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Abstract

Top-down models usually include piecewise-smooth functions to describe marginal cost curves, while bottom-up models describe those curves with a step function. When a bottom-up cost curve is available, we can explicitly represent this curve with a top-down model in order to replicate its shape instead of arbitrary assumptions. We propose methods to approximate a piecewise function from a step function using constant elasticity of substitution technologies. Specifically, we consider a pollution abatement sector and calibrate the parameters of the abatement function in order to be able properly to assess the economic effects of an environmental policy. Our methodology can be applied to any sector characterized by decreasing returns to scale technologies. We conclude that the elasticities of substitution need not be estimated only on the basis of historical data, but can be precisely calibrated on the basis of engineering estimates of technology potential.

Keywords:
elasticity of substitution, calibration, abatement, top-down and bottom-up modeling

JEL:
D24, Q53, C60

1 Introduction

There is a conventional notion that elasticities of substitution are always estimated on the basis of historical data. It is a critical parameter in top-down modeling and it provides a good approximation of prospective technology options. When elasticities are estimated from historical data, there is no guarantee that the parameter values will remain valid into the future under different abatement policies (Jaccard et al., 2004). We propose a methodology to determine the elasticity of substitution on the basis of engineering studies. Instead of an econometric estimation, we calibrate a bottom-up cost curve.

Top-down models usually include piecewise-smooth functions to describe marginal cost curves, while bottom-up models describe those curves with a step function. When bottom-up cost curve is available, we can explicitly represent this curve with a top-down model in order to replicate its shape instead of using arbitrary assumptions. However, there is a lack of information about the range of alternative activities to which the producer can switch, implying that elasticity of substitution must be assumed. Judgments about the scope of substitution possibilities are discussed in Wing (2006) and Baker et al. (2008). We show how to identify the elasticity of substitution with bottom-up data. The piecewise-smooth approximation method is explained using a pollution abatement sector, but our methodology can be applied to any sector characterised by decreasing returns to scale technologies.

Relatively few top-down models explicitly specify the production function of pollution abatement activities. Initially Jorgenson and Wilcoxen (1990) assumed that an industry's production function for pollution abatement directly mirrors the production function for its good output. Later Nordhaus and Yang (1996) implemented a quadratic abatement cost curve and calibrated the intercepts of the estimated marginal abatement cost (MAC) curve. Ellerman and Decaux (1998) fitted simple analytical curves to a set of MAC curves and investigated the robustness of MACs with respect to abatement levels among regions. Hyman et al. (2002) implemented a constant elasticity of substitution (CES) abatement function. The authors chose a value of elasticity of substitution and compared it to the bottom-up MAC to allow for an arbitrary adjustment of the fit. Gerlagh et al. (2002) proposed an ordinary least square estimation to cover as much information as possible on the technical measures underlying the abatement options. Boehringer et al. (2006) used an activity analysis to directly incorporated bottom-up function into a top-down model. Revesz and Balabanov (2007) defined an average abatement cost function using a degree of abatement possibilities and a scaling factor. The GEM-E3 model for the European Union (Capros et al., 2008) explicitly specifies MAC as an isoelastic exponential function and installations of abatement technologies are considered as an input for the firms rather than as an investment.

In this paper we show an algorithm for a smooth representation of bottom-up cost curve which enables us to portray the isoquant defined by the activity analysis formulation. The benchmark equilibrium describes prices and quantities at a reference point. Properly calibrated, this point will be the same in both the smooth and the step curves. What functional form should we consider? Yu (2005) proposes to capture abatement activity similar to iceberg cost together with standard constant returns to scale production function. However, the abatement process is characterised by decreasing returns to scale technologies. For equilibrium analysis a function like CES is well suited for studying production process and it is relatively easy to calibrate. We consider a CES function and decreasing returns to scale. The best fit for the CES elasticity will be that which minimizes the weighted deviation from the bottom-up curve.

The inclusion of the bottom-up information on abatement options into a top-down model in a traditional way[1] involves (i) piecewise-smooth approximation that best describes the bottom-up cost curve, (ii) integration of the results of the approximation into a top-down model. We are not going to analyse the ways to include abatement function in top-down models, but we show how to evaluate parameters of the abatement function to be used in top-down models. We discuss and compare four methods assuming decreasing returns to scale technology. A rational polluter, when faced with the necessity to reduce pollution, will first take the cheapest options and then turn to more costly ones. The marginal cost curve will therefore be non-decreasing. In addition, complete emission reduction is not possible via technical measures and a reduction of economic activity is required. Thus the cost curve approaches a vertical asymptote while the marginal cost approaches infinity.

A discussion of the importance to analyze marginal, rather than total or average abatement cost will be presented. We consider a combination of three cost curves to verify that targeted cost matters during the approximation procedure. We verify this hypothesis using abatement cost curves for greenhouse gases in the Czech Republic, Poland and Switzerland estimated by McKinsey & Company (McKinsey study, 2008, 2009a,b). The results for all three curves suggests that it does not matter whether we target marginal or total cost, but it might matter when average cost is targeted. We present the details of this experiment for Switzerland.

Finally, we address the issue of negative bottom-up cost. A McKinsey type cost curve gives the illusion that part of pollution abatement can be done for free. The construction of the cost curve implies that each action is independent from every other action and the probability of adopting is the same for all new technologies. A wide discussion of the free lunch problem can be found in Holmes (2010). We correct these negative costs using rescaling and compare three approaches, as results of top-down models are sensitive in this respect. In any event, below zero costs are inherently problematic.

[1] Integration of bottom-up cost with top-down modelling is possible either using smooth abatement function (traditional approach) or activity analysis approach for step abatement function (hybrid approach).

The rest of the paper is organised as follows. Section 2 explains how to represent a decreasing returns to scale technology with a top-down modeling. We show a relationship between the Marshallian concept of supply function and the Arrow-Debreu production function. We complete this section with alternative calibration strategies to approximate an abatement curve. The details of the algorithm are available in Appendix. In section 3, we use Swiss data to approximate both the marginal and the total abatement cost curves for greenhouse gases. Several rescaling methods are applied in order to avoid negative cost. Section 4 concludes.

