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Midquotes or Transactional Data? The Comparison of Black Model on HF Data

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Midquotes or Transactional Data?
The Comparison of Black Model on HF Data*

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Abstract

The main idea of this research is to check the efficiency of the Black option pricing model on the basis of HF emerging market data. However, liquidity constraints - a typical feature of an emerging derivatives market - put severe limits for conducting such a study. That is the reason why Kokoszczynski et al., 2010, have conducted their earlier research on midquotes data treating them as potential transactional data. They have got some intriguing conclusions about implementing different volatility processes into the Black option model. Nevertheless, taking into account that midquotes do not have to be the proper representation of market prices as probably transactional data do, we decide to compare in this paper the results of the research conducted on HF transactional and midquotes data. This comparison shows that the results do not differ significantly between these two approaches and that BIV model significantly outperforms other models, especially BRV model with the latter producing the worst results. Additionally, we provide the discussion of liquidity issue in the context of emerging derivatives market. Finally, after exclusion of spurious outliers we observe significant patterns in option pricing that are not visible on the raw data.

Keywords:

option pricing models, financial market volatility, high-frequency financial data, midquotes data, transactional data, realized volatility, implied volatility, microstructure bias, emerging markets

JEL:

G14, G15, C61, C22

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1. Introduction

The Black's 1976 futures option pricing model began a new era of futures option valuation theory. The rapid growth of option markets in the 1970s[1] brought soon a lot of data and stimulated an impressive development of research in this area. Quite soon numerous empirical studies put in doubt basic assumptions of the Black model: they strongly suggest that the geometric Brownian motion is not a realistic assumption. Many underlying return series display negative skewness and excess kurtosis (see Bates 1995, Bates 2003). Moreover, implied volatility calculated from the Black-Scholes model often vary with the time to maturity of the options and the strike price (cf. Rubinstein 1985, Tsiaras 2009). These observations drove many researchers to propose new models that each relax some of those restrictive assumptions of the Black-Scholes model (Broadie and Detemple 2004, Garcia et al. 2010, Han 2008, Mitra 2009). Basing on Han 2008, we can divide these researchers in a few groups. The first one engage in extending Black-Scholes-Merton framework by incorporating stochastic jumps or stochastic volatility (Amin and Jarrow 1992, Hull and White 1987), another one goes into estimating the stochastic density function of the underlying asset directly from the market option prices (Derman and Kani 1994, Dupire 1994) or using other distribution of the rate of return on the underlying asset rather than normal distributions (Jarrow and Rudd 1982, Corrado and Su 1996, Rubinstein 1998, Lim et al. 2005). On the other hand, the Black-Scholes model is still widely used not only as a benchmark in comparative studies testing various option pricing models, but also among market participants. Christoffersen and Jacobs 2004 show that much of its appeal is related to the treatment of volatility – the only parameter of the Black-Scholes model that is however not directly observed.

Detailed analysis of literature (An and Suo 2009, Andersen et al. 2007, Bates 2003, Brandt and Wu 2002, Ferreira et al. 2005, Mixon 2009, Raj and Thurston 1998) seems to suggest that the BSM model with implied volatility calculated on the basis of the last observation performs quite well even when compared with many different pricing models (standard BSM model, BSM with realized volatility, GARCH option pricing models or various stochastic volatility models).

In an earlier paper Kokoszczyński et al. 2010 use high-frequency (10-seconds) data for WIG20[2] index options to check whether the same observation applies also to the Polish market. The results show that the Black model with implied volatitility (BIV) gives the best results, the Black model with historical volatility (BHV) is slightly worse and the Black models with realized volatility give clearly the worst results. This ranking, based on four different types of error statistics, is rather robust with respect to different times to maturity and moneyness ratios.

The only important aspect that has not been varied in that research is the nature of data – market prices are represented in that paper by midquotes calculated on the basis of bid and ask quotes. As we know these quotes do not represent actual prices at which transactions take place. Nevertheless, most papers we know that test alternative option pricing models and include the Black-Scholes model among models tested therein use bid-ask quotes (midquotes) as they allow to avoid microstructural noise effects (cf. Dennis and Mayhew 2009). In addition, Ait-Sahalia and Mykland 2009 state explicitly that quotes "contain substantially more information regarding the strategic behaviour of market makers" and they "should be probably used at least for comparison purposes whenever possible" (p. 592). However, Beygelman 2005 and Fung and Mok 2001 argue that midquote is not always a good proxy for the true value of an option. Thus, this paper investigates whether the results presented in Kokoszczyński et al. 2010 apply to both transactional data and quotes. This is one of the major foci of this paper.

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[1] The Chicago Board of Options Exchange was founded in 1973 and it adopted the Black-Scholes model for option pricing in 1975.

[2] The WIG20 is the index of twenty largest companies on the Warsaw Stock Exchange (further detailed information may be found at www.gpw.pl).

The structure of this paper has been thus planned in such a way as to answer the following research questions:

- Can we treate midquote data as a representation of market prices similar to transactional data in order to reveal specific market features?
- Are there any differences between the results for these two sets of data concerning the efficiency of option pricing models we test?
- Can we distinguish any patterns of liquidity behavior in emerging markets using transactional data?
- What is the real influence of outliers or "spurious outliers" on final results and how can we identify these observations which can be later excluded from the dataset?

The rest of our paper is organized as follows. The second part describes the methodological issues. Next section presents data and the fluctuations of volatility processes derived from transactional data. The following part discusses the liquidity issues. Results are presented in section five and the last section concludes.

