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THE SYSTEMIC RISK APPROACH BASED ON IMPLIED AND REALIZED VOLATILITY

PAWEŁ SAKOWSKI
RAFAŁ SIERADZKI
ROBERT ŚLEPACZUK

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The systemic risk approach based on implied and realized volatility

*Paweł Sakowski[a], Rafał Sieradzki[b], Robert Ślepaczuk[a]**

[a] University of Warsaw, Faculty of Economic Sciences, Department of Quantitative Finance, Quantitative Finance Research Group
[b] New York University Stern School of Business; Cracow University of Economics
Corresponding author: rslepaczuk@wne.uw.edu.pl

Abstract: We propose a new measure of systemic risk to analyze the impact of the major financial market turmoils in the stock markets from 2000 to 2021 in the USA, Europe, Brazil, and Japan. Our Implied Volatility Realized Volatility Systemic Risk Indicator (IVRVSRI) shows that the reaction of stock markets varies across different geographical locations and the persistence of the shocks depends on the historical volatility and long-term average volatility level in a given market. The methodology applied is based on the logic "the simpler is always better than the more complex, if it leads to the same results". Such an approach significantly limits the model risk and substantially decreases computational burden. Robustness checks show that IVRVSRI is a precise measure of the current systemic risk in the stock markets. Moreover, IVRVSRI seems to be a valid indication of current systemic risk in equity markets and it can be used for other types of assets and high-frequency data.

Keywords: systemic risk, implied volatility, realized volatility, volatility indices, equity index options, market volatility

JEL codes: G14, G15, C61, C22

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1. Introduction

The magnitude and the speed of the contagion of the financial market turmoils is the main point of interest in numerous studies. This topic is of special importance because the reactions of the financial markets to any existing or forthcoming crisis are fast, and it is hard to identify them on time based on the real economic measures, as they are announced with a delay. The main aim of this paper is to analyze and compare the systemic impact of the major financial market turmoils in the equity markets in the USA, Europe, Brazil, and Japan from 2000 to 2021. For this purpose, we construct an indicator based on implied and realized volatility measures (IV and RV, respectively) for each market, which are easily available to all market participants. Moreover, we construct a general indicator at the worldwide level. Our partial motivation to undertake this study is to show that such Systemic Risk Indicators can be constructed from simple metrics, and there is no need to use any sophisticated risk models for this purpose (Caporin et al. [12]). In other words, we want to show that the model risk can be significantly reduced while the results are similar to the ones obtained by the use of much more sophisticated tools. We set three research hypotheses:

- RH1: *It is possible to construct a robust Systemic Risk Indicator based on the well-known concepts of realized and implied volatility measures.*
- RH2: *The indication of the proposed Systemic Risk Indicator depends on the geographical location of a given equity market.*
- RH3: *The robustness of the proposed Systemic Risk Indicator depends on various parameters selected: the memory parameter for RV, time to expiration for IV, the percentile selected for the risk map, the length of the history selected for the calculation of percentile in case of risk map.*

The latter aspect is particularly important, as in many studies researchers do not consider extent to which the initial parameters of the model affect the final results, especially those regarding the speed of reaction to unexpected market turmoils. We check the sensitivity of the proposed Systemic Risk Indicator to the change of the selected parameters like: the memory parameter for the realized volatility (RV), time to expiration for the implied volatility (IV), the percentile selected for the risk map, and the length of the history selected for the calculation of percentile in case of risk map.

The structure of this paper is as follows. The second section presents a literature review. The third section describes Data and Methodology. The fourth section presents the Results, and the fifth one includes Conclusions.

2. Literature review and classification the selected systemic risk indicators

2.1. Literature review

The major approach in the literature to measure systemic risk is based either on market data or a mix of market and balance sheet data. Those combined risk indicators use i.a. such metrics as VaR and CoVaR. The results obtained for one country, market segment, or economic sector are aggregated to get a general measure of risk. In general, various methods yield almost the same results as in Engle and Ruan [18], Brownlees and Engle [10], Acharya et al. [2], Bisias et al. [7] or Caporin et al. [12].

One of the first attempts focusing on systemic risk was Brimmer [9] who reminded the last resort lending function of the central bank, which has digressed from its overall strategy of monetary control to also undertake a tactical rescue of individual banks and segments of the financial market. De Bandt and Hartmann [15] developed a broad concept of systemic risk, the basic economic concept for the understanding of financial crises. They claimed that any such concept must integrate systemic events in banking and financial markets as well as in the related payment and settlement systems. At the heart of systemic risk are contagion effects, and various forms of external effects. The concept also includes simultaneous financial instabilities following aggregate shocks. They surveyed the quantitative literature on systemic risk, which was evolving swiftly in the last couple of years.

Eisenberg and Noe [17] considered a default by firms that were part of a single clearing mechanism. The obligations of all firms within the system are determined simultaneously in a fashion consistent with the priority of debt claims and the limited liability of equity. They first show, via a fixed-point argument, that there always exists a "clearing payment vector" that clears the obligations of the members of the clearing system and that under mild regularity conditions, this clearing vector is unique. Next, they develop an algorithm that clears the financial system in a computationally efficient fashion and provides information on the systemic risk faced by individual firms. Finally, they produce qualitative comparative statics for different financial systems. These comparative statics imply that in contrast to single-firm results, even unsystematic, non-dissipative shocks to the system will lower the total value of the system and may lower the value of the equity of some of the individual system firms.

Bisias et al. [7] point out that systemic risk is a multifaceted problem in an ever-changing financial environment, any single definition is likely to fall short and may create a false sense of security as financial markets evolve in ways that escape the scrutiny of any one-dimensional perspective. They provide an overview of over 30 measures of systemic risk in the economics and finance literature, chosen to address key issues in measuring systemic risk and its management. The measures are grouped into six various categories including: macroeconomic measures, granular foundations and network measures, forward-looking risk measures, stress-test measures, cross-sectional measures, measures of illiquidity and insolvency. They analyze these measures from the supervisory, research, and data perspectives, and present concise definitions of each risk measure. At the same time, they point out that the system to be measured is highly complex, and the measures considered were largely untested outside the GFC crisis. Indeed, some of the conceptual frameworks that they reviewed were still in their infancy and had yet to be applied.

Schwarcz [26] agreed that governments and international organizations worried increasingly about systemic risk, under which the world's financial system could have collapsed like a row of dominoes. There is widespread confusion, though, about the causes and even the definition of systemic risk, and uncertainty about how to control it. His paper offers a conceptual framework for examining what risks are truly "systemic," what causes those risks, and how, if at all, those risks should be regulated. Scholars historically have tended to think of systemic risk primarily in terms of financial institutions such as banks. However, with the growth of disintermediation, in which companies can access capital-market funding without going through banks or other intermediary institutions, greater focus should be devoted to financial markets and the relationship between markets and institutions. This perspective reveals that systemic risk results from a type of tragedy of the commons in which market participants lack sufficient incentives, and absence of the regulation to limit risk-taking in order to reduce the systemic danger to others.

In this light, Acharya [1] models systemic risk is modeled as the endogenously chosen correlation of returns on assets held by banks. The limited liability of banks and the presence of a negative externality of one bank's failure on the health of other banks give rise to a systemic risk-shifting incentive where all banks undertake correlated investments, thereby increasing economy-wide aggregate risk. Regulatory mechanisms such as bank closure policy and capital adequacy requirements that are commonly based only on a bank's own risk fail to mitigate aggregate risk-shifting incentives, and can, in fact, accentuate systemic risk. Prudential regulation is shown to operate at a collective level, regulating each bank as a function of both its joint (correlated) risk with other banks as well as its individual (bank-specific) risk.

