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Abstract

In this paper the application of several option pricing models has been tested on the basis of options traded on the Warsaw Stock Exchange. At first, theoretical option prices have been calculated according to the models chosen. Next, the models have been tested by comparing their option prices estimates to prices observed on the market. The models chosen are: a few alternative versions of the Duan (1995) GARCH Option Pricing Model, and two versions of the model by Black (1976). A separate section is devoted to the impact of the implied dividend yield on prices of options. The study covers a period from January 2006 to March 2012. Results show that the most accurate models are the Black model with a volatility term structure, and the Duan GARCH Option Pricing Model with implied dividend yield and Student's T random errors.

Keywords:
GARCH models, Duan methodology, Black model, implied volatility, option pricing

JEL:
C15, C53, G17, G23

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1. Introduction

Option trading dates back long before adequate pricing formulas became available, that is, as far as in the seventeenth century, when options played an important role in the creation of the tulip bubble in the Netherlands. However, the options market started growing rapidly in size only since the 1970s, when Black and Scholes (1973) and Merton (1973) derived their famous equation for the pricing of vanilla call and put options. The new tool became useful amongst corporate treasurers, who had now available a new interesting instrument to hedge different kinds of market risks that their companies were facing, and amongst speculators, who were now able to use options as a highly risky, but potentially rewarding instrument.

However, when the Black-Scholes equation became available, there was still another question remaining. The volatility, as one of the parameters of the equation, was not readily observable on the market, and it was not clear how to measure it. Therefore, the volatility itself became a field of research of many scientific papers. Since volatility is not directly observable, an estimator has to be provided, and researchers disagree on which one is the most accurate. However, even if the volatility estimate inserted into the Black-Scholes formula is accurate enough, the formula itself has its own drawbacks. The first one is the fact that the volatility is not constant and an accurate option pricing model has to take that into account. The volatility not only changes through time, but it is different for options representing different maturities and different strike prices on the same underlying asset, creating volatility smiles and volatility surfaces. That makes the problem even more complex and requires more sophisticated approaches. For example, one popular way to price new options on the basis of already existing options is to use interpolation or extrapolation basing on the volatility surface. When pricing an option when no market prices are available, one needs to plot a whole volatility path for every day until the maturity of the option. Unfortunately, the Black-Scholes model does not allow that. One of the possible alternatives is to use Monte-Carlo methods, but their drawback is that they are very time consuming.

There are a lot of papers about the pricing of options focusing on the US market, especially on options on the S&P500. This is the oldest, and the most liquid market of options, therefore it allows to perform accurate research on a long portion of data, without worrying about such market anomalies like the lack of liquidity present in emerging markets. On the other hand, the number of papers studying options markets in CEE countries is limited, which is to a significant extent due to the fact that most stock exchanges of the region have a very young derivatives market, which in many cases is not liquid enough to perform a broad research on it. However, as the Warsaw Stock Exchange is growing, and introducing more and more sophisticated derivatives products, the options market becomes more and more worth studying.

This paper concentrates on estimating different option pricing models for options on WIG20 in order to verify which one is the most accurate. Several papers already tackled a similar problem, including studies by Fiszeder (2007), Piontek (2009), Kokoszczyński et al. (2010). However, this study tests the most recent data, and is assumed to be the first to cover all the following periods: the pre-2007 bull market, the credit crunch period, the recovery of 2010 and the recent crisis in the European Union. The study focuses on GARCH option pricing models, and compares their empirical performance to benchmark Black models. The main model that I use in my study is the Duan option pricing model (Duan 1995), which assumes that the path of the instrument underlying the option follows a Brownian motion with dynamic volatility. This volatility follows a GARCH process, which is modeled by three parameters, which allows to reduce the effect of volatility smiles in the prices of options.

The goal of this paper associated with its practical applications is to create a basis for the valuation of non-standard path-dependent derivatives on stocks and stock indices in Poland, like barrier options, lookback options, Asian options, etc. The popularity of these products will be certainly growing together with the overall size of the stock market, which will create a demand for sophisticated pricing tools. The amount of types of exotic options is by definition infinite, and many of these products are tailor-made to customer's needs, that is why accurate pricing tools are essential. Apart from that, the models I use to price options on WIG20 can be also used to price non-traded options on other indices, like the WIG or the mWIG40. These models can also be extended to index components, for example, calculate prices of options on single stocks.

2. Literature Review

The issue of option pricing has been widely discussed in the literature. The amount of scientific publications devoted to that issue boomed when an equation that was able to price options accurately appeared (Black & Scholes 1973). The famous Black-Scholes equation made options available for mass trading, both for corporate treasurers willing to hedge their risk and for speculators willing to establish a specific exposure to the market. The Black-Scholes model was soon extended to options on futures (Black 1976).

When it became clear that an accurate tool for option pricing has become available, there was still another question remaining. Academics and practitioners were asking themselves the question how to make profits on making bets on the direction in which the volatility would go, or simply how to price options accurately. The simplest approach consists of using the historical volatility and fit it into the Black-Scholes formula for futures prices (Black 1976). However, this model tends to be inaccurate as it gives equal weights to all past rates of returns, regardless how much in the past they are. The second approach is the realized volatility estimator proposed by Black (1976) and Taylor (1986), which is based on squared log returns summed over a certain interval of time. This measure is also backward-looking, therefore it exhibits significant biases.

An enormous amount of literature is devoted to option pricing using GARCH models. This branch flourished when Engle (1982) first proposed the ARCH model. That model states that the conditional variance of tomorrow is a function of the unconditional long-term variance, and of the returns of the previous days. The ARCH model was later extended to the GARCH model, which contained one more parameter that takes into account the fact that there are longer periods of high variance, as well as longer periods of low variance (Bollerslev 1986). The question remains, how to implement these volatility estimators in the theory of options pricing. An answer to that question is given by Duan (1995), which proposed the GARCH option pricing model. In his model options are priced using the Monte-Carlo approach. In this model, the simulated asset prices follow a GARCH process and therefore capture the heteroscedasticity incorporated in stock returns, which allows to reduce the pricing bias caused by the volatility smile. However, Duan only shows that his model corrects certain biases resulting from the Black-Scholes model and does not perform empirical tests of his approach.

The articles in the above paragraph describe only the theoretical option pricing models, with their rationale and mathematical derivation. Very often in the option pricing literature, empirical studies are conducted separately from theoretical derivations of models. These empirical studies focus on comparing real market option prices (transactional, closing or mid quote data) to prices estimated by different models, and sorting them into maturity and moneyness classes. Since Duan published his article about the GARCH option pricing model in 1995, various researchers used that model, or invented their own extensions, in order to obtain the closest to reality prices of options.

Most studies focus on the US market, especially on options on the S&P500 index. This is a very liquid and mature market, and therefore results are not biased by market anomalies. Hsieh and Ritchken (2000) compare different GARCH option pricing models to prices of options on the S&P500, for a period from January 1991 to December 1995. they cover options with a time to maturity from 6 days to 2 months. Results are compared to a standard Black-Scholes model. The models involved in the study are the Duan's NGARCH model, and the Heston and Nandi model. The research finds out that GARCH models are much more accurate in terms of option pricing than the Black-Scholes model. A similar but slightly different model is developed by Heston and Nandi (2000). That model also tests options on the S&P500, from 1992 to 1994. The results are consistent with the previous study – deep OTM call and put options are highly mispriced, while deep in-the-money (ITM) contracts are very well priced. This study is different from the previous one in a way that it treats call and put options separately. An interesting feature then emerges – percentage errors for OTM contracts are much larger for call options than for put options. Errors for ITM options tend to be equally small regardless of the time to maturity of options, while errors for OTM options tend to increase with decreasing time to maturity, which suggests that very cheap options are highly speculative and their theoretical price may not reflect their true value.

Another paper discussing the pricing of options on S&P500 is by Christoffersen and Jacobs (2004) covering the period from June 1988 to May 1992. The study compares the accuracy of several types of GARCH models. However, they use the Root Mean Square Error as the statistic which compares option pricing models. Unfortunately, this approach does not allow comparing accurately different options within the same model – it is rather used for comparing models between each other. According to the paper, all GARCH models seem to yield very similar results across all moneyness and maturity classes.

Another model used in option pricing, the Heston model, takes into account the bias of the Black-Scholes model coming from the heaviness of tails and the skewness of returns distribution (Heston 1993). Contrary to other studies, in the Heston model the volatility is not a deterministic, but stochastic process, that takes into account the fact that financial returns exhibit leptokurtosis. According to recent studies, the Heston model tends to perform better than the GARCH model in terms of accuracy of option pricing (Kokoszczyński et al. 2010). An older study by Engle, Kane and Noh examines the profits from straddle trading[1] on the basis of GARCH forecasts. It shows that trading at-the-money (ATM) straddles with GARCH forecasts leads to positive excess returns, higher than for the implied volatility regression model.

There exist also some studies examining the accuracy of option pricing models on the Polish market. Despite the fact that the market is still immature and illiquid, several researchers attempted to describe the behavior of option prices. Fiszeder (2007) uses

[1] The straddle option strategy involving buying (selling) simultaneously a call and a put option with equal strike prices so that the delta of the net position is equal to 0. This is a way to create a pure bet on the future direction of the volatility.

transactional data on options on WIG20 to test several GARCH option pricing models according to the approach proposed by Duan. His sample ranges from January 2001 to June 2007. The models used are: GARCH, GARCH-M, NGARCH-M, GJR and GJR with Student-t conditional distribution. Fiszeder's findings are not much different from those obtained by studies on the American and German markets. All classes of GARCH models beat the Black-Scholes model, but are worse than the model used commonly by financial markets practitioners, which uses the implied volatility from the previous period in order to price options. However, Fiszeder's results are difficult to assess as he compares models only with the RMSE statistic, without showing absolute percentage errors. Still, the paper contains the percentage of overpredictions, to which I will be able to directly compare my own results. Another similar research on options on WIG20 is conducted by Piontek (2009), who uses an AR-GARCH model to analyze options maturing in 2007. However, he assumes a constant dividend yield and a constant risk-free rate, which may create a certain bias in his research. Still, he finds out that for ITM options, GARCH models and the Black-Scholes model yield similar pricing errors, but for OTM options, GARCH models perform much better, despite bigger errors than for ITM options. For ATM options, there is no clear difference between the performance of the two models.

The most recent study on options on WIG20 has been conducted by Kokoszczyński et al. (2010). Their approach to the problem differs from the previous papers in a sense that it involves high frequency, 10-second mid quote data[2]. The study is performed on a period from January to June 2008. Despite the use of different data, similarly to the study of Fiszeder, they find that the Black model with implied volatility as an estimate of the sigma parameter yields the smallest errors, while the worst model is the Black Realized Volatility model. Also, they find out that the biggest percentage absolute errors occur for OTM options, which as the authors explain is due to a significant number of outliers that led to extremely high errors (even up to 40 000%). Similarly to the previous studies, errors for ITM and deep ITM options are very small.