2 Calibration of marginal cost function

Given the abatement technologies within a bottom-up model, the MAC curve represents the marginal loss in profits from avoiding the last unit of emission given some level of output. In a top-down model, the MAC curve is defined as the shadow cost that is produced by the constraint on pollution emissions. Thus the MAC for a given economy represents the social cost of the last unit of emissions abated. The question is how to calibrate this social cost function. We explain this issue using a CES technology. First, the integration of Marshallian concept in the Arrow-Debreu models is presented. Next, different calibration approaches are explained.

2.1 Decreasing returns to scale

The marginal abatement cost is nondecreasing, a strictly convex technology represents the pollution abatement processes where the output increases by less than a proportional change in inputs. We have to find out a function that will be able to describe such a curve. Let us describe the pollution abatement service Q using a technical potential X and expenditures K, where expenditures includes capital, labour, and materials necessary for the abatement process once the abatement technology has been chosen (Figure 1a). The potential to reduce pollution through technical abatement activities provides an upper bound on the abatement in the model. The remaining part of pollution can be reduced only by decreasing the economic activity.

When abatement capacity X is in fixed supply, a production function $Q = f(K, \bar{X})$ exhibits decreasing returns to scale in the variable input K (Figure 1b). The variable input includes capital, labour, and materials necessary for the abatement process. Following Cretegny and Rutherford (2004), there is therefore no loss of generality by formulating the model on the basis of a constant returns to scale CES technology with a fixed factor:

$$Q = \phi \left(\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)\bar{X}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$$

where ϕ is a scale factor, α is a distribution parameter, and σ is a constant elasticity of substitution between abatement capacity and required expenditures on abatement. It gives a linear expansion path of the cost minimization problem on the Figure 1a. To be able to abate one unit of emission, we need an abatement technology and maintenance. Once we have decided what technology to apply, the abatement level will be determined by the input K. The decreasing returns to scale technology implies that the abatement level increases less than proportional to this input.

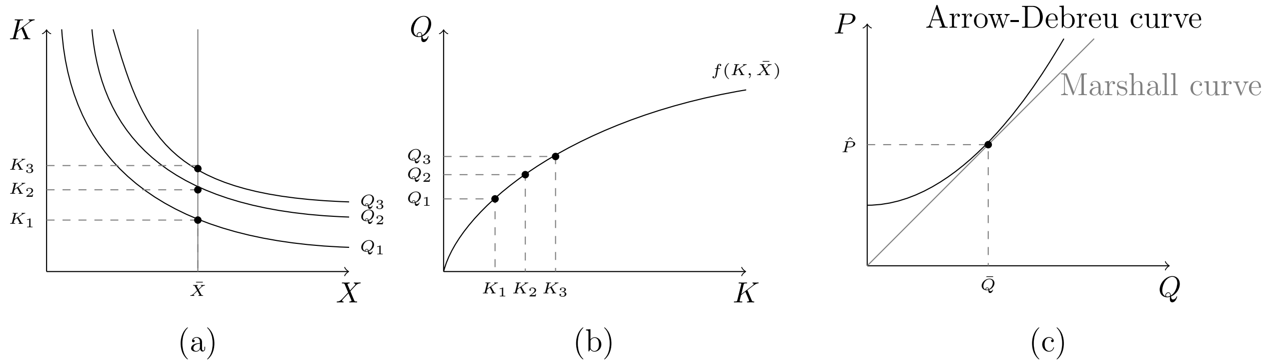

Figure 1: (a) Isoquant map for abatement; (b) Abatement production; (c) Abatement supply

The total cost function for the decreasing returns to scale technology is determined only by the variable input (Figure 1b). Assuming competitive market for K, the input price P_k is given at a fixed level. Choosing units such that $\bar{X} = 1$ and $\bar{P}_k = 1$, the scale of output at a given output price P is consistent with the profit maximization problem:

$$\max \Pi(Q, K) = PQ - K \quad \text{s.t.} \quad Q = \phi \left(\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)\right)^{\sigma/(\sigma-1)} \tag{1}$$

Using the marginal product constraint $\partial Q/\partial K = 1/P$, the conditional demand on the variable input becomes:

$$K = \frac{Q}{\phi} \left(\alpha \phi P\right)^{\sigma} \tag{2}$$

Substituting (2) back into the production function (1), the supply CES function becomes:

$$Q = \left(\frac{1 - \alpha^{\sigma}\phi^{\sigma-1}P^{\sigma-1}}{1-\alpha}\right)^{\sigma/(1-\sigma)} \tag{3}$$

We can solve this equation, but the specification of the CES function parameters is complicated and error-prone when represented in this classical form because α and ϕ

depend on the assumed value of σ. Alternatively we may define (3) in the calibrated share form (see Appendix for details):

$$Q = \bar{Q}\left(\frac{1-\theta(P/\hat{P})^{\sigma-1}}{1-\theta}\right)^{\sigma/(1-\sigma)} \tag{4}$$

where θ is a value share parameter for K. Both formulas, (3) and (4), describe the same curve, but it is easier to evaluate the last one because it has one less parameters. Using an arbitrary calibration point, the value share parameter θ is determined by the variable input K, but the elasticity of substitution σ can be determined by the supply elasticity η. Applying the Harberger calibration point ($\bar{Q} = 1, \hat{P} = 1$), we can find the relationship between supply elasticity and two other parameters:

$$\eta = \sigma\frac{\theta}{1-\theta}$$

Thus there is a direct positive relationship between σ and η, but a negative relationship between σ and θ. This relationship, according to Rutherford (2008), can be used in a variety of ways to calibrate the supply function. We will choose the level of the fixed factor θ to match the base year supply, and then assign the elasticity of substitution between abatement capacity and the abatement cost accordingly. When σ is less than one, then the inputs are essential and the supply curve has the shape shown on the Figure 1c. When σ is close to one, the supply curve represented by (3) and (4) matches the isoelastic Marshallian supply curve.