Brownlees and Engle [10] introduce SRISK to measure the systemic risk contribution of a financial firm. SRISK measures the capital shortfall of a firm conditional on a severe market decline and is a function of its size, leverage and risk. They use the measure to study top financial institutions in the recent financial crisis. SRISK delivers useful rankings of systemic institutions at various stages of the crisis and identifies Fannie Mae, Freddie Mac, Morgan Stanley, Bear Stearns, and Lehman Brothers as top contributors as early as 2005-Q1. Moreover, aggregate SRISK provides early warning signals of distress in indicators of real activity.

The CoVaR method proposed by Adrian and Brunnermeier [3] estimates the systemic risk of a financial system conditional on institutions being in distress based on publicly traded financial institutions. They define an institution's contribution to systemic risk as the difference between CoVaR conditional on the institution being in distress and CoVaR in the median state of the institution. They quantify the extent to which characteristics such as leverage, size, and maturity mismatch predict systemic risk contribution.

The micro-level methods have been criticized by Allen and Tang [4]. They base their research on the assumption that financial intermediaries including commercial banks, savings banks, investment banks, broker/dealers, insurance companies, mutual funds, etc. are special because they are fundamental to the operation of the economy. The specialness of banks is reflected in the economic damage that results when financial firms fail to operate properly. They proposed a new measure to forecast the likelihood that systemic risk-taking in the banking system as a whole, called CATFIN. It measures the tail risk of the overall banking market using VaR methodology at a 1% level with monthly data. This early warning system should signal whether aggressive aggregate systemic risk-taking in the financial sector presages future macroeconomic declines. Gao et al. [19] showed that among 19 different risk measures, CATFIN performs the best in predicting macro-level shocks.

Romer and Romer [24] examine the aftermath of postwar financial crises in advanced countries. Through the construction of a new semiannual series on financial distress in 24 OECD countries for the period 1967–2012. The series is based on assessments of the health of countries' financial systems from a consistent, real-time narrative source, and classifies financial distress on a relatively fine scale. They find that the average decline in output following a financial crisis is statistically significant and persistent, but only moderate in size. More importantly, the average decline is sensitive to the specification and sample, and that the aftermath of the crises is highly variable across major episodes. Following this research, Engle and Ruan [18], using a crisis severity variable constructed by Romer and Romer [24], estimated a Tobit model for 23 developed economies. They developed a probability of crisis measure and SRISK capacity measure from the Tobit estimates. These reveal the important global externality whereby the risk of a crisis in one country is strongly influenced by the undercapitalization of the rest of the world.

Acharya et al. [2] present an economic model of systemic risk in which undercapitalization of the financial sector as a whole is assumed to harm the real economy, leading to a systemic risk externality. Each financial institution's contribution to systemic risk can be measured as its systemic expected shortfall (SES), that is, its propensity to be undercapitalized when the system as a whole is undercapitalized.

The research by Wang et al. [27] addresses the measurement of the systemic risk contribution (SRC) of country-level stock markets to understand the rise of extreme risks in the worldwide stock system to prevent potential financial crises. The proposed measure of SRC is based on quantifying tail risk propagation's domino effect using CoVaR and the cascading failure network model. While CoVaR captures the tail dependency structure among stock markets, the cascading failure network model captures the nonlinear dynamic characteristics of tail risk contagion to mimic tail risk propagation. The validity test demonstrated that this method outperforms seven classic methods as it helps early warning of global financial crises and correlates to many systemic risk determinants, e.g., market liquidity, leverage, inflation, and fluctuation. The results highlight that considering tail risk contagion's dynamic characteristics helps avoid underestimating SRC and supplement a "too cascading impactive to fail" perspective to improve financial crisis prevention.

The micro-level methods have been criticized by Allen and Tang [4]. They base their research on the assumption that financial intermediaries including commercial banks, savings banks, investment banks, broker/dealers, insurance companies, mutual funds, etc. are special because they are fundamental to the operation of the economy. The specialness of banks is reflected in the economic damage that results when financial firms fail to operate properly. They proposed a new measure to forecast the likelihood that systemic risk-taking in the banking system as a whole, called CATFIN. It measures the tail risk of the overall banking market using VaR methodology at a 1% level with monthly data. This early warning system should signal whether aggressive aggregate systemic risk-taking in the financial sector presages future macroeconomic declines. Gao et al. [19] showed that among 19 different risk measures, CATFIN performs the best in predicting macro-level shocks.

Caporin et al. [12] introduced TALIS (TrAffic LIght System for Systemic Stress) that provides a comprehensive color-based classification for grouping companies according to both the stress reaction level of the system when the company is in distress and the company's stress. level. This indicator can integrate multiple signals from the interaction between different risk metrics. Starting from specific risk indicators, companies are classified by combining two loss functions—one for the system and one for each company—evaluated over time and as a cross-section. An aggregated index is also obtained from the color-based classification of companies.

Kielak and Ślepaczuk [21] compare different approaches to Value-at-Risk measurement based on parametric and non-parametric approaches for different portfolios of assets, including cryptocurrencies. They checked if the analyzed models accurately estimate the Value-at-Risk measure, especially in the case of assets with various returns distribution characteristics. Buczyński and Chlebus [11] checked which of the VaR models should be used depending on the state of the market volatility. They showed that the best of the models that is the least affected by changes in volatility is GARCH(1,1) with standardized student's t-distribution. Non-parametric techniques or FHS with skewed normal distribution have very prominent results in testing periods with low volatility but are relatively worse in turbulent periods. Woźniak and Chlebus [28] point out that under the conditions of sudden volatility increase, such as during the global economic crisis caused by the Covid-19 pandemic, no classical VaR model worked properly even for the group of the largest market indices. In general, there is an agreement between market risk researchers that an ideal model for VaR estimation does not exist, and different models' performance strongly depends on current economic circumstances.

Some spectacular crash events, including the FTX collapse in November 2022, followed by a dramatic slump in prices of most of the cryptocurrencies triggered a question about the resiliency of this financial market segment to shocks and the potential spillover effect. In one of the latest research, Jalan and Matkovskyy [20] studied systemic risk in the cryptocurrency market based on the FTX collapse. Using the CATFIN measure to proxy for the systemic risk they claimed that the FTX crisis did not engender higher systemic and liquidity risks in this market compared to previous negative shocks.

Various rigorous models of bank and payment system contagion have now been developed, although a general theoretical paradigm is still missing. Direct econometric tests of bank contagion effects seem to be mainly limited to the United States. Empirical studies of the systemic risk in foreign exchange and security settlement systems appear to be non-existent. Moreover, the literature surveyed reflects the general difficulty to develop empirical tests that can make a clear distinction between contagion in the proper sense and joint crises caused by common shocks, rational revisions of depositor or investor expectations when information is asymmetric ("information-based" contagion) and "pure" contagion as well as between "efficient" and "inefficient" systemic events.

Bearing in mind the huge dynamics of the recent shocks (e.g. the Covid-19 pandemic, and the FTX collapse), we claim that the monthly data frequency (like in the case of CATFIN) is not enough to create a valid early warning indicator. At the same time, we claim that the existing indicators of systemic risk are over sophisticated and some of them require huge computing power or access to paid datasets. Therefore, there is a need to create a precise and simple indicator of systemic risk based on a publicly available date with relatively high frequency. In this study, we base on the macro-level data which is easily accessible to the general public to construct a robust systemic risk indicator. We show that our simple metrics can yield similar (or better) results than complex methods and can be computed with a relatively high-frequency using publicly available data, which is a great advantage.