To sum up, the amount of available literature on option pricing is huge. However, only a few studies examine the Polish options market. My study would be the first one to incorporate together the bull market period of 2006-2007 and 2009-2010, the bear market of 2008 and the crisis in the European Union. It would be very interesting to check whether the options market behaves somehow differently, depending on the level of market volatility.

However, the problem with the literature on option pricing models is that different researchers use different kinds of testing statistics, present their findings in many different manners, which makes the results much more difficult to compare. Still, some general principles governing the options market may be verified, and I will refer as frequently as necessary to previous studies during the discussion of my results. Moreover, as researchers use in their studies different datasets, it is particularly interesting to test the persistence of the results.

3. Methodology

This paper tests the application of different option pricing models on the basis of data from the Warsaw Stock Exchange. The goal will be to see which model prices options most accurately. Different versions of the Black (1976) model and of the Duan (1995) option

[2] Mid quote=(bid+ask)/2

pricing model will be used. Theoretical option prices obtained using the chosen methods will be compared with prices observed on the real market of options on WIG20 traded on the Warsaw Stock Exchange. Next subchapters describe the methodology in details.

3.1. Selection of options

Options tested in the study are selected according to a rule of thumb which takes into account the turnover and significance of the volatility parameter in the price of the option. In other words, options with a very long maturity (due to small turnover) and very short maturity (due to extreme percentage moves caused by small prices) are removed from the dataset. The methodology for the selection of options is described in details in section 3.2, which is devoted to descriptive statistics for options on WIG20.

3.2 Statistics for the measurement of results

The statistics used in the paper for the measurement of results are the Mean Absolute Pricing Errors (MAPE) and the Percentage of Overpredictions (OP). Both of them will be calculated, and analyzed separately according to the moneyness and time to maturity of options. The equations for those statistics go as follows:

$$OP_m = \frac{\sum_{i=1}^{f(m)} x_i}{f(m)} \qquad MAPE_m = \frac{\sum_{i=1}^{f(m)} \left| \frac{P_{MOD_i}}{P_{MKT_i}} - 1 \right|}{f(m)}$$

where:
P_{MOD}– price of the option calculated using a specific option pricing model
P_{MKT} – price of the same option observed on the market
m – designation of a set containing options with a specific time to maturity and moneyness ratio
$f(m)$ – function that returns the number of observations according to the set m chosen.
$x_i = 1$ if $P_{MOD_i} > P_{MKT_i}$, 0 otherwise

3.3 Option prices estimation – benchmark models

1. *Black model with historical volatility*

As the first benchmark model, I use the Black (1976) model to price options on futures on WIG20. I decided not to choose the standard Black-Scholes model in order to avoid making assumptions about the dividend yield. According to the Black model, if a futures contract has the same maturity as an option, the option on an asset can be priced using the futures contract on that asset, without making any assumptions about the dividend yield. In other words, we use the dividend yield implied in the prices of futures (Hull 2008). The problems connected with the assumptions about implied dividend rates will be discussed in details in the section devoted to the models involving Monte-Carlo simulations. The equation for the Black model goes as follows:

Theoretical price of a call option on futures:

$$c = e^{-rT}[F_0 \times N(d_1) - K \times N(d_2)]$$

Theoretical price of a put option on futures:

$$p = e^{-rT}[K \times N(-d_2) - F_0 \times N(-d_1)]$$

where:

$$d_1 = \frac{\ln\left({F_0}/{K}\right) + (\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where:

F_o – price of the futures contract on an asset
K – strike price of the option
r – risk-free rate
σ – standard deviation of the underlying asset, expressed yearly
T – time to maturity of the option, expressed in years

For the first benchmark model, the volatility estimator will be equal to a 20-day historical standard deviation, expressed in yearly terms. The equation for that estimator goes as follows:

$$\sigma_t = \sqrt{\frac{\sum_{i=1}^{20}(r_{t-i} - \bar{r})^2}{20}} \times \sqrt{252}$$

where:

σ_t – historical standard deviation of the underlying asset for day t

$\bar{r}$ –historical mean

r_{t-i} – daily return on WIG20 at day $t - i$

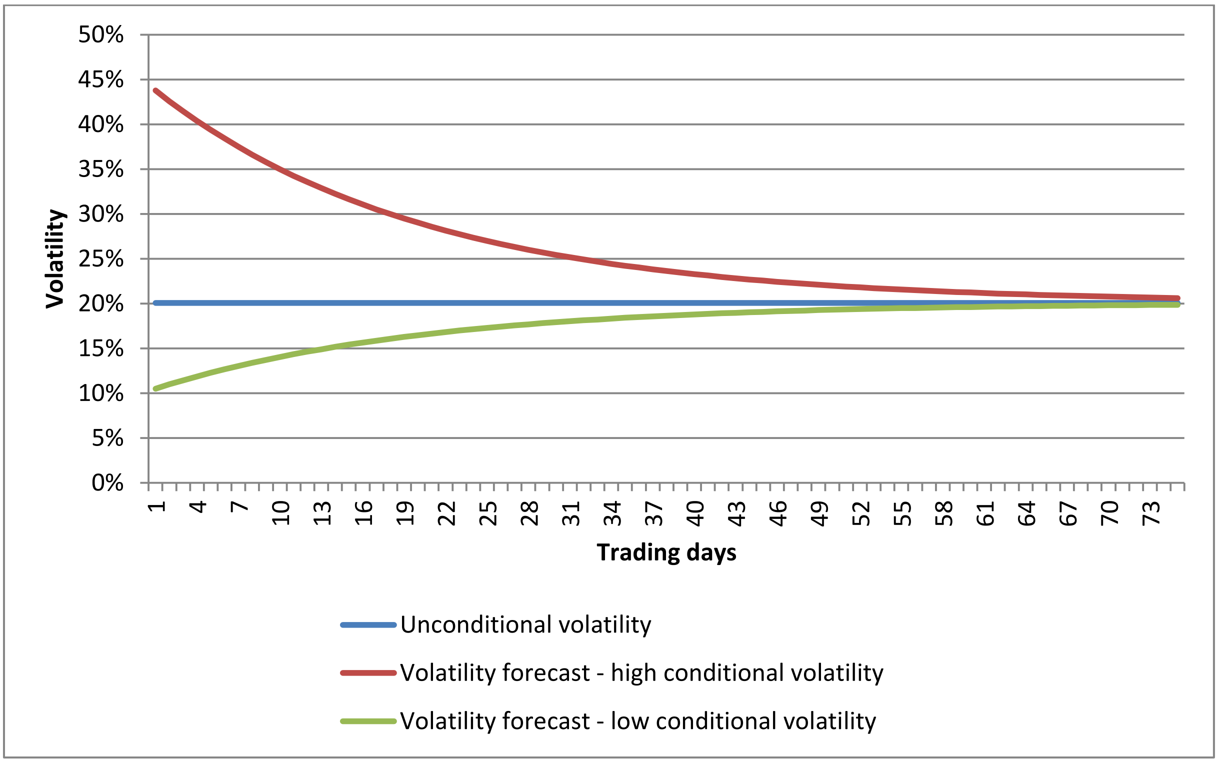
2. *Black model with volatility derived from a GARCH estimation*

As GARCH models by definition forecast future variance, intuitively one would like to use the variance forecasts in option pricing models in order to make the theoretical option price closer to its market counterpart. Details regarding GARCH models are described in the section devoted to methods involving Monte-Carlo simulations.

The problem with using GARCH volatility forecasts within the Black formula is that the volatility of the asset underlying an option in the Black model is constant through the whole life of the option. However, the GARCH volatility is by definition dynamic. Therefore, using the conditional volatility in the Black formula may not be conceptually correct, as the GARCH volatility is a mean-reverting process and will in a certain time converge to the unconditional volatility. On the other hand, a well-observed phenomenon on financial markets is volatility clustering (proved by a high beta factor in a GARCH estimation), and especially for short-maturity options the probability that the conditional volatility will return to its unconditional value is much smaller than the probability than it will persist. Thus, the conditional volatility in this case seems a much better volatility proxy to be inserted into the Black model, but still its use can by questionable.

Fortunately, there exists a third approach. One can extend the GARCH forecast of volatility to more periods than one day. Figure 1 illustrates how that kind of volatility forecast is going to look like.

Figure 1. Long-term conditional volatility forecasts using GARCH models



The figure represents long-term forecasts of volatility using a standard GARCH(1,1) model with its parameters equal to 0.000008 for gamma, 0.1 for alpha and 0.85 for the beta. The starting values, equal to 45% for the high conditional volatility, and 10% for the low conditional volatility, are chosen arbitrarily. Source: own calculations.

Of course, the prediction power of the above estimates decreases with the increase of the time horizon, and converges exponentially to the long-term volatility rate. However, for option pricing purposes, taking the average of all daily volatility estimates for the whole period to maturity tends to be a more accurate volatility estimator than the one that predicts the volatility only for day $t+1$, or the unconditional volatility. It is however important to note that this approach has not been scientifically proved, but represents rather a heuristic view. Ideally, we would like to have the whole volatility term structure in our option pricing model. Unfortunately, the Black model allows us to insert only one measure of volatility. That is why we need to take an average value. The equation for the variance rate until the maturity of an option is therefore the following:

$$\frac{1}{T}\int_0^T V(t)dt = V_L + \frac{1-e^{-aT}}{aT}[V(0) - V_L]$$

where:

$a = \ln \frac{1}{\alpha + \beta}$

$V(t) -$ estimate of the variance rate in t days

$V_L -$ unconditional variance

$T -$ time to maturity of the option, expressed in days

$\alpha, \beta -$ parameters of the GARCH model

This approach has one advantage – it allows to obtain different volatility estimates depending on the maturity of the option, potentially reducing the pricing bias. The accuracy of such approach in practice is however a topic for a separate study.

3.4 Option prices estimation – models involving Monte-Carlo simulations

Another popular type of models widely used in the literature as well as in practice are those that involve Monte-Carlo simulations in a GARCH volatility context. This section describes some of them.

The model I will focus on is the Duan (1995) option pricing model. The Duan approach develops an option pricing model and its corresponding delta formula in the context of the GARCH asset return process, developed by Bollerslev (1986). Duan introduced a mean-reverting GARCH process into the simulations of the behavior of the instrument underlying an option, eliminating one significant drawback of the standard Black model: the assumption that the volatility stays constant until maturity. This approach allows also to reduce the pricing bias caused by the volatility smile. The following paragraph describes the concepts behind the Duan option pricing model in details.

Let X_t be the asset price at day t. Under, physical probability measure P, the rate of return of that asset is assumed to be described according to the following measure:

$$ln \frac{X_t}{X_{t-1}} = r_t = r - q - \delta\sqrt{h_t} - \frac{1}{2} h_t + \varepsilon_t$$

where ε_t is a process following a normal distribution with mean 0 and conditional variance h_t:

$$\varepsilon_t = \eta_t \sqrt{h_t}, \qquad \eta_t \sim N(0,1)$$

and h_t is assumed to follow a GARCH(p, q) process of Bollerslev (1986):

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

where:

$p, q -$ orders of the GARCH model

$\alpha, \beta, \omega -$ parameters of the GARCH model

In other words, the conditional variance is a function of q past returns and p past conditional variances, which makes h_t a predictable measure. To ensure stationarity, the model is estimated in a way that $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j$ is smaller than one. It is worth to remark that, for $p = q = 0$, the model reduces to the standard homoskedastic lognormal process, assumed in the Black model.