2. Option pricing methodology

The basic pricing model we choose is the Black-Scholes model for futures prices, i.e. the Black model (Black, 1976). We call it further the BHV model – the Black model with historical volatility. Below are formulas for this model:

$$c = e^{-rT}[FN(d_1) - KN(d_2)] \tag{1}$$

$$p = e^{-rT}[KN(-d_2) - FN(-d_1)] \tag{2}$$

$$where:$$

$$d_1 = \frac{\ln(F/K) + \sigma^2 T/2}{\sigma\sqrt{T}} \tag{3}$$

$$d_1 = \frac{\ln(F/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{4}$$

where c and p are respectively valuations of a call and a put option, T is time to maturity, r is the risk-free rate, F – the futures price, K – underlying strike, σ – volatility of underlying and $N(.)$ is the cumulative standard normal distribution.

There are two reasons why we use the Black model instead of the standard Black-Scholes model. First, we are able to omit the problem of calculating the dividend ratio for the index and hence to eliminate one possible source of pricing error. Second, we can use additional data from the period between 9.00 a.m. and 9.30 a.m. each day, even though index quotation starts only at 9.30 a.m.

To further justify such an approach, we assume that we can price a European style option on WIG20 index applying the Black model for futures contract, where WIG20 index futures contract is the basis instrument. This is possible due to following facts:

- WIG20 index futures expire at exactly the same day as WIG20 index options do,
- the expiration prices are set exactly in the same way,
- WIG20 index options are European-style options, hence early expiration - like in the case of American options - is impossible[3].

[3] Early expiration of American-style option could result in the significant error in the case of such a pricing, because of the difference in prices of index futures and of WIG20 index before the expiration date (the basis risk).

One of the most important issues about option pricing is the nature of an assumption concerning the specific type of volatility process. Therefore, we check the properties of the Black model with three different types of volatility estimators: historical volatility, realized volatility and implied volatility. Below we provide a brief description of each of those estimators.

The **historical volatility (HV) estimator** is based on the formula

$$VAR_{\Delta}^{n} = \frac{1}{(N_{\Delta} * n) - 1} \sum_{t=1}^{n} \sum_{i=1}^{N_{\Delta}} (r_{i,t} - \bar{r})^2 \tag{5}$$

where:

VAR_{Δ}^{n} – variance of log returns calculated on high frequency data on the basis of last *n* days,

$r_{i,t}$ - log return for *i*-th interval on day *t* with sampling frequency equal to Δ, which is calculated in the following way:

$$r_{i,t} = \log C_{i,t} - \log C_{i-1,t} \tag{6}$$

$C_{i,t}$ – close price for ***i***-th interval on day *t* with sampling frequency equal to Δ,

N_{Δ} – number of Δ intervals during the stock market session,

n – memory of the process measured in days, used in the calculation of respective estimators and average measures.

$\bar{r}$ – average log return calculated for last *n* days with sampling frequency Δ, which is calculated in the following way:

$$\bar{r} = \frac{1}{N_{\Delta} * n} \sum_{t=1}^{n} \sum_{i=1}^{N_{\Delta}} r_{i,t} \tag{7}$$

In this research, we use $N_{\Delta}=1$ and hence the HV estimator is simply standard devation for log returns based on the daily interval. This approach is commonly used by the wide range of market practitioners.

The second approach is the **realized volatility (RV) estimator** proposed early by Black (1976) and Taylor (1986) and firmly popularised by Bollerslev (cf. Andersen et al. 2001). It is based on squared log returns summed over the time interval of N_{Δ} .

$$RV_{\Delta,t} = \sum_{i=1}^{N_{\Delta}} {r_{i,t}}^2 \tag{8}$$

The **implied volatility (IV) estimator** is based on the last observed market option price. It assumes that all parameters (with the exception of sigma) are also known. We calculate the implied volatility for the last market price for each option and then average them separately for each class of TTM and moneyness ratio, and for both call and put options[4]. Hence, for each observation we have 50 different IV values (5 * 5 * 2). These values are then treated as an input variable for volatility parameter in calculations of the theoretical options price for the Black model with the implied volatility (BIV) for the next observation.

Before entering into formula of the Black model, the HV and RV estimators have to be annualized and transformed into standard deviation. The formula for the annualization of the HV estimator is as follows:

$$HV = {}^{annual\_std}SD_{\Delta}^{n} = \sqrt{252 * N_{\Delta} * VAR_{\Delta}^{n}} \tag{9}$$

Contrary to HV estimator, which is based on information from many periods ($n>1$), RV estimator requires information only from a single period (time interval of Δ). Therefore, the

[4] We devide 128 options into 5 moneyness ratio classes and 5 time-to-maturity classes. The details of this classification are presented in Section 3.

procedure of averaging and annualizing realized volatility estimator is slightly different from that presented in formula (9):

$$ {}^{annual\_std}[RV]^{n}_{\Delta} = \sqrt{252}\sqrt{\frac{1}{n}\sum_{t=1}^{n}[RV]_{\Delta,t}} \quad (10) $$

Having these volatility estimators we study several types of option pricing models:

- BHV - the Black model with historical volatility (sigma as standard deviation, n=21),
- BRV - the Black model with realized volatility (realized volatility as an estimate of sigma parameter; RV calculated on the basis of observations with several different Δ intervals and different values for parameter n applied in the process of averaging),
- BIV - the Black model with implied volatility (implied volatility as an estimate of sigma; IV calculated for the previous observation, separately for each TTM and moneyness class, and for both call and put options, hence for 50 different groups).