2.2. *A comparison of the selected systemic risk indicators*

Following the Cleveland Fed's commentary on the performance of their systemic risk indicator (Craig 2020), we agree that a good financial-stress indicator (we may also say a good systemic risk indicator) is reliable, timely, straightforward, valid, and ongoing. Most of the indicators miss some of those features. For example, indicators that base on the balance-sheet data are neither timely nor ongoing, as financial data is provided on a monthly basis to the regulators and it is publicly released on a quarterly basis and with a delay. This means that those indicators can be computed by market regulators with a higher frequency than by the wide public, which is a disadvantage for the market participants. Moreover, some of the indicators are complex and thus they involve a significant model risk. In other words, if two indicators perform the same, the better one is the simpler one. In Table 1 we provide an overview of the selected systemic risk indicators.

Table 1: **A comparison of the selected systemic risk indicators**

Indicator	Data frequency*	Scope /Focus	Markets	Description /Methodology	Links	Real-time frequency	Type of data	References
IVRVSRI	Daily	Equities	Global, USA, Europe, Japan, Brazil	The methodology is based on the combination of the information hidden in the latent process of volatility using the concept of implied and realized volatility. This indicator includes the publicly available index of implied volatility and realized volatility on the underlying equity index.		Possible	Public, available in real-time	(this study)
Talis3	Monthly	Financial companies	USA	A TrAffic LIght System for Systemic Stress (TALIS-cube) provides a color-based classification for grouping financial companies according to the system's stress reaction level when the company is in distress. TALIS3 integrates multiple signals from the interaction between different risk metrics. Starting from specific risk indicators, companies are classified by combining two loss functions: one for the system and one for each company that is evaluated over time and as a cross-section. An aggregated index is presented in the form of a color-based classification of companies.		Not possible	Public, available with delay	[12]
Systemic Expected Shortfall (SES)	Monthly	Banks	USA	The idea of systemic risk is rooted in the undercapitalization of the financial sector as a whole as it is assumed to harm the real economy, leading to a systemic risk externality. Each financial institution's contribution to systemic risk can be measured as its systemic expected shortfall (SES), that is, its propensity to be undercapitalized when the system as a whole is undercapitalized. SES increases in the institution's leverage and its marginal expected shortfall (MES), that is, its losses in the tail of the system's loss distribution.	https://vlab.stern.nyu.edu/docs/srisk	Not possible	Public, available with delay	[2]
Srisk	Monthly	Depositories including banks, Insurance companies, broker-dealers, and others	USA	Srisk measures the systemic risk contribution of a financial firm. Srisk measures the capital shortfall of a firm conditional on a severe market decline and is a function of its size, leverage, and risk. Moreover, aggregate Srisk provides early warning signals of distress in indicators of real activity.	https://vlab.stern.nyu.edu/docs/srisk	Not possible	Public, available with delay	[10]
CoVaR		Banks		CoVaR measure of the systemic risk is defined as the change in the Value at Risk of the financial system conditional on an institution being under distress relative to its median state. Such characteristics as leverage, size, maturity mismatch, and asset price booms significantly predict CoVaR.		Not possible	Public, available with delay	[3]
Cleveland Fed's Systemic Risk Indicator	Monthly	Banks	USA	This indicator combines measures of balance-sheet strength, volatility, and correlation of the asset values of the major banks with the forward-looking characteristics of option prices. This method uses the concept of the distance to default, a measure developed by Merton (1974) for firms such as banks that are highly leveraged. It is based on the calculation of two measures of insolvency risk, one an average of default risk across individual banking institutions (average distance-to-default) and the other a measure of risk for a weighted portfolio of the same institutions (portfolio distance-to-default). The systemic risk indicator is the difference (spread) between the two. When the insolvency risk of the banking system as a whole rises and converges to the average insolvency risk of individual banking institutions—the narrowing of the spread—it reflects market perceptions of imminent systematic disruption of the banking system	https://www.clevelandfed.org/indicators-and-data/systemic-risk-indicator	Not possible	Public, available with a delay	[14], [25], [22]
CATFIN	Monthly	Banks	USA, Europe, Asia	The CATFIN measure of aggregate systemic risk complements bank-specific systemic risk measures by forecasting macroeconomic downturns six months into the future using out-of-sample tests conducted with US, European and Asian bank data. This measure is based on the concept of the bank "specialness" in the economy. High levels of systemic risk in the banking sector impact the macroeconomy through aggregate lending activity. A conditional asset pricing model shows that CATFIN is priced for financial and non-financial firms.		Not possible	Public, available with delay	[4]

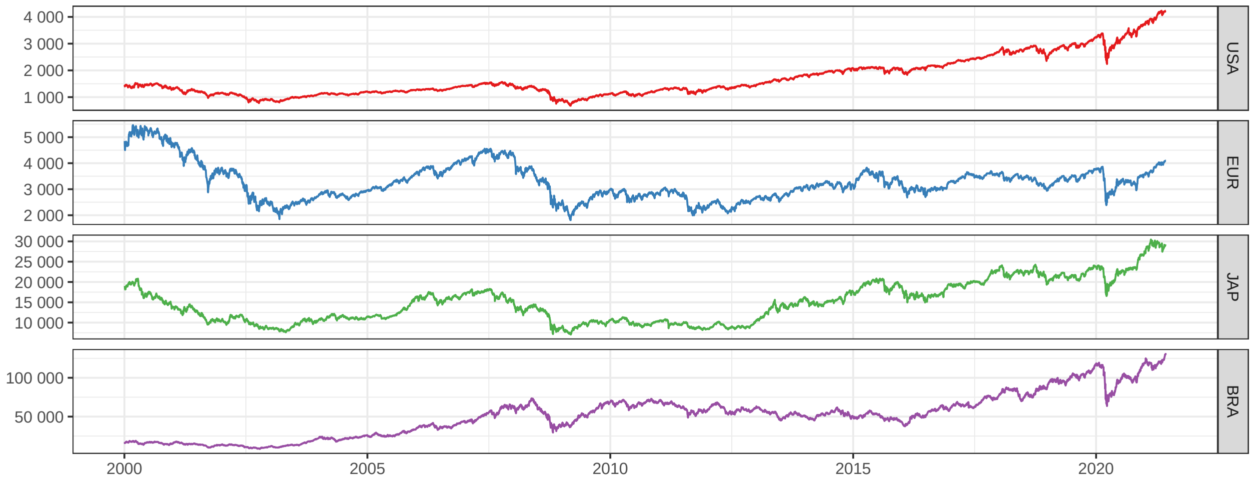
Source: Author's own. Data frequency is determined by the lowest frequency of the data used in the calculation of a systemic risk indicator. Some indicators combine daily (market) and monthly data (balance sheet), and the indicators are presented on a daily basis. We claim that it is not appropriate, and in fact, such indicators are monthly. Moreover, one should bear in mind that balance sheet data is available with a delay, which further reduces the indicators' timeness.

