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Quantile regression analysis to predict GDP distribution using data from the US and UK

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Abstract: This paper aims to find the best models to forecast one-quarter-ahead and one-year-ahead US and UK real GDP growth distributions by employing quantile regression with skewed-t distribution on different sets of relevant near-term predictors. The research data period starts in 1947Q1/1955Q1 for US/UK data and ends in 2021Q3/2020Q4 for one-quarter-ahead/one-year-ahead prediction. The out-of-sample period ranges from 1996Q3 to 2021Q3 for one-quarter-ahead prediction and to 2020Q4 for one-year-ahead forecasting. The author applies a two-step testing procedure, in which models with the lowest average error in out-of-sample period are selected to the next step where the cumulative distribution functions of probability integral transforms are computed for the out-of-sample period, to select the best models. The improvement in the final forecasts of the tested models results, among others, from the use of new macroeconomic data with a higher frequency and focusing on the specific properties of the tested models separately for the US and UK. The chosen best models indicate that there exist better models than the model proposed by Adrian et al. (2016) to predict US growth distributions and that near-term predictors can produce good UK growth forecasts. Additionally, some simplified models associated with significantly lower portion of model risk are detected to produce meaningful forecasts for both US and UK case. For the US data, there exist several models that can produce timely predicted results.

Keywords: GDP growth, density forecast, quantile regression, US GDP, UK GDP, cumulative distribution function, probability integral transform, out-of-sample forecasting

JEL codes: E01, E17, C15, C31, C52, C53, C54, C58, F43

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INTRODUCTION

Density forecast has gained huge popularity during the last few decades among academic researchers and industry practitioners in modeling different macroeconomic variables. Specifically, numerous researches are conducted to predict the inflation or GDP growth distributions (Tay and Wallis 2000). These attempts were led by macroeconomic researchers in the United States with the "Survey of Professional Forecasters" to provide density forecasts for inflation and output growth, followed by the United Kingdom with "Panel of Independent Forecasters" to predict inflation distribution (Tay and Wallis 2000). This movement is further followed by various central banks in Sweden, Chile, Hungary, Slovakia, Czech Republic, etc. to arrive at fan charts for a varieties of economic variables of interests (Pońsko and Rybaczyk 2016). Practically, different banks apply methods proposed by relevant literature to model GDP growth distributions in various lengths of horizon forecasts under mandatory stress testing exercises such as Comprehensive Capital Adequacy Review (CCAR), Current Expected Credit Losses (CECL), International Financial Reporting Standards (IFRS) or numerous internal stress testing exercises like Internal Capital Adequacy Assessment Process (ICAAP).

In fact, GDP growth is one of the core macroeconomic variables that attract huge interests from policy institutions and banks as the variable affects almost all the estimations on the firm-wide level in majority of banks in their financial planning and analysis. Plenty of literature have tried to predict the GDP growth uncertainties in addition to the classical point forecast approaches. Typically, Adrian et al. (2016) employed a two-step procedure with quantile regression and skewed-t distribution to derive US real GDP distributions in one-quarter ahead and one-year ahead horizons conditioned on two variables: namely, the National Financial Conditions Index (NFCI) and real GDP growth. The study successfully explained the dynamics of the left tails in the forecasted densities of one-quarter-ahead and one-year-ahead real GDP growth, which is mainly attributed to the NFCI. Additionally, the authors strengthened the importance of the NFCI by running additional models on alternative financial indicators: term spread, credit spread, and equity market option-implied volatility. However, the paper suffers from one drawback: other economic variables were not checked such as unemployment rate, initial jobless claim, inflation, output gap, etc. to produce more robust results. Figueres and Jarociński (2020) leveraged the result of Adrian et al. (2016) to explore which financial indicator is the most appropriate for European area in predicting the tail risks of GDP growth by adopting quantile regressions, which confirms the practical application of quantile regression in modeling GDP growth density forecasts. With respect to the United Kingdom, quantile regression was checked to measure tail risks to UK GDP growth in Aikman et al. (2018). The research focused mainly on the left tails of predicted distributions by using 3 medium-term composite measures of private non-financial leverage, assets valuation, and terms of credit indices. So far, there has been no literature conducted to explore the applicability of quantile regression on predicting the density forecast of real GDP growth for Great Britain using different near-term indicators.

Therefore, this paper aims to investigate the power of different economic variables in predicting the US and UK real GDP growth distributions by utilizing quantile regression. Specifically, the author tries to find the answers to the following smaller research questions:

RQ1. By employing the same methodology, are there any better models to predict

one-quarter-ahead US real GDP growth distributions, considering the models suggested by Adrian et al. (2016) as benchmark models?
RQ2. Similarly, will there exist better explanatory variables for the prediction of one-year ahead US real GDP growth density?
RQ3. How do the tails of calibrated growth distributions of US growths behave?
RQ4. Can quantile regression be applied to UK data to produce meaningful results using near-term measures or indicators?
RQ5. What are the behaviors of the tails in the UK forecasted real GDP distributions?
RQ6. Which measure of financial condition can provide informative results on the tail risks to the UK real GDP growth?
RQ7. Should simpler models be chosen in favor of simplicity among the models chosen to predict the dependent variables for the US and UK?
RQ8. If the publication frequencies of predictors are taken into account, which model should be chosen to predict the US and UK growth distributions in order to ensure the timeliness of forecasts?

In order to answer the above-mentioned research questions, relevant data are collected for both US and UK. In case of US data, in addition to real GDP levels, time series data are also gathered for NFCI, unemployment rate, 10-year treasury yields, 2-year treasury yields, 3-month treasury rates, S&P500 index levels, CPI, output gap, and initial jobless claims. These US series are mainly extracted from the Federal Reserve Bank of ST. Louis' Economic Research website except for NFCI, S&P500 index levels, and output gap whose data are taken from the website of Federal Reserve Bank of Chicago, Yahoo Finance, and Congressional Budget Office. For UK data, data for real GDP levels together with unemployment rates, CPI, 2-year bond yields, 10-year bond yields, and PNFC's effective loan rates, are extracted from the website of Bank of England whereas FTSE index levels are taken from Yahoo Finance. Transformation to quarterly data is needed whenever a collected time series is not in quarterly frequency. Additional independent variables are created using the extracted data and tested in different quantile regression models to predict the US and UK one-quarter or one-year ahead real GDP growth distributions. The lengths of these variables are different depending on the availability of series (The longest series of US/UK start in 1947Q1/1955Q1 whereas the shortest variables starts in 1981Q3/1984Q1.) However, all researched series for US and UK end in December, 2021. Both US and UK data are split dynamically into in-sample and out-of-sample data on a rolling basis. Specially, for the first in-sample data, the start point is fixed at the point where all the variables used in each model are available and ends in 1996Q2. The in-sample data is then rolled by adding one quarter at a time until 2021Q3 (for predicting one-quarter-ahead GDP growth) and 2020Q4 (for predicting one-year-ahead GDP growth). Consequently, the out-of-sample data is the remaining data period, dynamically.

In total, 1274 models are checked to find the best models to predict one-quarter and one-year ahead US real GDP growth's density distributions from a set of 10 predictors: NFCI, unemployment rate, S&P500's realized volatility, CPI, output gap, real GDP one-quarter growth, spread between 10-year treasury rate and 2-year treasury rate, spread between 10-year and 3-month T-bill, initial jobless claim levels, growth of initial jobless claims. In the UK's case, 238 models are investigated to select the best forms to forecast the similar growth densities from the pool of 7 independent variables: FTSE realized volatility, unemployment rate, CPI, real GDP 1-quarter growth, spread between 10-year bond yields and 2-year bond-yields, PNFC's effective loan rate, spread between PNFC's effective loan rate and 2-year bond yields. Best models are selected using the average error measure calculated in the out-of-sample testing: the lower the average error is, the better prediction power the model has. Additionally, the empirical cumulative distributions (CDF) of the probability integral transforms (PITs) are computed for the best models to assess the reliability of the calibrated density forecasts.

In this study, results show that there exist better models to predict one-quarter and one-year US real GDP growth's density distributions. Specifically, for one-quarter ahead US real GDP growth, different sets variables are recommended to produce better results based on specific needs of modelers. If one modeler needs to have the best distribution, the model with 5 independent variables: NFCI, output gap, 10y-2y treasury yield spread, 10y-3m treasury yield spread, and annualized quarterly growth of initial jobless claims should be selected. However, if simplicity of a model is considered while having better result than that of the benchmark model, a model with NFCI and annualized quarterly growth of initial jobless claims can do the job. In terms of one-year ahead US real GDP growth, the best model with 5 predictors consists of NFCI, CPI, output gap, 10y-2y treasury yield spread, annualized quarterly growth of initial jobless claims whereas the simplest model is the same as for predicting one-quarter ahead growth distributions. It is also indicated that the left tails of the empirical distributions exhibit stronger dynamics than the right tails for all the best selected models, which is in line with the results from Adrian et al. (2016).

In terms of the UK data, the study reveals that quantile regression on near-term predictors combined with skewed-t distribution can be applied to predict both one-quarter ahead and one-year-ahead real GDP growth distributions. In more details, model comprising of FTSE realized volatility, unemployment rate, CPI, quarterly real GDP growth rate and 2-year bond yield provides the best result for predicting one-quarter growth distributions whereas FTSE realized volatility, and quarterly real GDP growth rate are proved to be the most important predictors in the simplest model with 2 variables. For the one-year ahead real GDP growth distributions, the simplest model that can be applied to predict density forecasts is the model with 3 variables: FTSE realized volatility, PNFC's effective loan rate, and quarterly real GDP growth rate. The most complicated model in this case consists of FTSE realized volatility, CPI, 10y-2y bond yield spread, PNFC's effective loan rate, and quarterly real GDP growth rate. It is also observed that the predicted distributions for one-quarter-ahead and one-year-ahead real GDP growth exhibits similar behaviors to what was observed in US data in terms of dynamics of the left tails and right tails.

The remainder of this study is organized as follows. Section 1 reviews empirical literature on forecasting the GDP distribution and paper motivation. Section 2 presents the data selection process and methodology, while Section 3 provides empirical results and further discussion. After that, the conclusion is made, followed by sections of references and list of abbreviations.

1. LITERATURE REVIEW

This section presents some important aspects of density forecasting, especially GDP growth distribution forecasting, in macroeconomics and finance. The section also provides several empirical researches performed to apply quantile regression to forecast GDP growth distribution in different countries over the previous periods, and explains how the idea of this paper is developed.