Thus the Marshallian concept of decreasing returns to scale technologies can be integrated in the Arrow-Debreu model with constant returns to scale technologies using a sector specific factor. Once we have defined the supply function, we are now able to approximate a bottom-up abatement function.

2.2 Approximation methods

The key formula for the calibration of an abatement function is defined by the supply function (4). Using it, we try to approximate a bottom-up MAC curve as shown in Figure 2a. For a given set of technologies i, the MAC curve represents the relation between potential abatement level q_i and its cost c_i. It has the shape of a step function, increasing each time the next cheapest technology is introduced. The emission abated by technology i is $q_i = Q_i - Q_{i-1}$, while the aggregate abatement level Q_i is an emission reduction cumulative to the upper point of step i. The black line represents our targeted approximated curve.

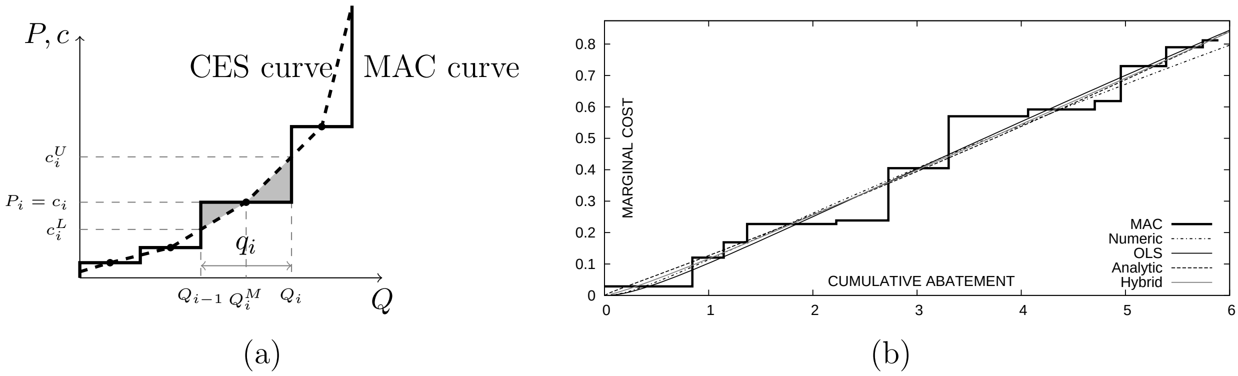

Figure 2: (a) Step versus smooth marginal cost curve; (b) Approximation results for four methods

Let us match the abatement schedule close to the middle point of step Q_i^M. The objective is to minimize distance between the step and the smooth curves. The difference between the observed and the approximated abatement cost should be weighted with the pollution reduction achieved by the technical measure. This ensures that the technologies that reduce a lot of pollution are given sufficient weight in our algorithm. This weighting factor ω_i can be calculated in different ways. For example, assuming a normal distribution we can calculate the factors for each of the components of the approximated function: $\omega_i = exp\left[-(c_i - c^*)^2/2s^2\right]$, where s is the standard deviation in costs, c_i is the marginal cost of abatement described by the bottom-up curve, c^* is an arbitrary point on the bottom-up curve where we want to approximate the level of the constant elasticity of substitution (for example, we may try to match the MAC schedule close to the middle of the range: $c^* = \max_i c_i/2$). Another way to define the factor is simply: $\omega_i = q_i$. Both definitions imply that the weighting factor depends on how far each components of the approximated curve departs from the targeted curve and they provide a similar precision of the following approximation methods. For simplicity, we suggest to use the second formula. Using the dual approach and the heuristic weighting factor ω_i, we will examine several methods to approximated bottom-up curve.

2.2.1 Method 1 - numeric fit

The objective is to minimize shaded triangulares shown on Figure 2 for each step of the curve:

$$\min_{\sigma,\theta,c_i,\hat{c},\bar{Q}} \sum_i \omega_i \int_{q_i^L}^{q_i^U} (c_i^U - c_i^L)^2 \mathrm{d}q_i \quad \text{s.t.} \quad c_i = \hat{c}\left(\frac{1-(1-\theta)(Q_i^M/\bar{Q})^{(1-\sigma)/\sigma}}{\theta}\right)^{1/(\sigma-1)} \quad \forall i \quad (5)$$

where the labels L, M and U stand for low, middle, and upper point of the step i. The constraint is the inverse supply CES function defined by formula (4) assuming competitive supply ($c_i = P_i$). The algorithm choose a calibration point at the bottom-up cost curve and attempts to minimize the distances between the two curves. We can solve it as a standard optimization problem (see Appendix for the algorithm in GAMS).

The results of this approximation are presented in Figure 2b as a blue line. We use a hypothetic MAC curve (the red line) to verify our algorithm. The endogenous calibration point $(\bar{Q}, \hat{c}) = (1.4, 0.17)$ gives the result $\sigma = 1.095, \theta = 0.43$.

2.2.2 Method 2 - ordinary least square

The objective is to minimize the sum of squared distances between the bottom-up step function and the targeted CES function using the same constraint as in formula (5):

$$\min_{\sigma,\theta,c_i,\hat{c},\bar{Q}} \sum_i \omega_i \left(\bar{c}_i - c_i\right)^2 \quad \text{s.t.} \quad c_i = \hat{c}\left(\frac{1-(1-\theta)(Q_i^M/\bar{Q})^{(1-\sigma)/\sigma}}{\theta}\right)^{1/(\sigma-1)} \quad \forall i \tag{6}$$

where $\bar{c}_i$ and c_i represent the observed and the evaluated marginal cost of abatement associated with technology i, while $\hat{c}$ is the reference marginal cost that will be determined endogenously. OLS method uses a calibration point close to the previous method and both approaches give a similar result (see Figure 2b, the blue line): $\sigma = 1.08, \theta = 0.42$ for the endogenous calibration point $(\bar{Q}, \hat{c}) = (1.39, 0.16)$.

