Computable General Equlibrium

Intro

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Plan of action

- What is CGE?
- Modelling rules:
  - models
  - data
  - calibration
- Introduction to GAMS and its syntax.
- Simple pure exchange model and modelling convention.

- We want to be able to analyze the effects of changes in economic policy or economic environment:
    - we want to see how, for example, a tax on good A affects the the economy:
    - it will, for sure, affect the price of good A that consumers pay and also his consumer surplus (partial equilibrium).
    - BUT: something else will happen
        - the consumer will switch to consuming some other goods (substitution).
        - changes in demand for goods will affect the demand for factors of production and their prices
        - these will in turn affect the consumer's income.
        - which in turn will affect the consumption of goods again.
    - These are called general equilibrium effects. We do not want to ignore them.

General equilibrium modelling

- We model the economy (it can be a single country economy, but also a single village or city economy or even world economy) as a complete system
  - We assume how many agents, goods and markets there are.
  - We assume the form of consumer preferences and production technology.
  - In the simplest case we assume price taking and constant returns to scale (more advanced models assume monopoly power and IRS).
  - We assume that in our economies there are no "leaks". The economy behaves like in a circular diagram from a first year textbook - all consumer income is generated in the model and spent. The total spendings of consumers is equal to the value of total consumption.
- We write down the model similarly to what we did in class: our model describes conditions needed for the Walrasian equilibrium to hold: market clearing, feasibility etc.
- This leads us to the data problem: the Social Accounting

The Social Accounting Matrix

- The SAM is a snapshot of the economy at a given point in time.
- It shows all the flows of incomes and value between all the agents in the economy: households, firms, government, the abroad.
- It has to be compatible with our model - we need to have the same set of agents in the model and in the SAM.
- The SAM is based on a double accounting principle: the sum of elements of every row (incomes) has to equal the sum of all elements in a column (expenditures).

The Social Accounting Matrix

- Next slide: an example SAM from the NBP model.
    - For example: the household (INSTH) is a sum of factor income and transfers from other institutions.
    - And it has to equal to the household consumption expenditures, outgoing transfers, income taxes and savings. (ROW=COLUMN).

The Social Accounting Matrix

	SEC	FAC	INSTH	G	TAX	INV	INSTF
SEC	Intermediate use		Private consumption	Public consumption	Subsidies	Investment demand	Exports
FAC	Value added						
INSTH		Factor income	Transfers	Transfers			Transfers
G				Tax revenues			Transfers
TAX	Indirect taxes	Factor taxes	Income tax				
SAVE			Household savings	Budget surplus			Foreign savings
INSTF	Imports		Transfers	Transfers			

The data (1)

- Construction of the Social Accounting Matrix is probably the most tedious excercise.
- First of all, we need an input-output table to know how much factors and intermediate use is required for production of each good.
- Then, we need data on consumption by various institutions (households, government, exports).
- For example: the NBP SAM combines data from the input output table, National Accounts, Household Budgets Survey, international trade data etc.

The data (2)

- Usually each of that piece of data comes in a different classification and it has to be made compatible (eg. international trade uses a commodity-based classification and production data uses the activity-based classification).
- Once you have all the data in compatible form, you have to balance it, so that it fits the ROW=COLUMN requirement. There are formal procedures to do that but still there are many assumptions you have to make on the way.
- There are some ready-made solutions eg. the GTAP database.

Calibration (1)

- Let's assume we have the data ready and the model written down. How do we combine the two?
- Here is where the calibration process starts. **Calibration** is the process of "fitting" the model to the data.
- How do you do that:
  - First, we make a very important assumption: we assume that the database describes an economy in equilibrium. We call it **benchmark equilibrium**.
  - Then we choose all the parameters of the model to replicate that **equilibrium.** We assume that all prices are 1 so the nominal values in equilibrium reflect the real flows.

Calibration (2)

- This is opposite to solving a homework-like question, when I ask: what is the equilibrium given the parameters.
- Here we ask: what are the parameters if the SAM is the equilibrium.
- Usually there is more that one set of parameters that supports the equilibrium. Some parameters we directly take from econometric estimates or estimate yourself outside the model. Sometimes we guess a parameter and then do a sensitivity test to see how the choice of that parameter affects the results of your simulations (**sensitivity testing**).

Example of calibration

- Let us assume that the utility function of a consumer is Cobb-Douglas $U = a^{\alpha} b^{1-\alpha}$. a and b are consumption levels of the two goods present in the economy.
- We know from our SAM, that the total spending on consumption goods is 100.
- We also know that in the benchmark (SAM), the spending on goods is 30 and 70 respectively. So with prices equall to 1, the benchmark consumption levels are $a0 = 30$ and $b0 = 70$ (note the notation).
- What is our calibrated parameter α to have our model replicate the benchmark consumption provided that prices are 1 and total spending on consumption is 100? Answer: ......

Example of calibration

- This can be easily generalized to many goods. Same applies to production functions (shares in the cost of production define the C-D parameters).
- Remember that by doing the above we already assume that the elasticity of substitution is 1. If we had a CES function, we would have another parameter to calibrate (and this we cannot do using the data, we have to have external sources, such as econometric estimates).

Model calibrated, what's next?

- Once your model replicates the equilibrium, you can start doing simulations or **counterfactuals**.
- For example, you can impose a change in the tax rate or a change in the level of endowments.
- The model is pushed out of equilibrium. If you wrote everything correctly, the optimization software is going to find another equilibrium (unless it is infeasible).
- You can compare the equilibrium activity levels (quantities) and prices from the benchmark with you counterfactual.
- You will see how your policy shock affected the economic activity. From the utility function you can look at welfare changes.

