation used in this work corresponds to the rate of change between close prices of consecutive days. The MA part forecasts the next data point based on the previous q values of error terms. Overall, values p, d, and q define the order of the ARIMA model, and this work uses order (1, 1, 1). The objective behind using such order is to reduce the risk of overfitting. Thus, the equation of our model can be defined as:

$$y_t = ay_{t-1} + e_t + be_{t-1} \tag{12}$$

where:

$y_t - forecast\ value\ at\ time\ t$

$y_{t-1} - given\ value\ at\ time\ t-1$

$e_t - error\ term\ at\ time\ t$

$e_{t-1} - error\ term\ at\ time\ t-1$

In order to estimate parameters a and b maximum likelihood estimator is used. Model is fitted thoroughly every 63 days. Additionally, the model performs several new iterations to update the model parameters every day. This extra iterations are done to update the previously found optimum parameters.

To determine the position of the strategy, the forecasted value for the following day is compared to the actual price. If forecast is higher than the actual price plus the transaction costs a long position is set. On the other hand, the strategy sets a short position if the forecasted price is lower than the actual price minus transaction costs. If transaction costs are larger than the predicted gain, a neutral position on the market is used. Overall, the procedure of this strategy can be summarized as:

- Split dataset in IS and an OOS periods
- For each day:
  - Fit model parameters if last time it was done was 63 days ago.
  - Forecast next Close price.
  - Set the strategy position.
  - Update IS window with the new value.
  - Update model with a small number of iterations.
  - Calculate returns from such day.
- Combine all returns from the original OOS window.

The length of each IS window is 504 days, and each OOS window is 1 day. All parameters are changed in the sensitivity analysis to test their robustness.

3.3.4 Macro-economic factor

This strategy is based on the idea that unemployment and equity prices are negatively correlated. As unemployment in the United States goes down, American equities should go up, and the opposite. It uses the logarithmic differences of Initial Claims Seasonally Adjusted (ICSA) as the macro-economic predictor of S&P 500. ICSA is released every Thursday by the Federal Reserve of St. Louis with a weekly frequency.

Historically those two factors have been negatively correlated, and this strategy aims to create signals based on that. More specifically, it uses a rolling window from ICSA time series, and two quantiles are calculated. Every new macro-economic data is released, it is compared with such values to generate a long, neutral or short signal for the following week. Quantile values are optimised in the WFO process. The possible parameter combinations from this work are:

- Buying quantile: 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9.
- Selling quantile: 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95.

Buying quantile refers to the upper threshold for generating a long signal. Thus, if the difference between the new ICSA value and the previous one is below the quantile *buying quantile* of the last 1260 days, the strategy creates a buy signal on S&P 500. On the other hand, if the value is above the quantile *selling quantile*, the strategy creates a sell signal. Additionally, if the new value is between both quantiles, no signal is generated.

The range of *buying quantile* goes from 0.5 to 0.9 because the intention is to generate more long than short and neutral signals. One constraint for this strategy is that the *buying quantile* has to be lower or equal the *selling quantile*, so not all combinations are run. In total, this optimisation runs a total of 51 parameter combinations on each IS period. The length of the rolling window to calculate the distribution of differences in ICSA is 1260 days (around five years). Each IS window is one year, and each OOS window is six months. All parameters are changed in the sensitivity analysis to test their robustness.

3.3.5 Summary of crucial assumptions

Overall, the assumptions and logic of all strategies can be summarized in the following table:

Table 2: Summary of logic from investment strategies

Strategy	MA Crossover	Sell in May and go away	ARIMA model	Macro-economic factor
Logic definition	Strat. avoids downtrend markets	Strat. avoids worse performing months of the year	Strat. predicts next return based on a linear combination of previous returns, and on the error term of the last forecasted return	Strat. goes together with the macro-economic situation
Long signals	True	True	True	True
Neutral signals	True	True	True	True
Short signals	False	False	True	True
Long signals logic	if fast MA ≥ slow MA: Long	if month not in (sell_month + sell_duration): Long	if predicted_price >(price+costs): Long	if (ICSA_t - ICSA_t-1) ≤ buy_quantile: Long
Neutral signals logic	if fast MA <slow MA: Neutral	if month in (sell_month + sell_duration): Neutral	if ((price-costs) <(predicted_next_price) <(price+costs)): Neutral	if buy_quantile <(ICSA_t - ICSA_t-1) <sell_quantile: Neutral
Short signals logic	-	-	if predicted_price <(price-costs): Short	if sell_quantile <(ICSA_t - ICSA_t-1): Short

Note: Summary of the logic of all investment strategies used in this work.

3.4 Portfolio of strategies - Signal combination

Reducing portfolio risk through diversification is seeked by many investors. Diversification is achieved by investing on different assets with a low correlation among them, which reduces the risk in a portfolio.

This work seeks that diversification by investing in different strategies. Since all strategies from this work invest in the same asset, S&P 500 index, their signals are combined and they generate one signal altogether. The method for combining them is by calculating the mean from each signal individually. Since two of the strategies (MA crossover and *Sell in May and go away*) can only generate a position of 0 (neutral) or +1 (long), and the other two strategies (ARIMA and Macro-economic factor) can generate a position of -1 (short), 0 (neutral) or +1 (long), the overall portfolio will generate signals from -0.5 to +1 in S&P 500 futures. Thus, every strategy has the same weight on the overall portfolio.

4 Empirical results

This section shows the performance of all strategies on the S&P 500 from 1990-01-01 until 2021-04-23. There is a plot with the equity line of each investment strategy compared to the benchmark, which is the Buy & Hold strategy on the S&P 500 index. On the right axis of each plot, the strategy´s position on each day is shown. Additionally, some strategies include the "Only long position allowed", which is activated when Buy & Hold performed better than all parameter combinations in the previous IS period. The initial capital for all plots is the open price of the S&P 500 index in 1990. Additionally, there is a table with performance metrics from each strategy.

The last part of the section includes the results of a portfolio built from combining investment signals from all individual strategies.

4.1 Individual strategies

4.1.1 Moving average crossover

The result from the MA crossover strategy is shown in Figure 6 and Table 3. The main objective of this trend-following strategy is to capture most of the upper trends and exit the long position when the two moving averages confirm a downtrend regime. It is possible to see that this goal is well achieved. The strategy exits a long position in the three major downtrends (the early 2000s, 2008-2009, and March of 2020). Additionally, the strategy is able to buy the index when it detects the start of an upper trend (during all 1990s decade, from 2003 to 2008, and for most of the time after 2010).

MA crossover strategy obtained an IR* of 0.68, which outperforms the benchmark (IR* of 0.45). Furthermore, all risk metrics are significantly better than in Buy & Hold.

Figure 6: MA crossover strategy compared to S&P 500

Note: Equity line of MA crossover and Buy & Hold strategy. MA crossover strategy uses an IS window of 3 years and an OOS window of 1 year.