Initially, we calculate BRV models with four different Δ values: 10s, 1m, 5m, and 15m. Then, we check the properties of averaged RVs with different values of parameter n in pricing models. Similarly, like Kokoszczyński et. al. (2010), we find no significant differences between different averaged RVs. As a result, we calculate BRV models based only on Δ=5m interval with different values of averaging parameter (n=1, 2, 3, 5, 10, and 21) and hence we obtain the following seven BRV models: BRV5m (non-averaged one), BRV5m_1, BRV5m_2, BRV5m_3, BRV5m_5, BRV5m_10, and BRV5m_21[5].

Finally, in order to verify our research hypothesis we use three different error statistics:

- Root Mean Squared Error (RMSE):

$$ RMSE = \sqrt{\frac{1}{N_{\Delta}n}\sum_{i=1}^{N_{\Delta}n}(Black_i - close_i)^2} \quad (11) $$

- Heteroscedastic Mean Absolute Error (HMAE):

$$ HMAE = \frac{1}{N_{\Delta}n}\sum_{i=1}^{N_{\Delta}n}\left|\frac{Black_i - close_i}{close_i}\right| \quad (12) $$

- Percentage of overprediction (OP):

$$ OP = \frac{1}{N_{\Delta}n}\sum_{i=1}^{N_{\Delta}n}OP_i, where: OP_i = \begin{cases} 1 & if \quad Black_i > close_i \\ 0 & if \quad Black_i < close_i \end{cases} \quad (14) $$

where $close_i$ is the option price (midquote or last observed transaction price) for the i-th interval and $Black_i$ is the Black model price (BHV, BRV or BIV) for the i-th interval. We calculate these statistics for all models, for different TTM and MR classes, and for both call and put options.

3. Data and the description of volatility processes

3.1 Data description

In empirical analysis we apply transaction data for the WIG20 index options and WIG20 futures contracts, obtained from the DM BOŚ provider[6]. The sample covers the period from January 2, 2008 to June 20, 2008. We have aggregated the options transactional prices data from the original frequency of 1 second into 1 minute interval, and use the WIG20 futures prices data have 10 second interval[7]. Each trading day, sessions begin at 9 am and finish at 4:30 pm[8]. Hence,

[5] It is common approach in financial research to set the interval between 5 minutes and 15 minutes because they constitute the good trade-off between the nonsynchronous bias and other microstructure biases.
[6] Dom Maklerski Banku Ochrony Środowiska, http://www.bossa.pl
[7] We do not aggregate WIG20 futures data into 1 minute interval, because we also want to include RV estimators with Δ parameter of frequency higher than 1 minute.

we have 53 218 observations (118 trading days, 451 observations for each trading session). The risk free interest rate is approximated by the WIBOR interest rate, also converted into 1-minute intervals.

The whole data set comprises transaction prices for 65 call index options and 63 put index options expiring in March, June and September 2008 (C, F and I series for call options, and O, R and U series for put options). In order to present the results of analysis, we order them according to:

- 2 types of options (call and put),
- 5 classes of moneyness ratio, for call options: deep OTM (0-0.85), OTM (0.85-0.95), ATM (0.95-1.05), ITM (1.05-1.15) and deep ITM (1.15+), and for put options in the opposite order[9],
- 5 classes for time to maturity: (0-15 days], [16-30 days], [31-60 days], [61-90 days], [91+ days).

This categorization allows us to compare different pricing models along numerous dimensions.

3.2 The descriptive statistics for WIG20 futures time-series.

Table 3.1 presents the descriptive statistics for 1-minute interval data. They were calculated for two samples: with (sample denoted R_f) and without opening jumps effects ($R_{f'}$)[10]. This values show properties of returns distribution in both samples.

Table 3.1. The descriptive statistics for index futures returns (with and without opening jump effect).

		R_f [a]	$R_{f'}$ [b]
N		53216	53099
Mean		-0.000004569	-0.00000250
Median		0	0
Standard Deviation		0.0009491	0.00079783
Range		0.07528	0.02826
Minimum		-0.0456339	-0.01065966
Maximum		0.0296495	0.0175979
Kurtosis		231.8	19.9
Skewness		-3.00	1.08
Kolmogorov-Smirnov	**Statistic**	0.16595	0.15499
	p-value	<0.01	<0.01
Jarque-Berra	**Statistic**	119 272 818	884 434
	p-value	<0.00001	<0.00001

[a] full sample, [b] sample without opening jump effect.

---

[8] Actually, the continuous trading stops at 4:10 p.m. Between 4:10 p.m. and 4:20 p.m. close price is settled and then, till 4:30 p.m. investors can trade only on the basis of close price.

[9] Moneyness ratio is usually calculated according to the following formula:

$$\text{moneyness ratio} = \frac{S}{K / e^{rT}} = \frac{F}{K} \qquad (15)$$

where K is the option strike price, S is the price of underying, F is the futures price of underlying, r is the risk free rate and T is time to maturity.

[10] By *opening jump effects* we mean returns between 16:30:00 and 9:00:00 on the next day. Thus, sample without opening jump effects does not include observations with these returns.

Both samples have high kurtosis and are asymmetric. The distribution for the full sample has strong negative skewness while removing jumps effects makes the distribution right skewed. Overall, both Jarque-Bera and Kolmogorov-Smirnov statistics indicate that returns in both samples are far from normal. Nevertheless, we see that for the modified sample the kurtosis coefficient and the skewness coefficient (in absolute terms) are both substantially smaller. Therefore, we can assume that opening effect is to the large extent responsible for the non-normality of returns, although returns with opening jumps excluded are still obviously not normal.