2.2.3 Method 3 - analytic fit

The bottom-up cost can be also approximated through a partial equilibrium approach (closed form) instead of a general equilibrium (open form) as above methods. This method uses the same objective function as the first method, but a different constraint. Instead of a standard optimization problem, we will evaluate deviations for each of the anchor points (Q_i, c_i) within technology i using a loop. This requires that we define a variable parameter θ_i instead of a fixed θ:

$$\theta_i = \frac{\sum_j c_j q_j + c_i q_i^M}{c_i Q_i^M} \quad \text{for} \quad c_j < c_i$$

The calibration within the partial equilibrium approach is a source of imprecision, because we are not able to apply a fixed σ in a closed form. We can use a variable elasticity of

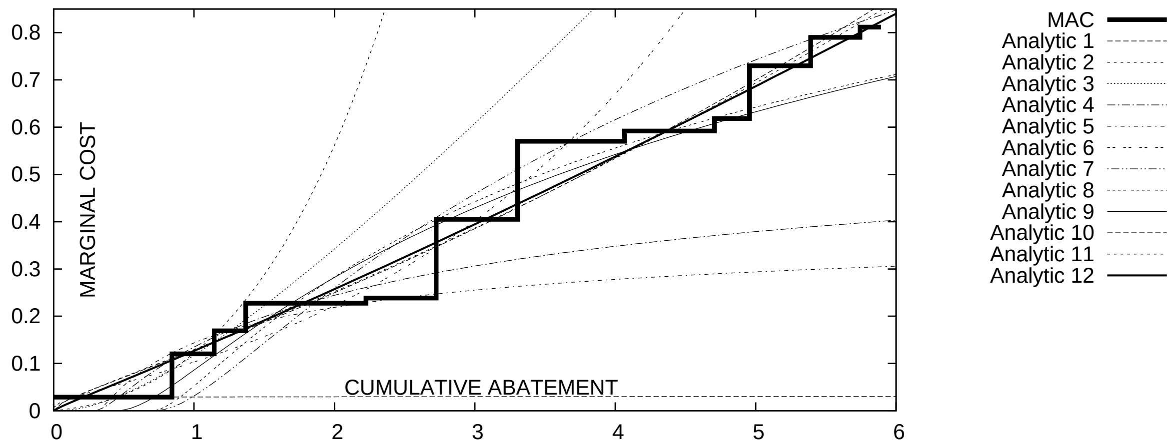

Figure 3: Analytic method

substitution σ_i and rely on the following approximation:

$$\sigma_i = \frac{\sum_j \omega_j \sigma_{ij}}{\sum_j \omega_j} \quad \text{for} \quad j \neq i \tag{7}$$

where $\sigma_{ij} = \frac{\ln(Q_j^M/Q_i^M)}{\ln(c_j/c_i)}$ is a direct elasticity of substitution. Substitution elasticities describe the adjustment potential in cost minimizing inputs with respect to factor prices. In order to determine which of the estimated elasticity values σ_i best determines the targeted MAC curve, we evaluate the deviations for each of the anchor point and choose the minimum one:

$$\min_{\sigma,\alpha,c_j} \sum_j \omega_j \int_{q_j^L}^{q_j^U} (c_j^U - c_j^L)^2 \mathrm{d}q_j \quad \text{s.t.} \quad c_j = c_i \left(\frac{1-(1-\theta_i)(Q_j^M/Q_i^M)^{(1-\sigma_i)/\sigma_i}}{\theta_i} \right)^{1/(\sigma_i - 1)} \quad \forall j \tag{8}$$

The results of this approximation are presented in the Figure 3. For each technology of the 12-step bottom-up MAC curve, the elasticity of substitution and the value share parameters are evaluated taking into account previous steps of the curve but ignoring the following steps. The minimum deviation provides the last step (the green line) with $\sigma = 0.97, \theta = 0.48$. This result differs from the previous two methods. The other eleven approximations are shown as gray lines and we ignore them.

2.2.4 Method 4 - hybrid fit

This method is a combination of partial and general equilibrium approaches. We will evaluate deviations for each of the anchor point (Q_i, c_i) using a loop like in method 3, but we will evaluate σ better. Start from the optimization procedure:

$$\min_{\sigma_{ij},\theta_i} \left(\left(\frac{1-(1-\theta_i)(Q_j^M/Q_i^M)^{(1-\sigma_{ij})/\sigma_{ij}}}{\theta_i} \right)^{1/(\sigma_{ij}-1)} - \frac{c_j}{c_i} \right)^2 \quad \text{s.t.} \quad \theta_i = \frac{\sum_j c_j q_j + c_i q_i^M}{c_i Q_i^M} \text{ for } c_j < c_i \tag{9}$$

and solve it within the loop for the reference technology i to technology j. Next, adopt the exact values of σ_i in (7) by applying new σ_{ij} from (9). Finally, proceed with method 3 to define and solve (8).

For each technology of the 12-step bottom-up MAC curve, the elasticity of substitution and distribution parameters are evaluated, similar to the analytic fit. The minimum deviation gives the last step with $\sigma = 1.02, \theta = 0.48$. This is a pink line on the Figure 2b. The result for θ is the same as in the previous method, but σ is better evaluated and comparable to the numeric fit and the OLS approximation.

2.2.5 Comparison

The four alternative methods to approximate a bottom-up marginal cost curve allow us to determine the values for elasticity of substitution and for three other parameters. The weakness of the hybrid method is that it is time-consuming. On the other hand, the numeric fit does not cover exactly the whole step curve as shown on Figure 2b. Analytic fit may also deviate because this method does not allow for a precise evaluation of the elasticity of substitution. Comparing all four methods, the OLS is the simplest and most precise method to approximate the MAC curve.