Table 3: Performance metrics of MA crossover strategy

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All Risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	0.45	18.19	56.78	14.04	7.17	0.07	10.4	0.14	0.59	1	0
MA crossover	8.16	**0.68**	**12.06**	**19.34**	**8.07**	**3.76**	**0.29**	**0.71**	**0.42**	**1.01**	172	2059

Note: Performance metrics of MA crossover and Buy & Hold strategy. MA crossover strategy uses an IS window of 3 years and an OOS window of 1 year.

4.1.2 Sell in May and go away

The result from *Sell in May and go away* strategy is shown in Figure 7 and Table 4. The objective of this strategy was to avoid worse-performing months of each year, and it was not well accomplished. Sharp drawdowns do not happen in the same months, and the strategy is not able to stop them. This investment strategy seems to give random signals for buying and exiting trading positions.

Overall, this strategy delivers worse risk-adjusted returns (IR* of 0.36) than the Buy & Hold (IR* of 0.45). Risk metrics are better in our strategy but are mainly due to the fact that the strategy spends less time on an active position on the market than Buy & Hold.

Figure 7: Sell in May and go away strategy compared to S&P 500

Note: Equity line of *Sell in May and go away*, and Buy & Hold strategy. *Sell in May and go away* strategy uses an IS window of 10 years and an OOS window of 1 year.

Table 4: Performance metrics of Sell in May and go away strategy

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	14.04	7.17	**0.07**	10.4	**0.14**	**0.59**	1	0
Sell in May	5.72	0.36	**15.71**	**48.12**	**12.25**	**7.11**	0.04	**6.58**	0.12	0.47	65	2371

Note: Performance metrics of *Sell in May and go away* and Buy & Hold strategies. *Sell in May and go away* strategy uses an IS window of 10 years and an OOS window of 1 year.

4.1.3 ARIMA

The result of the ARIMA strategy is shown in Figure 8 and Table 5. The objective of this strategy is to predict the next daily return based on a linear combination of the previous return, and on the error term of the last forecasted return. This strategy obtains the best performance during periods of downtrends with high volatility. This may be due to the fact that during sharp falls of S&P 500 index returns show autocorrelation and the ARIMA model is able to forecast with higher accuracy. On the other hand, market returns seem to be more efficient during long and non-volatile upper trends, and this strategy performs worse than the benchmark.

Overall, the ARIMA strategy obtained an IR* of 0.42, which is slightly worse than Buy & Hold. On the other hand, risk metrics are slightly better in our strategy.

Figure 8: ARIMA strategy compared to S&P 500

Note: Equity line of ARIMA and Buy & Hold strategies. ARIMA strategy uses a training window of the last 504 days to forecast each day. Model parameters are fitted every 63 days.

Table 5: Performance metrics of ARIMA strategy

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	14.04	**7.17**	**0.07**	10.4	0.14	**0.59**	1	0
ARIMA	7.31	0.42	**17.57**	**42.69**	**13.66**	8.37	0.07	**8.57**	**0.17**	0.54	3555	653

Note: Performance metrics of ARIMA and Buy & Hold strategies. ARIMA strategy uses a training window of the last 504 days to forecast each day. Model parameters are fitted every 63 days.

4.1.4 Macro-economic factor

The result from the Macro-economic factor strategy is shown in Figure 9 and Table 6. The main objective of this strategy is to create a signal based on a macroeconomic predictor, ICSA. Figure 2 shows that there are four main periods when ICSA was increasing (1990, 2000-2002, 2008-2009, and early 2020). Figure 9 shows when the strategy generated selling signals and how it performed. On the other hand, this strategy performed much worse than Buy & Hold from 1995 to 2000. This may be because S&P 500 index tripled its price in just 5 years, while Initial Claims did not decrease at such a constant rate. This correlation is exactly where the strategy generates alpha from.

Macro-economic factor strategy obtained an IR* of 0.19, which is much lower than Buy & Hold (0.45). In terms of risk, performance metrics are also significantly worse than in Buy

& Hold.

Figure 9: Macro-economic factor strategy compared to S&P 500

Note: Equity line of MacroFactor and Buy & Hold strategies. MacroFactor strategy uses an IS window of 1 year, an OOS window of 6 months, and a rolling window of 1260 days to calculate the distribution of ICSA differences.

Table 6: Performance metrics of macro-economic factor strategy

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	**14.04**	**7.17**	**0.07**	**10.4**	**0.14**	**0.59**	1	0
MacroFactor	3.2	0.19	**17.12**	**53.34**	14.59	8.77	0.01	11.69	0.06	0.22	1093	980

Note: Performance metrics of MacroFactor and Buy & Hold strategies. MacroFactor strategy uses an IS window of one year, an OOS window of 6 months, and a rolling window of 1260 days to calculate the distribution of ICSA differences.

4.2 Portfolio of algorithmic strategies - Signal combination

The combination of strategies can be understood as a portfolio of investment strategies on a single asset, the S&P 500 index. The final position of the portfolio is calculated as the mean of the position from all strategies. It was shown before how different strategies are better at forecasting during different market regimes.

MA crossover strategy delivers very consistent results. It can go long during periods of upper trends (during the 1990s; from 2003 to 2008; and after 2009), and generally exits the market on periods of downtrends (from 2001 to 2003; during 2008; and in March of 2020).

Sell in May and go away seems to generate random signals and spends less time being long on the underlying asset. This makes the strategy to be better than Buy & Hold strategy only during downtrends which last for more than one year (from 2000 to 2002; from 2008 to 2010).

ARIMA model is better during periods of high volatility (2008-2009; 2020) and bad with lower volatility (during the 1990s; from 2003 to 2008; from 2009 to 2020).

The *macro factor strategy* is better after 2000 when the negative correlation between ICSA and S&P 500 becomes stronger.