3.3 The description of volatility processes

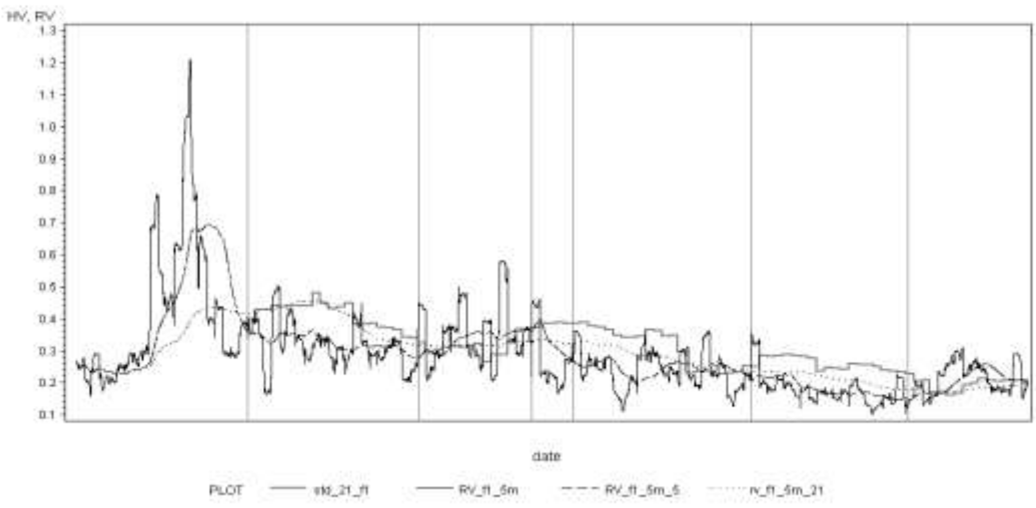
We consider three different volatility measures: historical, realized and implied volatility. Obviously, this is the reason for differences between theoretical option prices we compare.

In the case of the historical volatility estimator N_{Δ}=1 for every $r_{i,t}$ (daily log returns) and $C_{i,t}$ in formulas (5), (6) and (7). Moreover, we use the constant value of parameter n=21, because we want to reflect the historical volatility from the last trading month.

Realized volatility was initially calculated for Δ=10s, 1m, 5m and 15m. However, Kokoszczyński *et al.* (2010) show that differences between theoretical options prices from the Black model wih RV calculated with these four Δ parameters are negligible. Therefore, we present our results for RV calculated only with Δ=10s and Δ=5m[11]. However, the procedure of averaging has been applied only for 5-minute interval and *n* days, where *n*=1, 2, 3, 5, 10, and 21.

Figure 3.1 presents realized volatility compared to historical volatility time series. The distinguishing fact is that the not-averaged RV time series is much more volatile than the averaged RV or HV time series. Obviously, such high volatility of volatility can strongly influence theoretical prices of the BRV model and its stability over time. One can thus expect that in periods of high returns volatitlity the BRV model with non-averaged RV estimator may produce high pricing errors.

Figure 3.1. Historical and realized volatility (5m, 5m_5, 5m_21). [a]

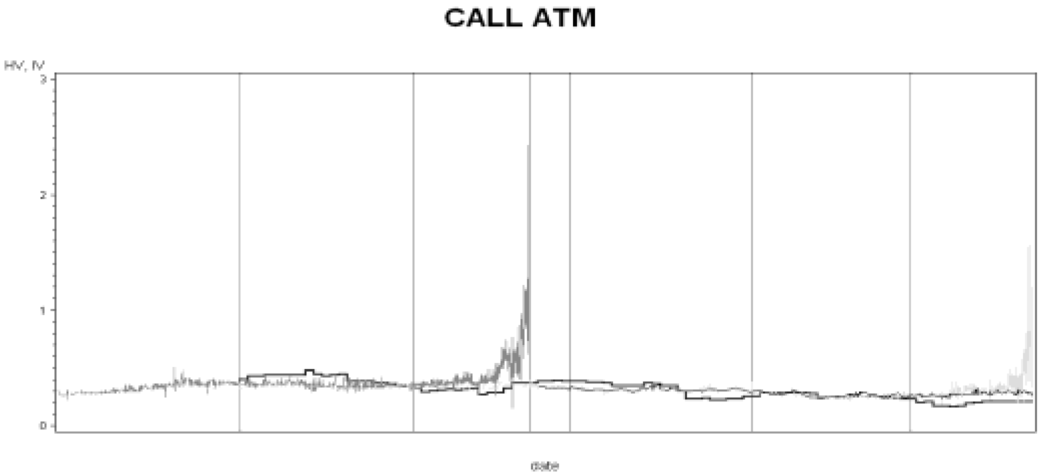


[a] The volatility time series cover the data period between January 2, 2008 and June 19, 2008. Vertical lines represent end of month and additionally the day of March 20th, when option series C (call) and O (put) matured.

[11] As mentioned before, although transactional data is of 1 minute frequency we have decided to include RV estimator also with Δ higher than 1 minute (i.e. 10-seconds).