The software

- We will use GAMS - General Algebraic Modeling System, a high-level modeling system for mathematical programming problems.
- It is commercial software, however it can run small models in the free version.
- It can solve linear and non-linear mathematical programming problems.
- It is very useful as it allows the user to use the indexed notation, for example $X(INSTH, SEC)$ is a variable x that is indexed over households $INSTH$ and sectors SEC.

The GAMS IDE

- Normally you will use the GAMS IDE or the development interface for GAMS. It is fairly simple.
- Once you fire it up for the first time, you have to start a new project (File/Project/New Project) in a directory of your choice. Give it some name.
- Then you start a new program file where you can put your code.
- However, I prepared for you two files with some starting code.

- SETS Declares the indexes of the model
- PARAMETERS Declares the constants, parameters and the dummy variables to be used in the program. The elements under this component are described by a brief descriptive sentence following the declaration statement,
- CALIBRATION Data is used to generate the structural parameters. The purpose here is to impose the algebraic system of equations onto the data, so that the first "solution" of the model will reflect exactly the "base-year" SAM data.
- VARIABLES All variables, endogenous, or exogenous are declared here with brief descriptive identifiers.
- VARIABLE INITIALIZATION The Variables are initialized with the base-year data values. This enables the algorithm to start its search for equilibrium from a "sufficiently close" data point. EQUATIONS The algebraic equations of the model are declared with descriptive statements.

Equations, initialization and solution

- EQUATION SPECIFICATIONS The declared EQUATIONS are specified in the GAMS language.
- CLOSURE AND RESTRICTIONS Exogenous variables are "fixed" using the .FX appends. Bounds are introduced using .LO and .UP. The overall macro closure recognized in the model is implemented here.
- SOLVE AND DISPLAY The MODEL statement declares the model by specifying which EQUATIONS form the MODEL. SOLVE statement commands GAMS to interface with the algorithm to search for a numerical solution. The solution values are inputted to the display tables and are documented using the DISPLAY command.

SUM and PROD

- If you want to sum some variables or parameters, you can do it manually or use the SUM syntax:
- $SUM(SEC, X(SEC))$ - this command sums up all the X's from a set SEC.
- Same applies to *PROD* where you can have a product instead of the sum.

What we will do.

- Our first classrom model is:
  - Closed economy
  - Pure exchange with two agents and two goods.
  - Utility function of both consumers is a Cobb Douglas function.

Our simplistic SAM

	X1	X2	CONS1	CONS2	
X1			40	70	110
X2			60	30	90
CONS1	50	50			100
CONS2	60	40			100
	110	90	100	100	

Value of consumers wealth	(yellow)
Consumption of goods	(blue)

Lets introduce some variables

$C_{1,x1}$ - demand for $x1$ by consumer 1. $C_{1,x2}$, $C_{2,x1}$, $C_{2,x2}$ respectively.
Utility functions:

$$W_1 = A_1 C_{1,x1}^{\alpha_1} C_{1,x2}^{1-\alpha_1}$$

$$W_2 = A_2 C_{2,x1}^{\alpha_2} C_{2,x2}^{1-\alpha_2}$$

Assume endowments (from SAM at prices==1): $\omega_{1,x1} = 50$, $\omega_{1,x2} = 50$, $\omega_{2,x1} = 60$, $\omega_{2,x2} = 40$,

What are we looking for?

- In a classroom excercise, you will be normally looking for equilibrium consumption. Here we know the equilibrium consumption (from SAM).
- We need to find parameters of our production function that support our equlibrium given the endowments.
- This process is called **calibration.**

What are we looking for?

- Some facts about the Cobb-Douglas function:
- Demand: $C_{1,x1} = \alpha_1 \frac{Income_1}{p_1}$ and respectively for other consumers, goods.
- Income is the value of endoments: $Income_1 = p_1\omega_{1,x1} + p_2\omega_{1,x2}$ - if we assume prices=1, then we can read it from SAM.
- From demand: $\alpha_1 = \frac{p_1 C_{1,x1}}{Income_1}$. If we look at the SAM: $\alpha_1 = \frac{40}{100}$. We can do it for all other $\alpha' s$.

We still need A_1 and A_2

- We want our utility to correspond to total consumption of the good.
- In this way we have a measure of both aggregate consumption of the household and utility at the same time.
- In the SAM, the aggregate consumption of consumer 1 is 100.
- So using the utility function

$$100 = A_1 40^{\alpha_1} 60^{1-\alpha_1}$$

- Now we only have to find A_1 given the α_1 we found previously. We will do it in GAMS.

How does the model look like

- We need two market clearing equations:
  - Aggregate demand for each good = Aggregate supply (endowment)
- Income balance equation (budget constraint) for both consumers

Aggregate consumption

We will introduce the price index for each consumer (his CPI)

$$PW_1 = \frac{1}{A_1}(\frac{p_1}{\alpha_1})^{\alpha_1}(\frac{p_2}{1-\alpha_1})^{1-\alpha_1}$$

This is expenditure for each unit of utility - unit expenditure function $e(p,u)$ at $u = 1$. Think about the resemblance of the above to the (unit) cost function.
In that case, the budget constraint for the consumer is:

$$PW_1 * W_1 = INC_1$$

We have our model calibrated. What now?

- We can run counterfactuals: impose shocks on the model by changing exogeneous variables, for example
  - change endowments
  - introduce taxes (ad-valorem, specific) or lump sum
  - introduce transfers
- We can observe what happens with consumption and more importantly, we want to see how those changes affect welfare.