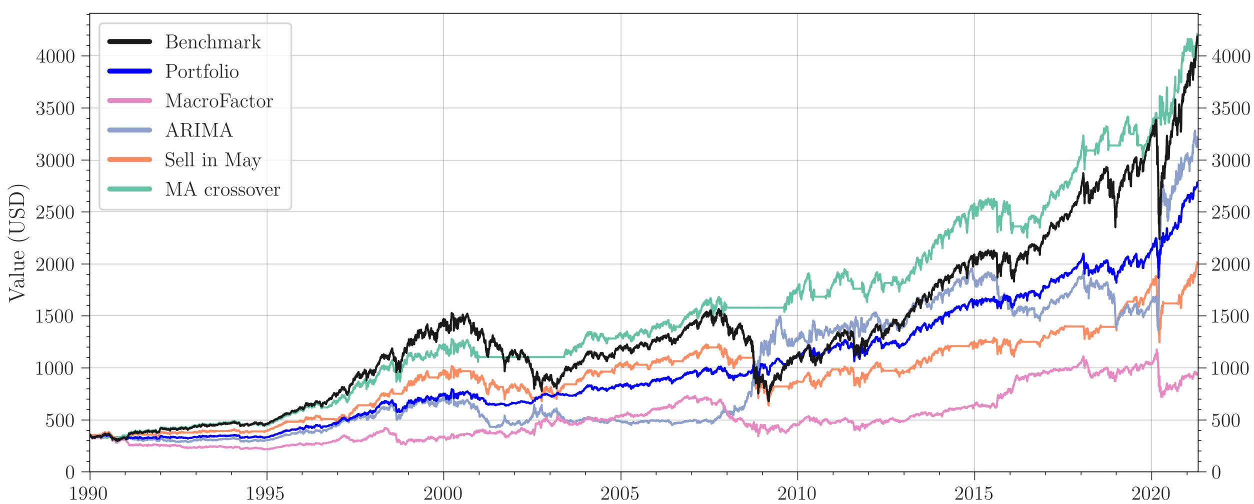
From the analysis of results of all individual strategies, we can expect stable results of the portfolio, as each of the strategies is better during different periods, as seen in Figure 10. The result from the combination of signals from all strategies is shown in Figure 11, and performance metrics are included in Table 7. We can see that it performs better than the Buy & Hold strategy in terms of risk-adjusted returns, with an IR* of 0.61, versus 0.45 of Buy & Hold. It is significant to notice that risk metrics of the portfolio are much better than the ones from Buy & Hold, and most individual strategies independently.

Having such risk metrics allows us to add leverage to the portfolio and have a similar level of risk to the Buy & Hold strategy. Figure 12 shows the capital of the investment in the portfolio of strategies with a leverage of 200%. Its performance metrics are also included in Table 7. This leverage causes the portfolio to end on the level of 14819, while Buy & Hold reaches 4183, with a starting capital of 353. The absolute return of the leveraged portfolio is 4193 %, and the one from Buy & Hold is 1183 %, which is almost 4 times less. It is also important to notice that several risk metrics are still better than in Buy & Hold strategy even with a leverage of 200% for our ensemble model.

Figure 10: Portfolio of strategies compared to S&P 500

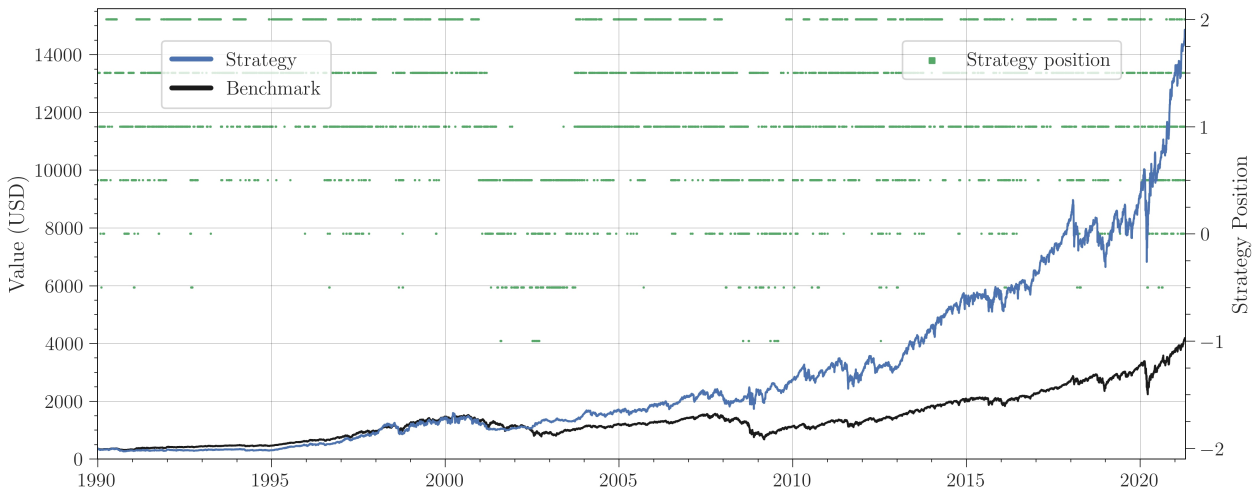
Note: Strategy represents the equally weighted combination of signals from the four algorithmic strategies described before.

Figure 11: Individual strategies and portfolio compared to S&P 500



Note: Equity lines of the portfolio of strategies and each of the strategies and Buy & Hold applied to S&P 500.

Figure 12: Portfolio of strategies with 200% leverage compared to S&P 500



Note: Strategy represents the equally weighted combination of signals, multiplied by 2, from the four algorithmic strategies described before.

Table 7: Performance metrics of the portfolio of strategies on S&P 500

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	8.21	0.45	18.19	56.78	14.04	7.17	0.07	10.4	0.14	0.59	1	0
Portfolio Lev. x1	6.81	0.61	**11.13**	20.01	8.08	4.71	0.21	0.85	0.34	0.84	1124	465
Portfolio Lev. x2	**12.68**	0.57	22.27	37.92	15.74	4.89	0.19	6.5	0.33	0.81	2248	465
MA crossover	8.16	**0.68**	12.06	**19.34**	**8.07**	**3.76**	**0.29**	**0.71**	**0.42**	**1.01**	172	2059
Sell in May	5.72	0.36	15.71	48.12	12.25	7.11	0.04	6.58	0.12	0.47	65	2371
ARIMA	7.31	0.42	17.57	42.69	13.66	8.37	0.07	8.57	0.17	0.54	3555	653
MacroFactor	3.2	0.19	17.12	53.34	14.59	8.77	0.01	11.69	0.06	0.22	1093	980

Note: Performance metrics of the portfolio of investment strategies, with and without leverage of 200%, all individual strategies, and Buy & Hold strategy.

5 Sensitivity Analysis

This section performs an analysis to test the robustness of all strategies that belong to the portfolio of algorithmic strategies. Each strategy was optimised with default parameters such as the IS and OOS window lengths. If strategies are robust, changing such parameters should not have a big impact on the strategies´ performance.

The sensitivity analysis consists of changing each of the parameters one by one. Generally, each parameter was doubled and divided by 2, while others remained constant on each test.

Additionally, the list of parameters tested on each IS was either shortened or lengthened.

Furthermore, this work analyzes the performance of the portfolio built from the algorithmic strategies on the NASDAQ Composite, which is highly correlated to the S&P 500.