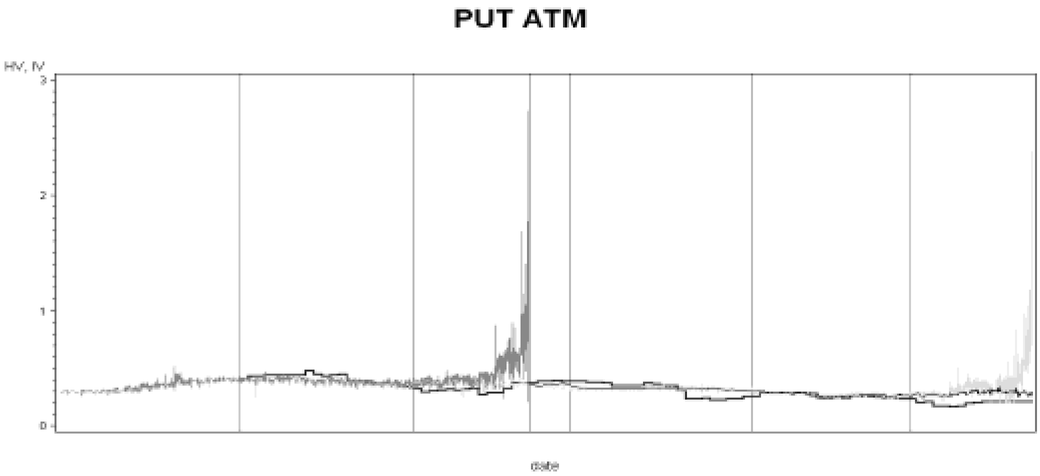
On the other hand, the implied volatility time series exhibits substantially different trajectories than RV or HV time series. Figure 3.2 and Figure 3.3 present how IV time series evolve in time. Similarly to Kokoszczyński et al. (2010) we observe that for the short TTM (10-5 days) IV tends to increase and for the very short TTM (less than 5 days) it explodes, reaching the level of over 200% (annualized). This happens mostly for the (deep) OTM and ATM options, for both call and put options. For that reason, some researchers often exclude from various comparisons options with short TTM and market prices lower than 5-10. However, we have consciously decided to conduct our research on the full sample, believing that such an approach would allow us to better answer the question what kind of observation should be treated as outliers.

Figure 3.2. Implied volatility for ATM call option. [a]



[a] The volatility time series cover the data period between January 2, 2008 and June 19, 2008. Vertical lines represent end of month and additionally the day of March 20th, when option series C (call) and O (put) matured.

Figure 3.3. Implied volatility for ATM put option. [a]



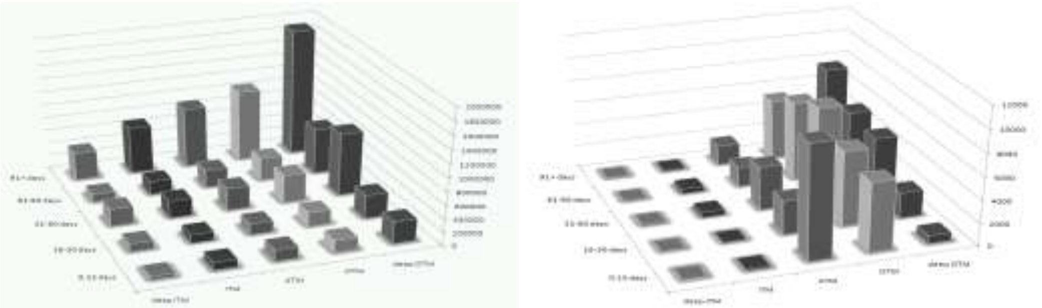
[a] The volatility time series cover the data period between January 2, 2008 and June 19, 2008. Vertical lines represent end of month and additionally the day of March 20th, when option series C (call) and O (put) matured.

4. The liquidity issue

As we mentioned earlier, liquidity constraints - a typical feature of an emerging derivatives market - put severe limits for conducting such a study as we present here. It was the reason why Kokoszczyński et al. 2010 conducted their research using midquotes data. Therefore currently we verify their previous results using transactional data for the same time period. We provide detailed discussion of liquidity of the WIG20 index option market with respect to volume, turnover and open positions for both midquotes and transactional data.

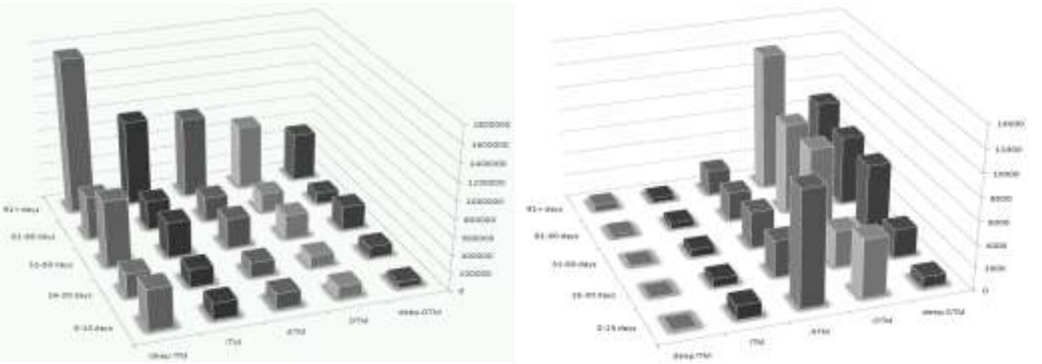
Comparing the distribution of call volume for midquotes and transactional data, presented on Figure 4.1, we can observe the following patterns of volume fluctuations. Both data sets show the lowest volume for low TTM and MR equal to ITM and deepITM. Midquotes data suggest clearly that volume increases gradually along both dimensions (MR from deepITM to deepOTM, TTM from 0-15 days to 91+ days). Practically no volume is observed for call options with MR equal to deepITM and ITM, though other values of MR show significantly higher volume (that feature is not so regular for different TTM values). One may infer from this that investors rarely trade highly valued options (deepITM and ITM).