The conventional risk-neutral valuation relationship has to be generalized in order to allow for heteroskedasticity of the process for the return of the asset. This can be done through the *locally risk-neutral valuation relationship* (LRNVR). A risk-neutral pricing measure Q is said to satisfy the LRNVR if that measure is absolutely continuous with respect to measure P, $\frac{X_t}{X_{t-1}} | \phi_t$ distributes lognormally under Q,

$$E^Q \left(\frac{X_t}{X_{t-1}} \middle| \emptyset_{t-1} \right) = e^r,$$

and

$$Var^Q \left(ln \frac{x_t}{x_{t-1}} \middle| \emptyset_{t-1} \right) = Var^P \left(ln \frac{x_t}{x_{t-1}} \middle| \emptyset_{t-1} \right)$$

almost surely with respect to measure P, where $\emptyset_{t-1}$ stands for the past information known at t.

In other words, the conditional variance under P has to be equal to the conditional variance under Q. This model does not locally depend on preferences, but local risk neutralization is not sufficient to eliminate the preference parameters. Therefore, all preference considerations are reduced to a single parameter – the unit risk premium δ. In the case of a process with $p = q = 0$, the LRNVR is reduced to the standard risk-neutral valuation relationship[3].

Under the pricing measure Q, it is true that:

$$ln \frac{X_t}{X_{t-1}} = r_t = r - q - \frac{1}{2} h_t + \xi_t$$

$$\xi_t = \eta_t \sqrt{h_t}, \qquad \eta_t \sim N(0,1)$$

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i (\xi_t - \delta \sqrt{h_{t-i}})^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

Then, the terminal price of an asset underlying an option can be derived as:

$$S_T = S_t \times \exp[(T - t) \times (r - q) - \frac{1}{2} \sum_{i=t+1}^{T} h_i + \sum_{i=t+1}^{T} \xi_i]$$

[3] See Duan (1995) for more details.

Monte-Carlo simulations are used in order to obtain a reasonable amount of terminal asset prices calculated according to the above equation. In my study, I simulate 10 000 different paths for S_t every day. Under the pricing measure Q, the price of the option is equal to the discounted average payoff at every simulation:

$$CALL_x = e^{-r(T-t)} \times E^Q[\max(S_T - K, 0)|\phi_t]$$

$$PUT_x = e^{-r(T-t)} \times E^Q[\max(K - S_T, 0)|\phi_t]$$

where $n = 10\ 000$, and K is the strike price of the option.

There remains the question whether the assumption of a continuous dividend yield is reasonable, and can be used as a part of the drift in the Monte-Carlo simulation. If we decide to take the implied dividend yield, we have to be sure that futures on WIG20 are priced correctly, which may not always be the case. The reasons behind such market failures may be diverse: high transaction costs, restrictions regarding short selling or the inability of market participants to borrow at the risk-free rate. The implied dividend yield used in the study must be chosen in such a way that minimizes arbitrage opportunities in order to make the results reliable. The methods used in the calculation of the implied dividend yield are discussed in subsection 3.4.

3.5 Volatility at day t

Every day that I price an option, I require an initial conditional variance h_{t-1} and coefficients α, β, ω as parameters of the Monte-Carlo simulation. In order to obtain them, I need to perform every day a GARCH optimization on past data. Details regarding the GARCH process used are presented in subsection 3.3.

I decided to use a GARCH(1,1) model because it gives the most accurate estimations, and differences between all other GARCH models tend to be insignificant. However, in order to reflect the fact that returns on WIG20, as typical financial returns, exhibit fatter tails than a normal distribution has, a separate simulation will be done, with the errors of the process η_t having a Student's t distribution. The hypothesis is that this simulation will be more accurate than the one with normal errors.

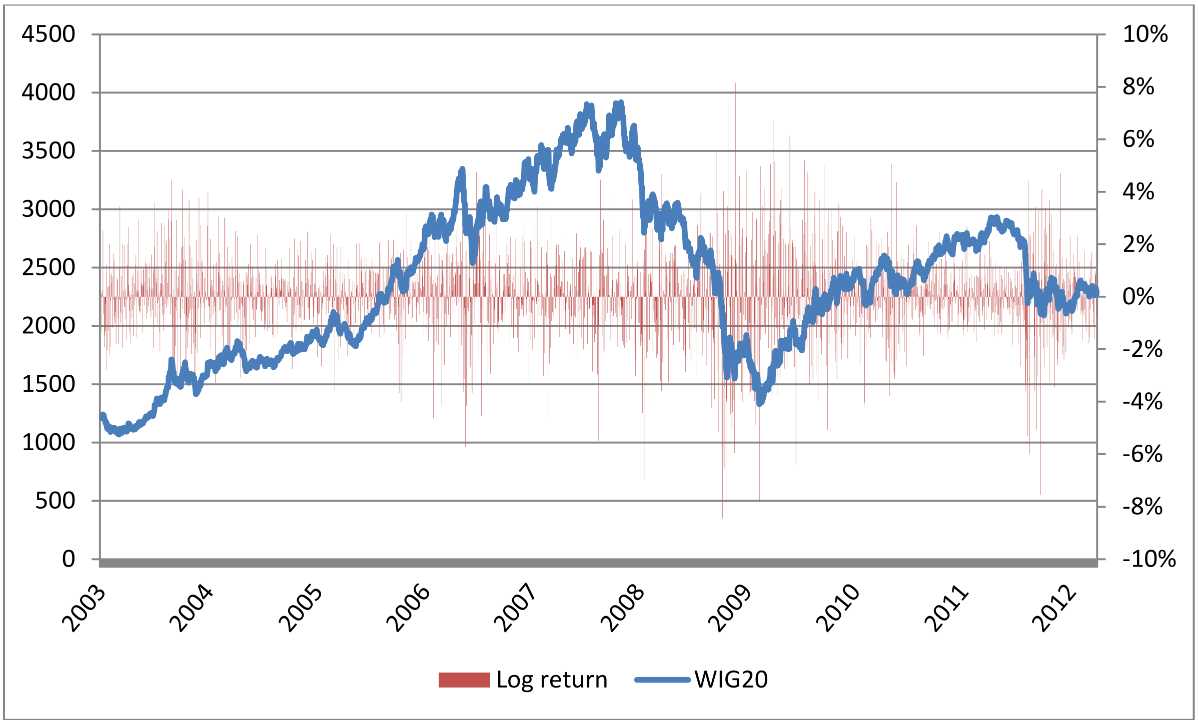
4. Data

Prices of options have been downloaded from the website of the official data provider of the Warsaw Stock Exchange wseinfospace.pl. I considered options traded since 01/01/2006 with all strike prices and maturities available, both call options and put options. From the same website I have also downloaded values for the WIG20 Index, prices of futures contracts on WIG20 and values of the WIBOR rate, which I used as a proxy for the risk-free rate. I use three different WIBOR rates every day, depending on the time to maturity of the options – 1M, 3M and 6M, which leads to a total of 4 713 observations. I use prices of futures, options, and WIBOR for the period from 01/01/2006 to 30/03/2012, which is the estimation window chosen for the study. Prices for the WIG20 Index concern a longer period, from 01/01/2003 to 30/03/2012. The reason is that the study starts on the 01/01/2006, and at that moment it needs past data in order to estimate a GARCH model. I consider a 3-year period sufficient in order to obtain reliable and accurate results. All prices used in the study are closing prices.

4.1. Returns on WIG20 – descriptive statistics

Figure 2 presents the evolution of the WIG20 index together with the returns on WIG20 for the study period. As it can be seen from Figure 2, the study covers a period of the bull markets of 2006-2007 and 2011, the bear market of 2008, and the recent turmoil. The period from the 1st of January 2003 to the 31st of December 2005 was used solely for the estimation of the parameters of the GARCH model and no options have been priced during that period. The length of the period used to estimate the parameters of the GARCH models has enabled to capture periods of both high and low volatility, which allowed for more realistic results. Table 1 presents descriptive statistics for the returns on WIG20 during the study period.

Figure 2. WIG20 together with logarithmic returns on WIG20



The figure shows the value of the WIG20 index and its corresponding logarithmic daily returns for the period from the 1st of January 2003 to the 31st of March 2012. Source: own calculations on the basis of data provided by wseinfospace.pl.

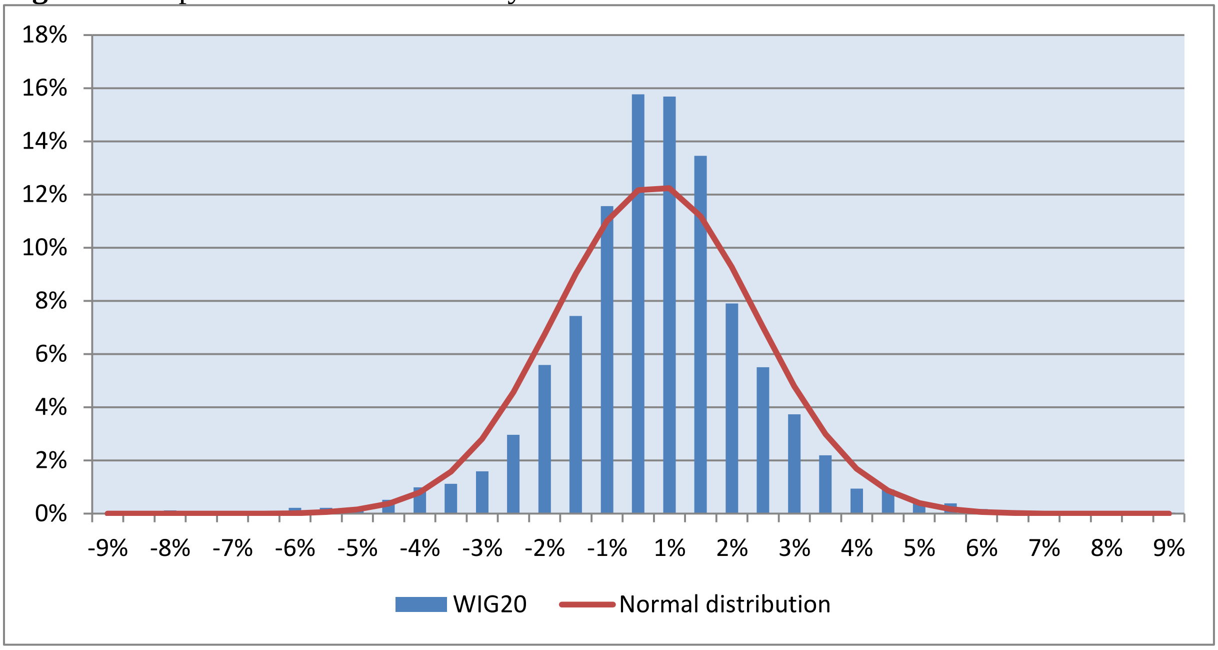
Table 1. Descriptive statistics for returns on WIG20

Number of observations	2 327
Average return	0.03%
Standard deviation	1.61%
Minimum	-8.44%
Maximum	8.15%
Skewness	-0.24
Kurtosis	5.62

The table contains descriptive statistics for daily logarithmic returns on WIG20 for the period from the 1st of January 2003 to the 31st of March 2012. Source: own estimates on the basis of data provided by wseinfospace.pl

A daily average return of 0.03% corresponds to a yearly return of 7.20%, which confirms that the period chosen for the study is appropriate as it mirrors well enough the behavior of the market. A daily standard deviation of 1.61% corresponds to a yearly volatility of 25.54%. It may seem high for a stock index, but it is to a certain extent due to the end 2008 - beginning 2009 crisis period, where several 3- and 4-sigma events took place. Excess kurtosis and negative skewness are also typical for financial returns. From the above statistics we can clearly see that returns on WIG20 are clearly not normal. To confirm this, Figure 3 represents a histogram of these returns, plotted together with a normal distribution with mean and standard deviation equal to, respectively, 0.03% and 1.61%. The figure clearly shows that the series of returns on WIG20 exhibit fatter tails than a normal distribution has, as well as a higher peak. Especially, the left tail of the empirical distribution is thicker.