5.1 Individual strategies

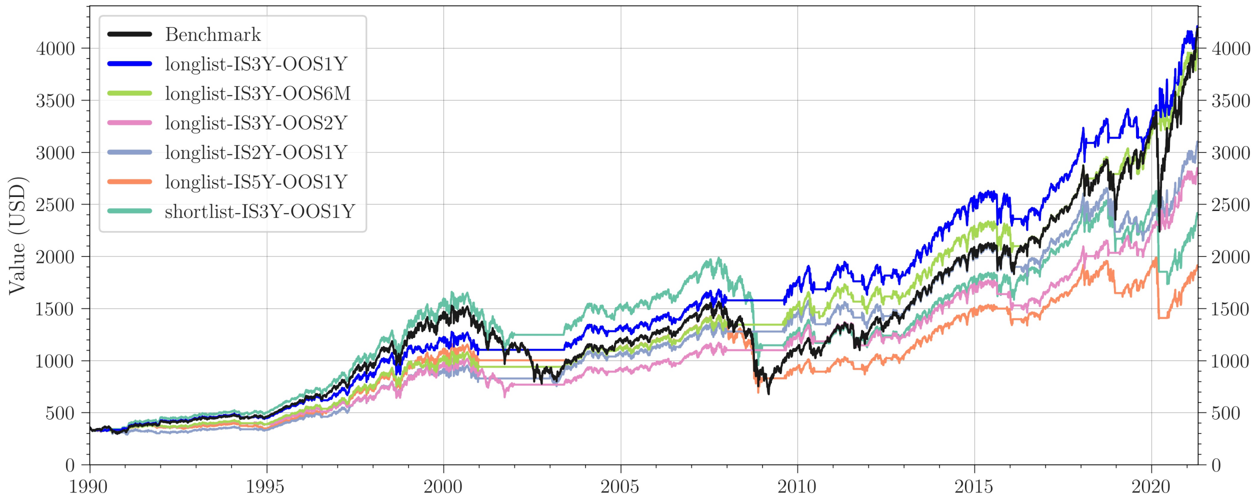
5.1.1 Moving average crossover

The original parameters, along with the newly tested combinations of parameters for the WFO of this strategy were:

1. List of parameters to be optimised on each IS window:
   - Original list (longlist) of parameters to be optimised on each IS window:
     - Fast MA: 1, 3, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65.
     - Slow MA: 5, 10, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300.
   - Newly tested list (shortlist) of parameters to be optimised on each IS window:
     - Fast MA: 5, 10, 20, 30, 40, 50
     - Slow MA: 50, 100, 150, 200, 250, 300
2. Length of IS window: it is changed from three years (IS3Y) to two years (IS2Y) and five years (IS5Y).
3. Length of OOS window: it is changed from one year (OOS1Y) to six months (OOS6M) and two years (OOS2Y).

Figure 7 and Table 8 show the performance of the strategy after changing the default parameters. It is visible that the strategy does not perform well after reducing the number of possible MA combinations. Additionally, changing the IS and OOS window lengths worsens risk-adjusted returns. However, by analyzing the equity lines, we can see how all of them enter long positions at similar moments after drawdowns. Differences in performance are mainly caused by the moment that they exit such buy positions when starting drawdowns.

Figure 13: Sensitivity analysis of MA crossover strategy



Note: Equity lines of the strategies derived from changing default parameters of the MA crossover strategy (long list of fast MA and slow MA combinations, 3 years of IS window, 1 year of OOS window).

Table 8: Performance metrics from the sensitivity analysis of MA crossover strategy

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	0.45	18.19	56.78	14.04	7.17	0.07	10.4	0.14	0.59	1	0
longlist IS3Y OOS1Y	8.16	**0.68**	**12.06**	**19.34**	**8.07**	**3.76**	**0.29**	**0.71**	**0.42**	**1.01**	172	2059
longlist IS3Y OOS6M	7.98	0.66	12.19	**19.34**	8.19	**3.76**	0.27	0.72	0.41	0.98	134	1962
longlist IS3Y OOS2Y	6.82	0.53	12.75	36.77	9.17	6.55	0.1	2.81	0.19	0.74	116	1879
longlist IS2Y OOS1Y	7.18	0.57	12.71	21.37	9.17	4.02	0.19	1	0.34	0.78	103	1776
longlist IS5Y OOS1Y	5.54	0.37	14.86	51.9	10.39	6.87	0.04	5.51	0.11	0.53	71	1709
shortlist IS3Y OOS1Y	6.33	0.4	15.69	51.93	11.77	9.3	0.05	8.91	0.12	0.54	29	968

Note: Performance metrics of the strategies derived from changing default parameters of the MA crossover strategy (long list of fast and slow MA combinations, 3 years of IS window, 1 year of OOS window).

5.1.2 Sell in May and go away

The original parameters along with the newly tested combinations of parameters for the WFO of this strategy were:

1. List of parameters to be optimised on each IS window:

   - Original list (shortlist) of parameters to be optimised on each IS window:

  - Selling month: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
  - Selling duration: 1, 2, 3, 4, 5, 6.
- Newly tested list (longlist) of parameters to be optimised on each IS window:
  - Selling month: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
  - Selling duration: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

2. Length of IS window: it is changed from ten years (IS10Y) to five years (IS5Y) and twenty years (IS20Y).

3. Length of OOS window: it is changed from one year (OOS1Y) to six months (OOS6M) and two years (OOS2Y).

Figure 14 and Table 9 show the performance of the strategy after changing the default parameters. It is visible that the strategy has very consistent risk-adjusted returns when changing each of the parameters. The combination which has the best performance uses the longest IS period (20 years), which may suggest that, if any seasonality exists, it needs long training windows.

Figure 14: Sensitivity analysis of Sell in May and go away strategy

Note: Equity lines of the strategies derived from changing default parameters of the *Sell in May and go away* strategy (short list of selling duration parameter, 10 years of IS window, 1 year of OOS window).

Table 9: Performance metrics from the sensitivity analysis of Sell in May and go away strategy

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	14.04	7.17	**0.07**	10.4	**0.14**	**0.59**	1	0
shortlist IS10Y OOS1Y	5.72	0.36	15.71	48.12	12.25	7.11	0.04	6.58	0.12	0.47	65	2371
shortlist IS10Y OOS6M	5.73	0.38	15.15	48.12	11.68	11.25	0.05	9.58	0.12	0.49	73	2671
shortlist IS10Y OOS2Y	5.52	0.36	15.37	48.12	11.71	**6.09**	0.04	**5.28**	0.11	0.47	65	2597
shortlist IS10Y OOS3Y	4.69	0.33	14.41	48.12	11.07	7.12	0.03	5.46	0.1	0.42	62	2601
shortlist IS5Y OOS1Y	5.19	0.33	15.92	54.08	12.48	17.8	0.03	19.13	0.1	0.42	69	2640
shortlist IS20Y OOS1Y	3.7	0.25	14.64	53.23	11.94	8.76	0.02	8.15	0.07	0.31	63	3132
longlist IS10Y OOS1Y	4.86	0.36	**13.35**	**39.5**	**9.5**	11.32	0.04	5.67	0.12	0.51	67	3978

Note: Performance metrics of the strategies derived from changing default parameters of the *Sell in May and go away* strategy (short list of selling duration parameter, 10 years of IS window, 1 year of OOS window).