We have developed our methods in the spirit of the work by Rob Dellink in Gerlagh et al. 2002 and by Robert Hyman in Hyman et al. 2002. Both authors propose to use CES technologies to calibrate an abatement function. Robert Hyman proposes to treat the MAC curve as the inverse of the input demand function of emission and to estimate the elasticity of substitution between the emission level and the other input aggregate for each sector. This method requires an assumption of non-zero abatement level in the benchmark and it requires a sectoral bottom-up abatement cost curve. Our methods does not need these requirements and assumptions.

Rob Dellink proposes to use a standard OLS method to approximate the total abatement cost (TAC) curve, instead of the MAC. The Author calls the approximated curve the

Table 1: Approximated curve versus targeted cost

Approximated curve	Targeted cost	Abbreviation
marginal	marginal	$MAC(mc)$
average	marginal	$AAC(mc)$
total	marginal	$TAC(mc)$
marginal	average	$MAC(ac)$
average	average	$AAC(ac)$
total	average	$TAC(ac)$
marginal	total	$MAC(tc)$
average	total	$AAC(tc)$
total	total	$TAC(tc)$

Pollution-Abatement-Substitution (PAS) curve. The basic idea is to split the sectoral emission into abatable and unabatable and to apply OLS estimation to evaluate the emission level before abatement took place in a benchmark assuming that we observe the emission level after abatement. This method assumes non-zero abatement level in the benchmark and it requires a sectoral bottom-up abatement cost curve, the same as the method by R. Hyman.

3 Targeted cost

The results of the bottom-up cost approximation (elasticity of substitution σ, value share parameter θ, reference output $\bar{Q}$, and reference marginal cost $\hat{c}$) can be implemented into a top-down model in order to take the abatement possibilities into account. However, as Morris et al. (2008) note, unless one takes grate care in understanding the exact conditions under which bottom-up cost curves are constructed, it is easy to misuse them. For example, is it matter for top-down modelling which cost is approximated: marginal or total? The total cost curve is piece-wise linear, with the slopes for individual segments determined by the costs of applying the various technologies. The marginal cost curve takes the shape of a step function, indicating the marginal costs at various reduction levels. The marginal conditions are essentials for top-down modelling, but Dellink (2005) suggests to approximate a bottom-up total cost curve instead of a marginal cost curve.

Using the OLS method, we consider a combination of the three cost curves (marginal, average, and total) to verify how important the targeted cost is. Table 1 shows the details for nine options. The last option corresponds to a PAS curve, while the first option corresponds to our approach presented in the Section 2.2. We have to adjust our approach in order to consider the total and the average cost in addition to the marginal cost. The total cost is equal to the value of variable input when technology exhibits decreasing returns to scale. There is a unit benchmark price of the variable input $\bar{P}_k$, but $P_k \neq \bar{P}_k$. We can define the total cost based on the capacity constraint (A4) defined in the Appendix:

$$K = \bar{K} \left(\frac{c}{\hat{c}}\right)^{\sigma} \left(\frac{Q}{\bar{Q}}\right) \tag{10}$$

where $\hat{c} = P_k$, while $\bar{K}$ can be expressed in terms of the value share parameter using definition (A2): $\bar{K} = \theta \bar{Q}$. Thus the total cost becomes:

$$P_k K = \hat{c} \theta \bar{Q} \left(\frac{c}{\hat{c}}\right)^{\sigma} \left(\frac{Q}{\bar{Q}}\right) \tag{11}$$

This allow us to determine the PAS curve using the definition of marginal cost (6) and total cost (11):

$$\min_{\sigma,\theta,tc_i,\hat{c},\bar{Q}} \sum_i \omega_i \left(\bar{tc}_i - tc_i\right)^2 \quad \text{s.t.} \quad tc_i = \hat{c}\theta Q_i^M \left(\frac{1-(1-\theta)(Q_i^M/\bar{Q})^{(1-\sigma)/\sigma}}{\theta}\right)^{\sigma/(\sigma-1)} \quad \forall i \tag{12}$$

where $\bar{tc}_i$ is the bottom-up total cost, tc_i is the approximated total cost. Alternatively we may define the approximation of the average cost curve:

$$\min_{\sigma,\theta,tc_i,\hat{c},\bar{Q}} \sum_i \omega_i \left(\frac{\bar{tc}_i - tc_i}{Q_i^M}\right)^2$$

with the same constraint (12). We will now combine the nine options described in Table 1 in order to compare the results of an approximation. For this experiment we take the data for greenhouse gas (GHG) abatement cost in Switzerland estimated in the McKinsey study (2009b). The curve takes into account 21 possibilities for a GHG emissions reduction in Switzerland for the base scenario. The shape of the bottom-up marginal cost curve is shown in Figure 4a. The cost estimates are negative with huge net savings provided by switching to light-emitting-diodes from incandescents. Eleven other technologies also provide net economic savings, and most of these are associated with transport. Taken these data, we have calculated the corresponding total cost (Figure 5a) and average cost (Figure 4a). The bottom-up costs consist of capital and operational costs, but exclude transaction costs (for example, costs for implementation, enforcement or monitoring). Another important issue that may explain the negative cost is the level of discount rate and the way it is applied (differentially or uniformly). In any case, a negative cost is inconsistent with top-down modelling because it implies a free lunch. The importance of a proper representation of an abatement cost curve in CGE modelling is highlighted by Klepper and Peterson (2006).