Figure 3. Empirical distribution of daily returns on WIG20 vs. normal distribution



The above figure shows the empirical distribution of daily logarithmic returns on WIG20 for the period from the 1st of January 2003 to the 31st of March 2012 (blue bars), together with the density of a normal distribution with mean 0.03% and standard deviation 1.61% (red line). Daily returns have been aggregated into 50 percentage points groups. Source: own calculations on the basis of data provided by wseinfospace.pl.

A total of 1 571 observations for futures on WIG20, and 2 328 observations for the WIG20 index are used.

4.2. Descriptive statistics – options on WIG20

As options on the WIG20 index are concerned, the dataset contains 50 515 valid call option prices, and 50 648 valid put option prices, with all available strikes and maturities. In my study I select options with a specific time to maturity, and specific strike prices.

Options tested in the study are selected according to the following methodology. During the whole estimation period, options on WIG20 were quoted every 100 index points. Every trading day, the value of the WIG20 index has been rounded to the nearest hundred. The number obtained is the strike of the option that will be considered as ATM. Then, eight option series with the same maturity are picked up, four ITM and four OTM, with a space of 100 index points of strike price between them. In all tables in the text, options are ranked into categories numbered from 1 to 9. 1 represents the options being the most OTM, while 9 represents options being the most ITM.

What concerns the time to maturity, only options with a maturity between 1 and 4 months are included in the study. The 4-month upper maturity limit of the option comes from the fact that options on the Warsaw Stock Exchange with a higher time to maturity tend to be very illiquid, and during many trading days simply no transactions are made, and including them in the research might have led to significant biases. The 1-month lower boundary is used as a filter for outliers. As the WSE options market is still very immature, most players are pure speculators, very few of them buy options for risk management purposes. Therefore, options that are significantly OTM, and thus cheap, exhibit high percentage moves by definition, and their volatility is enhanced by speculators. So, prices of cheap options tend to deviate from their fundamentals, and volatility estimators become useless.

In my study, a total of 28 278 different call and put options should be tested out of 1 571 trading days in the estimation period. In reality, at the beginning of the study period, for options that were strongly OTM and strongly ITM, no market price was provided as no trades have yet been executed. These observations have been removed in order not to disrupt overall results, decreasing the amount of options tested to 27 210, for all strike prices. Amongst the whole sample, there are 13 712 call options, and 13 498 put options. Tables 2 and 3 show how the number of observations is decomposed according to the moneyness and the time to maturity.

Table 2. Number of observations according to their time to maturity and moneyness – call options

	CALL									
Time to maturity	1	2	3	4	5	6	7	8	9	**TOTAL**
21 days < < 30 days	141	150	155	155	155	155	155	154	151	**1371**
30 days < < 60 days	629	674	696	719	719	718	719	714	683	**6271**
60 days < < 92 days	639	685	696	697	695	693	680	664	621	**6070**
TOTAL	**1 409**	**1 509**	**1 547**	**1 571**	**1 569**	**1 566**	**1 554**	**1 532**	**1 455**	**13 712**

The table shows how many observations have fallen into which moneyness and time to maturity class. The third line contains much smaller numbers than the other two simply because the time window for this class is smaller. The time to maturity is expressed in trading days. Total number of observations: 13 712. Source: own calculations on the basis of data provided by wseinfospace.pl.

Table 3. Number of observations according to their time to maturity and moneyness – put options

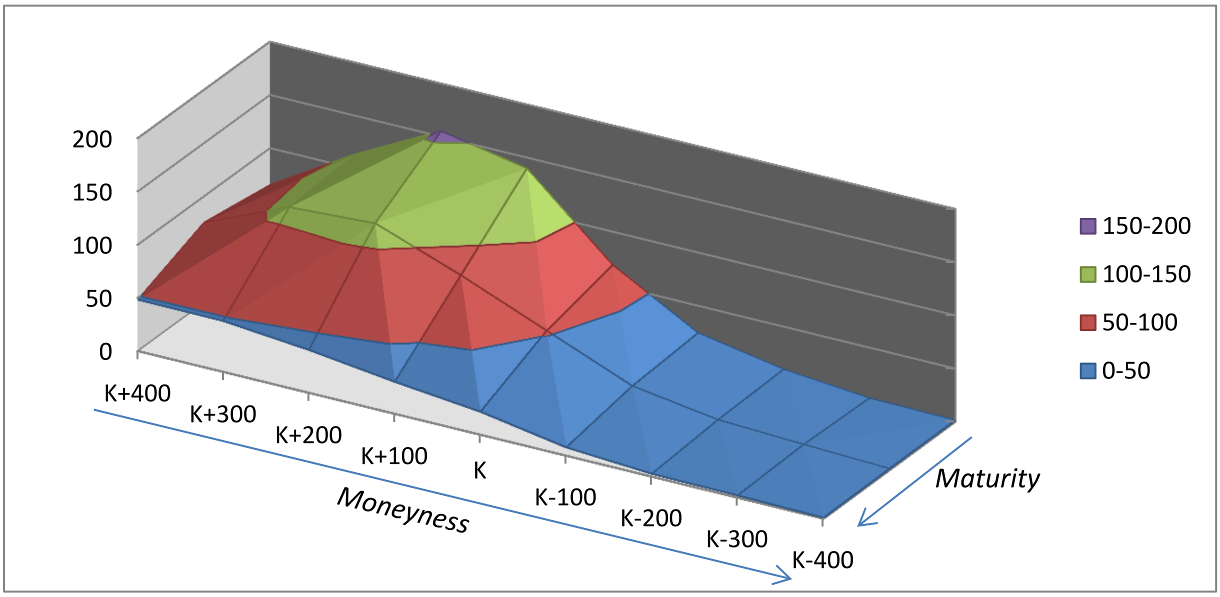
	PUT									
Time to maturity	1	2	3	4	5	6	7	8	9	**TOTAL**
21 days << 30 days	151	154	155	155	155	155	149	136	131	**1341**
30 days < < 60 days	672	714	719	719	719	697	674	653	602	**6169**
60 days < < 92 days	646	670	686	695	697	693	672	633	596	**5988**
TOTAL	**1 469**	**1 538**	**1 560**	**1 569**	**1 571**	**1 545**	**1 495**	**1 422**	**1 329**	**13 498**

The table shows how many observations have fallen into which moneyness and time to maturity class. The third line contains much smaller numbers than the other two simply because the time window for this class is smaller. The time to maturity is expressed in trading days. Total number of observations: 13 498. Source: own calculations on the basis of data provided by wseinfospace.pl.

From the two tables above one can see that the number of options used in the study is not equal when considering different maturity classes. The reason for that is that at the beginning of the study period, strong ITM and strong OTM options were very illiquid. As a result, no trades have been executed at all on some option series and these cases have been removed from the study. As for call options, the biggest number of observations is for options with a time to maturity between 30 and 60 trading days, ATM and slightly ITM. As for call options, the most frequently traded series are options also with a maturity of 30 to 60 trading days, but being ATM or slightly OTM. The first line in both tables yields much smaller values simply because the time window used in this class is much smaller than the other two ones.

Figure 4 shows the average daily trading volume, expressed in the amount of options traded, of calls and puts during the estimation period.

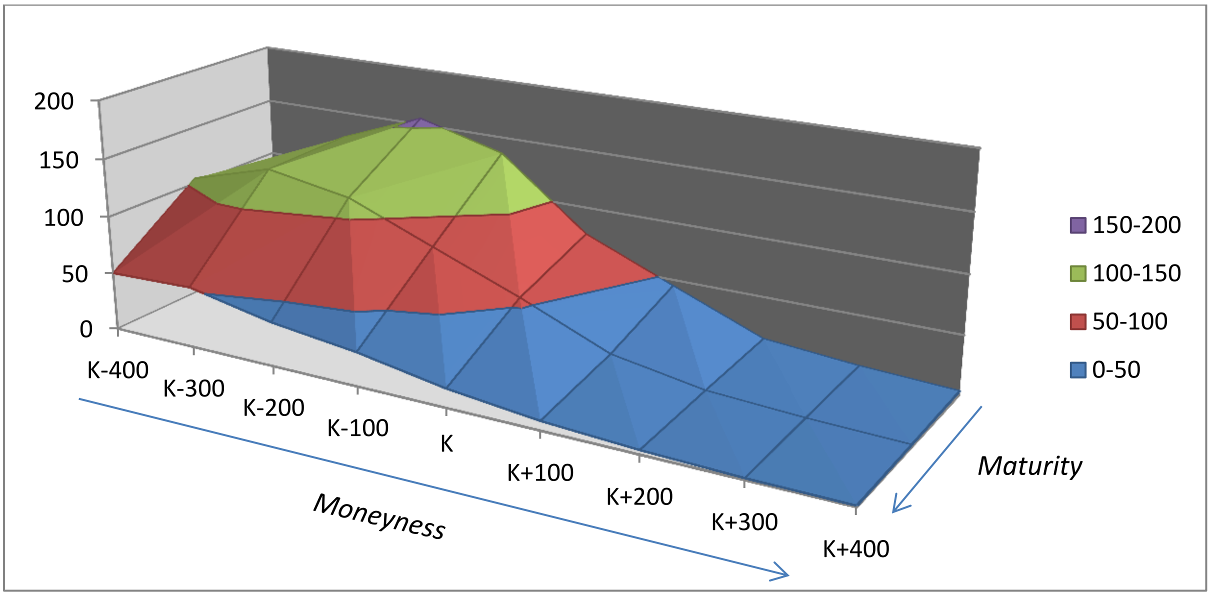
Figure 4. Average daily trading volume for options on WIG20, decomposed into maturity and moneyness classes – CALL options



The figure represents the average daily turnover for call options decomposed into maturity and moneyness classes. There are 9 moneyness classes, quoted every 100 index points starting from low moneyness and reaching high moneyness. K is the strike price of the option being ATM on a specific day. K+100, K-100, etc. indicate the strike price of s series of options relatively to the option which is on a certain day ATM. There are 3 maturity classes – the nearest represents options with a maturity between 60 and 92 trading days, the deepest options with a maturity below 30 trading days, and the one between represents options with a maturity between 30 and 60 trading days, Source: own estimates on the basis of data provided by wseinfospace.pl.