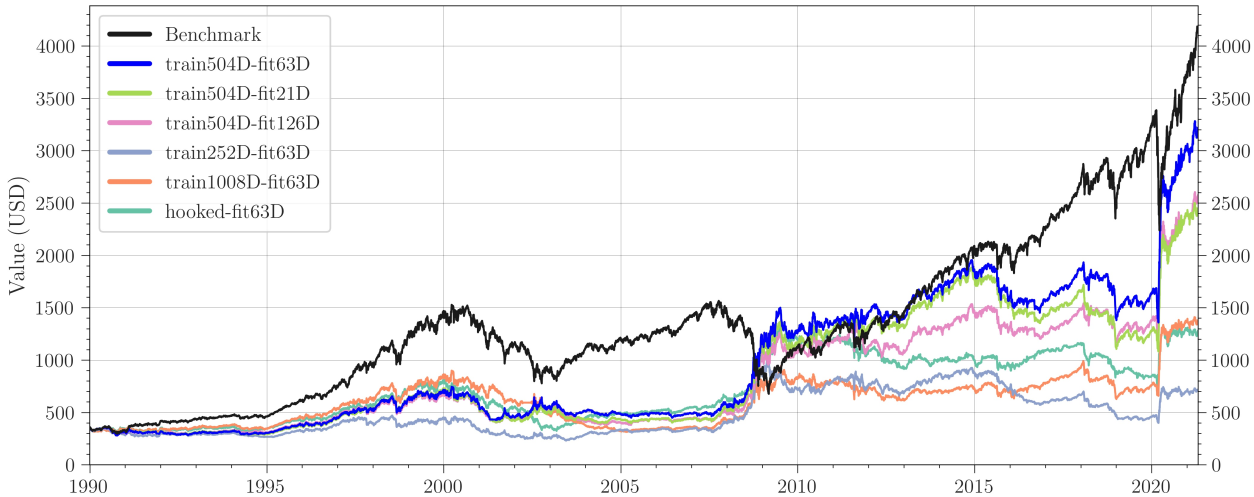
5.1.3 ARIMA

The original parameters along with the newly tested combinations of parameters for this strategy were:

1. Training window: it is changed from two years (train2Y) to one year (train1Y), four years (train504D), and hooked to the beginning of the data (hooked).
2. Frequency of fitting the model: it is changed from three months (fit63D) to one month (fit21D), and six months (fit126D).

Figure 15 and Table 10 show the performance of the strategy after changing the default parameters. It is visible that the strategy obtains worse results after changing the training window length. However, it shows robustness in the parameter for the frequency of fitting the model. This may be caused because the algorithm performs several additional iterations to update the model parameters every day. However, the performance of the strategies is very similar during periods of the high volatility of the market, which adds value to the overall portfolio.

Figure 15: Sensitivity analysis of ARIMA strategy



Note: Equity lines of the strategies derived from changing default parameters of ARIMA strategy (training window of 504 days, fitting frequency of 63 days).

Table 10: Performance metrics from the sensitivity analysis ARIMA strategy

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	14.04	**7.17**	**0.07**	10.4	0.14	**0.59**	1	0
train504D fit63D	7.31	0.42	17.57	**42.69**	13.66	8.37	**0.07**	**8.57**	**0.17**	0.54	3555	653
hooked fit63D	4.2	0.25	**16.98**	60.3	**13.49**	11.78	0.02	16.27	0.07	0.31	4871	1312
train1008D fit63D	4.45	0.25	17.5	65.19	14.35	9.28	0.02	15.18	0.07	0.31	3627	818
train252D fit63D	2.28	0.13	17.5	61.71	15.47	12.12	0	20.25	0.04	0.15	4065	779
train504D fit126D	6.55	0.37	17.61	46.91	13.67	8.48	0.05	9.57	0.14	0.48	3709	647
train504D fit21D	6.38	0.36	17.52	45.99	13.84	8.7	0.05	9.7	0.14	0.46	3487	667

Note: Performance metrics of the strategies derived from changing default parameters of ARIMA strategy (training window of 504 days, fitting frequency of 63 days).

5.1.4 Macro-economic factor

The original parameters along with the newly tested combinations of parameters for the WFO of this strategy were:

1. List of parameters to be optimised on each IS window:

   - Original list (shortlist) of parameters to be optimised on each IS window:
     - Buying quantile: 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9.

  - Selling quantile: 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95.
- Newly tested list (longlist) of parameters to be optimised on each IS window:
  - Buying quantile: 0.1, 0.2, 0.3, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9.
  - Selling quantile: 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1.

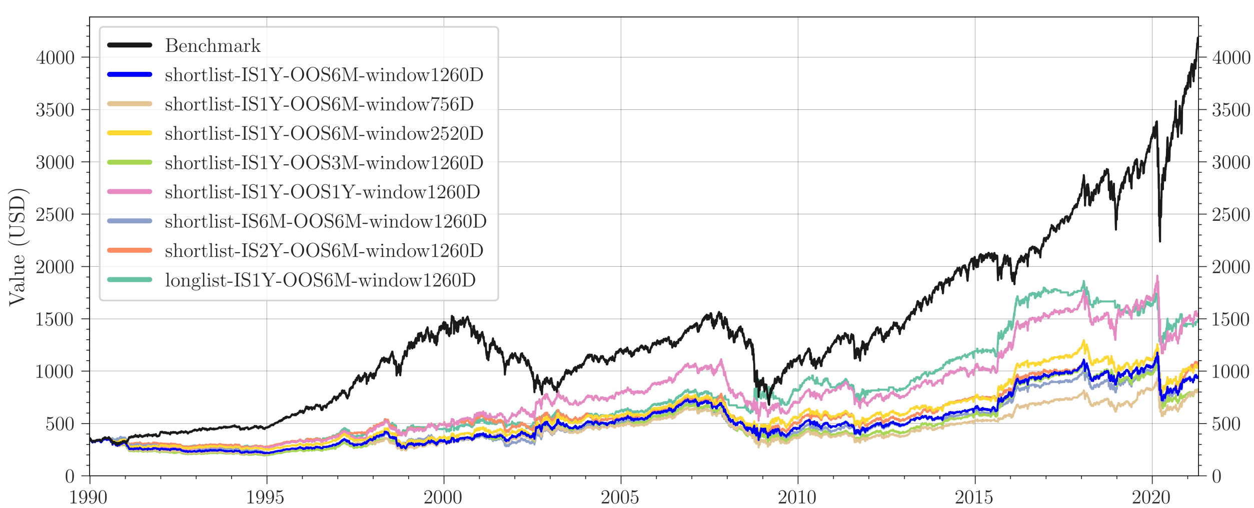
2. Length of IS window: it is changed from one year (IS1Y) to six months (IS6M) and two years (IS2Y).

3. Length of OOS window: it is changed from six months (OOS6M) to three months (OOS3M) and one year (OOS1Y).

4. The length of the rolling window to calculate the distribution of differences in ICSA: it is changed from five years (1260D) to three years (756D) and ten years (2520D).