Macroeconomic modeling has always been associated with point forecasts, which provide no information on the uncertainties around the point prediction. Density forecasting is, therefore, introduced to compensate for this disadvantage of point forecasting in a way that density forecast provides a full picture of the uncertainty surrounding the point forecasts and that density forecast allows a researcher to derive a probability associated with an event. According to Zarnowitz (1969), density forecast first appeared in the survey of macroeconomic forecasters in the United States under the name of "ASA-NBER survey" in a joint effort of the American Statistical Association and the National Bureau of Economic Research. The survey is known as the Survey of Professional Forecasters nowadays. Under this survey, the forecasters are asked to provide density forecasts for inflation and output growth, together with other interested variables. Specifically, probabilities are required to be submitted for corresponding intervals of inflation or output growth values across the forecasters. The mean probability distributions are then published by averaging the probabilities provided by the respondents. Similar idea was adopted in the United Kingdom in 1996 to arrive at the density forecasts of inflation. The density forecasts in the UK are calibrated using two-piece normal distribution (Britton et al. 1998), which incorporates subjective judgement of the Monetary Policy Committee of the Bank of England by adjusting the degree of skewness of the distribution. Many other central banks around the world subsequently applied fan charts to communicate forecasting uncertainties: the Central Bank of Chile published fan charts of inflation and GDP growth since 2000 (Fornero et al. 2020); the National Bank of Poland applies fan chart to predict inflation or GDP growth distribution in a way that mode is used for central path with asymmetric distribution where variance of the current distribution is estimated based on historical errors and variance of conditioning variables; In Canada, densities' central path is derived from expected values whereas variance is calculated based on historical errors and expert judgement, etc. Practically, the concept of modeling distributions of important macroeconomic variables make its way to different stress testing exercises in banks such as CECL, ICAAP or IFRS. These stress testing exercises include different scenarios representing various conditions of the economy. With the support of forecasted distributions, banks can quantify the probabilities at which specific values of an interested variable will fall into and therefore, have appropriate actions of reserve or strategic policies.

In addition to the central banks' effort, literature space has evolved with a variety of researches on quantitative methods to predict conditional GDP distributions. Hamilton (1989) proposed a two-state Markov process to calculate the parameters of an autoregression model by employing maximum likelihood method. Occasional and discrete shifts are assumed for the mean GDP growth rates in this Markov process and the shape of the distribution is decided based on the behaviors of the growth series. Chen et al. (2006) suggested the idea of semi-parametric multivariate approach: different distributions are mixed by evaluating a parametric copula model at non-parametric marginal distributions.

Clark (2011) proved that adding stochastic volatility to Bayesian vector autoregression models can improve the accuracy of the density forecasts of different variables such as US GDP growth, inflation, and unemployment. Especially, a parametric approach using quantile regression introduced by Adrian et al. (2016) to predict the real GDP conditional distributions received huge attention from academic researchers. The authors chose NFCI to represent financial conditions of the economy and real GDP one-quarter growth as an economic variable to explain the behaviors of predicted one-quarter ahead and one-year ahead real GDP growth distributions. In their approach, quantile regression (Koenker and Bassett Jr 1978) was first used to estimate values corresponding to 5 different percentiles of the target density. The densities were then calibrated using skewed-t distribution and the estimated percentile values. The paper found that the left tails of the predicted distributions expressed significantly stronger dynamics over researched time than the right tails, which indicated higher variation in the downside risk to the US economy. The dynamics of the left tails were proved to be explained mainly by the NFCI, which confirmed the strong relationship between financial stability and overall macroeconomic conditions. Additional important finding from this paper was that the conditional mean of the predicted distribution had negative correlation with the conditional volatilities. In order to support for the use of NFCI in their model, the authors presented results for other models using several alternative financial variables like term spread, credit spreads or equity volatility and showed that NFCI produced better calibrated distributions. Although the framework proposed in this paper delivered different important insights, the paper failed to explore other combinations of different independent variables in the quantile regression models to provide more robust results for the chosen variables.

Quantile regression was additionally leveraged by Aikman et al. (2018) to measure the downside risks to UK GDP growth. In comparison to Adrian et al. (2016), this paper is different in 3 aspects: the calibration of predicted distributions, the characteristics of variables used, and the forecasting horizons. Specifically, the author applied quantile regression on 9 deciles, 5th and 95th percentiles and then, fitted kernel density functions on the generated quantile estimates. In terms of variables, the author created 3 composite measures representing private non-financial indebtedness, asset valuations, and household-credit terms. These composite measures were anticipated to contain more information on more-distant growth prospects than the variables used in Adrian et al. (2016). Finally, Aikman et al. (2018) attempted to predict the UK GDP growth distribution in 1, 4, 8, and 12 quarters ahead. The main purpose of the research was to quantify how much risk is in the left tails of the growth distributions given specific changes in the composite measures. However, it was not clear on how all the predicted distributions for those horizons evolved over time except for distributions at 4-quarter horizon where large variation in the left tail was observed over the researched periods.

Aikman et al. (2019) expanded the previous research to predict GDP growths at 5th percentile at 3 to 5 year horizons on a panel data of 16 advanced economies by employing quantile regression on different variables for credit, house prices, current account deficits, volatility, and financial conditions. The authors found that periods of high credit and house price growth, together with large current account deficits were associated with huge tail risk in 3 to 5 years ahead. Additionally, reduction in volatility was found to have small negative impact on medium-term GDP growths. Figueres and Jarociński (2020) applied similar technique for the aggregate euro area data to identify which variable was the most suitable to be used as a financial indicator to forecast the tail risks of European GDP growth: the Composite Indicator of Systemic Stress was proved to be the most informative financial indicator. The paper also observed the similarity in the movements of the lower quantiles and upper quantiles which were discussed for US real GDP growth; tightened financial conditions typically were followed by bad outcome of GDP growth; and negative relationship between conditional mean and conditional variance are also detected.

Therefore, this paper is strongly motivated to explore whether there exists a better combination of predictors to explain the behaviors of US one-quarter or one-year ahead real GDP growth other than the model suggested by Adrian et al. (2016). The author tries to achieve this purpose by running multiple models on a large set of financial and economic variables and comparing the results. Especially, the author wants to analyze the behaviors of the upper and lower quantiles of the chosen model to see if there is any difference than the behaviors obtained from US data and what factors can explain the observed behaviors. Additionally, the author aims to explore the power of the proposed framework of combining quantile regression and skewed-t distribution to predict the similar dependent variables for the UK data using near-term independent variables and check how the tails of the distributions behave in this case. With the aim to have more robust results, the paper adopts a 2-step out-of-sample testing process, where models with the lowest out-of-sample average error will be selected to the next step where the cumulative distribution functions of probability integral transforms are computed using predicted distributions in out-of-sample period to assess whether the calibrated distributions match with the unobserved data generating process.

2. DATA AND METHODOLOGY

In this section, firstly, data and variables for the United States and United Kingdom are presented. Secondly, quantile regression is introduced and details are provided on how quantile regression is employed to produce results for percentile values for one-quarter ahead and one-year ahead real GDP growth. Finally, methodologies for out-of-sample testing are discussed to evaluate the empirical results and to choose the best models.

2.1. Data and Variables

All data series used for models in this paper are quarterly data and end in December, 2021. However, the starting points of the series vary across different variables due to the availability of the data. Starting points of the series can be found in Table 1 below together with other summary statistics.

Table 1: The descriptive statistics of US variables

Statistic	Min	1st quartile	Median	3rd quartile	Max	Median	SD	Start	End	Pub freq
r_gpd_1q_gr_ahead	−35.944	1.292	3.054	5.145	29.915	3.054	4.661	1949Q1	2021Q3	
r_gpd_4q_gr_ahead	−9.033	1.814	3.117	4.610	13.366	3.117	2.750	1949Q1	2020Q4	
NFCI	−1.029	−0.625	−0.371	0.146	4.505	−0.371	0.984	1971Q1	2021Q4	Weekly
URATE	2.500	4.500	5.500	6.725	11.000	5.500	1.664	1948Q1	2021Q4	Monthly
realized_vol_SNP	1.669	4.117	5.197	6.908	29.371	5.197	3.231	1947Q1	2021Q4	Daily
CPI	−11.314	1.601	3.233	4.874	15.803	3.233	3.255	1960Q1	2021Q4	Montly
output_gap	−10.536	−2.082	−0.734	0.873	5.687	−0.734	2.482	1949Q1	2021Q4	Quarterly
r_gdp_1q_gr	−35.944	1.292	3.054	5.145	29.915	3.054	4.661	1949Q2	2021Q4	Quarterly
spread_10Y_2Y	−1.990	0.255	0.880	1.545	2.820	0.880	0.895	1976Q2	2021Q4	Daily
spread_10Y_3M	−0.770	0.795	1.725	2.618	3.850	1.725	1.116	1981Q3	2021Q4	Daily
ICSA	185,385	300,981	344,577	402,827	2,729,308	344,577	194,466	1967Q1	2021Q4	Weekly
ICSA_gr	−247.208	−17.902	−2.188	14.953	1,072.485	−2.188	101.176	1967Q2	2021Q4	Weekly

Note: Descriptive statistics are calculated for all variables for the periods from "Start" to "End". All series are presented in percentage except for NFCI and ICSA which are in decimal. "Pub freq": Publication frequency.

2.1.1. US

US one-quarter ahead and one-year ahead real GDP growth are the dependent variables for US case. Additionally, there are 10 independent variables that are used to produce different forms of models to estimate the dependent variables, which are described in details below.

National Financial Condition Index (NFCI): The National Financial Condition Index is a measure of U.S. financial conditions which incorporates a broad set of measures in risk, liquidity, and leverage. The index is collected from the website of the Federal Reserve Bank of Chicago and is constructed as a weighted average of 105 measures of financial activity, which is described in details by Brave and Butters (2018). Generally, the weights are calculated using Principal Component Analysis to incorporate the cross-correlations of the component indicators and historical dynamics of measures. Historically, positive values of NFCI indicates that the financial condition is tighter than average, whereas looser-than-average financial conditions are associated with negative NFCI values. The data for NFCI starts from 1971 and is provided on a weekly basis. The weekly series is turned into quarterly in this paper's model estimation by calculating the average value of weekly values in a specific quarter. Especially, if a week starts in this quarter and ends in the next quarter, the NFCI value of that week is assigned to this quarter for the average calculation. For example, if a week ranges from March 28, 2022 to April 3, 2022, this week is considered to be in March. As a result, NFCI value of this week is included in the average calculation of Q1 instead of Q2.

Unemployment rate (URATE): The unemployment rate is collected from the website of Federal Reserve Bank of St. Louis, which originates from the "Current Population Survey (Household Survey)". The series starts in 1948Q1 on monthly basis and is seasonally adjusted. In order to create the quarterly series, the unemployment rate as of the last month of a quarter is selected.

Realized volatility of S&P index (realized_vol_SNP): The S&P500 index is constructed by using the adjusted close values extracted from Yahoo Finance, which is available from 1947Q1. The annualized volatilities for corresponding quarters are then calculated from this series based on the following formula:

$$Realized\ volatility = \sqrt{4\sum_{i=1}^{N} r_t^2} \tag{1}$$

where N is the number of daily returns in a specific quarter and r_t is the daily return of S&P 500 index calculated as:

$$r_t = log(S\&P_t) - log(S\&P_{t-1}) \tag{2}$$

Consumer price index (CPI): The consumer price index is downloaded from the website of Federal Reserve Bank of St. Louis. The data is expressed in terms of quarterly growth rate for total all items in the United States and is not seasonally adjusted. The data starts in 1960Q1.