Correcting for these negative costs is not straightforward. We can either implement

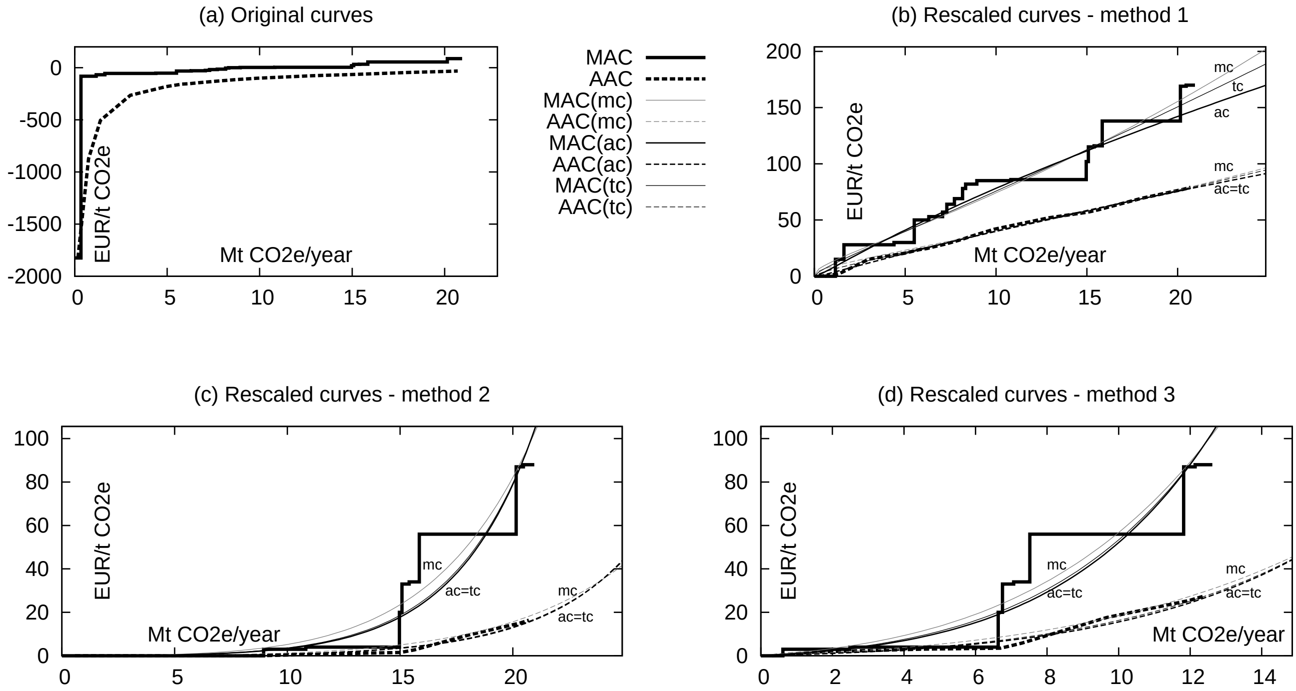

Figure 4: Calibration of marginal and average cost curves for GHG abatement in Switzerland

markup in top-down model or rescale the bottom-up curve. We compare three ways to rescale the curves:

1. One could proportionally rescale the original curve starting from zero. It will not change the relative marginal cost. The Figures 4b and 5b show rescaling results as red lines.[2].

2. Another option is to rescale only the negative cost and set it to zero (Figures 4c and 5c). This method assumes that the negative costs implicitly reflect the hidden cost and thus the total cost should be raised with the hidden cost leading to zero net cost.

3. Finally, we may exclude the negative cost from the curves (Figures 4d and 5d). This method can be implemented when a restrictive environmental policy is applied. However, removing the negative cost from the curve means removing the associated potential for a pollution reduction.

[2]In this particular case, we have ignored the first technology because the next step of the curve is 22 times higher. When a considerable difference between technologies exists, our algorithm is not able to approximate the curve properly. However, we have verified that our algorithm works properly with other curves described by McKinsey study (2008, 2009a).