Figure 16 and Table 11 show the performance of the strategies after changing the default parameters. It is visible that the strategy is robust to changes in default parameters. Risk-adjusted returns and risk metrics are very similar in all variations to the original strategy.

Figure 16: Sensitivity analysis of Macro-economic factor strategy



Note: Equity lines of the strategies derived from changing default parameters of the MacroFactor strategy (short list of buying and selling quantiles, 1 year of IS window, 6 months of OOS window, 1260 days of rolling window to calculate the distribution).

Table 11: Performance metrics from the sensitivity analysis of Macro-economic factor strategy

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	14.04	7.17	**0.07**	10.4	0.14	**0.59**	1	0
shortlist IS1Y OOS6M window1260D	3.2	0.19	17.12	53.34	14.59	8.77	0.01	11.69	0.06	0.22	1093	980
shortlist IS1Y OOS6M window756D	2.71	0.16	16.98	58.55	14.54	9.25	0.01	13.37	0.05	0.19	1061	1080
shortlist IS1Y OOS6M window2520D	3.52	0.21	17.11	52.39	14.19	8.75	0.01	11.13	0.07	0.25	1135	1120
shortlist IS1Y OOS3M window1260D	2.75	0.16	17.05	56.54	14.68	8.86	0.01	12.54	0.05	0.19	1183	1044
shortlist IS1Y OOS1Y window1260D	4.83	0.28	17.2	52.74	14.5	7.82	0.03	10.28	0.09	0.33	1043	803
shortlist IS6M OOS6M window1260D	3.2	0.18	17.34	55.93	14.71	8.64	0.01	12.33	0.06	0.22	1157	838
shortlist IS2Y OOS6M window1260D	3.64	0.21	16.98	53.21	13.87	7.95	0.01	9.97	0.07	0.26	1025	1089
longlist IS1Y OOS6M window1260D	4.7	0.32	**14.5**	**31.73**	**11.19**	**5.96**	0.05	**3.07**	**0.15**	0.42	925	2906

Note: Performance metrics of the strategies derived from changing default parameters of the MacroFactor strategy (short list of buying and selling quantiles, 1 year of IS window, 6 months of OOS window, 1260 days of rolling window to calculate the distribution).

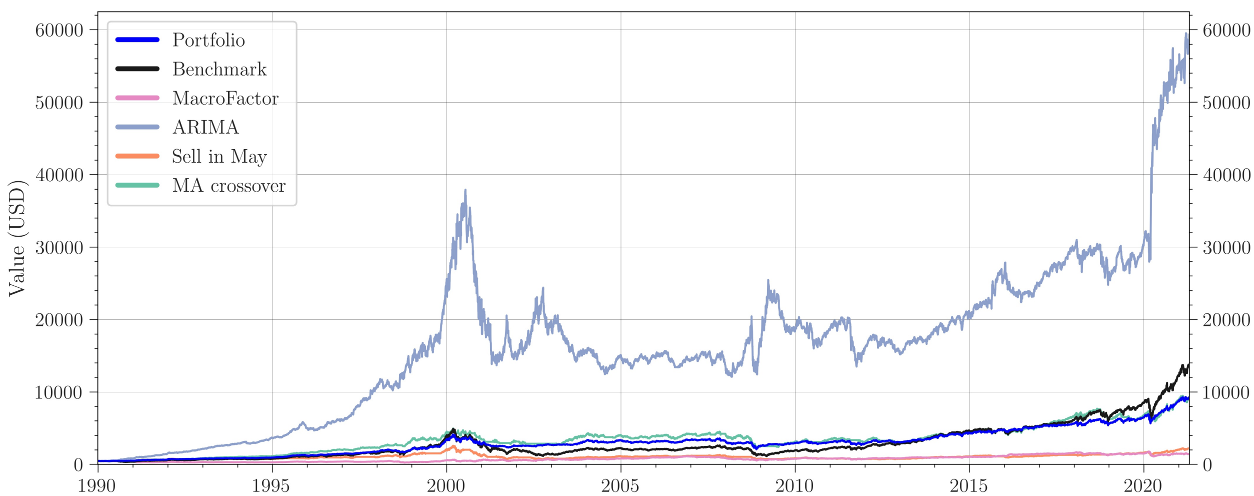
5.2 Underlying instrument - NASDAQ Composite

This work also proposes to test the robustness of the overall model of algorithmic strategies by applying them to another American stock market index, Nasdaq Composite (IXIC). Since IXIC is highly correlated to S&P 500, and the macro-economic strategy uses macro-economic data from the same country, the portfolio should also improve the Buy & Hold strategy performance in terms of risk-adjusted returns.

Figure 17 and Table 12 show the performance of the portfolio of strategies applied to the Nasdaq Composite. As in the S&P 500 index, we obtain more stable results than the benchmark. We can see that it performs better than the Buy & Hold strategy in terms of risk-adjusted returns, with an IR* of 0.7, versus 0.50 for Buy & Hold. It is also significant to notice how all risk metrics from the portfolio are much better than from Buy & Hold. This proves the robustness of the portfolio of investment strategies for obtaining considerably better results than Buy & Hold on two different American stock market indices.

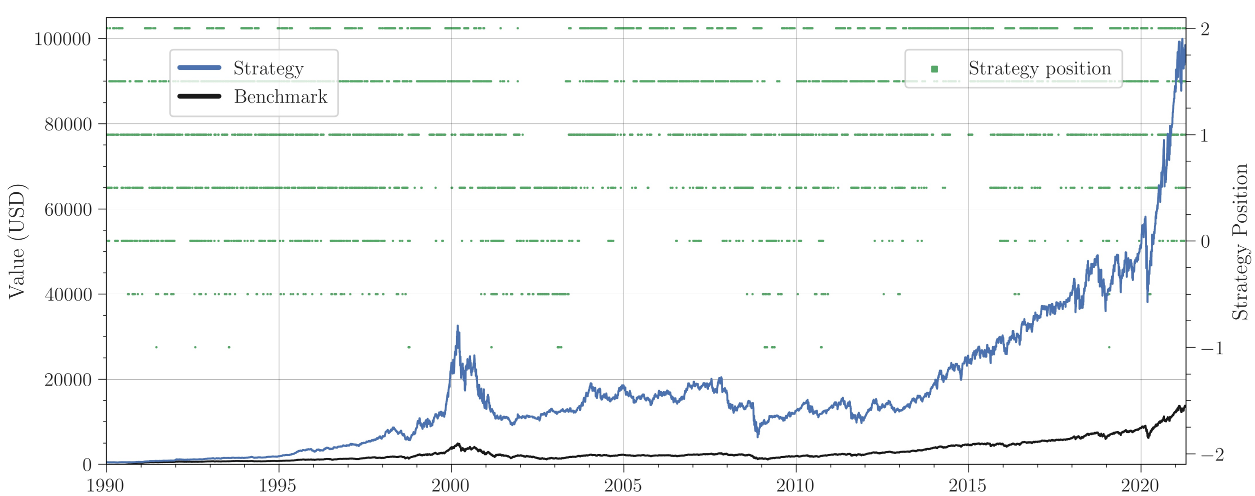
Similarly to the case of S&P 500, having such risk metrics allows us to add leverage to the portfolio and have a similar level of risk to the Buy & Hold strategy. Figure 18 shows the capital of the investment in the portfolio of strategies on Nasdaq Composite with a leverage of 200%. Its performance metrics are also included in Table 12. This leverage causes the portfolio to end at the level of 97355, while Buy & Hold reaches the level of 14016, with a starting level of 455. The absolute return of the leveraged portfolio is 21406 %, and the one from Buy & Hold is 3081 %, which is around seven times less. It is also important to notice that several risk metrics are similar than in Buy & Hold strategy even with a leverage of 200%.