Output gap (output_gap): The output gap measures the difference between the actual output of an economy and its potential output which is the maximum amount of goods and services an economy can produce when it is at full capacity. The quarterly output gap is collected through the "Budget and Economic Data" section in the webpage of Congressional Budget Office and starts from 1949Q1.

One-quarter real GDP growth (r_gdp_1q_gr): The real GDP growth is extracted from the "Budget and Economic Data" section in the webpage of Congressional Budget Office. The growth series is available since 1949Q2. The one-quarter real GDP growth is then calculated based on this series and annualized.

Term spread (spread_10Y_2Y & spread_10Y_3M): The term spread is generally defined as the difference between the interest rates of 2 government securities with different maturities. In this paper, 2 measures of term spread for the US are calculated: 10y-2y spread and 10y-3m spread. The 10y-2y spreads are the difference between the market yields on U.S. Treasury Securities at 10-year constant Maturity and at 3-year constant maturity. The calculation of 10y-3m spread is performed in the same fashion. The data for 10-year, 2-year, 3-month yields are daily series extracted from the website of Federal Reserve Bank of St. Louis and start from the second quarter of 1976. The quarterly data of these series are drawn by taking the yield as of the last working date of a corresponding quarter.

Initial jobless claims (ICSA & ICSA_gr): Initial jobless claim allows an unemployed worker to be eligible for unemployment insurance in the United States. This series is constructed on weekly basis from 1967Q1 and is extracted from the website of Federal Reserve Bank of St. Louis. The quarterly level series (ICSA) are calculated by averaging the weekly claims in a specific quarter. If a specific week starts in this quarter and ends in the next quarter, the weekly number is assigned to this quarter for the average calculation. The growth series is subsequently constructed from the level series by computing the annualized quarter-on-quarter growth of the average claims.

2.1.2. UK

UK one-quarter ahead and one-year ahead real GDP growth are the dependent variables for UK data. In order to have models forecasting these 2 dependent variables, 7 independent variables are used, which are described in details below. Table 2 summarizes different information on UK variables.

Table 2: The descriptive statistics of UK variables

Statistic	Min	1st quartile	Median	3rd quartile	Max	Median	SD	Start	End	Pub freq
r_gpd_1q_gr_ahead	−77.713	0.634	2.345	3.931	70.383	2.345	7.631	1955Q1	2021Q3	
r_gpd_4q_gr_ahead	−21.138	1.591	2.341	3.532	24.493	2.341	3.162	1955Q1	2020Q4	
realized_vol_FTSE	3.218	4.810	5.963	7.679	25.337	5.963	3.349	1984Q1	2021Q4	Daily
URATE	3.400	5.000	5.900	8.300	11.900	5.900	2.369	1971Q1	2021Q4	Quarterly
CPI	−2.822	1.568	3.567	6.278	37.804	3.567	5.451	1960Q1	2021Q4	Monthly
r_gdp_1q_gr	−77.713	0.634	2.345	3.931	70.383	2.345	7.631	1955Q2	2021Q4	Quarterly
spread_10Y_2Y	−1.956	0.020	0.861	1.599	4.042	0.861	1.099	1970Q1	2021Q4	Daily
eff_loan_rate	2.440	3.255	7.508	11.870	19.210	7.508	4.636	1977Q2	2021Q4	Daily
loan_2Y_spread	−1.027	1.606	2.289	2.911	6.156	2.289	1.174	1977Q2	2021Q4	Daily

Note: Descriptive statistics are calculated for all variables for the periods from "Start" to "End". All series are presented in percentage except for NFCI and ICSA which are in decimal. "Pub freq": Publication frequency.

Realized volatility of FTSE (real_vol_FTSE): The FTSE index is established by taking the adjusted close values extracted from Yahoo Finance, which is available from January, 1984. The annualized volatilities for corresponding quarters are then calculated from this series based on the following formula:

$$Realized\ volatility = \sqrt{4\sum_{i=1}^{N} r_t^2} \tag{3}$$

where N is the number of daily returns in a specific quarter and r_t is the daily return of FTSE index calculated as:

$$r_t = log(FTSE_t) - log(FTSE_{t-1}) \tag{4}$$

Unemployment rate (URATE): The UK unemployment rate is collected from the website of UK's Office for National Statistics. The series starts from the first quarter of 1971 on quarterly basis and is seasonally adjusted.

Consumer price index (CPI): The consumer price index is downloaded from the website of Federal Reserve Bank of St. Louis. The data is expressed in terms of quarterly growth rate for total all items in the United Kingdom and not seasonally adjusted. The data starts from January, 1960.

One-quarter real GDP growth (r_gdp_1q_gr): The real GDP growth is extracted from the website of Federal Reserve Bank of St. Louis. The growth series is available since 1955Q2. The one-quarter real GDP growth is then calculated from this series and annualized.

Term spread (spread_10Y_2Y): The term spread for the UK in this paper is calculated as the difference between yields of 10-year government bonds and 2-year government bonds whose data are extracted from the website of Bank of England. The quarterly data for 10y-2y spread is constructed by taking the yield as of the last working date of a corresponding quarter and starts from 1970Q1.

PFNC's effective loan rate (eff_loan_rate): According to the Bank of England, PNFC's effective loan rate is the monthly average interest rates of loans that financial institutions offers to resident and private non-financial corporations. This rate has its history since 1977Q2. The quarterly data for this series is constructed by taking the rate of the last month of a corresponding quarter.

Spread between effective loan rate and 2y bond yield (loan_2y_spread): The spreads between the PNFC's effective loan rates and the 2-year government bond yields are also calculated to create additional predictor for UK models. The data is available since 1977Q2 based on the history of the component series.

2.2. Methodology

The methodology section explains in details specific methods used to predict the density forecasts for the dependent variables and how to select the best models among generated models.

2.2.1. Model estimation

For the purpose of predicting the GDP growth distributions, the paper first applies quantile regression to forecast the one-quarter-ahead and one-year-ahead GDP growth values for corresponding chosen percentiles (5th, 25th, 50th, 75th, and 95th percentiles) for every quarter in out-of-sample periods. The predicted percentile values, subsequently, serve as inputs to calibrate the forecasted densities using skewed-t distributions.

Quantile regression

Quantile regression was introduced by Koenker and Bassett Jr (1978) to model different points in a specific conditional distribution rather than the conditional mean. Unlike linear regression in which the model parameters are chosen using ordinary least squares (OLS), in quantile regression, the optimizer is performed in a different way where the quantile weighted absolute value of errors is minimized. Additionally, the weights are different depending on whether the value of the error term is larger or smaller than the interested quantile. The idea of quantile regression can be expressed mathematically as below.

Let x and β be the vectors of independent variables and model parameters. The equation for estimating a τ_{th} quantile value with quantile regression with the n number of estimates is:

$$Q_\tau(y_i) = \beta_0(\tau) + \beta_1(\tau)x_{i1} + ... + \beta_p(\tau)x_{ip} \qquad i = 1, 2, ..., n \tag{5}$$

The model parameters of quantile regression are estimated by minimizing the mean absolute deviation (MAD):

$$weighted\ MAD = \frac{1}{n}\sum_{i=1}^{n}\rho_\tau(u_i) \tag{6}$$

where u is the error term and :

$$\rho_\tau(u) = \tau \max(u, 0) + (1 - \tau)\max(-u, 0) \tag{7}$$

Davino et al. (2013) reaffirms that quantile regression is robust to outliers or existence of heteroskedasticity in data. However, the book also mentions that the existence of serial correlation can lead to incorrect inference of model results although model estimates are not biased. Additionally, Davino et al. (2013) shows that limit number of distinct quantiles should be used in quantile regression if dataset is small to provide noticeable differences in coefficient estimates.

Skewed-t distribution

The densities for the dependent variables are assumed to follow skewed-t distribution which was developed by Azzalini and Capitanio (2003). Therefore, after the above-mentioned percentile values are estimated for the real GDP growth rates, these values are treated as inverse cumulative distribution functions to estimate the parameters of the corresponding skewed-t distributions.

Let y, μ, σ, α, ν be the interested dependent variable, location parameter, scale parameter, skewness parameter, and kurtosis parameter of the skewed-t distribution, respectively. The probability distribution function under skewed-t distribution is calculated as follows:

$$f(y; \mu, \sigma, \alpha, \nu) = \frac{2}{\sigma} t(\frac{t-m}{\sigma}; \nu) T(\alpha \frac{y+\mu}{\sigma} \sqrt{\frac{v+1}{\nu + \frac{y+\mu}{\sigma}}}; \nu + 1) \tag{8}$$

where $t(.)$ and $T(.)$ are the probability distribution function and cumulative distribution function of Student-t distribution, correspondingly. The above equation shows that the skewed-t distribution is drawn from the Student-t distribution where its shape get adjusted by its own cumulative distribution and rescaled by its skewness parameter.

In order to estimate the set of parameters (μ, σ, α, ν) for the skewed-t density for each quarter, the total of squared distances between each quantile function estimated in the previous step and the quantile function of the skewed-t distribution is minimized. The quantile functions selected in the distribution calibration process are for 5th, 25th, 75th, and 95th. The total of squared distances are described in the formula below.

$$Total\ squared\ distances = \sum_\tau (\hat{Q}(\tau|x_i) - F^-1(\tau; \mu, \sigma, \alpha, \nu))^2 \tag{9}$$

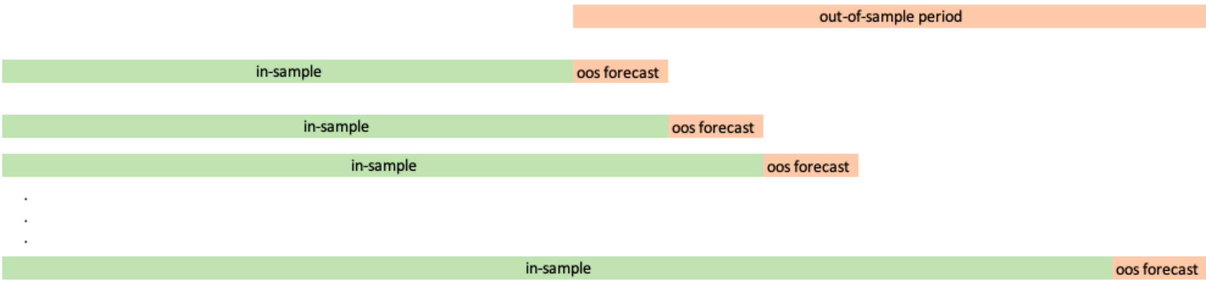
where τ has values of 0.05, 0.25, 0.5, 0.75, and 0.95.

2.2.2. Out-of-sample testing

In order to perform out-of-sample testing, out-of-sample forecasts are produced dynamically by using models estimated on rolling in-sample data. The process is presented in details in Figure 1. Specifically, the out-of-sample period is fixed from 1996Q3 to

2021Q3 for predicting one-quarter-ahead growth and 2020Q4 for one-year-ahead growth for both US and UK data. The first in-sample data starts from the point where all the independent variables of a model form are available and ends in 1996Q2. Then, the first out-of-sample forecasts for all percentiles are calculated using the models estimated on this first in-sample data. The next in-sample period is constructed by adding 1 quarter to the previous in-sample data and consequently, the next out-of-sample forecasts for all percentiles are produced by the new models estimated on this new in-sample data. This process continues until the out-of-sample forecasts are estimated for all percentiles for the final quarter in the out-of-sample period.