3. Data and Methodology

3.1. Data

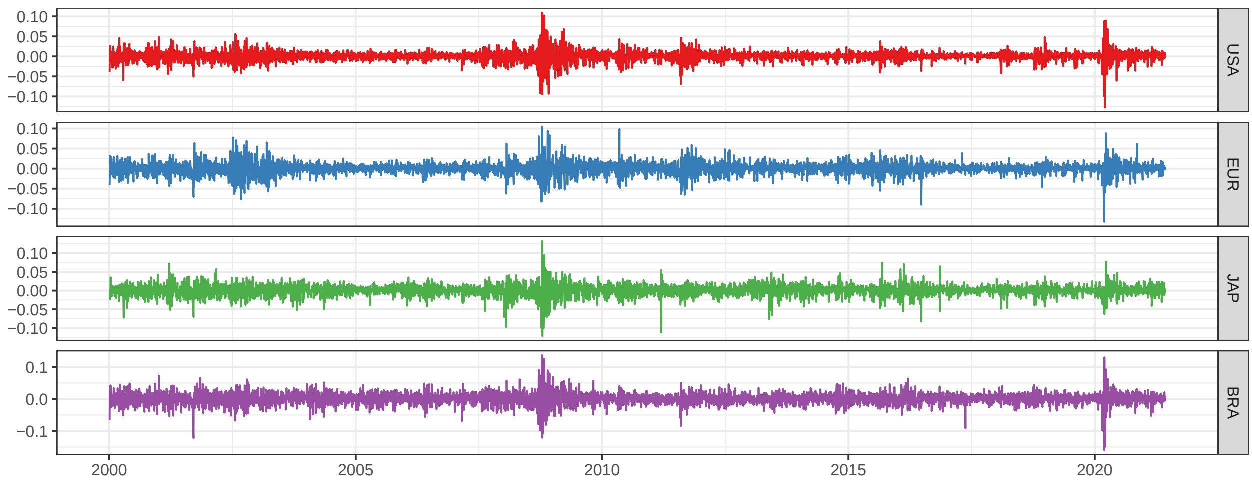
Our data set is based on daily data for volatility indices (VIX, VSTOXX, VNKY, and VXEWZ) and daily price and market cap data for equity indices (S&P500, EuroStoxx50, Nikkei 225, Bovespa) in the period between 2000 to 2021. Figure 1 presents the fluctuations of the analyzed times series, while Figure 2 fluctuations of returns. Figure 1 informs us about different magnitude of upward and downward movements on analyzed markets, while Figure 2 additionally visualize volatility clustering with high and low volatility periods indicating calm and more stressfull periods of time.

Figure 1: The fluctuations of S&P500, EuroStoxx50, Nikkei225 and Bovespa indices between 2000 and 2021.



Note: The main equity indices for USA, Europe, Japan and Brazil in the period between 2000 and 2021.

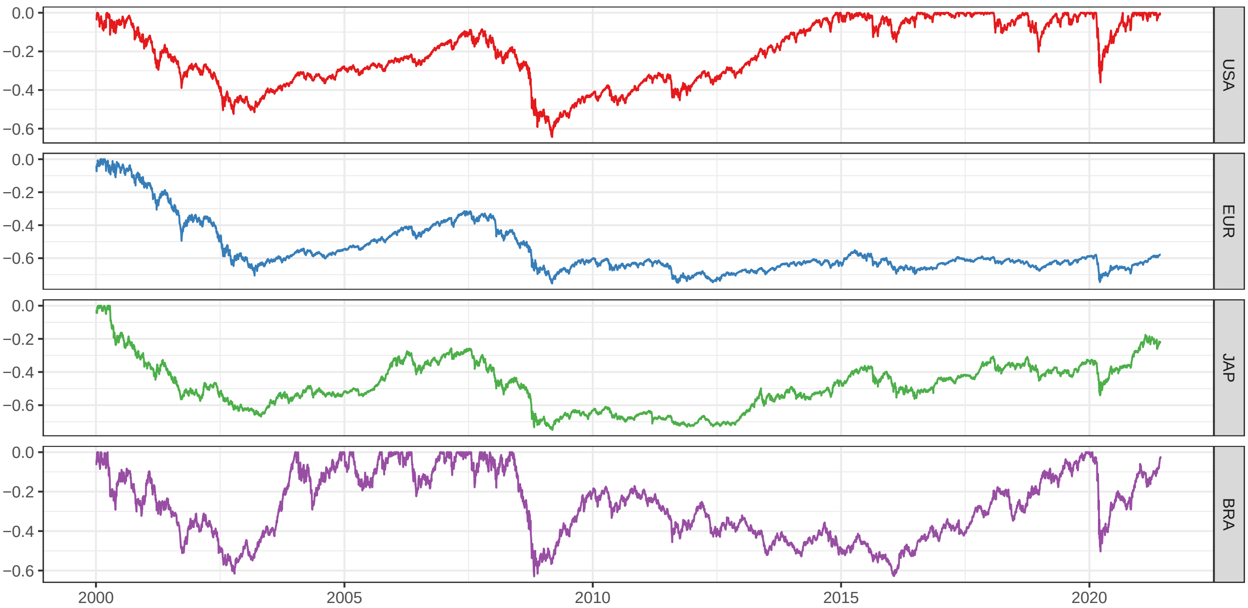
Figure 2: Returns of S&P500, EuroStoxx50, Nikkei225 and Bovespa indices between 2000 and 2021.



Note: Returns of the main equity indices for USA, Europe, Japan and Brazil in the period between 2000 and 2021.

Drawdowns of analyzed equity indices, depicted on Figure 3, show the length of the most important turmoils and additionally visualize their speed and magnitude.

Figure 3: Drawdowns of S&P500, EuroStoxx50, Nikkei225 and Bovespa indices between 2000 and 2021.



Note: Panel (1) presents drawdowns for S&P500 index prices. Panel (2) presents drawdowns for EuroStoxx50 index prices. Panel (3) presents drawdowns for Nikkei225 index prices. Panel (4) presents drawdowns for iShares Brazil ETF (EWZ) index prices.

Table 2: Descriptive statistics of equity indices returns.

statistic	USA	EUR	JAP	BRA
nobs	5392	5392	5392	5392
NAs	1	1	2	1
Minimum	-0.13	-0.13	-0.12	-0.16
1. Quartile	0	-0.01	-0.01	-0.01
Mean	0	0	0	0
Median	0	0	0	0
3. Quartile	0.01	0.01	0.01	0.01
Maximum	0.11	0.1	0.13	0.14
Stdev	0.01	0.01	0.01	0.02
Skewness	-0.4	-0.2	-0.42	-0.4
Kurtosis	10.95	6.09	6.92	7.06
Norm.	0	0	0	0

Note: 'Norm.' denotes p-value of the Jareque-Bera test for normality.

Descriptive statistics of returns, presented in Table 2, confirm the well-known fact about equity returns, i.e. high kurtosis, negative skewness and associated non-normality of returns.

3.1.1. Market Capitalization

In order to calculate the proper weights in IVSRI, RVSRI and IVRVSRI indicators, we decided to use market capitalization data for each of the equity indices used (Table 3).

Table 3: Market capitalisation for S&P500, EuroStoxx50, Nikkei225 and Bovespa indices.

Indices	MarketCap	Weights
S&P500	35.6T	77.7%
EuroStoxx50	3.7T	8.1%
Nikkei225	5.5T	12%
Bovespa	1T	2.2%
TOTAL	45.9T	100%

Note: Market Capitalization for equity indices were downloaded on 2021-06-21 from:
S&P500 index: https://ycharts.com/indicators/sp_500_market_cap
EuroStoxx50: https://en.wikipedia.org/wiki/EURO_STOXX_50
Nikkei 225: https://www.bloomberg.com/quote/NKY:IND
Bovespa: https://en.wikipedia.org/wiki/B3_(stock_exchange)

3.2. Methodology

Our methodology is based on the combination of the information hidden in latent process of volatility using the concept of implied and realized volatility. We did it by utilizing the methodology for volatility indices based on Demeterfi et al. [16] and CBOE [13] and the concept of realized volatility for various frequency of data introduced by Andersen et al. [6] and Andersen et al. [5].