Figure 17: Portfolio of strategies on Nasdaq Composite index



Note: Equity lines of the portfolio of strategies and each of the strategies and Buy & Hold applied to Nasdaq Composite index.

Figure 18: Portfolio of strategies with 200% leverage on Nasdaq Composite index



Note: Equity lines of the portfolio of strategies with leverage of 200% and Buy & Hold applied to Nasdaq Composite index.

Table 12: Performance metrics of the portfolio of strategies on Nasdaq Composite

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	11.47	0.49	23.57	77.34	19.02	15.09	0.07	52.32	0.15	0.6	1	0
Portfolio Lev. x1	10.07	0.7	**14.29**	**49.93**	**10.83**	14.23	0.14	**10.99**	0.2	0.93	1455	646
Portfolio Lev. x2	**18.7**	0.65	28.57	80.61	20.42	16.35	0.15	76.87	**0.23**	0.92	2910	646
MA crossover	9.9	0.59	16.65	61.23	12.7	16.35	0.1	21.17	0.16	0.78	587	2247
Sell in May	5.43	0.27	19.96	75.73	16.39	21.11	0.02	52.3	0.07	0.33	67	2636
ARIMA	16.62	**0.73**	22.83	71.05	16.85	19.67	**0.17**	53.78	**0.23**	**0.99**	4501	523
MacroFactor	4.08	0.19	21.55	53.08	19.4	**9.89**	0.01	21.96	0.08	0.21	1065	1388

Note: Performance metrics of the portfolio of strategies and Buy & Hold applied to Nasdaq Composite index.

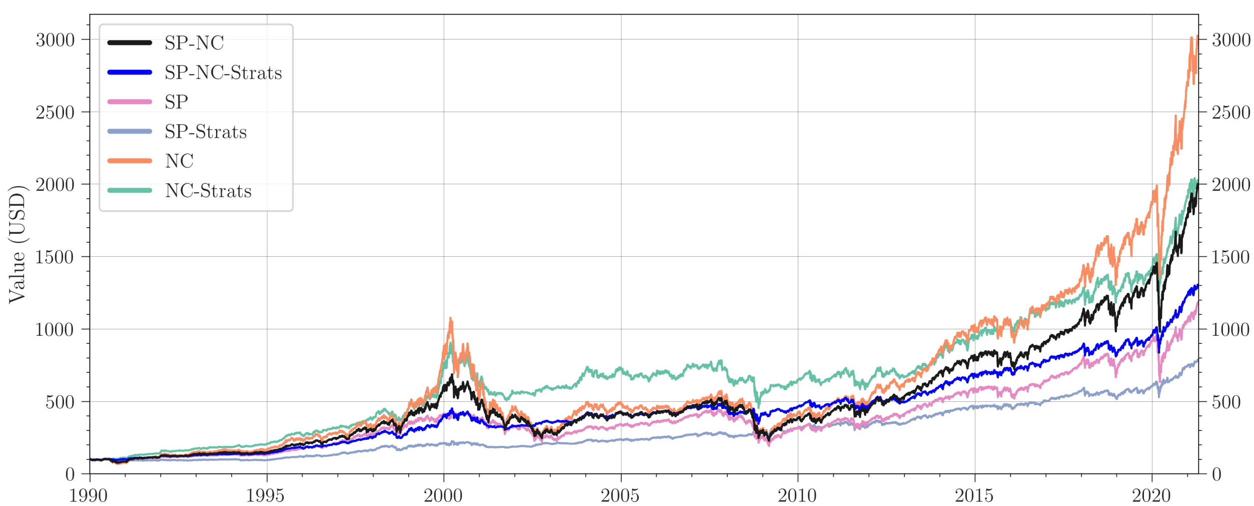
5.3 Portfolio with ensemble model applied to S&P 500 and Nasdaq indices

To conclude this work, we constructed an ensemble model that consists of the application of the algorithmic strategies on both American indices, S&P 500 and Nasdaq Composite. This ensemble model will be based on individual strategies applied together to two assets. Each model of strategies is applied individually to each index producing an investment position on each asset every day. The portfolio is rebalanced every day, and each model has a weight of 50% on each index. Such an ensemble model produces one extra layer of diversification, intending to increase risk-adjusted returns and reduce the risk.

Figure 19 and Table 13 show the performance of the portfolio of strategies applied to both indices (SP-NC-Strats). It also shows the performance of a portfolio built from the S&P 500 and Nasdaq Composite, with daily rebalancing and a weight of 50% to each index (SP-NC). Additionally, it shows the equity line of investment on S&P 500 (SP), Nasdaq Composite (NC), and the portfolio of strategies applied to each index individually (SP-Strats and NC-Strats). All equity lines have an initial level of 100. As in the previous sections, it is visible that we obtain better results than the benchmark (SP-NC) in terms of risk-adjusted returns (IR* of 0.71 versus 0.5), with a much lower risk level (All risk of 2.21 versus 30.15). Every risk measure is half the value of the benchmark.