Figure 1: Rolling process of in-sample estimation and out-of-sample forecast



Note: The main reason for such a procedure, in comparison to simple division on one in-sample and one out-of-sample period, is the possibility to analyze very long total out-of-sample period which is built with the forecasts from the models which use the most recent macroeconomic data.

By using all the out-of-sample forecasts generated during the above process, the out-of-sample testing process is performed in 2 steps: firstly, "average error" is calculated for all models in out-of-sample period; in the second step, models with the lowest average errors from the previous step are chosen to calibrate skewed-t distributions for every quarter from 1996Q3 to 2021Q3 for one-quarter-ahead growth or to 2020Q4 for one-year-ahead growth using the methodology described above and the cumulative distribution functions (CDFs) of probability integral transforms (PITs) are computed. The calculation of "average error" and the CDF of PITs are described in the following subsections.

Average error

As described above, for each quarter in the out-of-sample period, quantile regression models are estimated for all 5 percentiles: 5th, 25th, 50th, 75th, and 95th using the rolling in-sample data. Then, GDP growth values for the 5 percentiles for the next quarter are forecasted using those 5 regression models. Leveraging these forecasts, in this step, the absolute deviation is firstly calculated for each quantile as the absolute difference between the estimated value from regression model and realized value. The absolute deviations of all 5 quantiles (5th, 25th, 50th, 75th, and 95th) are then averaged for each quarter. This process is repeated on all the out-of-sample forecasts to calculate the average absolute deviations for all the quarters in the out-of-sample period. These average absolute deviations are then averaged to have the mean value, which is hereafter defined as average error.

$$Average\ error = \frac{1}{P}\sum_{i=1}^{P}(\frac{1}{5}\sum_{\tau}(|\hat{Q}(\tau|x_i) - y_i|)) \tag{10}$$

where τ has values of 0.05, 0.25, 0.5, 0.75, and 0.95, and P is the number of out-of-sample observations starting from 1996Q3 to 2021Q3 for one-quarter-ahead growth and to 2020Q4 for one-year-ahead growth.

Cumulative distribution function of probability integral transform

The concept of probability integral transform was introduced by Diebold et al. (1998) to evaluate the reliability of a forecasted density. In other words, the PITs serve as a tool to check whether the predicted distribution of an interested variable is statistically close to the underlying data generating process. The probability integral transform, by its own definition, is a cumulative density function corresponding to a conditional predictive distribution evaluated at the realized value of the dependent variable. Rossi and Sekhposyan (2016) expanded this concept by calculating the empirical CDF of PITs and compare with the computed confidence bands to assess the correctness of the calibration of the whole predicted distribution. The predicted distribution is considered to be correctly specified if the empirical CDF of PITs lies within the proposed confidence bands. Additionally, the closer the empirical CDF of the PITs to the diagonal line, which plots the CDF of PITs of a uniform distribution, is, the higher reliability that the forecasted densities can offer.

This work follows the process described by Rossi and Sekhposyan (2016) to assess the reliability of the prediction distributions. Specifically, using the rolling process described in Figure 1, the author first uses the quantile regression to estimate the models for the selected quantiles (5th, 25th, 75th, and 95th) for the first in-sample data. The skewed-t growth distribution is then calibrated for 1996Q3 in case of one-quarter ahead growth and for 1997Q2 in case of one-year ahead growth by using the estimated growth values from the quantile models for the mentioned quantiles. Next, the PIT is calculated using the calibrated distribution based on the following formula:

$$PIT = \int_{-\infty}^{y_t} \phi_t \, dy \tag{11}$$

where ϕ_t[1] is the calibrated predictive distribution for the dependent variable at time t and y_t is the realized value of the dependent variable at time t.

This process is iterated by expanding the in-sample data one quarter at a time until the end of the data (2021Q3 for one-quarter ahead and 2020Q4 for one-year ahead). Subsequently, the empirical CDF is computed on the series of PITs calculated for all the quarters in the out-of-sample period. Finally, the confidence bands are calculated using the formula proposed by Rossi and Sekhposyan (2017):

$$Confidence\ band = r \pm K_\alpha \sqrt{P} \tag{12}$$

where r is the corresponding CDF of the uniform distribution, K_α is the critical value extracted from Rossi and Sekhposyan (2017), and P is the number of out-of-sample estimates.

Out-of-sample testing plays a critical role in selecting better models for US case and in identifying good models for UK case to forecast one-quarter and one-year ahead

[1] ϕ_t can be any type of distribution. In this work, ϕ_t is of skewed-t distribution.

growth distributions. A model is considered to be better than the benchmark model if it has higher predictive power (measured by lower "average error") and the model's CDF of PITs is closer to the diagonal line than that of benchmark model. Likewise, a model is defined as a good model if it has reasonable "average error" and its CDF of PITs is close to the diagonal line.

3. EMPIRICAL RESULTS AND DISCUSSION

This section firstly analyzes the correlations between the independent variables and predictors in US and UK data sets. Secondly, estimated quantile models are discussed with analysis performed on model results. Finally, out-of-sample testing results are presented with the focus on choosing the best models explaining the underlying data generating process.

3.1. Correlation analysis

This subsection can provide some initial ideas on how different variables used in researched models can interact. However, the relationship between the variables can be different on specific quantile regressions corresponding to chosen quantiles.

3.1.1. US Variables

The correlations between US dependent variables and different explanatory variables are plotted in Figure 2 and Figure 3. It can be seen in Figure 2 that the dependent variable r_gdp_1q_gr_ahead has negative correlation of -0.3 with realized_vol_SNP, which is the realized volatility of S&P500 index. The next highly correlated variables are ICSA, ICSA_gr and NFCI with correlations of 0.29, -0.28, and -0.22, respectively. Figure 3 shows a slightly different story where ICSA has the strongest correlation of 0.4 with r_gdp_4q_gr_ahead, followed by output_gap, URATE, spread_10y_3m, and NFCI. Among the independent variables, output_gap and URATE are very highly correlated, with correlation of -0.91; spread_10y_3m and spread_10y_2y also have very significant positive correlation of 0.84. These highly correlated relationships might potentially impact the predicted results. The author wants to include these highly correlated variables in the set of investigated variables to check if there will be any risk to model results of quantile regression if highly correlated variables appear in the same model form. It is also surprising to notice that ICSA and ICSA_gr have opposite relationships with r_gdp_1q_gr_ahead while they seem have similar directional impact on r_gdp_1q_gr.

Figure 2: Correlations between US 1-quarter ahead real GDP growth and predictors

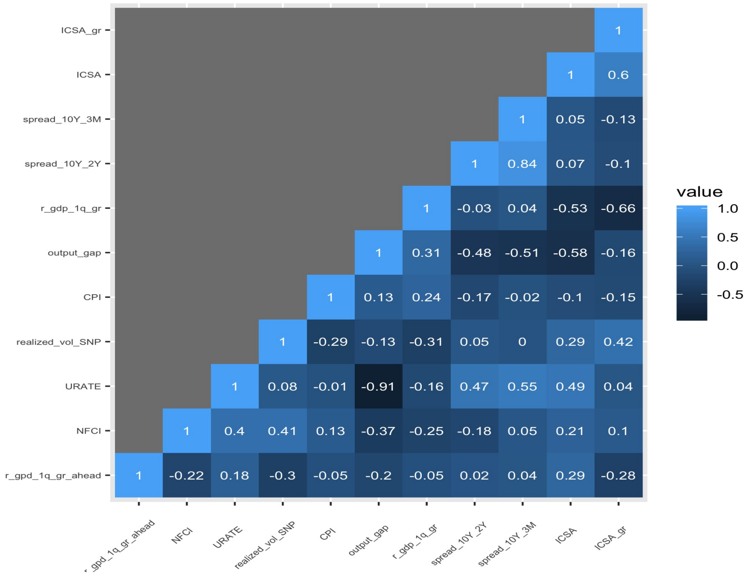
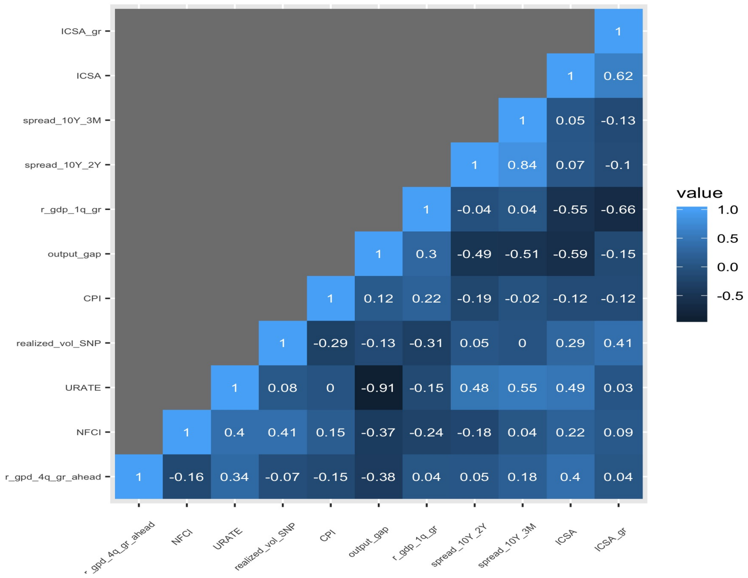


Figure 3: Correlations between US 4-quarter ahead real GDP growth and predictors



3.1.2. UK Variables

Figure 4 and Figure 5 plot the correlations between UK dependent variables and their predictors. It is noticed that r_gdp_1q_gr and realized_vol_FTSE are the most correlated variables with r_gdp_1q_gr_ahead with the correlations of -0.31 and -0.18 respectively. For the relationship with r_gdp_4q_gr_ahead , URATE, r_gdp_1q_gr, and loan_2Y_spread are the most connected variables with the correlations of 0.26, -0.22, and -0.19 respectively. For the UK data, the highest correlation is observed between effective_loan_rate and r_gdp_1q_gr, in which the correlation is -0.72.