Similarly to Caporin et al. [12], we decided to construct the gray-scale dynamic historical ranking evaluating the systemic risk day by day on the global and on the country level. What is more important, our methodology can be simply transformed and adapted for the use of the high-frequency data and then such high-frequency systemic risk indicator can monitor the risk on the real-time basis.

The general formula of IVRVSRI consist of two component indices based on implied (IVSRI) and realized (RVSRI) volatility. However, before we present the final formulas we have to introduce the concept of volatility indices and realized volatility measure.

3.2.1. Implied volatility - Volatility indices

One of the first and the best known volatility index is VIX index, introduced by CBOE in 2003, and additionally recalculated backward to 1987. Its formula based on the seminal paper of Demeterfi et al. [16] and then described in detail in CBOE [13] can be summarized by the following equation:

$$\sigma^2 = \frac{2}{T}\sum_{k=i}\frac{\Delta K_i}{K_i^2}e^{RT}Q(K_i) - \frac{1}{T}\Big[\frac{F}{K_0}-1\Big]^2 \tag{1}$$

where:
$\sigma = \frac{VIX}{100}$
T - time to expiration
K_i - strike price of i-th out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$
R - risk-free interest rate to expiration
F - forward index level derived from index option prices
K_0 - first strike below the forward index level (F)

The formulas for other volatility indices used in this study (VSTOXX, VNKY, and VXEWZ) are based on the similar methodology and their details can be found in Borse [8], Nikkei [23], and CBOE [13].

3.2.2. Realized volatility measure

In the case of historical volatility measure we use the realized volatility concept (Andersen et al. [5]) based on summation of log returns during the given period of time and then we annualize it in order to be able to combine it with IV. The formula used in this paper is as follows:

$$\mathrm{RV}_{t,i}^{1M} = \sqrt{\frac{252}{21}\sum_{k=0}^{20} r_{t-k,i}^2} = \mathrm{RVSRI_i}, \qquad r_{t-k,i} = \log\left(\frac{P_{t,i}}{P_{t-k,i}}\right) \tag{2}$$

where:

$\mathrm{RV}_{t,i}^{1M}$ - the realized volatility for i-th equity index on day t with the memory of 1 calendar month (i.e. 21 trading days)

P_{t-k} - the price of i-th equity index on day $t-k$

The memory of the realized volatility estimator was set to 21 days (trading days) in order to make it comparable with 30 calendar days in case of VIX.

3.2.3. IVRVSRI - Implied Volatility Realized Volaitlity Systemic Risk Indicator

Our methodology has significant advantages compared to other approaches presented in the literature (Caporin et al. [12], Buczyński and Chlebus [11]). Firstly, IVRVSRI uses systemic risk indication based on two simple and heavily grounded amongst market participants risk measures (IV and RV). Secondly, we analyze various financial market turmoils from 2000 until 2023 unhiding the characteristics and severity of major market crisis during the last 23 years. Thirdly, we construct a dynamic ranking (day by day) showing the current level of stress on the global level and additionally separately for USA, Europe, Brazil and Japan. Finally, our methodology can be simply extending by using high-frequency price data for the selected equity indices and the same frequency for volatility indices in order to mimic the systemic-risk on real time basis.

In order to accomplish this task we construct two component systemic risk indicators based on implied (IVSRI) and realized volatility measures (RVSRI) for each country separately and additionally on the aggregated level for all countries.

3.2.3.1. Implied Volatility SRI.

IVSRI is based on the separate volatility index for each country or geographical area and its share in the total market cap. The formula for IVSRI is as follows:

$$\mathrm{IVSRI} = \sum_{k=1}^{N} w_i * \mathrm{IV}_i \tag{3}$$

where:

N - the number of analyzed countries

IV_i - the implied volatility index for the i-th country, where: $\mathrm{IV}_i = \mathrm{IVSRI}_i$, for example VIX index for the USA or VSTOXX index for Europe

w_i - the weight of the given country in SRI, calculated according to:

$$w_i = \frac{\mathrm{MC}_i}{\sum_{k=1}^{N} \mathrm{MC}_i} \tag{4}$$

where:

MC_i - the market capitalization fo the given country

Based on Table 1 and Equation 4 we were able to construct weights vector $w = \{77.7\%, 8.1\%, 12\%, 2.2\%\}$ which then will be used for the purpose of our risk metrics calculations.

3.2.3.2. Realized Volatility SRI.

RVSRI is based on similar concept as IVSRI (section 3.2.3.1):

$$\mathrm{RVSRI} = \sum_{k=1}^{N} w_i * \mathrm{RV}_i \tag{5}$$

where:
$RV_i = RVSRI_i$

*3.2.3.3. IVRVcSRI - Implied Volatility Realized Volaitlity Systemic Risk Indicator on the country level (*IVRVSRI$_i$*).*

IVRVSRI can be calculated on the country level (IVRVSRI$_i$) as the weighted sum of IV (IVSRI$_i$) and RV (RVSRI$_i$) measures for the given country and on the global level (explained in detail in section 3.2.3.4). Below please find the formula 6 for IVRVSRI on the country level (IVRVSRI$_i$):

$$\text{IVRVSRI}_i = w_{IV} * \text{IVSRI}_i + w_{RV} * \text{RVSRI}_i \tag{6}$$

3.2.3.4. IVRVSRI - Implied Volatility Realized Volaitlity Systemic Risk Indicator on the global level (IVRVSRI).

IVRVSRI can be calculated in both ways, based on IVSRI and RVSRI on the global level (formulas 3 and 5):

$$\text{IVRVSRI} = w_{IV} * \text{IVSRI} + w_{RV} * \text{RVSRI}, \qquad w_{IV} + w_{RV} = 1 \tag{7}$$

where:
w_{IV} - the weight of IVSRI component in IVRVSRI, currently equal to 50%
w_{RV} - the weight of RVSRI component in IVRVSRI, currently equal to 50%

Alternatively, we can calculate IVRVSRI measure based on country specific IVRVSRI (i.e. IVRVSRI$_\text{i}$)

$$\text{IVRVSRI} = \sum_{k=1}^{N} w_i * \text{IVRVSRI}_i \tag{8}$$

The various weights for RVSRI and IVSRI in final IVRVSRI will be additionally checked as some robustness check.