Figure 4.1. The distribution of volume for call option with respect to midquotes and transactional data



[a] active midquotes [a] transactional data

Figure 4.2. The distribution of volume for put option with respect to midquotes and transactional data



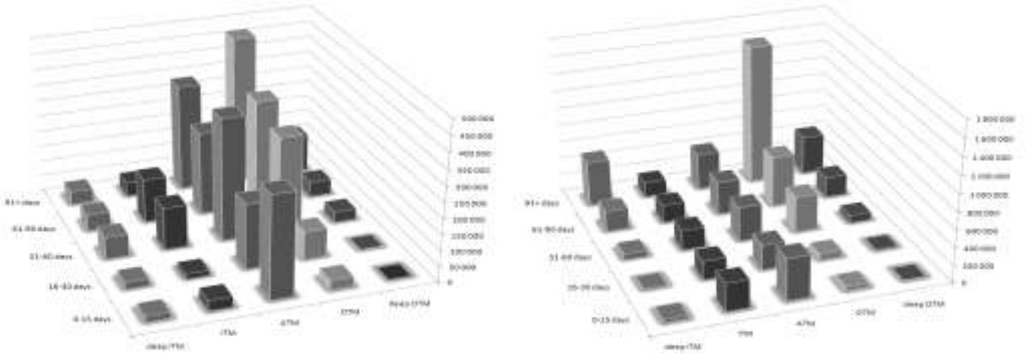
[a] active midquotes [a] transactional data

The distribution for put volume for midquotes data (Figure 4.2) is very similar to the distribution for the case of call options (Figure 4.1) with only one difference that the former volume increases with MR from deepOTM to deepITM. However, this is mostly due to the fluctuations of the basis instrument[12] and to the introduction of the new strikes for options employed by the WSE[13].

The behaviour of volume for put transactional data is almost the same as for call transactional data. Here investors focus their trades on low-valued options no matter what are the possibilities in terms of available bid-ask quotes. These results inform us that the volume based on real transactional data is rather robust to the behaviour of the market while the volume based on midquotes data is determined by current direction of the market.

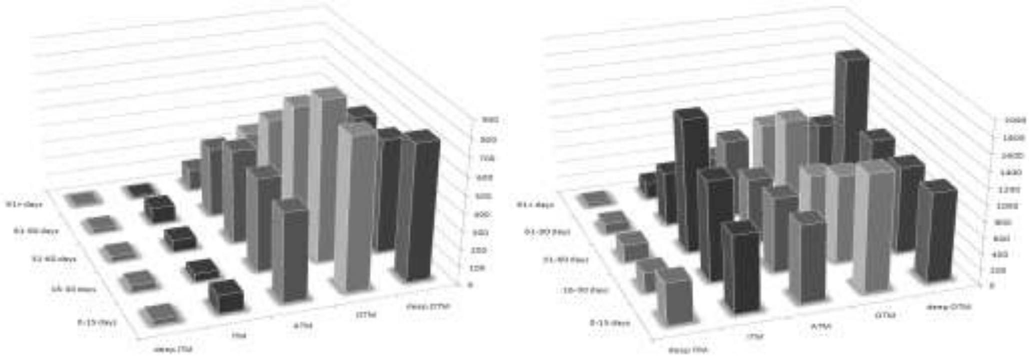
Next figure (Figure 4.3) presents the liquidity issue by focusing on volume of turnover increasing the importance of value of traded options relative to their number. We observe significant shift from deepOTM to ITM, ATM and OTM options for both midquotes and transactional data. It obviously means that most of investors involved in the option trades are focused on ATM range options. The same results are also observed for call and put options with only one difference indicating higher volume of turnover for put options. However, the last feature is once again connected with the behaviour of the basis instrument in the periods we study.

Figure 4.3. Volume of turnover for call and put option for transactional data



call options put options

Figure 4.4. Average number of open positions for call and put option for transactional data



call options put options

The next important characteristic of liquidity analyzed for our two different data sets is the number of open position which is once again presented separately for call and put options.

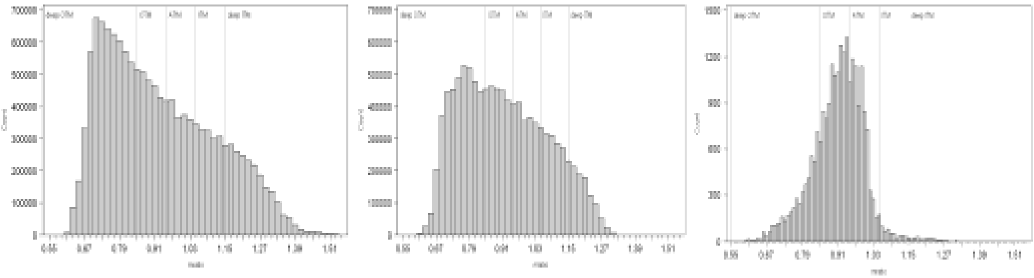
[12] WIG20 index futures was in sharp downward movement in the period we study.

[13] The procedure is based on the idea that in each moment several ITM, ATM and OTM option should be introduced for all expiration dates.

Analysing Figure 4.4 we notice high values of this number for MR equal to ATM, OTM and deepOTM, and additionally to ITM for put options. The average number of open positions was substantially higher for put options than for call options. This is once again partly caused by sharp downward movement of the basis instrument and the increase in demand for put options in speculative and hedging strategies.

The last point in the discussion of liquidity will focus on the comparison of our two data sets with respect to the posibility of trades. When we look at the Figure 4.5 we observe considerable difference between the opportunity to trade indicated in available strike prices or active midquotes and the actual trades revealed by transactional data. The similar patterns are observed in the case of call and put options and that is the additional confirmation for the results presented in the earlier figures.