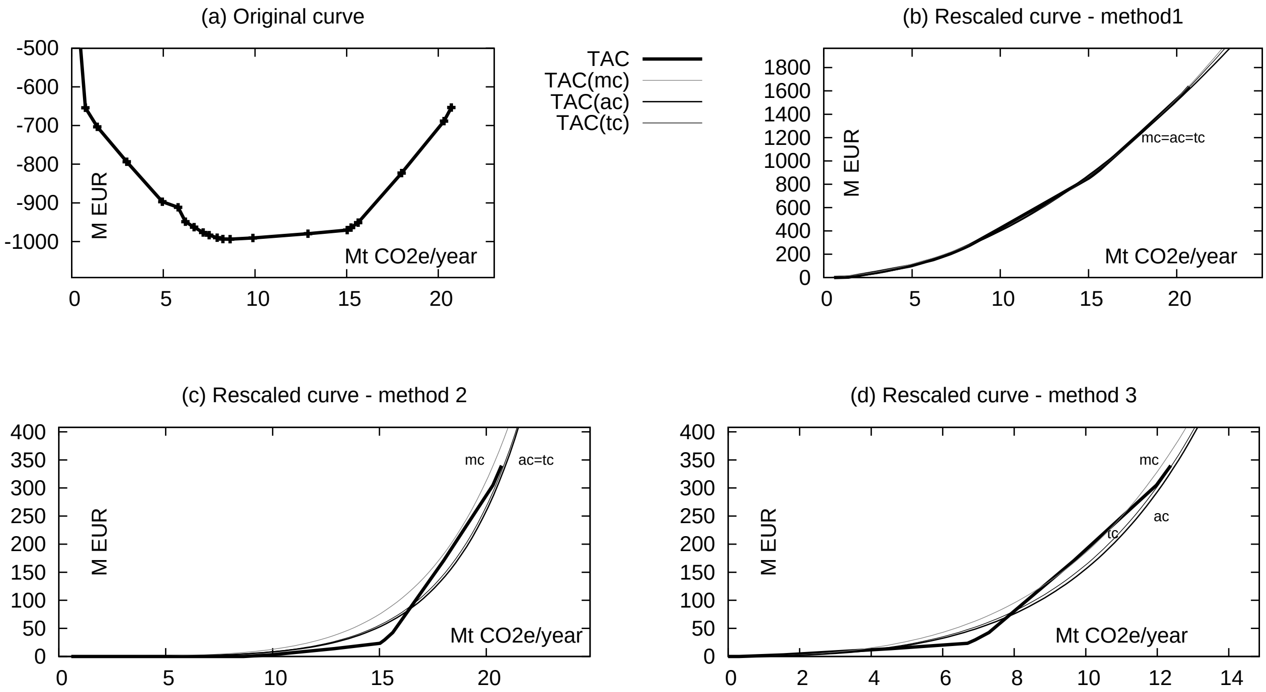

Figure 5: Calibration of total cost curve for GHG abatement in Switzerland

We have repeated the same experiment for the total cost curve and the results of the approximation are presented on Figures 4 and 5. For each rescaling method we have approximated three curves using three targeted costs. The green, blue, and pink lines show the results of approximation when marginal, average, and total cost is targeted, respectively. All 27 approximations give quite close results regarding the targeted curves. For the first method of rescaling the curve, the targeted cost does not make a difference for any curve within the domain, but it makes a difference for the marginal cost curve outside the domain (Figure 4b). For two other rescaling methods, targeting marginal cost (green line) gives more precise results within the flat part of the curve (Figures 5c,d) than targeting the total (pink line) or the average (blue line) cost, but the curvature is always lower (greater σ). This means that the PAS curve[3] (pink line in Figure 5) can be an alternative to calibrating the marginal cost curve.

These results suggest that targeted cost is not important for the calibration process. However, we have found cases when targeting average cost can create a problem (Figure 6a). It happens when the length of one step is considerably greater than of others. Thus depending on the shape of the abatement curve, the targeted cost could be matter. We would not be able to catch this deviation within the approximation of the total cost curve

[3]Our algorithm is different from the original PAS curve in Dellink (2005), but targeted cost, approximated curve, and CES technology are the same.

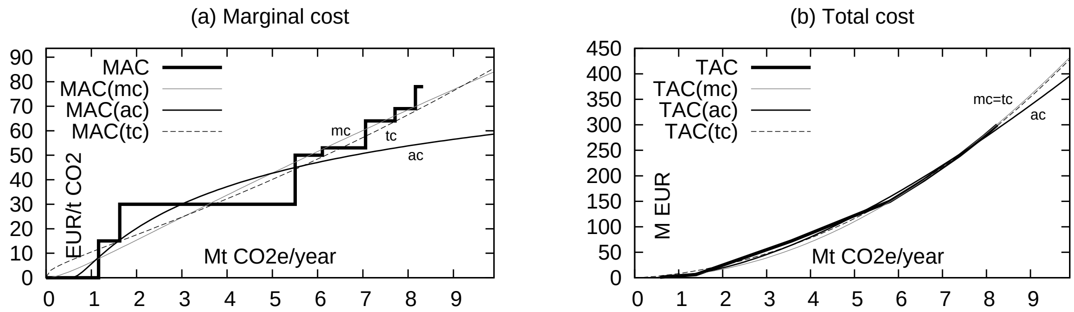

Figure 6: Deviation for calibrated marginal cost curve when average cost is targeted

(Figure 6b). For this reason it was important to verify the calibration process for different cost curves. Once we have fit the marginal cost curve, the total cost curve would also be properly approximated, but not vice versa.

4 Conclusion

Top-down models usually do not pay explicit attention to the characteristics of the technologies involved, but use smooth production functions. The objective of this paper was to develop a methodology for getting a precise approximation of a step function in order to implement it into a top-down model. We have demonstrated two general equilibrium methods, one partial equilibrium methods, and one hybrid method to approximate a bottom-up cost curve. The first two methods (Numeric and OLS) are solved as a standard optimization problem. The third method (Analytic) is solved using a loop. Finally, the forth method is a hybrid of the previous methods. The simplest and equally precise as other methods is ordinary least squares.

The results of the approximation include the elasticity of substitution, the parameter that provides a good approximation of the technology options. Options which may never have been employed, but which are assume to exist on the basis of engineering studies. In this paper we show that the elasticities need not be estimated from ex-post outcomes. Instead, we can calibrate the elasticities of substitution. Forward-looking engineering estimates of the technology potential are used in lieu of backward-looking econometric estimates of market outcomes.

Our methodology is based on a CES function, where one production factor is fixed. It allows us to simulate a decreasing returns to scale technology. The integration of Marshallian concept in Arrow-Debreu models shows the relationship between supply and production functions. We use the pollution abatement sector as an example for our methodology, but it can be applied to any sector with a fixed production factor. Our algorithm does not assume non-zero output level in the benchmark, unlike alternative algorithms. The GAMS code is available in the Appendix.

Generally, the CES approximation is precise, but precision decreases when we move outside of the domain of available abatement options. It does not matter what cost curve (marginal, average or total) is approximated, but rather what cost is targeted. We found no significant differences between targeting marginal or total cost, but targeting average cost can generate errors (when there is a considerable difference between length of steps of the bottom-up cost curve). It is also possible to obtain a poor approximation with any targeted cost when the height of the steps of the bottom-up cost curve vary considerably.

An application of our methodology is straightforward: fitting an abatement function into a top-down model should improve the precision of the simulated environmental policies. Applying it into other sectors should also improve the precision of a top-down model. For example, Schaefer and Jaccoby (2005) get inconsistency between energy use with bottom-up and top-down models because their calibration procedure is not able perfectly match bottom-up data. With the methods that we propose, a perfect match should be expected.