From Figure 4 one can see that most frequently traded call options on WIG20 are those with a shorter time to maturity, and being OTM. Investors mostly like trading options being 200 index points OTM, with time to maturity below 30 trading days, with an average daily trading volume equal to 155.2 options. Options being more OTM are less frequently traded. However, for the deepest OTM options the average daily turnover is still equal to 62.2 options. Generally, the trading volume decreases with increasing moneyness and maturity. Hardly any trades are executed for deep ITM options – the average daily trading volume for all maturity classes is equal only to 1.1 options. For ATM options, the average daily trading volume is equal to 40.3 options. If one would like to draw conclusions regarding the nature of trades, it can be said that market participants prefer cheap options, because of their speculative nature – they tend to be much more volatile than options being more ITM. Still, investors like less the cheapest options – the reason for this are probably wide bid-ask spreads observed for cheaper options that significantly increase the costs of entering into option trades.

Figure 5. Average daily trading volume for options on WIG20, decomposed into maturity and moneyness classes – PUT options



The figure represents the average daily turnover for put options decomposed into maturity and moneyness classes. There are 9 moneyness classes, quoted every 100 index points starting from low moneyness and reaching high moneyness. There are 3 maturity classes – the nearest represents options with a maturity between 60 and 92 trading days, the deepest options with a maturity below 30 trading days, and the one between represents options with a maturity between 30 and 60 trading days, Source: own estimates on the basis of data from wseinfospace.pl.

Similarly to call options, according to Figure 5 put options are also more frequently traded when their time to maturity is shorter and moneyness smaller. Like calls, the most frequently traded puts are those being 200 index points OTM and with a maturity below 30 days, with an average daily trading volume equal to 155.4 options. Options being deep in the money are less frequently traded, but still exhibit an average daily volume of 78.9 options for all maturity classes. Overall, the daily trading volume of put options decreases with increasing moneyness and maturity, just like it is the case for call options. ATM put options exhibit an average daily turnover of 39.6 options, and deep ATM options 1.53 options a day. To conclude, investors prefer OTM options because they are cheaper and therefore more volatile and speculative. However, the average turnover for deep OTM puts is bigger than for OTM calls – this may suggest that not only speculators want to buy these instruments, but also investors holding long positions in shares that want to protect themselves against extreme events, thereby hedging tail risk.

4.3. Data for the calculation of volatility

The way the historical volatility has been implemented is relatively simple and has been explained in the subsection 2.3.1. However, the way the GARCH volatility has been calculated requires some clarification.

The calculation of the GARCH α, β and γ parameters requires performing maximum likelihood estimation on past data. It is performed on the series described in subsection 3.1. I use 756 trading days preceding the pricing day for the GARCH estimation, to ensure that the series is long enough to reflect a sufficient number of periods with high volatility, as well as periods with low volatility. The obtained α, β, ω parameters are then used in the Duan process to obtain the final option price. As the variance of the return on the first simulation day r_1, I use the conditional variance h_{t-1}, which I plot into the GARCH equation in order to obtain h_t.

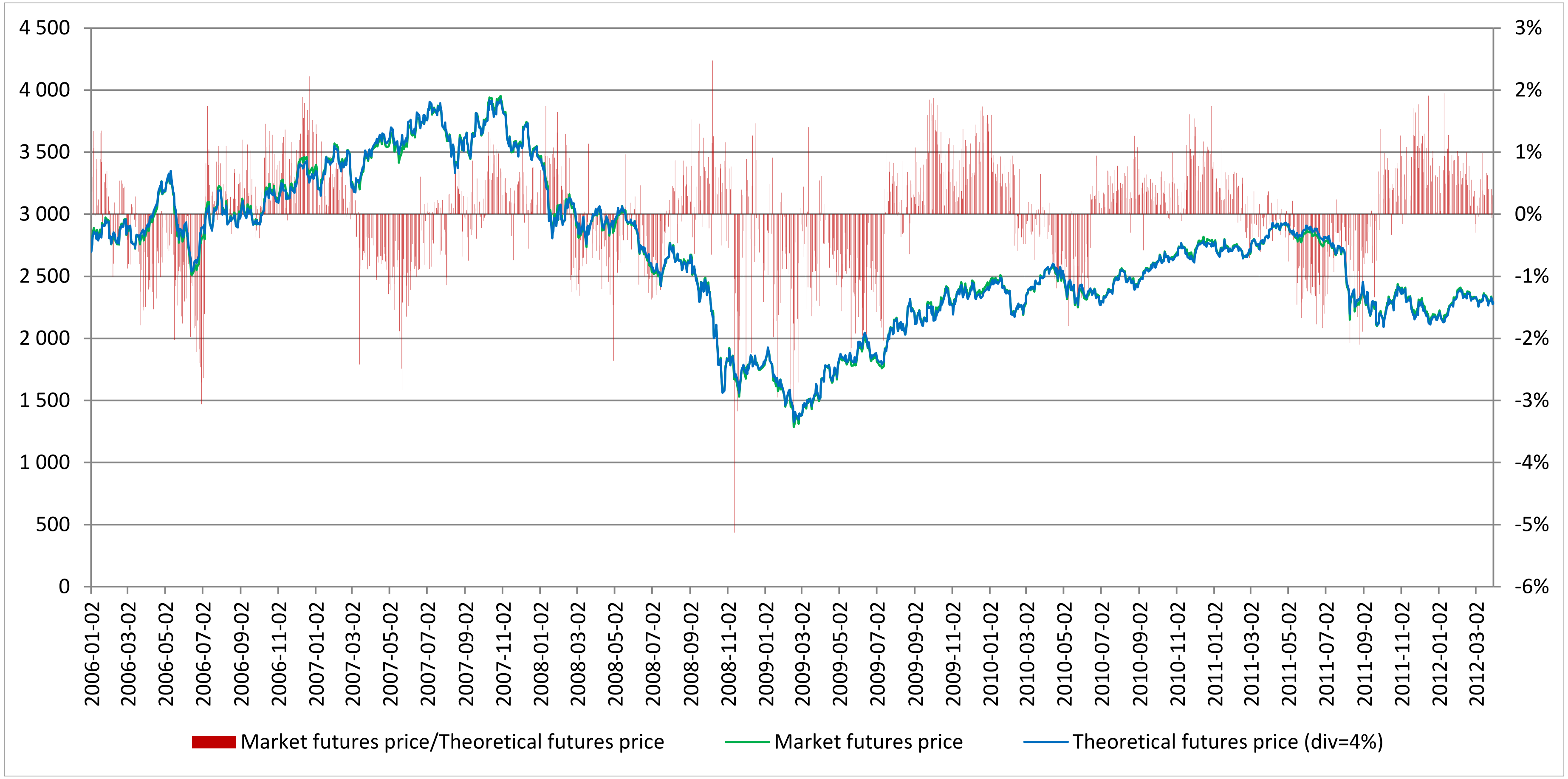
4.4. Additional issues

As it has been already mentioned in this paper before, some assumptions regarding the implied dividend yield have to be made in order to make the results of the estimation of the models accurate. The following paragraphs describe the methodologies used for the implied volatility.

Figure 6 represents futures prices observed on the market, together with hypothetical futures prices, assuming a constant dividend yield of 4%, and the percentage price difference between them. This figure shows some potential mispricings of futures contracts. One can see periods of significant positive errors, and periods of significant negative errors. These errors represent, at least in theory, potential arbitrage opportunities. When considering the dividend yield in the drift term of the Monte Carlo process, it appears that very often in the model we are going to have a significantly negative drift (when the implied dividend is high, say 10%) which may lead to a model which underprices call options and overprices put options, or, on the other hand, a significantly positive drift (when the implied dividend yield is negative, say -5%) which may in turn lead to a model which overprices call options and underprices put options. One may ask if it would not be more reasonable if in the model we would assume a dividend yield which is constant. For that reason, the model with constant dividend will be considered separately of the model with the implied dividend.

However, if we take a deeper look at Figure 6, we can see that, on average, the implied dividend yield is very high in the first six months of a year, and very low (negative) in the last six months of every year. This is due to the fact that dividends are normally paid around June, and not as we assumed, continuously during the whole year. For that reason, the possibilities of arbitrage discussed before may be much more limited. However, the question asked in Figure 6 regarding the drift of the Monte-Carlo process still remains.

Figure 6. Market prices of futures contracts vs. theoretical prices of futures assuming a constant dividend yield equal to 4%, plotted together with the percentage difference between them



The figure shows market prices of futures on WIG20 (green line), together with theoretical option prices assuming a constant dividend yield of 4% (blue line). The red bars show the percentage price difference between the market price and the hypothetical price. Futures were selected according to their maturity using the same methodology as for options, see the subsection 'Selection of options' for more details. The dividend yield of 4% was chosen arbitrarily, but is very close to the average implied dividend yield, which is equal to 4.08%. Source: own calculations on the basis of data provided by wseinfospace.pl.

5. Results

The results have been presented in two separate parts. Tables 4 to 15 represent the results for the Mean Absolute Pricing Errors (MAPE). Tables 16 to 27 show the results for the second statistic, namely the Percentage of Overpredictions (OP). Tables 28 to 31 show results for median absolute errors for the best models. Results have been divided into three maturity classes[4]: from 21 days to 29 days to maturity, from 30 to 59 days, and from 60 to 92 days. Nine different classes of moneyness are shown, ranging from deep OTM options on the left side to deep ITM options on the right side. Moneyness increments are presented every 100 points of strike price. Call options and put options are treated separately.

5.1. List of models

As a reminder, the following models will be included in the paper:

1. Black model with 20-day historical volatility
2. Black model with conditional GARCH volatility
3. Duan approach with implied dividend yield
4. Duan approach with dividend yield equal to 4%
5. Duan approach without dividends
6. Duan approach with implied dividend yield – GARCH with Student's t errors

5.2. Results for the Mean Absolute Pricing Errors

At the beginning, the two benchmark Black models will be discussed. I will start with the Black model with 20-day historical volatility, presented on Table 4 and 5. As this is by definition a backward-looking measure while the volatility observed in the market is forward-looking, it is expected that this estimator will be performing poorly when comparing to other models. A MAPE of 18.23% for ATM call options and 18.10% for ATM put options confirms the above hypothesis. When analyzing the results across moneyness classes, similar results to previous studies are obtained. MAPE are the smallest for deep ITM options (8.01% for call options and 7.15% for put options for the whole sample), and the biggest for deep OTM options (64.71% for call options and 63.89% for put options across the whole sample). This is consistent with previous studies, and similar patterns are observed across all other models used. The reasons for those patterns will be explained at the end of the chapter devoted to results. When analyzing the results across time to maturity clusters, it is noticeable that the best performing class for call options varies depending on the moneyness. As far as ATM and ITM options are concerned, the smallest MAPE appears for short-term options. For OTM options, the best performing maturity class is the one with the longest time to maturity. This should not astonish, as short-term OTM options are cheap and very often the model value is very different from the value observed on the market. For put options, similar patterns are observed across all moneyness classes.