Figure 4: Correlations between UK 1-quarter ahead real GDP growth and predictors

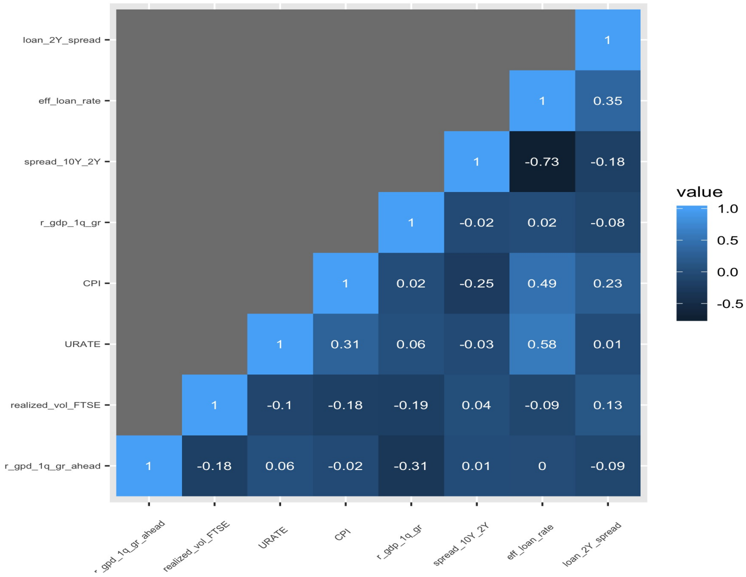
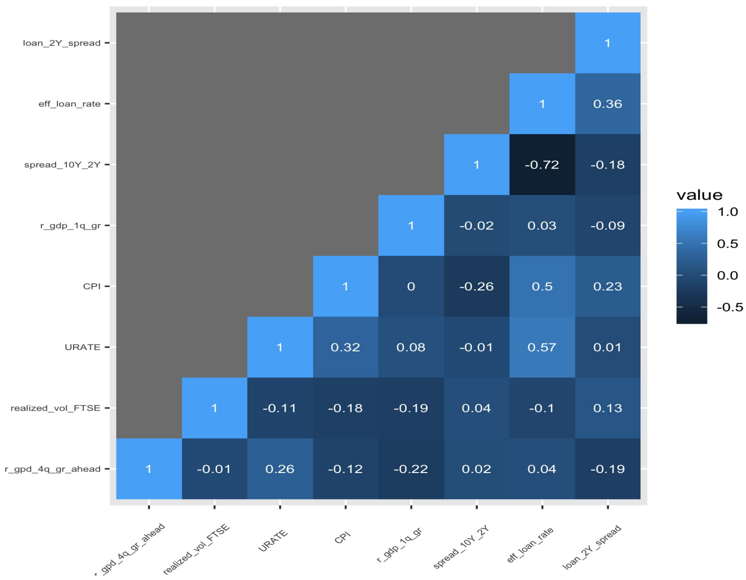


Figure 5: Correlations between UK 4-quarter ahead real GDP growth and predictors



3.2. US results

The main purpose of this sub-section is to investigate whether there exist better models than the models proposed by Adrian et al. (2016), hereinafter called "benchmark models", in which there are two independent variables: NFCI as a financial indicator and r_gdp_1q_gr as an economic indicator. In such cases, the author would like to check if there is any difference between the behaviors in the tails of the better models and those of the benchmark models.

Using the pool of 10 proposed independent variables, the author attempts to check all possible combination of these predictors where each model consists of maximum 5 independent variables. Therefore, 1274[2] models, in total, are investigated to find the best models to predict the US one-quarter ahead and one-year-ahead real GDP growths. The best models are searched in 4 categories: model with 2 predictors, model with 3 predictors, model with 4 predictors, and model with 5 predictors. The purpose of this selection is to investigate whether model with higher complexity can improve the forecasted results or if simpler model can offer equivalent results with lower maintenance cost.

3.2.1. One-quarter ahead real GDP growth

Following the procedure described in "Methodology" section, the 4 best models presented in Table 3 below have the lowest average errors and therefore, are chosen to the second phase of out-of-sample testing process to be compared with the benchmark model. By its definition, model "best 5" is the best model that is chosen among all the models that have 5 predictors drawn from the pool of mentioned 10 variables. Model "best 4", "best 3", and "best 2" are defined in the same way. As can be seen in Table 3, all chosen models have lower "average error" than the "average error" of the benchmark model. Moreover, all best models include NFCI and the annualized growth rate of initial jobless claims. Additionally, the "best 5" and "best 4" models contain 2 highly correlated variables: spread_10Y_2Y + spread_10Y_3M.

Table 3: Best models to predict US 1-quarter-ahead real GDP growth

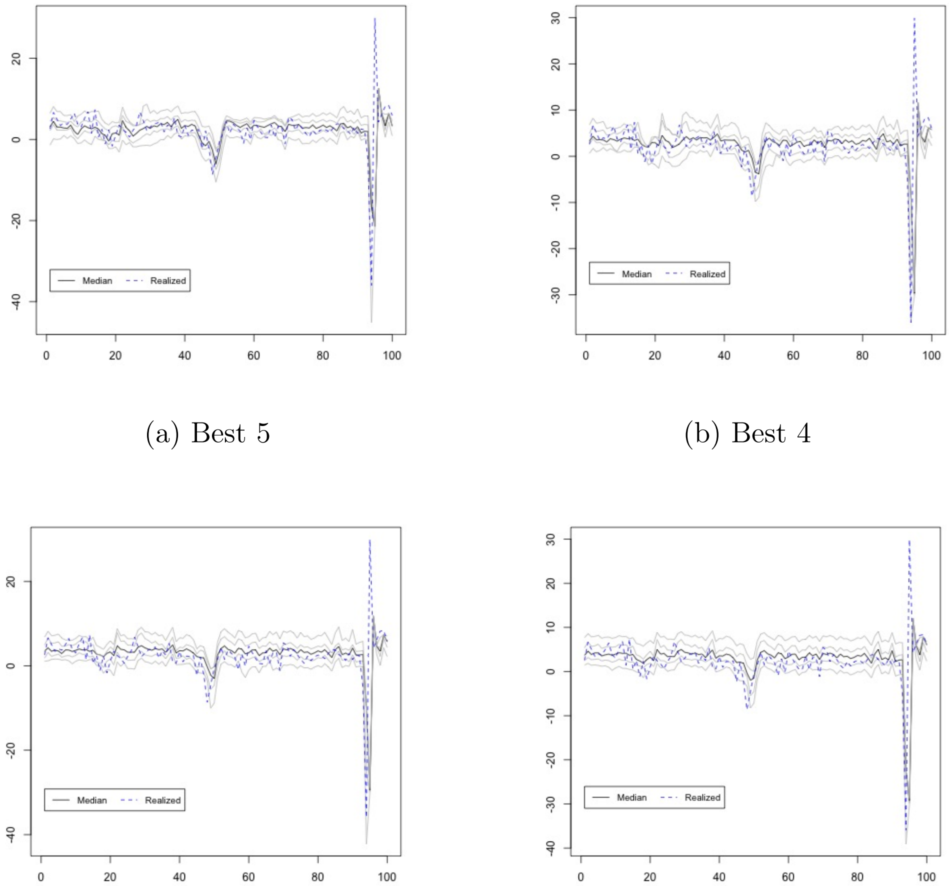
Model name	Model form	Average error
Benchmark	r_gpd_1q_gr_ahead ~ NFCI + r_gdp_1q_gr	3.581%
Best 5	r_gpd_1q_gr_ahead ~ NFCI + output_gap + spread_10Y_2Y + spread_10Y_3M + ICSA_gr	3.022%
Best 4	r_gpd_1q_gr_ahead ~ NFCI + spread_10Y_2Y + spread_10Y_3M + ICSA_gr	3.046%
Best 3	r_gpd_1q_gr_ahead ~ NFCI + spread_10Y_3M + ICSA_gr	3.235%
Best 2	r_gpd_1q_gr_ahead ~ NFCI + ICSA_gr	3.426%

Note: Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing.

Figure 6 plot the out-of-sample predictions of the researched quantiles: 5th, 25th, 50th, 75th, and 95th for US one-quarter-ahead real GDP growth together with the realized time series in the out-of-sample period starting from 1996Q3. It can be seen from the plot that all the median series predicted using the chosen models follow the trends of realized values, especially during crisis times. Specifically, during the episodes of financial crisis 2007-2008 and the covid period, the models can predict almost in real-time the 1-quarter-ahead real GDP growth. Additionally, it is observed that the distributions' left tails express more strong dynamics than the right tails where values tend to be more stable before the covid happened (RQ3), which is in line with the observation from Adrian et al. (2016) on the benchmark model and Figueres and Jarociński (2020) on model for EU data. The "best 4" model contains both spread_10Y_2Y and spread_10Y_3M but produces good results. This result indicates that highly correlated variables in quantile regression do not have serious impact on model results.

[2] 1274 models include 2 sets of 637 models generated to predict each dependent variable: C_{10}^1 models with 1 independent variable, C_{10}^2 models with 2 independent variables, C_{10}^3 models with 3 independent variables, C_{10}^4 models with 4 independent variables, C_{10}^5 models with 5 independent variables.

Figure 6: Out-of-sample plotting - US one-quarter-head real GDP growth



(a) Best 5

(b) Best 4

(c) Best 3

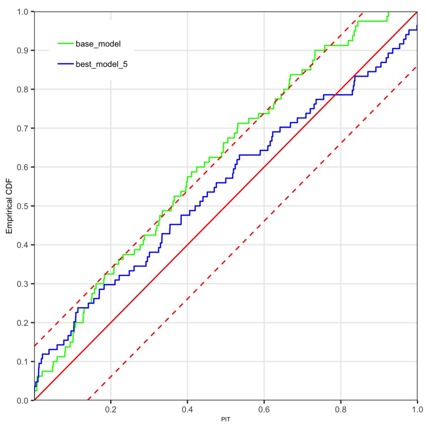
(d) Best 2

Note: The figures plot estimated values corresponding to 5 percentiles (5th, 25th, 50th, 75th, 95th) of the US 1-quarter-ahead real GDP growth density forecast together with realized GDP growths over time. The estimated values are calculated from quantile regression models for those percentiles with respective independent variables used in each model. Details of the estimated process can be found in Section 2.2.2. Out-of-sample testing. Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing.

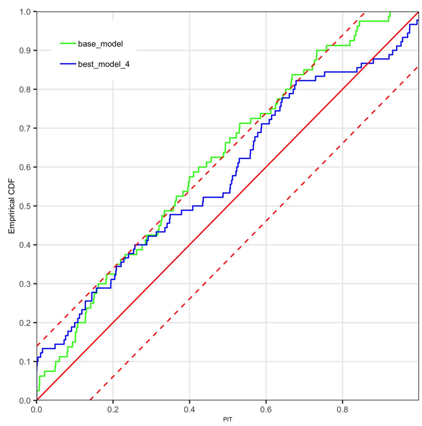
Figure 7 then plots the empirical CDFs of PITs of all the chosen models in Table 3 in comparison with that of benchmark model. The dash lines are the visualization of the confidence bands whose calculation is described in "Methodology" section. It is clearly seen that the empirical CDFs of "best 5", "best 4", and "best 3" lie within the bands and much closer than the benchmark model to the diagonal line which is the CDF of a uniform distribution. Additionally, the "best 2" model is also closer to the diagonal line in terms of the area under the line. These observations prove that the chosen models produce results that better describe the unobserved data generating process of the one-quarter real GDP distribution, which means that better models than the benchmark one do exist (RQ1). Additionally, the "best 2" model indicates that annualized growth of initial jobless claims serves better as an economic indicator than the annualized 1-quarter real GDP growth.