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Appendix

A. The calibrated share form

Alternatively to profit maximization problem (1), we can represent the marginal cost as the Lagrange multiplier on the output constraint in the cost minimization problem, if output is fixed at $Q = \bar{Q}$:

$$\min TC = K \tag{A1}$$

$$\text{s.t.} \quad \bar{Q} = \phi \left(\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)\right)^{\sigma/(\sigma-1)} = \bar{Q} \left(\theta \left(\frac{K}{\bar{K}}\right)^{(\sigma-1)/\sigma} + (1-\theta)\right)^{\sigma/(\sigma-1)}$$

The constraint is defined as a CES production function written in a classical form first, and then in a calibrated share form. It is helpful for numerical modelling to work with a calibrated share form of the function (Boehringer et al., 2003). Each variable in this form is defined in relation to its reference level. Parameter θ is calculated as a value share of input K at the benchmark point $(\bar{X}, \bar{K})$:

$$\theta = \frac{\bar{P}_k \bar{K}}{\bar{c}\bar{Q}} \tag{A2}$$

where $\bar{c} = 1$ is the benchmark marginal cost and $\bar{P}_k = 1$. When the price of variable input departs from its benchmark value ($P_k \neq \bar{P}_k$) the marginal cost function for CES technology is written as :

$$c = \left(\theta \left(\frac{P_k}{\bar{P}_k}\right)^{1-\sigma} + (1-\theta)\left(\frac{P_x}{\bar{P}_x}\right)^{1-\sigma}\right)^{1/(1-\sigma)} \tag{A3}$$

Compensated demand functions can be incorporated explicitly from the Shepard's lemma:

$$\frac{K}{\bar{K}} = \frac{\partial c}{\partial (P_k/\bar{P}_k)} = \left(\frac{c\bar{P}_k}{P_k}\right)^{\sigma} \frac{Q}{\bar{Q}} \quad \text{and} \quad \frac{X}{\bar{X}} = \frac{\partial c}{\partial (P_x/\bar{P}_x)} = \left(\frac{c\bar{P}_x}{P_x}\right)^{\sigma} \frac{Q}{\bar{Q}} \tag{A4}$$

If the abatement capacity is fixed ($X = \bar{X}$), the capacity constraint (A4) can be inverted to obtain an explicit expression for its price:

$$\frac{P_x}{\bar{P}_x} = c \left(\frac{Q}{\bar{Q}}\right)^{1/\sigma} \tag{A5}$$

Assuming competitive supply ($c = P$), we can obtain the following expression by substituting (A5) into the cost function (A3):

$$1 = \theta \left(\frac{P}{P_k} \right)^{\sigma - 1} + (1 - \theta) \left(\frac{Q}{\bar{Q}} \right)^{(1-\sigma)/\sigma} \tag{A6}$$

It gives us the supply function (4) in a Marshallian framework $Q = f(P)$. The price of the variable input P_k is the shadow price of abatement P because it replaces the benchmark price of abatement $\bar{P} = 1$. For the purpose of notations, we use $P_k = \hat{P} = \hat{c} \neq 1$ and we do not mix it with $\bar{P}_k = \bar{P} = \bar{c} = 1$ and $P = c$.

B. GAMS code of the algorithm

```
$title        Calibration of MAC function

set        i        Abatement technologies /1*12/
       pt        Middle and Upper points of the step /M,U/;

parameter        mc(i)        Marginal cost of technology
                q(i)         Maximum supply by technology,
                qref(i)         Reference quantity (cumulative to midpoint of step),
                mcstar        Target cost
                omega(i) Weight on the ith factor
                stddev        Standard deviation in prices (relative to mean) /0.75/;

*        Generate some random data in which marginal costs increase monotonically:
mc(i) = 0;
qref(i) = 0;
loop(i,
        mc(i) = mc(i-1) + (2/card(i))*uniform(0,1);
        q(i) = uniform(0,1);
        qref(i) = qref(i-1) + (q(i-1)+q(i))/2;     );
mcstar   = smax(i, mc(i))/2;
stddev   = stddev * mcstar;
omega(i) = exp(-sqr(mc(i)-mcstar)/(2*sqr(stddev)));
omega(i) = omega(i)*q(i)/sum(i.local, omega(i)*q(i));

variables        SIGMA        Elasticity of substitution
                THETA        Benchmark value share of the variable input
                QBAR        Reference output
                CBAR        Reference cost,
                OBJ        Objective function
                C(i,pt)        Cost estimates at different points;

$macro dev(cf,i)  (q(i)/2 * ( (mc(i)-cf(i-1,"U")) * (mc(i)-cf(i-1,"U") \
                + sqr(cf(i,"M")-cf(i-1,"U"))/(mc(i)-cf(i-1,"U")) * 1/3 \
                + cf(i,"M")-cf(i-1,"U")) + (cf(i,"U")-mc(i))  * (cf(i,"U")-mc(i) \
                + sqr(cf(i,"U")-cf(i,"M"))/(cf(i,"U")-mc(i)) * 1/3 + cf(i,"U")-cf(i,"M"))))

equation        objdef        Objective function,
                cdef        Cost function;

objdef..        OBJ =e= sum(i, omega(i) * dev(C,i));

cdef(i,pt)..        THETA * (C(i,pt)/CBAR)**(SIGMA-1) =e=
                1 - (1-THETA)*((qref(i)+(q(i)/2)$sameas(pt,"U"))/QBAR)**((1-SIGMA)/SIGMA);

model lsqr /objdef, cdef/;

THETA.LO = 0.01;      THETA.UP = 0.99;                        THETA.L  = 0.5;
```

```
CBAR.LO = 0;          CBAR.UP = 1.5*smax(i, mc(i));        CBAR.L   = 0.5;
C.LO(i,pt) = CBAR.LO; C.UP(i,pt) = CBAR.UP;                C.L(i,pt)= 0.5;  QBAR.L   = 0.5;

*        There is a singularity at SIGMA=1 which can create problems, so
*        we may need to solve this problem a couple of times:

SIGMA.LO = 0; SIGMA.UP = 0.99; SIGMA.L  = 0.5;
solve lsqr using nlp minimizing OBJ;

if (SIGMA.L  = SIGMA.UP,
    SIGMA.LO = 1.001; SIGMA.UP = +INF; SIGMA.L = 1.2;
    solve lsqr using nlp minimizing OBJ;  );
```

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