A surprisingly good performance in terms of MAPE for the Black model with conditional GARCH volatility is observed, which can be seen on Table 6 and 7.

[4] Results are presented in trading days to maturity.

Table 4. Mean Absolute Pricing Errors (MAPE) for CALL options – Black model with 20-day historical volatility

	CALL								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	**64,71%**	**49,16%**	**35,89%**	**25,30%**	**18,23%**	**13,98%**	**11,57%**	**9,62%**	**8,01%**
60 days << 92 days	59,78%	43,86%	34,17%	25,73%	20,03%	16,44%	13,62%	10,49%	8,57%
30 days << 60 days	65,15%	51,06%	36,08%	24,94%	17,09%	12,28%	10,05%	8,63%	7,67%
21 days << 30 days	85,03%	64,80%	42,75%	25,01%	15,45%	10,90%	9,58%	10,43%	7,29%

Table 5. MAPE for PUT options – Black model with 20-day historical volatility

	PUT								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	**63,89%**	**52,71%**	**39,75%**	**27,54%**	**18,10%**	**12,39%**	**9,86%**	**8,10%**	**7,15%**
60 days << 92 days	56,12%	46,31%	36,36%	26,98%	18,84%	13,43%	10,57%	8,85%	7,47%
30 days << 60 days	69,37%	57,22%	42,36%	28,22%	17,85%	11,89%	9,62%	7,52%	6,80%
21 days << 30 days	72,77%	59,62%	42,67%	26,94%	15,92%	9,98%	7,71%	7,35%	7,28%

Table 6. MAPE for CALL options – Black model with conditional GARCH volatility

	CALL								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	73,00%	57,62%	35,81%	21,33%	13,37%	10,67%	9,77%	8,86%	7,69%
60 days << 92 days	67,89%	46,17%	29,99%	20,58%	15,17%	12,96%	11,68%	9,52%	8,15%
30 days << 60 days	75,75%	62,11%	37,00%	21,26%	11,69%	8,65%	8,04%	7,96%	7,35%
21 days << 30 days	83,88%	89,77%	56,55%	25,00%	13,04%	9,77%	9,41%	10,19%	7,30%

Table 7. MAPE for PUT options – Black model with conditional GARCH volatility

	PUT								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	53,74%	39,23%	26,26%	16,97%	12,10%	10,03%	8,70%	7,68%	6,95%
60 days << 92 days	43,91%	32,72%	22,48%	15,99%	12,88%	11,07%	9,31%	8,43%	7,22%
30 days << 60 days	58,82%	41,77%	27,40%	16,93%	11,25%	9,26%	8,26%	7,00%	6,63%
21 days << 30 days	73,17%	55,78%	37,75%	21,58%	12,62%	8,80%	7,98%	7,42%	7,19%

Table 8. MAPE for CALL options – Duan approach with implied dividend yield

	CALL								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	**87,20%**	**64,35%**	**37,76%**	**22,51%**	**15,00%**	**11,94%**	**10,79%**	**9,40%**	**7,99%**
60 days << 92 days	80,30%	50,66%	31,87%	22,44%	17,26%	14,61%	12,95%	10,29%	8,61%
30 days << 60 days	91,30%	69,80%	39,11%	21,65%	12,88%	9,59%	8,93%	8,36%	7,54%
21 days << 30 days	100,17%	102,35%	58,17%	26,81%	14,70%	10,85%	9,92%	10,43%	7,45%

Table 9. MAPE for PUT options – Duan approach with implied dividend yield

	PUT								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	**54,93%**	**43,80%**	**32,09%**	**21,88%**	**14,60%**	**11,13%**	**9,09%**	**7,76%**	**6,84%**
60 days << 92 days	47,46%	38,05%	27,79%	20,10%	14,74%	12,20%	9,68%	8,42%	6,99%
30 days << 60 days	58,77%	45,90%	33,77%	22,38%	14,27%	10,42%	8,65%	7,12%	6,58%
21 days << 30 days	69,86%	59,05%	43,36%	27,52%	15,46%	9,53%	8,37%	7,72%	7,36%

Table 10. MAPE for CALL options – Duan approach with dividend yield equal to 4%

Time to maturity	CALL								
	1	2	3	4	5	6	7	8	9
Whole sample	**86,37%**	**64,05%**	**38,30%**	**23,65%**	**16,24%**	**13,01%**	**11,41%**	**9,94%**	**8,18%**
60 days << 92 days	80,63%	50,13%	32,23%	22,77%	17,41%	14,66%	12,94%	10,64%	8,78%
30 days << 60 days	89,63%	69,84%	40,08%	23,84%	15,31%	11,56%	10,11%	9,10%	7,77%
21 days << 30 days	97,85%	101,66%	57,58%	26,73%	15,39%	12,32%	10,76%	10,76%	7,61%

Table 11. MAPE for PUT options – Duan approach with dividend yield equal to 4%

Time to maturity	PUT								
	1	2	3	4	5	6	7	8	9
Whole sample	**54,79%**	**44,08%**	**32,49%**	**22,43%**	**15,77%**	**12,40%**	**9,88%**	**8,34%**	**7,14%**
60 days << 92 days	47,42%	38,58%	28,59%	21,31%	16,39%	13,37%	10,41%	9,00%	7,26%
30 days << 60 days	58,28%	45,75%	33,69%	22,23%	14,70%	11,40%	9,33%	7,63%	6,85%
21 days << 30 days	70,84%	60,30%	44,20%	28,42%	17,90%	12,64%	9,99%	8,74%	7,95%

Table 12. MAPE for CALL options – Duan approach without dividends

Time to maturity	CALL								
	1	2	3	4	5	6	7	8	9
Whole sample	**104,95%**	**77,37%**	**45,15%**	**26,80%**	**17,37%**	**13,08%**	**10,81%**	**9,29%**	**7,87%**
60 days << 92 days	101,33%	62,85%	39,54%	26,65%	19,12%	14,96%	12,41%	9,79%	8,38%
30 days << 60 days	108,02%	83,87%	46,76%	26,72%	16,09%	11,56%	9,46%	8,62%	7,57%
21 days << 30 days	107,73%	114,50%	63,11%	27,81%	15,45%	11,77%	10,06%	10,18%	7,14%

Table 13. MAPE for PUT options – Duan approach without dividends

Time to maturity	PUT								
	1	2	3	4	5	6	7	8	9
Whole sample	**60,31%**	**49,78%**	**38,36%**	**27,48%**	**19,20%**	**14,43%**	**11,36%**	**9,38%**	**7,90%**
60 days << 92 days	53,99%	45,24%	35,15%	26,51%	20,17%	15,94%	12,52%	10,39%	8,42%
30 days << 60 days	63,47%	51,23%	39,38%	27,50%	18,09%	13,22%	10,52%	8,53%	7,43%
21 days << 30 days	73,25%	62,78%	47,80%	31,72%	19,91%	13,18%	9,96%	8,71%	7,69%

Table 14. MAPE for CALL options – Duan approach with implied dividend yield – GARCH with Student's t errors

Time to maturity	CALL								
	1	2	3	4	5	6	7	8	9
Whole sample	86,94%	64,35%	37,58%	22,37%	14,91%	11,88%	10,75%	9,36%	8,00%
60 days << 92 days	79,83%	50,49%	31,61%	22,24%	17,15%	14,50%	12,94%	10,26%	8,65%
30 days << 60 days	90,41%	68,88%	38,61%	21,46%	12,77%	9,54%	8,86%	8,30%	7,53%
21 days << 30 days	103,67%	107,26%	59,77%	27,24%	14,80%	10,95%	9,94%	10,40%	7,40%

Table 15. MAPE for CALL options – Duan approach with implied dividend yield – GARCH with Student's t errors

Time to maturity	PUT								
	1	2	3	4	5	6	7	8	9
Whole sample	54,81%	43,81%	32,09%	21,87%	14,58%	11,17%	9,07%	7,76%	6,84%
60 days << 92 days	47,22%	37,99%	27,82%	20,04%	14,79%	12,28%	9,70%	8,46%	6,99%
30 days << 60 days	58,68%	45,93%	33,72%	22,39%	14,17%	10,41%	8,61%	7,10%	6,59%
21 days << 30 days	70,07%	59,24%	43,38%	27,66%	15,56%	9,59%	8,33%	7,67%	7,32%

MAPE for ATM call options for the whole sample is equal only to 13.37%, and for ATM put options only to 12.10%. Across maturity clusters, the smallest errors are observed, like for the Black model with historical volatility, for ITM and ATM options with a smaller time to maturity (the best class is the one which groups options with a time to maturity between 30 and 60 trading days). For OTM options, MAPE increase with decreasing time to maturity. Therefore, it is noticeable that this model can to a significant extent reduce the bias of the Black model caused by the assumption of constant volatility. However, it is still biased in the way that it assumes that stock returns follow a normal distribution.

I will now focus on the analysis of the models calculated using the Duan methodology, first on the models with GARCH residuals following a normal distribution. I will describe how a model performs with different assumptions about the dividend yield. As it has already been discussed in the chapter devoted to the description of the methodology, it is arguable what dividend yield should be used in the Monte-Carlo simulation, as it has not been confirmed that futures on WIG20 are priced well enough to allow a safe extraction of the dividend yield from them. The implied dividend yield can sometimes be irrationally high, and sometimes negative.

The first model discussed will be the one with the implied dividend yield, represented by Table 8 and 9. What is disappointing, results are worse than for the benchmark Black model with GARCH volatility. The MAPE for the whole sample for ATM call options is equal to 15.00%, and for ATM put options 14.60%. That makes this third best after the Black with GARCH volatility and the Duan with GARCH with Student's t residuals. Errors for ITM and deep ITM options are very similar to the ones observed in benchmark models, and are slightly in favor of the Black model with GARCH volatility, except for the deepest ITM put options, where MAPE for the Duan model is equal to 6.84%, while for the Black model it is equal to 6.95%, which creates an 11 percentage point advantage for the Duan model. When analyzing the Duan model with implied dividend yield across maturity classes, for ITM, deep ITM and ATM options the smallest errors are observed in the middle maturity cluster, both for call and for put options. The biggest MAPE for those options are those with the highest time to maturity. When it comes to the analysis of OTM and deep OTM put and call options, errors are decreasing with the increasing time to maturity, which is consistent with the intuition and with benchmark models. Even for OTM options that are the closest to ATM this pattern is observed. What is interesting, errors for deep OTM options are significantly bigger for call options than for put options, for the shortest maturity class MAPE for deep OTM calls these errors exceed slightly 100%. That makes the model far worse than the benchmark Black model with GARCH volatility, and worse even than the Black model with historical volatility. This appears illogical, as the Duan model which captures the volatility smile should in theory price options not being ATM better than Black models. An explanation to this phenomenon will be given during the analysis of the Percentage of Overpredictions statistic.