Figure 4.5. *Moneyness ratio* histogram for call options with respect to available strike prices, active midquotes and transactional data.

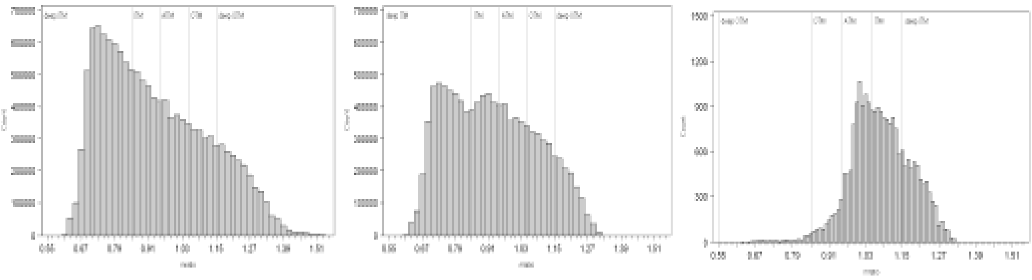


Available strike prices indicate options which were introduced by the WSE on the market.

Active midquotes indicate options which were quoted in the sample period.

Transactional data. Please note other scale of the vertical axis compared with two other diagrams.

Figure 4.6. *Moneyness ratio* histogram for put options with respect to available strike prices, active midquotes and transactional data.



Available strike prices indicate options which were introduced by the WSE on the market.

Active midquotes indicate options which were quoted in the sample period.

Transactional data. Please note other scale of the vertical axis compared with two other diagrams.

The most important outcome for the liquidity analysis is the major difference in the trade volume between presented figures for midquotes and transactional data. We can see that the number of actual trades is on average less than 0.2% of potential trades indicated by active midquotes. It obviously confirms the low liquidity phenomenon of emerging markets and it is the reason why we have decided to conduct addititonal study applying transactional data in order to verify results obtained for midquotes data. Some considerations we present earlier in this section inform us about significant differences between midquotes and transactional data. Nevertheless, we pose the hypothesis that major results of our research for transactional and midquotes data would not differ. This hypothesis is formally verified in the next section.

5. Results

Finally, we obtain three error statistics (RMSE, OP, HMAE) calculated for all pricing models (BRV10s, BRV5m, BRV5m_5, BRV5m_21, BHV, BIV and additionally for the BRV model with different values of parameter *n*) which are divided into 5 TTM classes and 5 MR classes. The detailed number of pricing errors calculated for each model is presented in the following table:

Table 5.1. Number of predicted premiums for different classes of MR and TTM for BRV model*

option	moneyness	0-15 days	16-30 days	31-60 days	61-90 days	91+ days	*Total*
CALL	deep OTM	205	304	1 726	1 670	985	***4 890***
CALL	OTM	1 586	2 037	3 280	2 285	1 161	***10 349***
CALL	ATM	3 403	1 235	1 437	771	409	***7 255***
CALL	ITM	85	35	165	157	59	***501***
CALL	deep ITM	15	26	81	31	36	***189***
	Total Call	**5 294**	**3 637**	**6 689**	**4 914**	**2 650**	***23 184***
PUT	deep OTM	368	857	2 134	1 014	1 011	***5 384***
PUT	OTM	1 615	1 170	2 345	1 694	917	***7 741***
PUT	ATM	3 416	1 215	1 559	1 005	423	***7 618***
PUT	ITM	450	144	283	215	107	***1 199***
PUT	deep ITM	19	8	48	61	102	***238***
	Total Put	**5 868**	**3 394**	**6 369**	**3 989**	**2 560**	***22 180***
	Total Call and Put	***11 162***	***7 031***	***13 058***	***8 903***	***5 210***	***45 364***

* 45.9 thousand for BIV, and 37 thousand for BHV.

We structure the results section into three subsections containing the description for HF transactional data results, the comparison between HF transactional data and midquotes results, and the discussion of results we would have got if the procedure of exclusion of the possible outliers has been introduced.

5.1. The description of results for HF transactional data.

The discussion of results for HF transactional data is based on two-dimensional charts presented as panels containing five or six boxes where we show error statistics (OP, RMSE or HMAE) for all models, all MR and TTM classes. Each chart is scaled with global minima and maxima and that enables simple and reliable comparison of presented results. Figures 5.1-5.6 present error statistics for call and put options separately, with individual boxes for different MR, albeit for all TTM and all models in one box. Analysing these figures we come to the same conclusions as in the case of midquotes data, but before we repeat them we will look more carefully at each error statistics.

Figure 5.1 - presenting OP for call options - indicates that the BIV model (number 6) is the best one. It is characterised by the same level of over- and underprediction (the value of OP is approximately equal to 0.5), and results for other models differ substantially. The second best models according to this metric are the BHV model and the BRV5m_21 model. Additionally, we can see that results for the BHV model vary strongly with changes in TTM. The worst results are observed for the non-averaged BRV models which mostly underpredict, in particular for ATM, ITM and deepITM ratios.

Figure 5.1. OP statistics for **call** options for all MR with respect to different pricing models and TTM.