3.2.4. Dynamic quartile ranking based on IVRVSRI (DQR_IVRVSRI)

3.2.4.1. DQR_IVRVSRI on the country level.

In the next step we decided to construct dynamic quartile ranking (DQR_IVRVSRI) based on RVSRI, IVSRI, and IVRVSRI indications on the country and on the global level. DQR_IVRVSRI on the country level is constructed based on the following steps:

1. We create quartile map chart based on IVRVSRI$_i$ for each country under investigation,
2. This map chart on the daily level shows colored systemic risk indicator,
3. Colors indicate the following:

- RED that IVRVSRI$_i$ was in its 4th quartile based on historical indications -> VERY HIGH country-systemic risk
- ORANGE that IVRVSRI$_i$ was in its 3rd quartile based on historical indications -> HIGH country-systemic risk
- LIGHT GREEN that IVRVSRI$_i$ was in its 2nd quartile based on historical indications -> LOW country-systemic risk
- GREEN that IVRVSRI$_i$ was in its 1st quartile based on historical indications -> VERY LOW country-systemic risk

3.2.4.2. DQR_IVRVSRI on the global level.

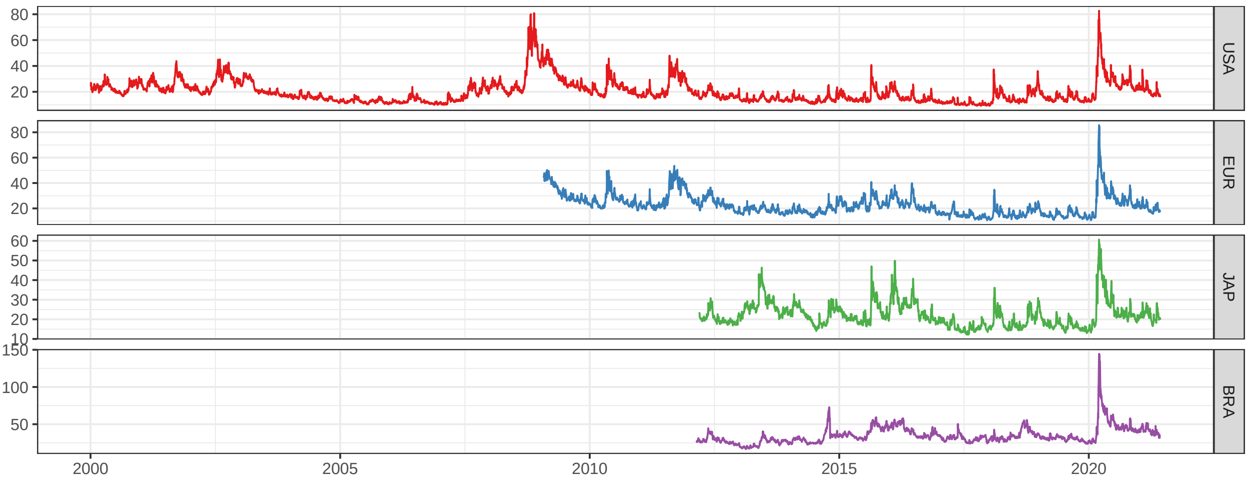
This time we use the same logic as for IVRVSRI$_i$ for each country separately, however the map is constructed on the global level.

4. Results

Based on the logic "keep it simple", we want to check if it is possible to create Systemic Risk Indicator based on widely available (most often publicly available and free of charge) volatility risk measures which can have similar properties as systemic risk indicators introduced in highly cited papers (Brownlees and Engle [10], Acharya et al. [2], Romer and Romer [24] or Engle and Ruan [18]) or in the most recent study of Caporin et al. [12]. In the Results section, we present Figures and map charts visualizing systemic risk indicators and theirs components.

Figure 4 shows the fluctuations of IV indices for each country separately and shows the most significant turmoils affecting the equity market in each country under investigation, i.e. GFC (20087-2009), COVID pandemic (March 2020), and a few of lower magnitude like Eurozone debt crisis (2009-2014), and turmoils in August 2015, February 2018 and November-December 2018.

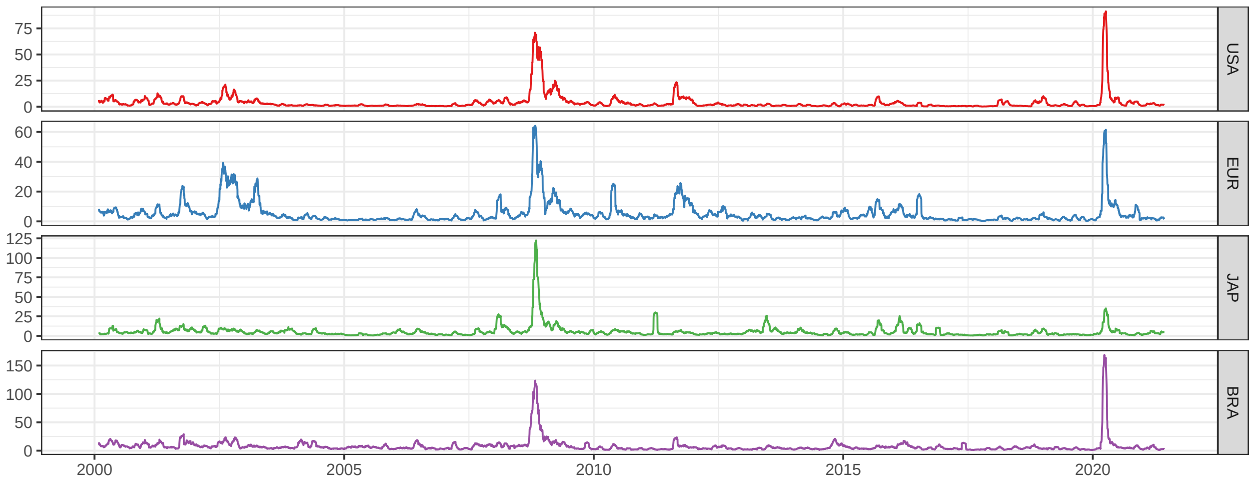
Figure 4: Implied Volatility indices for S&P500, EuroStoxx50, Nikkei225 and Bovespa between 2000 and 2023



Note: Panel (1) presents VIX index calculated based on S&P500 index options series. Panel (2) presents VStoxx index calculated based on EuroStoxx50 index options series. Panel (3) presents VNKY index calculated based on Nikkei225 index options series. Panel (4) presents VXEWZ index calculated based on the iShares Brazil ETF (EWZ) index options series.

On the other hand, Figure 5 presents RV indices for each country separately. The comparison of these two figures (4 and 5) informs us that the anticipated reaction (IV indices in Figure 4) to the current market stress is not always the same as the current reaction revealed in realized volatility of returns (IV versus RV for Japan during Covid pandemic in March 2020).

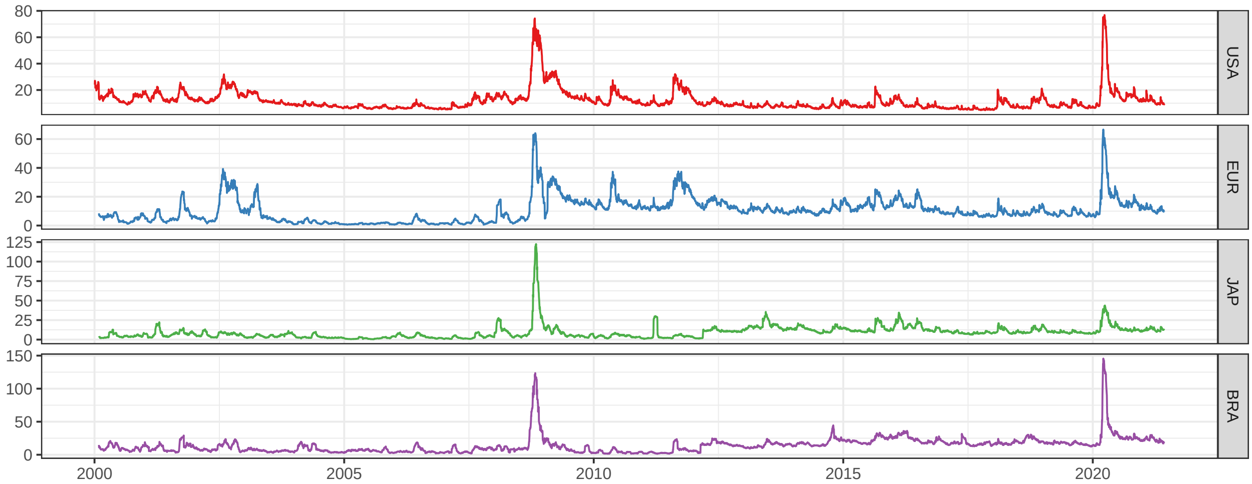
Figure 5: Realized Volatility indices for S&P500, EuroStoxx50, Nikkei225 and Bovespa between 2000 and 2023



Note: Panel (1) presents RV index calculated based on S&P500 index prices based on the formula 2. Panel (2) presents VStoxx index calculated based on EuroStoxx50 index prices based on the formula 2. Panel (3) presents VNKY index calculated based on Nikkei225 index prices based on the formula 2. Panel (4) presents VXEWZ index calculated based on the iShares Brazil ETF (EWZ) index prices based on the formula 2.