Observations from Figure 6 and Figure 7 together indicate that practically, the "best 3" model with lower complexity can be applied to predict the growth distributions as Figure 6 shows that predicted median series follows the trend of realized values very closely and Figure 7 shows that the CDF of PITs of predicted distributions lies very well within the confidence bands. Additionally, if simplicity is the most important factor to reduce all costs related to maintaining and operating of the model, "best 2" model can be chosen (RQ7) as the median series is close to the series of realized values and has acceptable CDF of PITs where it generally is closer to the diagonal line than the CDF of PITs of the benchmark model. Additionally, if the publication frequency of explanatory variables are considered, "best 4", "best 3", and "best 2" models can be chosen to predict one-quarter-ahead US growth distributions as these models include variables published in very high frequencies (RQ8).

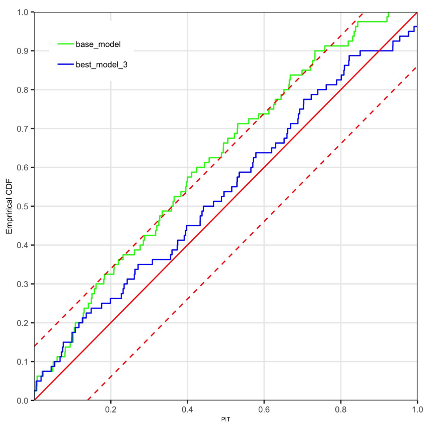
Figure 7: CDFs of PITs - US one-quarter-head real GDP growth



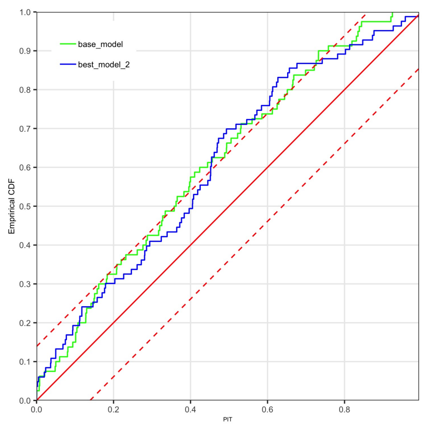
(a) Best 5



(b) Best 4



(c) Best 3



(d) Best 2

Note: The figure shows the empirical CDF of the PITs (solid lines), the theoretical CDF of the PITs (the 45 degree line) and the 5 % critical values bands based on the table reported in Rossi and Sekhposyan (2017). The process of arriving at empirical CDF of PITs for each model is described in details in Section 2.2.2. Out-of-sample testing. Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing. "base_model" is the benchmark model with NFCI and r_gdp_1q_gr as predictors.

3.2.2. One-year ahead real GDP growth

For predicting US one-year-ahead real GDP growth, the best four models in Table 4 are chosen using the "average error" criteria. Except for "best 2" model whose model form is the same as the form of the "best 2" model chosen for predicting one-quarter ahead growth, the model forms of remaining selected models are different with the presence of output gap. However, NFCI and ICSA_gr are also included in all chosen model specifications. Table 4 also shows that the four best models have lower "average error" than that of benchmark model.

Table 4: Best models to predict US one-year-ahead real GDP growth

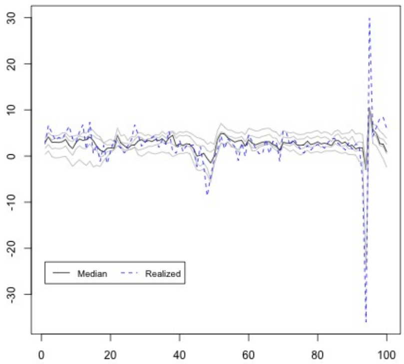
Model name	**Model form**	**Average error**
Benchmark	r_gpd_4q_gr_ahead ∼ NFCI + r_gdp_1q_gr	2.338%
Best 5	r_gpd_4q_gr_ahead ∼ NFCI + CPI + output_gap + spread_10Y_2Y + ICSA_gr	2.037%
Best 4	r_gpd_4q_gr_ahead ∼ NFCI + output_gap + spread_10Y_2Y + ICSA_gr	2.146%
Best 3	r_gpd_4q_gr_ahead ∼ NFCI + spread_10Y_3M + ICSA_gr	2.229%
Best 2	r_gpd_4q_gr_ahead ∼ NFCI + ICSA_gr	2.312%

Note: Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing.

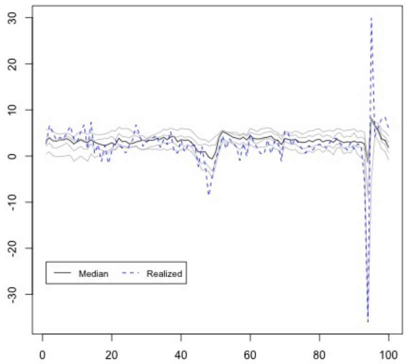
The plots for all predicted quantiles for one-year ahead growth in Figure 8 convey relatively similar story to what is observed in the plots for 1-quarter growth's quantiles that the upper quantiles express stronger movements than the right tails and the median series follow quite strongly the trend of the realized growth values. However, it is noticed that the models predicted not very well post covid period in comparison with the power of the best model in the case of 1-quarter-ahead forecast, which is confirmed by looking at Figure 9. In general, Figure 9 confirms that better models than the benchmark model are detected to predict the one-year ahead growth (RQ2) as the empirical CDFs of PITs of these models are much closer to the diagonal line and their "average errors" are lower than that of benchmark model. However, the results in Figure 9 are worse than results for one-quarter ahead as the empirical CDFs many times cross the confidence bands. It might be unintuitive to notice that the "average errors" of the best models for one-year-ahead growth forecasting are lower than those of the best models for one-quarter-ahead growth prediction while having worse results indicated by out-of-sample plotting and CDFs of PITs. However, one-quarter-ahead growth series and one-year-ahead growth series are totally 2 different series as shown in Table 1, where it is observed that the one-quarter-ahead growth series has much higher standard deviation and has different statistics. Figure 6 and Figure 8 also plot the realized values of one-quarter and one-year ahead series, in which the difference between 2 series can quickly be observed.

The answer is also "yes" to RQ7 when it comes to choosing the model with the highest simplicity - "best 2" model - to predict one-year-ahead US real GDP growth distributions as result of "best 2" model is better than that of the benchmark model and "best 2" model has short model form. Among the best models, there are 2 models - "best 3" and "best 2" models - comprise of only high-frequency variables, which can be leveraged if timeliness of forecasts is taken into account (RQ8).

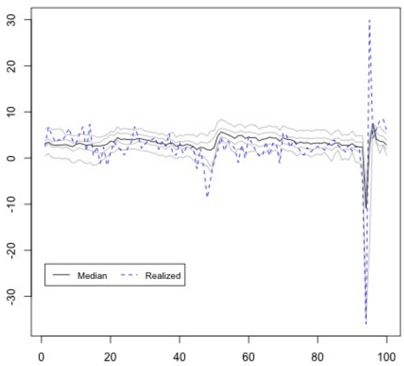
Figure 8: Out-of-sample plotting - US one-year-ahead real GDP growth



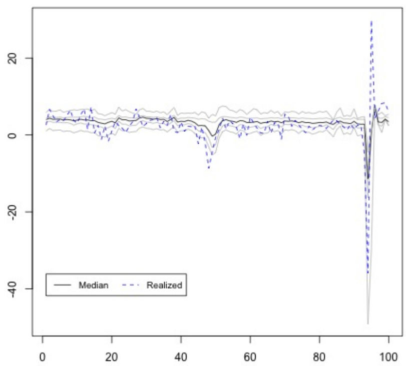
(a) Best 5



(b) Best 4



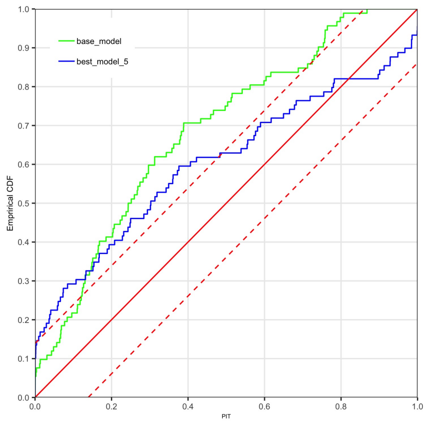
(c) Best 3



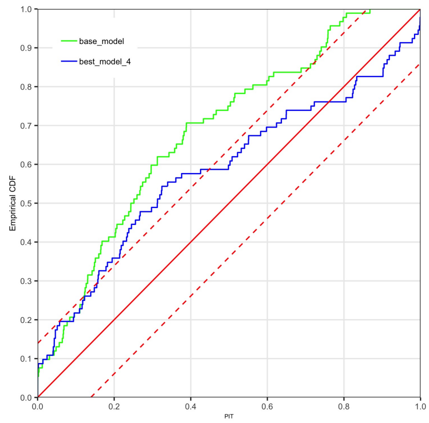
(d) Best 2

Note: The figures plot estimated values corresponding to 5 percentiles (5th, 25th, 50th, 75th, 95th) of the US one-year-ahead real GDP growth density forecast together with realized GDP growths over time. The estimated values are calculated from quantile regression models for those percentiles with respective independent variables used in each model. Details of the estimated process can be found in Section 2.2.2. Out-of-sample testing. Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing.

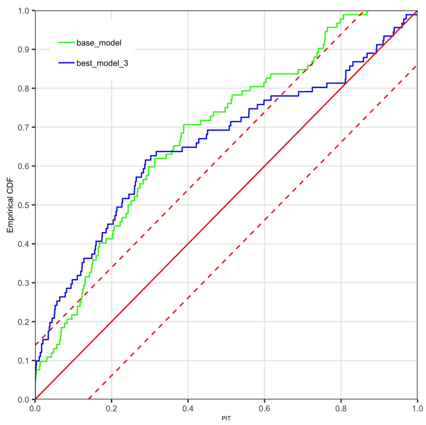
Figure 9: CDFs of PITs - US one-year-head real GDP growth



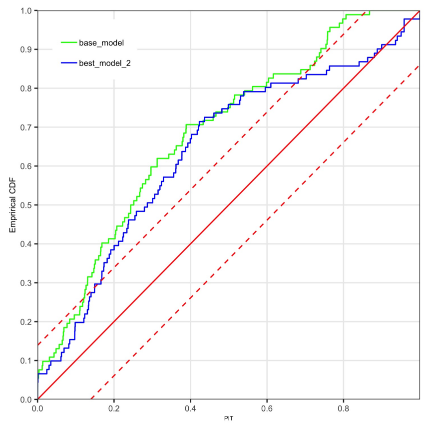
(a) Best 5



(b) Best 4



(c) Best 3



(d) Best 2

Note: The figure shows the empirical CDF of the PITs (solid lines), the theoretical CDF of the PITs (the 45 degree line) and the 5 % critical values bands based on the table reported in Rossi and Sekhposyan (2017). The process of arriving at empirical CDF of PITs for each model is described in details in Section 2.2.2. Out-of-sample testing. Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing. "base_model" is the benchmark model with NFCI and r_gdp_1q_gr as predictors.