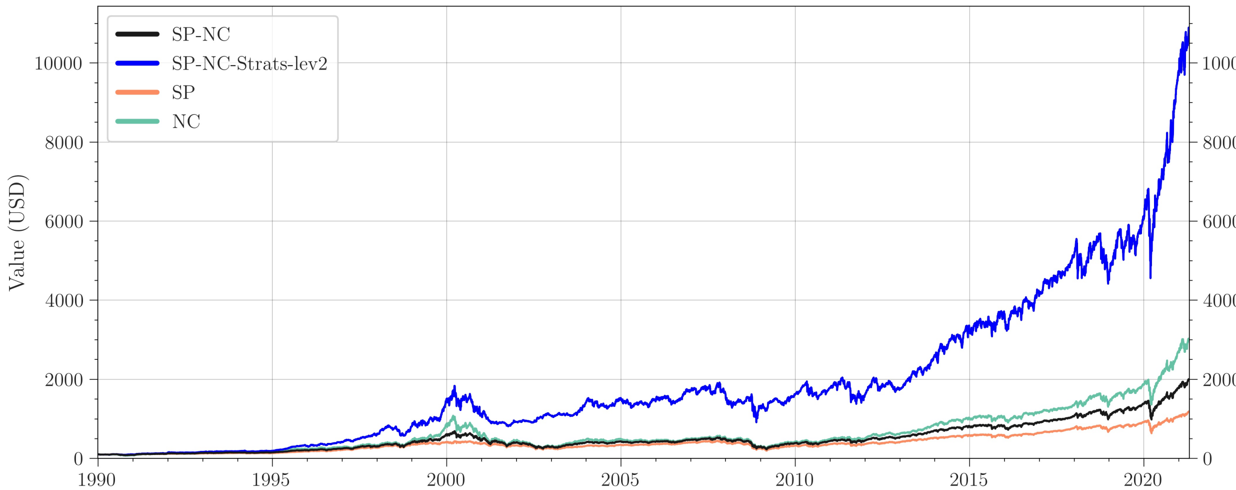
Having such risk metrics allows us to add leverage to the portfolio and have a similar level of risk to the Buy & Hold strategy with daily rebalancing. Figure 20 and Table 13 show the performance of the ensemble model of the portfolio of strategies applied to both indices, with a leverage of 200% (SP-NC-Strats-lev2), in comparison to the Buy & Hold strategy on the underlying assets (SP-NC, SP, NC). All equity lines have an initial level of 100. We can obtain much higher returns than the underlying assets (ARC of 16.14 versus 10.03, 8.21, and 11.47) with a similar or lower level of risk (All risk of 16.51 versus 30.15, 10.40, and 52.32). The ensemble model with leverage finishes at the level of 10816, while the portfolio of S&P 500 and Nasdaq Composite finishes at the level of 1990. Buy & Hold on S&P 500 finishes at the level of 1183, and Buy & Hold on Nasdaq Composite reaches the level of 2993. It is also important to notice that several risk metrics are still better than in Buy & Hold strategy on the underlying assets, even with a leverage of 200%.

Figure 19: Ensemble model on S&P 500 and Nasdaq Composite indices



Note: Equity lines of the portfolio of strategies applied to both indices (SP-NC-Strats), and each underlying asset individually (SP-Strats, and NC-Strats). SP represents Buy & Hold on S&P 500 index, NC represents Buy & Hold on Nasdaq Composite index, and SP-NC represents Buy & Hold on both indices with equal weights and daily rebalancing.

Figure 20: Ensemble model with 200% leverage on S&P 500 and Nasdaq Composite indices



Note: Equity lines of the portfolio of strategies with leverage of 200% applied to both indices (SP-NC-Strats-lev2), and Buy & Hold strategy applied to underlying asset individually (SP, NC), and on both indices with equal weights and daily rebalancing (SP-NC).

Table 13: Performance metrics of the ensemble model of the portfolio of strategies applied to both indices

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD
SP NC	10.03	0.5	20.19	66.98	16.24	13.73	0.07	30.15	0.15	0.62
SP NC Strats Lev. x1	8.54	**0.71**	11.96	31.8	8.78	6.62	0.19	2.21	0.27	**0.97**
SP NC Strats Lev. x2	**16.14**	0.67	23.92	55.86	16.91	7.31	0.19	16.51	0.29	0.95
SP	8.21	0.45	18.19	56.78	14.04	7.17	0.07	10.4	0.14	0.59
SP Strats	6.81	0.61	**11.13**	**20.01**	**8.08**	**4.71**	**0.21**	**0.85**	**0.34**	0.84
NC	11.47	0.49	23.57	77.34	19.02	15.09	0.07	52.32	0.15	0.6
NC Strats	10.07	0.7	14.29	49.93	10.83	14.23	0.14	10.99	0.2	0.93

Note: Performance metrics of the portfolio of strategies applied to both indices, without and with leverage (SP-NC-Strats and SP-NC-Strat-lev2), and each underlying asset individually (SP-Strats, and NC-Strats). SP represents Buy & Hold on S&P 500 index, NC represents Buy & Hold on Nasdaq Composite index, and SP-NC represents Buy & Hold on both indices with equal weights and daily rebalancing.

Conclusions

This work consists of the creation and testing of a portfolio of algorithmic strategies on the S&P 500 index. Such strategies are built on different assumptions and their aim is to perform well in different market conditions. This work explains in detail the process of creating, optimising, and performing a sensitivity analysis on each strategy individually. The final step of this work is to combine the signals generated from all strategies and produce a signal to trade on S&P 500 index. Additionally, the robustness of the portfolio of strategies is confirmed by applying it to the Nasdaq Composite index. The data period used in this research goes from 1980-01-01 to 2021-04-23. However the out-of-sample period starts on 1990-01-01.

Special attention is given to the optimisation process, which is based on a Walk-Forward procedure, which contains an algorithm that chooses the most robust combination of parameters in terms of risk-adjusted returns from the in-sample period, instead of simply picking the best performing one. This most robust combination of parameters is the one used for the out-of-sample period, and this process is performed on a rolling window basis.

The first hypothesis of the research was the Efficient Market Hypothesis, which was rejected as it was possible to obtain much higher risk-adjusted returns on a portfolio of strategies (IR* of 0.61) than for Buy & Hold (IR* of 0.45) on S&P 500 index. By applying leverage of 200% to our portfolio, it was possible to obtain an absolute return four times larger than the S&P 500 index during 31 years period with a similar level of risk than the benchmark. A very similar result was obtained by applying the same strategies on the Nasdaq Composite index (IR* of 0.70 vs IR* of 0.49 from Buy & Hold), even though all research had been done previously on S&P 500 index. By applying leverage of 200% it was possible to obtain an absolute return around 7 times larger than the Buy & Hold strategy.

The second hypothesis was whether it is possible to obtain better risk-adjusted returns by combining signals from several investment strategies, than in each of them individually. This hypothesis was also rejected. However, the average IR* from the strategies applied to S&P 500 index is 0.41, and the portfolio delivered an IR* of 0.61. Only one strategy has slightly larger risk-adjusted returns than the portfolio of strategies, which was MA crossover (IR* of 0.68). When applying the strategies to Nasdaq Composite a very similar result was obtained. The average IR* from the strategies is 0.44, and the portfolio delivered an IR* of 0.70. Again, only one strategy has slightly larger risk-adjusted returns than the portfolio of strategies, which was ARIMA (IR* of 0.73).

There are several possibilities in which this work can be extended. The first one is to create and develop more strategies on the same or on a different uncorrelated asset. This would allow the portfolio to be more diversified and achieve more consistent returns accross time, and higher risk-adjusted returns. The second option would be to allocate weights on a rolling window basis and assign them based on their most recent risk-adjusted returns. Another possibility would be to allocate the not invested capital on treasury bonds from the United States. This would allow the investor to obtain profits at the risk-free rate when the portfolio of strategies chooses not to be invested in the main asset, S&P 500.

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