The second model which uses the approach provided by Duan assumes a constant dividend yield of 4%[5], presented on Table 10 and 11. The value has been chosen arbitrarily, with a method of trials and errors which assumed minimizing errors provided by the green bars on Figure 6. This model performs slightly worse than the Duan's GARCH with implied dividend yield, which suggests that the hypothesis that futures prices reflect cash dividend payments, which are normally concentrated around June, holds. When comparing the performance of this model across maturities, very similar patterns to the model with implied

[5] This is very close to the average implied dividend yield for the whole estimation period – 4.08%.

dividend are observed. Only for deep OTM options with short maturity that model outperforms the previous one.

The last Duan GARCH option pricing model with normally distributed residuals dealt with assumes a dividend yield equal to 0. That model is depicted on Table 12 and 13. As expected, this model, as the least realistic is the worst performer amongst all other GARCHes. For put options, it even performs worse than the Black model with historical volatility. The bad performance of that model, especially for deep OTM and OTM call options, may be explained by the fact that the drift component of the Monte-Carlo simulation is, due to the lack of dividend yield, unreasonably high. That reason may also explain its relatively good performance for OTM and deep OTM put options.

The last class of GARCH models is the one that uses in the estimation of the GARCH process residuals following a Student's t distribution instead of a Gaussian one. Its performance is shown on Table 14 and 15. The amount of degrees of freedom is in this model estimated using the maximum likelihood method. It appears than this model is the best performing one for ATM options amongst all other GARCH models tested in the study. The MAPE for ATM call options is equal to 14.91% while the MAPE for ATM put options is equal to 14.58%. This outperforms the GARCH model with implied dividend by, respectively, 9 and 2 percentage points. Still, the model is beaten by the Black model with conditional GARCH volatility. The model performs slightly better than the Duan model with dividend for deep OTM options, which is logical because it captures the volatility smile better. However, results for ITM options are almost indistinguishable between the two models. Overall, the two models exhibit very similar patterns.

5.3. Results for the Median Absolute Pricing Errors

The mean as a statistic is very sensitive to single outliers. In my research there have been many of these outliers concerning the absolute errors, especially for OTM options. It is therefore worth comparing the analysis of mean error with the analysis of median errors. The hypothesis is that for the latter the errors will be smaller. However, I support the view that the mean analysis is better, as it "penalizes" models with lots of outliers, which is useful when models would be implemented in the real financial market, as outliers are those observations that may produce the biggest losses. I perform a median analysis for the two best models – the Black model with GARCH volatility, and the Duan approach with a GARCH model with Student's t residuals. Results are shown in Tables 28 to 31.

Table 16. Median Absolute Pricing Errors (MdAPE) for CALL options – Black model with conditional GARCH volatility

	CALL								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	**40,61%**	**29,50%**	**21,17%**	**15,16%**	**10,15%**	**8,13%**	**6,93%**	**6,59%**	**5,95%**
60 days << 92 days	38,22%	28,73%	20,51%	15,00%	11,37%	9,95%	8,57%	7,74%	6,52%
30 days << 60 days	38,70%	29,53%	20,81%	15,24%	9,41%	6,67%	5,99%	5,79%	5,55%
< 30 days	58,47%	33,51%	26,82%	16,56%	9,48%	8,13%	6,44%	6,03%	5,25%

Table 17. MdAPE for PUT options – Black model with conditional GARCH volatility

	PUT								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	**54,28%**	**38,28%**	**24,11%**	**14,03%**	**9,30%**	**7,61%**	**6,71%**	**5,79%**	**5,25%**
60 days << 92 days	41,85%	29,76%	18,37%	12,66%	9,79%	8,72%	6,79%	6,01%	5,25%
30 days << 60 days	58,27%	41,66%	26,36%	15,03%	8,76%	7,16%	6,74%	5,66%	5,29%
< 30 days	73,17%	53,84%	32,63%	18,20%	8,88%	6,85%	6,07%	5,14%	5,03%

Table 18. MdAPE for CALL options – Duan approach with implied dividend yield – GARCH with Student's t errors

	CALL								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	**40,84%**	**30,89%**	**22,36%**	**16,27%**	**11,96%**	**9,36%**	**7,86%**	**7,13%**	**6,18%**
60 days << 92 days	37,02%	29,80%	21,55%	17,39%	14,01%	11,63%	9,81%	8,38%	6,75%
30 days << 60 days	39,77%	30,42%	22,00%	14,96%	10,28%	7,73%	6,62%	6,05%	5,66%
< 30 days	58,59%	39,01%	29,47%	19,33%	12,52%	9,16%	6,26%	6,38%	5,61%

Table 19. MdAPE for PUT options – Duan approach with implied dividend yield – GARCH with Student's t errors

	PUT									
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**9**
Whole sample	**56,10%**	**44,57%**	**30,94%**	**20,02%**	**12,16%**	**8,98%**	**7,21%**	**6,04%**	**5,12%**	
60 days << 92 days	47,83%	38,04%	26,28%	18,78%	12,26%	10,19%	7,78%	6,47%	5,04%	
30 days << 60 days	60,26%	47,12%	33,17%	20,68%	12,13%	8,27%	6,90%	5,67%	5,22%	
< 30 days	73,92%	59,71%	41,23%	23,61%	11,39%	6,85%	6,22%	5,45%	5,11%	

It is now clearly seen in most cases median values for absolute pricing errors are significantly below their mean values. It is seen especially in deep OTM call options, where for both models medians are around 50% smaller than means. What is interesting, that pattern is not observed in deep OTM put options, where the median errors are very close to the mean errors. This suggests that the significance of outliers in the pricing of deep OTM put options is limited. However, as the models rather underpriced this type of options, it was often the case that the model returned an option price close to 0at a time when the market price was significantly higher; in that case, the maximum possible error could be very close to 100%. Thus, significant outliers may still exist.

As for call and put ITM and ATM options, medians are slightly smaller than means in all cases. This suggests an existing, but limited significance effect of outlier observations on the mean. When comparing the two models between each other in terms of medians, once again the Black model (Tables 28 and 29) beats the Duan one (Tables 30 and 31) in all cases, except one: deep ITM put options, where the Duan model outperforms the Black model with 13 percentage points across all maturity classes.

5.4. Results for the Percentage of Overpredictions

The analysis of the Percentage of Overpredictions statistic (OP) will enable to see which models overpriced and which underpriced market prices of options, as the MAPE did not give any information about the direction of errors[6]. Here, a general and interesting pattern is observed. In all the models tested, the percentage of call options overpriced decreases without almost any exceptions with the increasing level of moneyness. Exactly the opposite phenomenon is observed in the case of put options. Generally, call options are more often overpriced than put options.

I will first focus on the analysis of benchmark models. For both Black models, present in Tables 16 to 19, the OP for call options is a decreasing function of the level of moneyness. This suggests that the call options with the smallest implied volatility are those that are deeply OTM, or in other words, are the cheapest ones. According to the theory, the implied volatility should be higher for deep OTM and deep ITM options than for ATM options due to the existence of fat tails in the distributions of returns. Here, we clearly see that the implied volatility for deep OTM call options is small. This may suggest that investors do not believe in a fat right tail of the distribution of returns.

Table 20. Percentage of Overpredictions (OP) for CALL options – Black model with 20-day historical volatility

	CALL								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	**51,36%**	**49,25%**	**45,77%**	**39,84%**	**33,08%**	**30,70%**	**26,67%**	**31,11%**	**37,93%**
60 days << 92 days	52,40%	48,22%	44,19%	39,69%	30,23%	28,99%	26,20%	29,61%	35,50%
30 days << 60 days	51,61%	49,27%	46,63%	39,00%	34,02%	31,82%	26,69%	32,40%	40,32%
21 days << 30 days	45,32%	53,96%	48,92%	44,60%	41,73%	33,09%	28,78%	31,65%	37,41%

Table 21. OP for PUT options – Black model with 20-day historical volatility

	PUT								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	**15,14%**	**13,98%**	**18,49%**	**23,87%**	**31,92%**	**40,52%**	**46,59%**	**51,77%**	**59,41%**
60 days << 92 days	19,69%	20,62%	25,89%	28,37%	33,02%	41,40%	48,68%	53,95%	62,33%
30 days << 60 days	12,17%	8,50%	13,05%	19,50%	29,47%	38,86%	43,55%	49,27%	56,60%
21 days << 30 days	8,63%	10,07%	10,79%	24,46%	38,85%	44,60%	51,80%	53,96%	59,71%

Table 22. OP for CALL options – Black model with conditional GARCH volatility

	CALL								
Time to maturity	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole sample	**66,98%**	**63,64%**	**59,62%**	**55,53%**	**46,45%**	**38,34%**	**29,47%**	**32,81%**	**39,43%**
60 days << 92 days	67,13%	61,55%	58,29%	53,95%	43,41%	34,88%	27,91%	30,23%	36,90%
30 days << 60 days	68,91%	66,57%	62,02%	58,80%	49,85%	42,67%	31,67%	35,48%	42,38%
< 30 days	56,83%	58,99%	53,96%	46,76%	43,88%	33,09%	25,90%	31,65%	36,69%

[6] Another way to present the errors would have been the non-absolute percentage error. However, as the average values for this measure would be in many models concentrated around zero, they would be meaningless.

Table 23. OP for PUT options – Black model with conditional GARCH volatility

Time to maturity	PUT								
	1	2	3	4	5	6	7	8	9
Whole sample	**10,23%**	**8,94%**	**15,62%**	**27,69%**	**42,77%**	**51,91%**	**54,37%**	**57,50%**	**62,07%**
60 days << 92 days	14,88%	16,28%	22,95%	33,80%	45,74%	53,33%	59,07%	61,09%	67,60%
30 days << 60 days	7,18%	3,08%	10,12%	23,90%	40,76%	51,47%	50,00%	53,81%	57,18%
< 30 days	3,60%	3,60%	8,63%	17,99%	38,85%	47,48%	53,96%	58,99%	60,43%

Table 24. OP for CALL options – Duan approach with implied dividend

Time to maturity	CALL								
	1	2	3	4	5	6	7	8	9
Whole sample	**59,28%**	**52,59%**	**45,63%**	**40,93%**	**34,17%**	**29,95%**	**26,33%**	**32,13%**	**39,70%**
60 days << 92 days	55,81%	46,67%	42,79%	40,31%	32,87%	29,15%	25,43%	31,01%	38,14%
30 days << 60 days	63,93%	57,92%	48,39%	42,67%	36,07%	31,96%	28,01%	33,14%	41,94%
< 30 days	52,52%	53,96%	45,32%	35,25%	30,94%	23,74%	22,30%	32,37%	35,97%

Table 25. OP for PUT options – Duan approach with implied dividend

Time to maturity	PUT								
	1	2	3	4	5	6	7	8	9
Whole sample	**11,12%**	**9,21%**	**13,98%**	**20,87%**	**29,88%**	**41,34%**	**48,02%**	**53,27%**	**59,28%**
60 days << 92 days	14,88%	13,95%	18,76%	24,96%	32,25%	41,55%	50,23%	56,12%	62,48%
30 days << 60 days	8,50%	5,43%	10,56%	18,62%	28,74%	40,76%	45,16%	50,00%	55,72%
< 30 days	6,47%	5,76%	8,63%	12,95%	24,46%	43,17%	51,80%	56,12%	61,87%