3.3. UK results

For the UK data, the author is more interested in exploring the applicability of quantile regression in predicting the UK one-quarter-ahead and one-year-ahead real GDP growth by using near-term indicators and is interested in explaining how the tails of the predicted distribution behave in the UK case and what are the drivers behind the movements of the tails. With this, by combining all the independent variables in the set of 7 variables described previously, the author investigates 238[3] models where up to 5 predictors are included in each model to find out which model is the best to predict the UK growth distributions.

[3] 238 models include 2 sets of 119 models generated to predict each dependent variable: C_7^1 models with 1 independent variable, C_7^2 models with 2 independent variables, C_7^3 models with 3 independent variables, C_7^4 models with 4 independent variables, C_7^5 models with 5 independent variables.

3.3.1. One-quarter ahead real GDP growth

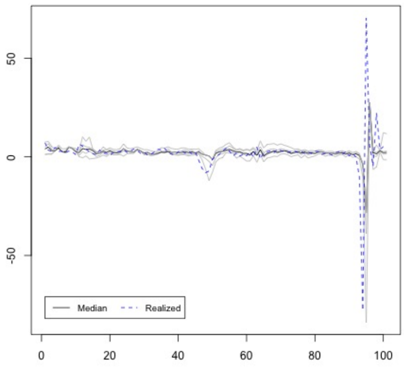
For the UK data, using the "average error" criteria, the chosen best models to predict one-quarter-ahead real GDP growth are recorded in Table 5. It is noticed that the realized_vol_FTSE and r_gdp_1q_gr are present in all chosen models. The predicted quantiles of these models are then plotted in Figure 10, where left tails of the distributions also exhibit stronger move than the right tails (RQ5). The plots of the empirical CDFs of PITs in Figure 11 prove that the predicted distributions are close to the unobserved underlying data generating process. These observations, together, indicate that quantile regression on near-term measures can be used to predict the densities for UK one-quarter-ahead real GDP growth (RQ4). The results indicates that the realized volatility of FTSE index can serve as a good financial indicator for predicting the one-quarter-ahead real GDP growth distributions (RQ6) while the UK real GDP growth rate can be considered a good economic indicator. However, if timeliness of forecasts is considered, no model in the best models to forecast one-quarter-ahead UK GDP growth distributions can be selected as no models contain only high-frequency predictors (RQ8).

Table 5: Best models to predict UK 1-quarter-ahead real GDP growth

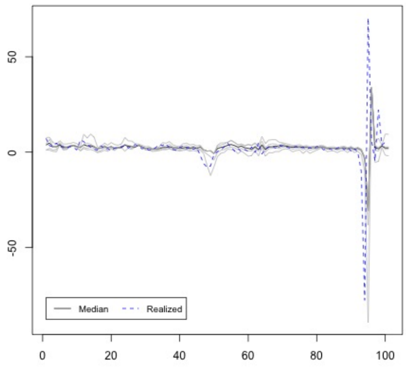
Model name	**Model form**	**Average error**
Best 5	r_gpd_1q_gr_ahead ∼ realized_vol_FTSE + URATE + CPI + r_gdp_1q_gr + loan_2Y_spread	4.256%
Best 4	r_gpd_1q_gr_ahead ∼ realized_vol_FTSE + URATE + r_gdp_1q_gr + eff_loan_rate	4.186%
Best 3	r_gpd_1q_gr_ahead ∼ realized_vol_FTSE + r_gdp_1q_gr + eff_loan_rate	4.329%
Best 2	r_gpd_1q_gr_ahead ∼ realized_vol_FTSE + r_gdp_1q_gr	4.94%

Note: Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing.

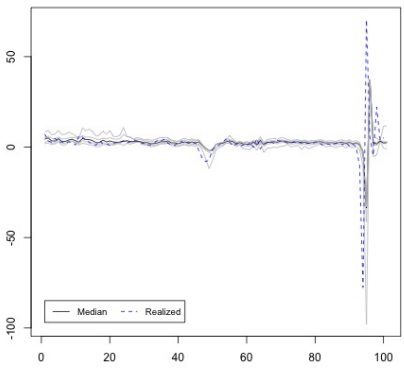
Figure 10: Out-of-sample plotting - UK one-quarter-head real GDP growth



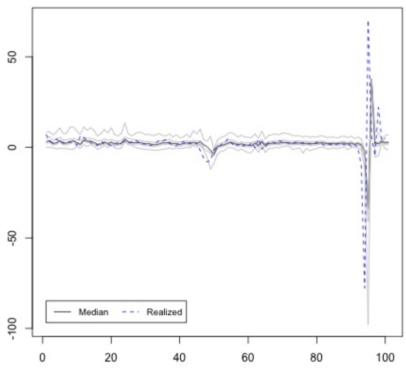
(a) Best 5



(b) Best 4



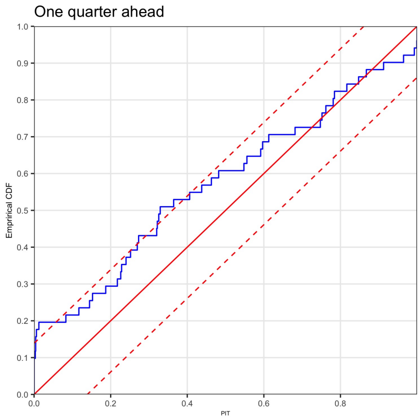
(c) Best 3



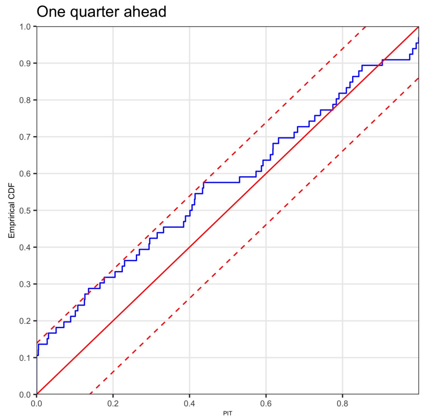
(d) Best 2

Note: The figures plot estimated values corresponding to 5 percentiles (5th, 25th, 50th, 75th, 95th) of the UK 1-quarter-ahead real GDP growth density forecast together with realized GDP growths over time. The estimated values are calculated from quantile regression models for those percentiles with respective independent variables used in each model. Details of the estimated process can be found in Section 2.2.2. Out-of-sample testing. Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing.

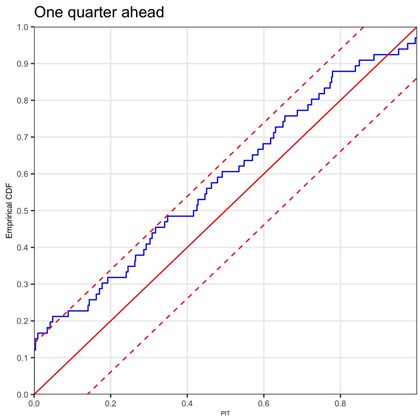
Figure 11: CDFs of PITs - UK one-quarter-head real GDP growth



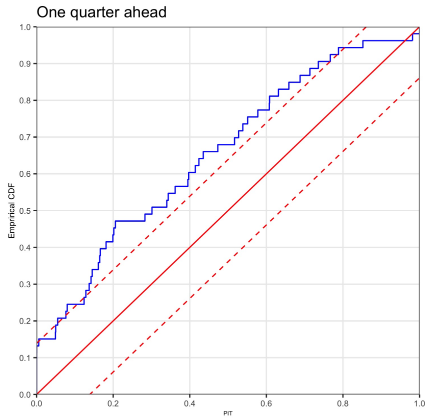
(a) Best 5



(b) Best 4



(c) Best 3



(d) Best 2

Note: The figure shows the empirical CDF of the PITs (solid lines), the theoretical CDF of the PITs (the 45 degree line) and the 5 % critical values bands based on the table reported in Rossi and Sekhposyan (2017). The process of arriving at empirical CDF of PITs for each model is described in details in Section 2.2.2. Out-of-sample testing. Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing.

3.3.2. One-year ahead real GDP growth

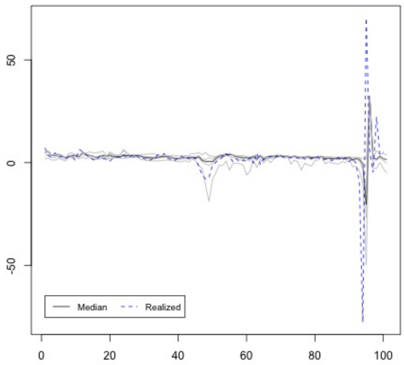
Table 6 reveals that the best models chosen to calibrate density forecasts for UK one-year-ahead real GDP growth also include realized_vol_FTSE + r_gdp_1q_gr but together with other variables: CPI, 10y-2y bond yield spread, and eff_loan_rate. Figure 12 shows that the stronger dynamics lie in the left tails of the predicted quantiles (RQ5). However, the empirical CDFs of PITs for the best models, which are presented in Figure 13, express worse results than the results for one-quarter-ahead growth. If simplicity is taken into account, the "best 3" model is the simplest form that should be chosen (RQ7) since the empirical CDFs of PITs for "best 2" model is very far away from the diagonal line which indicates the misspecification of the whole process of calibrating the predicted distributions. Results related to one-year-ahead real GDP growth distributions confirm that the realized volatility of FTSE index is a good financial indicator for predicting near-term UK real GDP growth distributions (RQ6). In terms of timeliness of forecasts, there is no model selected to forecast one-year-ahead UK GDP growth distributions as all the best models contain at least one quarterly variable (RQ8). The best models to forecast one-year-ahead UK growths also have lower “average errors” than the “average errors” of the best models to predict one-quarter-ahead UK growths. The explanations provided for similar observation in the US case are also applied in the UK case.

Table 6: Best models to predict UK one-year-ahead real GDP growth

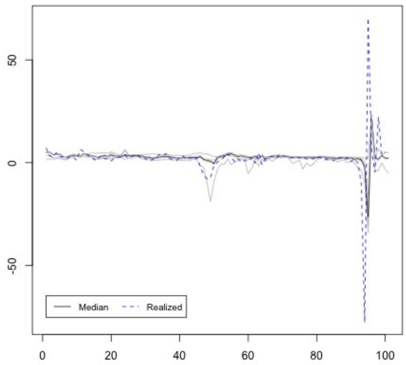
Model name	**Model form**	**Average error**
Best 5	r_gpd_4q_gr_ahead ∼ realized_vol_FTSE + CPI + spread_10Y_2Y + r_gdp_1q_gr + eff_loan_rate	2.507%
Best 4	r_gpd_4q_gr_ahead ∼ realized_vol_FTSE + spread_10Y_2Y + r_gdp_1q_gr + eff_loan_rate	2.426%
Best 3	r_gpd_4q_gr_ahead ∼ realized_vol_FTSE + r_gdp_1q_gr + eff_loan_rate	2.649%
Best 2	r_gpd_4q_gr_ahead ∼ realized_vol_FTSE + r_gdp_1q_gr	3.156%

Note: Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing.