Overall, our results show that the magnitude of reactions to the risk events varies across countries. Analyzing IVRVSRI indications on the country levels (Figure 6) we observe a very weak reaction of Japanese markets to COVID-19 pandemic in March 2020 in comparison to the USD and Eurozone, and literally no reaction of Japanese and Brazilian markets to the European sovereign debt crisis in 2009-2014. Only in the case of the GFC 2007-2009 all analyzed markets reacted strongly but the persistence of the crisis was not the same (Figure 6). Brazil and Japan recovered quickly with regard to the speed of the decrease of IVRVSRI indications while the USA and Europe were struggling much longer.

Figure 6: IVRVSRI on the country level separately for S&P500, EuroStoxx50, Nikkei225 and Bovespa between 2000 and 2023



Note: Panel (1) presents IVRVSRI for the USA calculated based on VIX index and S&P500 index prices based on the formula 6. Panel (2) presents IVRVSRI for Eurozone calculated based on the VSTOXX index and EuroStoxx50 index prices based on the formula 6. Panel (3) presents IVRVSRI for Japan calculated based on the VNKY index and Nikkei225 index prices based on the formula 6. Panel (4) presents IVRVSRI calculated based on the VXEWZ index and the iShares Brazil ETF (EWZ) index prices based on the formula 6.

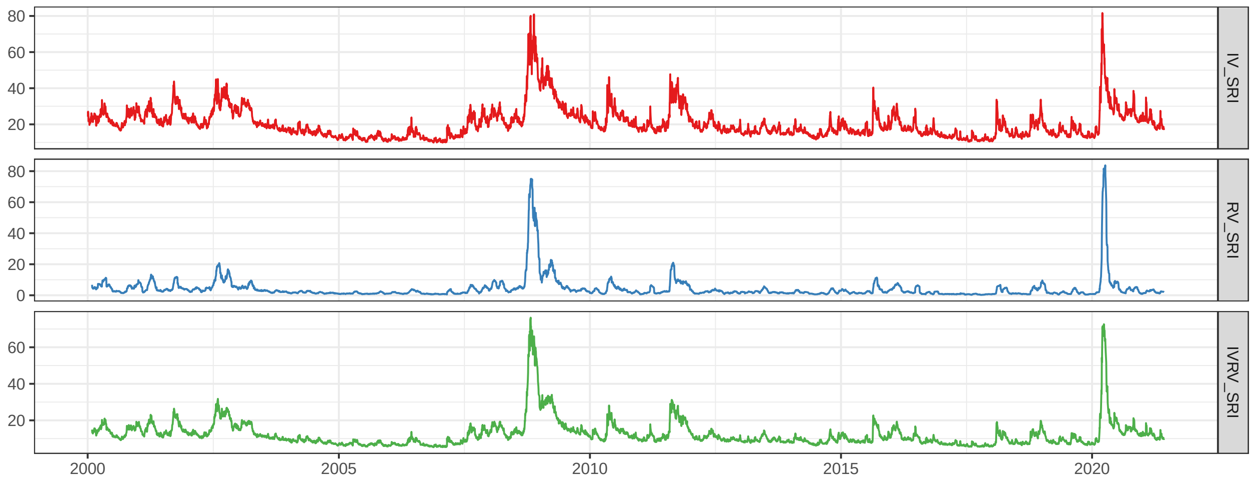
Next, Figure 7 shows the map chart with colored quartile levels of IVRVSRI indications on the country level. It shows that in the case of Eurozone, the GFC extended into the debt crisis and lasted with a small break in 2014 until 2016. In general, before the GFC the Eurozone, Japanese and Brazilian markets were more resilient than the American one to worldwide turmoils while the situation reversed after the Eurozone sovereign debt crisis, with Brazil and Japan being the least resilient in that period among all analyzed countries.

Figure 7: Quartile colour based IVRVSRI map.chart on the country level

Note: Panel (1) presents colored map.chart indicating quartiles of IVRVSRI$_i$ for the USA calculated based on VIX index and S&P500 index prices based on the formula 6. Panel (2) presents colored map chart indicating quartiles of IVRVSRI$_i$ for Eurozone calculated based on the VSTOXX index and EuroStoxx50 index prices based on the formula 6. Panel (3) presents colored map chart indicating quartiles of IVRVSRI$_i$ for Japan calculated based on the WNKY index and Nikkei225 index prices based on the formula 6. Panel (4) presents colored map.chart indicating quartiles of IVRVSRI$_i$ calculated based on the VXEWZ index and the iShares Brazil ETF (EWZ) index prices based on the formula 6. Quartalies on map.chart are indicated with green-red scale, where green indicates the 1st quartile (the lowest one) while red colour indicates the 4th quartile (the highest one).

Figure 8 shows the aggregated results for IVRVSRI and its components (IVSRI and RVSRI) on the global level. We can see that after aggregation of the country specific indices all the major financial crises are indicated and additionally we can observed their severity. GFC and Covid were the most severe turmoils, but other ones line the end of downward trend after the Dotcom bubble (2002-2003) and Eurozone debt crisis (2009-2014) are revealed as well. What is more, the reaction of IVSRI and RVSRI components on the global level to the above mentioned turmoils differs with regard to the magnitude of their reaction. Most often, the fear revealed in IVSRI (Panel (1) of Figure 8), especially in case of less severe turmoils (Eurozone debt crisis or the bottom of the Dotcom bubble), was not realized in the same magnitude of RVSRI indications (Panel (2) of Figure 8).