Table 26. OP for CALL options – Duan approach with dividend yield equal to 4%

Time to maturity	CALL								
	1	2	3	4	5	6	7	8	9
Whole sample	**60,44%**	**52,86%**	**46,86%**	**41,27%**	**37,31%**	**32,13%**	**29,95%**	**32,20%**	**38,74%**
60 days << 92 days	57,83%	49,46%	45,43%	41,24%	37,05%	31,63%	29,46%	30,85%	39,07%
30 days << 60 days	63,34%	56,16%	48,39%	42,82%	37,68%	32,70%	30,06%	32,99%	38,71%
< 30 days	58,27%	52,52%	46,04%	33,81%	36,69%	31,65%	31,65%	34,53%	37,41%

Table 27. OP for PUT options – Duan approach with dividend yield equal to 4%

Time to maturity	PUT								
	1	2	3	4	5	6	7	8	9
Whole sample	**11,60%**	**10,71%**	**13,51%**	**21,15%**	**32,61%**	**42,43%**	**46,79%**	**52,80%**	**58,32%**
60 days << 92 days	15,81%	15,35%	18,76%	26,05%	32,71%	40,62%	45,27%	52,40%	58,91%
30 days << 60 days	8,65%	7,33%	9,97%	18,18%	33,87%	45,01%	48,39%	52,93%	57,77%
< 30 days	6,47%	5,76%	6,47%	12,95%	25,90%	38,13%	46,04%	53,96%	58,27%

Table 28. OP for CALL options – Duan approach without dividends

Time to maturity	CALL								
	1	2	3	4	5	6	7	8	9
Whole sample	**69,51%**	**65,89%**	**61,73%**	**57,16%**	**54,23%**	**53,68%**	**51,84%**	**51,36%**	**57,98%**
60 days << 92 days	69,30%	65,58%	63,72%	60,16%	55,97%	54,42%	53,64%	53,18%	59,84%
30 days << 60 days	71,26%	66,42%	61,14%	55,43%	53,37%	54,11%	51,17%	50,73%	58,21%
< 30 days	61,87%	64,75%	55,40%	51,80%	50,36%	48,20%	46,76%	46,04%	48,20%

Table 29. OP for PUT options – Duan approach without dividends

Time to maturity	PUT								
	1	2	3	4	5	6	7	8	9
Whole sample	**9,75%**	**7,50%**	**8,53%**	**11,53%**	**16,92%**	**23,67%**	**29,06%**	**34,58%**	**40,45%**
60 days << 92 days	12,56%	10,85%	10,70%	13,33%	17,83%	22,02%	25,74%	32,40%	36,28%
30 days << 60 days	7,92%	4,69%	7,04%	10,56%	16,13%	24,78%	30,94%	34,60%	41,79%
< 30 days	5,76%	5,76%	5,76%	7,91%	16,55%	25,90%	35,25%	44,60%	53,24%

Table 30. OP for CALL options – Duan approach with implied dividend yield – GARCH with Student's t errors

Time to maturity	CALL								
	1	2	3	4	5	6	7	8	9
Whole sample	**59,28%**	**52,66%**	**45,29%**	**41,34%**	**33,29%**	**29,74%**	**26,60%**	**31,72%**	**38,68%**
60 days << 92 days	55,97%	47,44%	42,79%	40,62%	33,18%	28,68%	25,12%	31,01%	36,74%
30 days << 60 days	63,34%	57,62%	48,39%	43,26%	34,02%	32,11%	28,59%	32,55%	40,76%
< 30 days	54,68%	52,52%	41,73%	35,25%	30,22%	23,02%	23,74%	30,94%	37,41%

Table 31. OP for PUT options – Duan approach with implied dividend yield – GARCH with Student's t errors

Time to maturity	PUT								
	1	2	3	4	5	6	7	8	9
Whole sample	**10,91%**	**9,82%**	**13,71%**	**21,21%**	**30,01%**	**41,68%**	**48,09%**	**53,89%**	**59,82%**
60 days << 92 days	14,57%	14,57%	18,76%	25,43%	32,09%	42,02%	50,23%	56,12%	62,79%
30 days << 60 days	8,36%	6,16%	10,12%	18,91%	29,18%	41,20%	45,75%	50,73%	56,74%
< 30 days	6,47%	5,76%	7,91%	12,95%	24,46%	42,45%	49,64%	58,99%	61,15%

If we take another look at Figure 3, we see that the above is logical, as the excess kurtosis on the right tail of the distribution of returns hardly exists. On the other hand, models tend to underprice strongly OTM put options, which suggest that the probability of an extreme negative event on the market is already incorporated into the prices of options. When analyzing ITM options, we clearly see that call options become relatively expensive comparing to OTM call options. This is consistent with the behavior of OTM options, as with a high left tail in the distribution of returns there is a high probability that an ITM call will become OTM before maturity. On the other hand, ITM put options become cheaper relatively to OTM put options because due to the lack of a significant right tail in the distribution of returns there is only a small probability that the options will turn out to be OTM at maturity. Similar patterns are observed in other models than the Black ones. To check whether the probability distribution implied in option prices is consistent with the observed one and whether profits may be made on that basis is a topic for another study.

When analyzing the OP statistic for benchmark models across maturity classes, it is noticeable that the bigger the time to maturity, the smaller the OP, which suggests that the implied volatility grows together with the time to maturity. This is consistent with the theory and previous literature. This is visible especially for OTM and deep OTM call and put options, for other moneyness classes the pattern is not that evident, but still exists. The above holds for both benchmark models. What is different between the two models is that the Black model with GARCH volatility has a higher OP for call options across all maturities, while for put options the differences between the OP statistics depending on the moneyness between the two models are different.

I will now concentrate on the analysis of OP for all models using the Duan approach. Results can be seen on Tables 20 to 26. Clearly, in these models the same phenomenon concerning the volatility smile as in Black models, described two paragraphs before, is present. What changes is the degree of relative differences of the OP statistic between OTM and ITM options. We now see that the OP for call options is not a decreasing function of the level of moneyness any more – the smallest OP is now for ATM options and is equal to 34.17% for the Duan model with implied dividend (shown on Tables 20 and 21), while the OP for the same model for deep ITM options is equal to 39.70%. This suggests that this model captures slightly better the implied distribution of returns than benchmark models, which is consistent with the paper of Duan (1995). However, OP for put options for Duan models are still an increasing function of the level of moneyness. Across maturity classes, Duan models exhibit similar patterns as benchmark models – the OP statistic tends to decrease with the increasing time to maturity, which is especially visible for OTM options, and less visible for ITM options.

A deep separate analysis of Duan models with constant dividend yield and without dividend is senseless, as the sole component that changes in the parameters of the Monte-Carlo simulation is the drift. It is clearly seen in the results that both for call and put options, the model without dividends overprices call options and underprices put options more than models with dividends. This is logical, as the drift component is larger for the model without dividends, thus the average stock price at maturity of options is higher. The OP statistics for the model with constant dividend yield are very close to the one with implied dividend yield.

The last model left to analyze is the GARCH with Student's t residuals. The results are presented in Table 26 and 27. Its OP statistics are very close to the ones provided by the Duan model with implied dividend yield. Differences are so small than a deep discussion is meaningless.

6. Conclusions

The most interesting issue in option pricing is the question what should be the proper volatility estimate for the underlying instrument to be put into the model. The common approach amongst market practitioners is to use the implied volatility from the Black-Scholes formula to price new options. This practice leads to the smallest errors between prices predicted by models and those observed on the market. However, there is still left the question how to price options on instruments when options are not available for trading. Here, several possibilities are available – the Black model, the Heston model, or a model from the GARCH family, and many other ones.

The goal of this paper was to test some of these models, and weight their results against prices observed on the real market of vanilla options on WIG20. Two benchmark Black models have been used, and several other models from the GARCH family, using the approach first proposed by Duan (1995). Prices obtained using these models have been compared to prices observed on the options market of the Warsaw Stock Exchange. Two testing statistics have been used in order to assess the accuracy of every model – the Mean Absolute Pricing Error (MAPE), and the Percentage of Overpredictions (OP). A separate analysis compares the two best models with absolute median errors in order to assess the significance of outliers. The study covers a period from the 1st of January 2006 to the 31st of March 2012. It deals with call and put options with all moneyness classes, and with a time to maturity between 21 and 92 trading days. The study also compares how models behave with different assumptions about the continuous dividend yield.

The initial hypothesis of the study was that the best models would be all GARCH models. Amongst these GARCH models, the most accurate would be the one with Student's t residuals from the GARCH process. It was also expected that the Black model with historical volatility would be the worst, as it is by definition a backward-looking measure, while the volatility observed on the market is forward-looking.

Interestingly, the model which prices options in the most accurate manner is the Black model with a volatility term structure, which outperforms all other models in terms of both mean and median absolute errors. This is an approach which was not proposed in any previous literature, and as the results indicate, tends to be quite accurate. The second best model is the Duan approach with GARCH volatility with Student's t residuals. The good performance of the latter has been expected as a Student's t distribution has fat tails, which mirrors financial returns more accurately than a normal distribution does. The worst model was, as expected, the Black model with a 20-day historical volatility.

On average, all models return mean absolute errors that are very high for OTM options, and decreasing with increasing moneyness. This is consistent with previous literature and intuition. However, it is unfortunately not possible to compare the exact numbers due to the small amount of literature examining the Polish market available.

Three GARCH models with a different approach to dividends have been tested: a first model with a continuous dividend yield equal to 0, a second with a continuous dividend yield equal to 4%, and a third with a dividend yield equal to the one implied from prices of futures on WIG20. As the implied dividend yield sometimes turned out to be negative or irrationally high, the question was whether this measure was accurate. The results indicate that the assumption of a continuous implied dividend yield is the most accurate one.

When analyzing the Percentage of Overpredictions statistic, it is noticeable that all models overprice call options much more often than put options. This indicates that put options are relatively more expensive, which may arise from the fact that there exists a higher demand amongst long position holders that want to hedge their portfolios using options. This is especially visible for deep OTM put options, where only a few % of model prices exceed market prices. This suggests a significant fear occurrence of extremely negative events on the market amongst investors.

On the whole, it can be said that the goal of this paper associated with its practical application – the valuation of sophisticated path-dependent derivatives – has been to a significant extent fulfilled. This means that, for example, the Duan option pricing model with Student's t errors can be applied to the pricing of lookback, Asian or barrier options on WIG20. The Polish stock market will be, with a high degree of probability, growing in size and introducing new products, amongst which there will certainly be exotic options.

A certain bias of the study is the fact that the liquidity of options on WIG20 is significantly low. Therefore, such study may be repeated in some time, when the liquidity of the derivatives market in Poland will increase. Another interesting study would be to extend my work to other CEE countries and compare the results with those obtained for Poland.

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