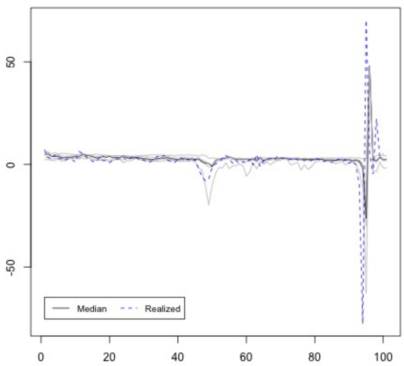
Figure 12: Out-of-sample plotting - UK one-year-ahead real GDP growth



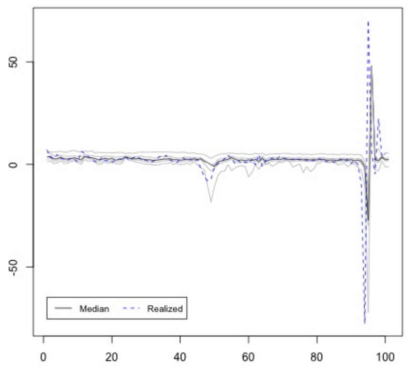
(a) Best 5



(b) Best 4



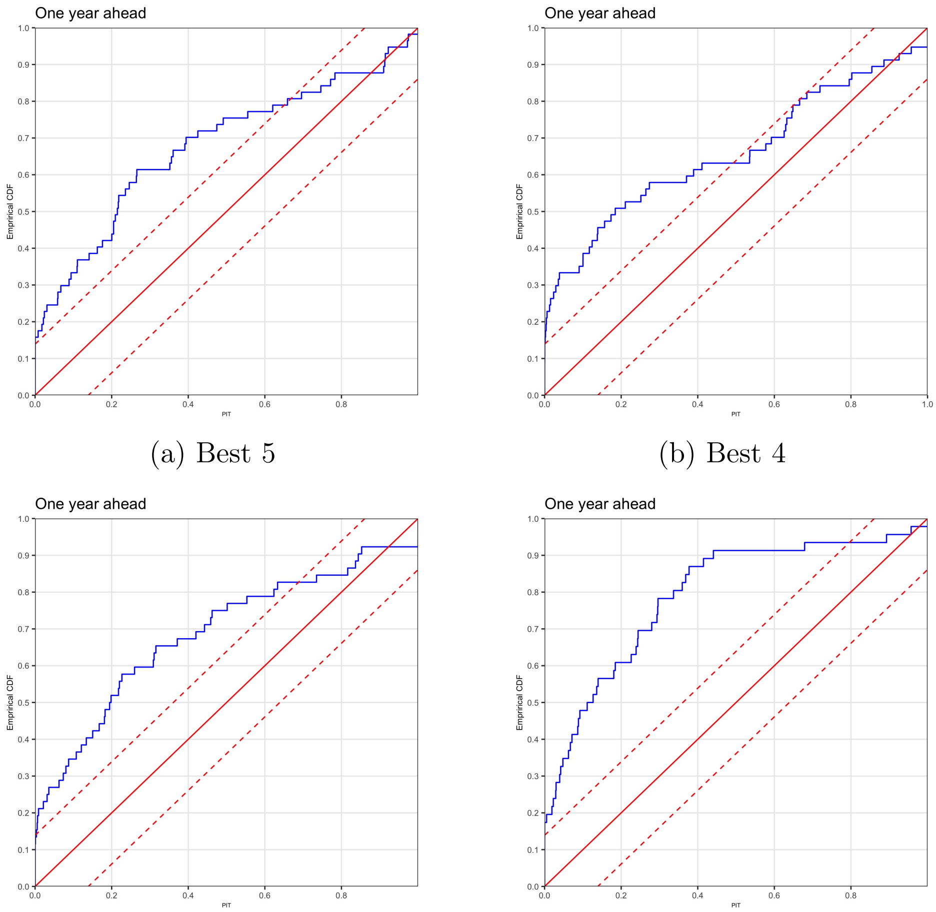
(c) Best 3



(d) Best 2

Note: The figures plot estimated values corresponding to 5 percentiles (5th, 25th, 50th, 75th, 95th) of the UK one-year-ahead real GDP growth density forecast together with realized GDP growths over time. The estimated values are calculated from quantile regression models for those percentiles with respective independent variables used in each model. Details of the estimated process can be found in Section 2.2.2. Out-of-sample testing. Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing.

Figure 13: CDFs of PITs - UK one-year-ahead real GDP growth



(a) Best 5

(b) Best 4

(c) Best 3

(d) Best 2

Note: The figure shows the empirical CDF of the PITs (solid lines), the theoretical CDF of the PITs (the 45 degree line) and the 5 % critical values bands based on the table reported in Rossi and Sekhposyan (2017). The process of arriving at empirical CDF of PITs for each model is described in details in Section 2.2.2. Out-of-sample testing. Best 5, 4, 3, 2: The model has the lowest average error among all models with 5, 4, 3, 2 independent variables, respectively. Average error for each model is calculated by the process described in details in Section 2.2.2. Out-of-sample testing.

CONCLUSIONS

This paper investigates the existence of better models than the model proposed by Adrian et al. (2016) to predict the distributions of US real GDP growths in the horizons of one quarter ahead and one year ahead. Additionally, the author attempts to explore the usefulness of quantile regression and skewed-t distribution in predicting the densities of one-quarter-ahead and one-year-ahead UK real GDP growths. The literature review presents various related empirical researches in different countries and explains the rationale for this paper.

In order to answer the main research questions, the author first employs the quantile regression to predict the dependent variables at 5 different quantiles, namely 5th, 25th, 50th, 75th, 95th quantile, together with skewed-t distribution to produce the calibrated distributions using those forecasted quantiles. Secondly, the author applies the methodology on different sets of variables whose all available data is used and data end in 2021Q1: 10 independent variables for US data and 7 predictors for UK data. Finally, a 2-step testing procedure is used to identify the best models for forecasting specific growth distributions: the average error for all out-of-sample forecasts is calculated for selecting the models with lowest average error and the cumulative distribution function of probability integral transforms is computed for forecasted distributions in out-of-sample period to assess on the correctness of the whole process of generating the real GDP distributions.

In this paper, 8 research questions were discussed:

RQ1: *By employing the same methodology, are there any better models to predict one-quarter-ahead US real GDP growth distributions, considering the models suggested by Adrian et al. (2016) as benchmark models?* Results on US data in subsection 3.2.1 show that the paper is able to find different models which are better than then benchmark models to predict the one-quarter-ahead US GDP growth distributions.

RQ2: *Will there exist better explanatory variables for the prediction of one-year ahead US real GDP growth density?* Results in subsection 3.2.2 also indicate that better models are detected to predict one-year-ahead US real GDP densities with different set of predictors. NFCI still serve as a strong financial indicator to predict the one-quarter-ahead and one-year-ahead US growth values. Additionally, ICSA_gr is identified to be better economic predictor to predict US growth series in comparison to real_gdp_1q_gr as proposed by Adrian et al. (2016).

RQ3: *How do the tails of calibrated growth distributions of US growths behave?* The results in subsection 3.2 on US results reaffirm the findings of Adrian et al. (2016) with the strong movements of left tails of the predicted distributions and more stability in the right tails although different predictors are used in the chosen models to predict one-quarter-ahead and one-year-ahead GDP growth distributions.

RQ4: *Can quantile regression be applied to UK data to produce meaningful results using near-term measures or indicators?* The answer to this question is an absolute "yes" as different model forms are identified to produce good results in subsection 3.3 on UK results.

RQ5: *What are the behaviors of the tails in the UK forecasted real GDP distributions?* Results in subsection 3.3 show that the left tails of the predicted distributions of UK's dependent variables express stronger movements than the right tails although the movement in the left tails is not as dynamic as that of the US GDP growth distributions.

RQ6: *Which measure of financial condition can provide informative results on the tail risks to the UK real GDP growth?* According to results in subsection 3.3, the realized volatility of FTSE index is identified as a good financial indicator for forecasting UK growth distributions because realized volatility of FTSE index is included in the best models to predict UK growth densities.

RQ7: *Should simpler models be chosen in favor of simplicity among the models chosen to predict the dependent variables for the US and UK?* Results in subsection 3.2 and subsection 3.3 indicate that simpler models can be chosen for both US and UK data as these simple models provide sufficiently good results.

RQ8: *If the publication frequencies of predictors are taken into account, which model should be chosen to predict the US and UK growth distributions in order to ensure the timeliness of forecasts?* Results in subsections 3.2 and 3.3 reveal that different models with high-frequency variables can be selected to predict US real GDP growth distributions in the horizon of 1 quarter and one year. However, no model should be selected when it comes to UK case as all best models contain at least one quarterly variable.

This work, in fact, presents some interesting findings in terms of better models for the prediction of US output growth distributions and in terms of the application of the 2-step procedure in UK data using near-term indicators. However, the study is open to different improvements. The first limitation is related to the chosen methodology. In this paper, only the 2-step procedure which includes quantile regression and skewed-t distribution is explored to find answers to the research questions. Although the author's desire is to leverage the methodology proposed by Adrian et al. (2016), other methodologies can be explored in the future to compare with the selected methodology and might produce more robust and better models to predict the distributions of output growth. Secondly, additional predictors, especially in the UK case, can be added to check for better combinations. Finally, other approaches of validating the calibrated distributions can be explored and compared to produce more robust results.

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List of abbreviations

BoE	Bank of England
CBO	Congressional Budget Office
CDF	Cumulative distribution function
CECL	Current Expected Credit Losses
Chicago Fed	Federal Reserve Bank of Chicago
CPI	Consumer Price Index
eff_loan_rate	PFNC's effective loan rate
FTSE index	Financial Times Stock Exchange 100 Index
GDP	Gross Domestic Product
ICAAP	Internal Capital Adequacy Assessment Process
ICSA	Initial jobless claims
ICSA_gr	Annualized quarter-over-quarter growth of ICSA
IFRS	International Financial Reporting Standards
loan_2y_spread	Spread between effective loan rate and 2y bond yield
MAD	Mean absolute deviation
NFCI	National Financial Conditions Index
output_gap	Output gap
PIT	Probability integral transform
PNFC	Private non-financial corporations cash
r_gdp_1q_gr	One-quarter real GDP growth
r_gdp_1q_gr_ahead	Annualized one-quarter-ahead real GDP growth
r_gdp_4q_gr_ahead	Annualized one-year-ahead real GDP growth
real_vol_FTSE	Realized volatility of FTSE
realized_vol_SNP	Realized volatility of S&P 500 index
RQ	Research question
S&P 500 index	Standard and Poor's 500 index
spread_10Y_2Y (US)	Difference between the market yields on U.S. Treasury Securities at 10-Year constant maturity and at 3-year constant maturity
spread_10Y_2Y (UK)	Difference between yields of 10-year government bonds and 2-year government bonds
spread_10Y_3M	Difference between the market yields on U.S. Treasury Securities at 10-Year constant maturity and at 3-month constant maturity
UK	United Kingdom
URATE	Unemployment rate
US	United States

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