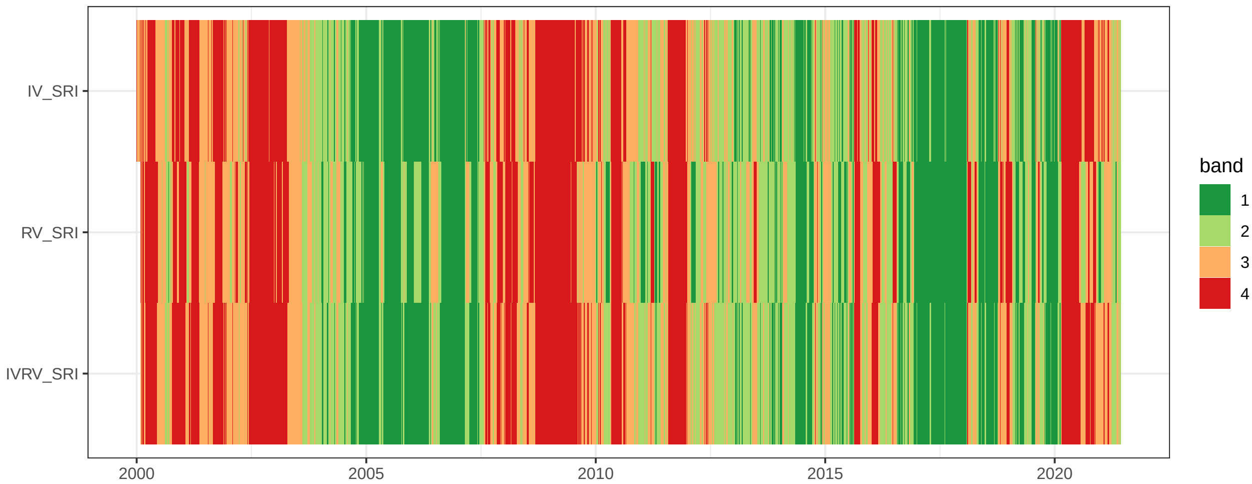
Figure 8: IVSRI RVSRI and IVRVSRI on the global level



Note: Panel (1) presents IVSRI on the global level calculated based on on the formula 3. Panel (2) presents RVSRI on the global level calculated based on the formula 5. Panel (3) presents IVRVSRI on the global level calculated based on the formula 7.

Figure 9 presents a colored map chart indicating quartiles of IVSRI, RVSRI, and IVRVSRI on the global level stressing the major turmoils on the aggregated level.

Figure 9: Quartile colour based IVSRI, RVSRI and IVRVSRI map on the global level

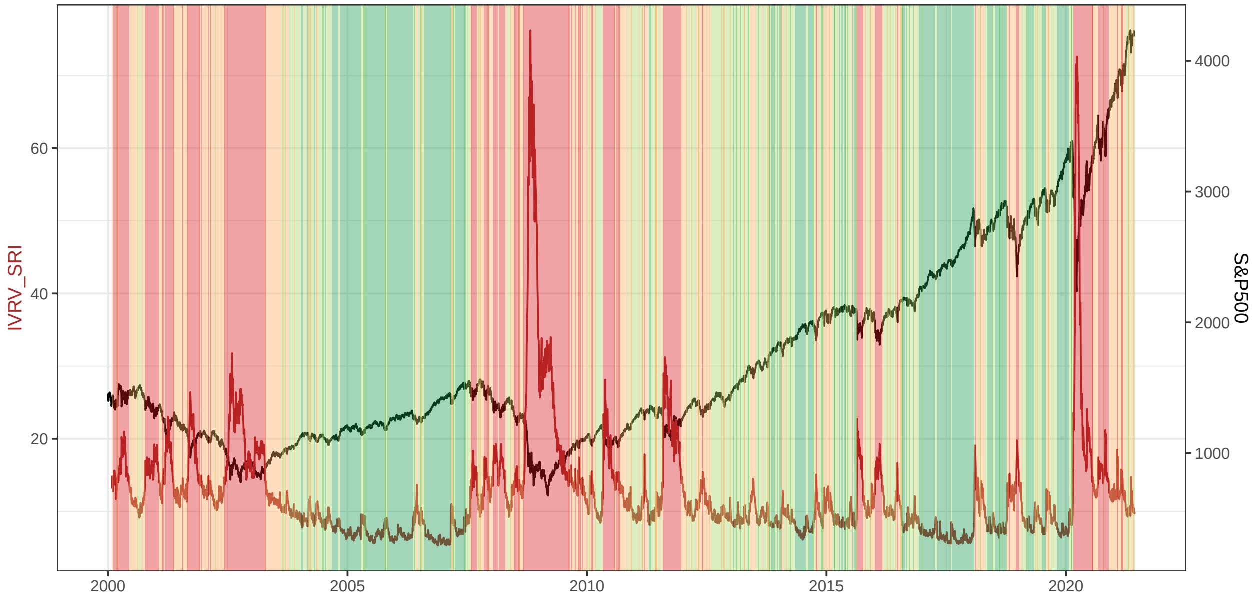


Note: Panel (1) presents colored map.chart indicating quartiles of IVSRI_i on the global level calculated based on the formula 3. Panel (2) presents colored map chart indicating quartiles of RVSRI_i on the global level calculated based on the formula 5. Panel (3) presents colored map.chart indicating quartiles of IVRVSRI_i on the global level calculated based on the formula 7. Quartalies on map.chart are indicated with green-red scale , where green indicates the 1st quartile (the lowest one) while red colour indicates the 4th quartiel (the highest one).

The $IVSRI$ and $RVSRI$ show slightly different risk levels in the "transition" periods when systemic risk changes. In general, we can state that the reaction of the implied-volatility-based metrics is faster than the realized volatility one, which is something we have expected. Moreover, the correlation between the IV-based indicator and the general systemic risk indicator (IVRVSRI) is higher than that of the RV-based ones. At the same time, the general systemic risk indicator (IVRVSRI) is a better indicator of systemic risk than any individual indicator based on only one measure of volatility (RVSRI or IVSRI), and this result is robust even after the change of the weights of the RVSRI and IVSRI in the general systemic risk measure.

Figure 10 depicts the comparison of fluctuations of S&P500 index and IVRVSRI on the global level. It clearly shows that each major financial turmoil was reflected on our IVRVSRI almost immediately informing market participants about increased level of stress.

Figure 10: IVRVSRI and S&P500 index on colored map.chart with quartiles of IVRVSRI.



Note: The flucuations of S&P500 index shows and IVRVSRI on the global level on the background of colored map chart with quartiles of IVRVSRI.

Referring to the main hypotheses, we were able to drawn the following conclusions. We can not reject RH1 as we show that it is possible to construct a robust Systemic Risk Indicator (IVRVSRI) based on the well-known concepts of realized and implied volatility measures. Moreover, we cannot reject RH2 as the indication of the proposed Systemic Risk Indicator ($IVRVSRI_i$) depends on the geographical location of a given equity market. As expected, the robustness of the proposed Systemic Risk Indicator depends on various parameters selected: the memory parameter for RV, time to expiration for IV, the percentile selected for the risk map, and the length of the history selected for the calculation of percentile in case of risk map, which supports RH3.

5. Conclusions

In this study, we propose a robust Systemic Risk Indicator based on the well-known concepts of realized and implied volatility measures. The main contribution of this paper to the broad bulk of studies of systemic risk indicators is the simplicity of the metrics that we propose, which at the same time yield similar results as more complex tools, thus significantly reducing the model risk. At the same time, the proposed methodology enables calculation of IVRVSRI on high-frequency data (even on the second level) which significantly decreases the time of response of our indicator to the starting point of each major financial turmoil. Moreover, in the case of many metrics, it is also much less computationally demanding and does not rely on paid data sets or data that is available only for market regulators. The indication of this measure depends on the geographical location of a given equity market. As expected, the robustness of the proposed Systemic Risk Indicator depends on various parameters selected: the memory parameter for RV, time to expiration for IV, the percentile selected for the risk map, and the length of the history selected for the calculation of percentile in case of the risk map.

This study can be extended by adding more countries to the analysis or other asset classes like currencies, commodities, real estate, cryptocurrencies, and hedge funds. Moreover, using high-frequency data would allow the construction of a real-time early implied volatility realized volatility systemic risk indicator (rteIVRVSRI) that would serve as an early warning indicator of systemic risk.

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University of Warsaw
Faculty of Economic Sciences
44/50 Długa St.
00-241 Warsaw
www.wne.uw.edu.pl