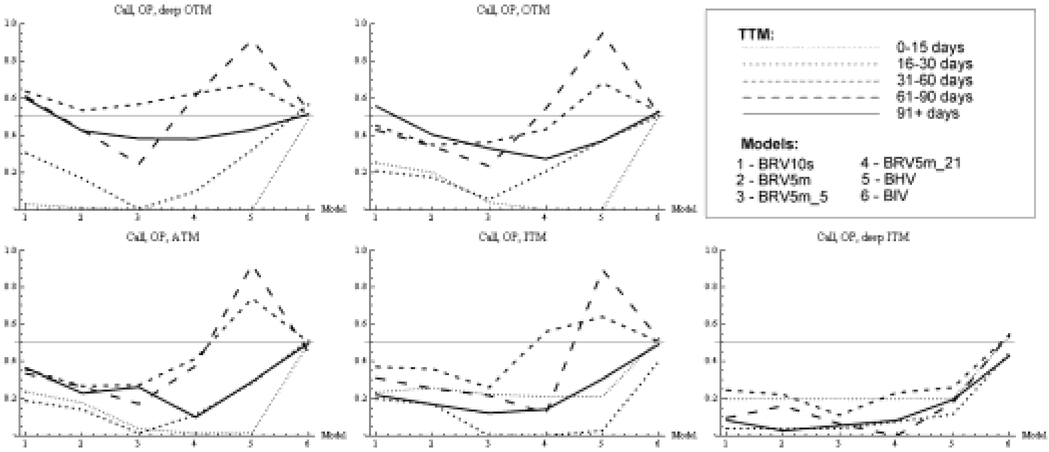
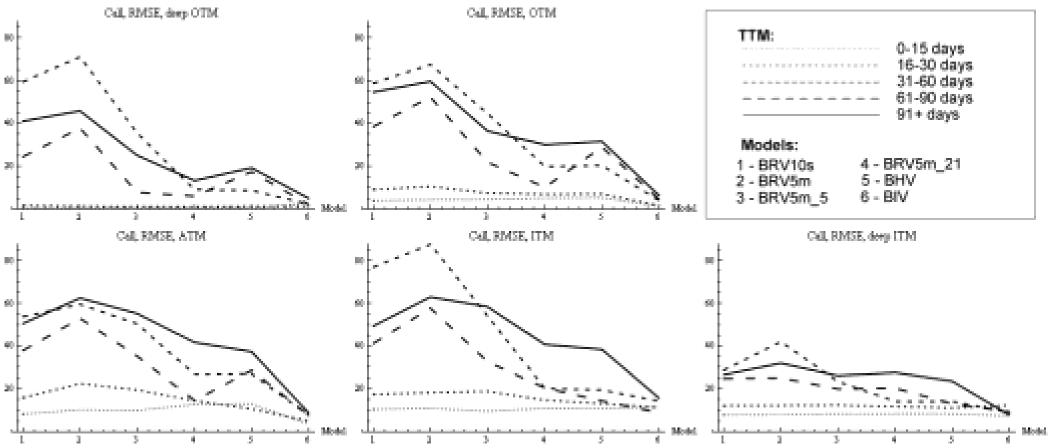


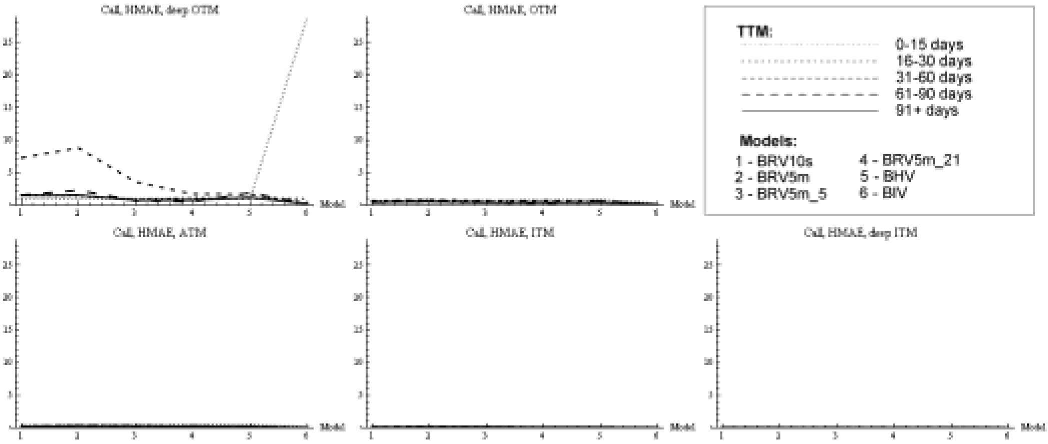
Figure 5.2 presents RMSE statistics for call options. Here again the best results are observed for the BIV model, but on the other hand we observe results as good as those for each model with TTM equal to 0-30 days. The latter means that for these values of TTM it does not matter which valuation model we choose. Analysing the results for the remaining values of TTM we observe gradual decrease of RMSE statistics while moving from the left hand side of each chart (model 1) to its right hand side (model 6). These observations confirm once again the ranking of our models: from the BIV model through the BHV and the BRV5m-21 ones to the non-averaged BRV model.

Figure 5.2. RMSE statistics for **call** options for all MR with respect to different pricing models and TTM.



The next figure (Figure 5.3) informs us about substantial outlier in our data which distorts results and makes it impossible to interpret them. These charts will be presented once again with the outlier excluded in the section 5.3.

Figure 5.3. HMAE statistics for **call** options for all MR with respect to different pricing models and TTM.



Our results for put options are shown in Figures 5.4 - 5.6. Figure 5.4 with OP statistics for put options confirms the ranking of models derived from the results for call options. The volatility of results for the BHV model with respect to various TTM values is again the highest one. It probably means that – when we use historical volatility with parameter *n* equal to 21 - the adjustment of historical volatility to its actual levels is too slow, and that results in strong over- or underprediction for specific TTM or MR ratios.

Figure 5.4. OP statistics for **put** options for all MR with respect to different pricing models and TTM.

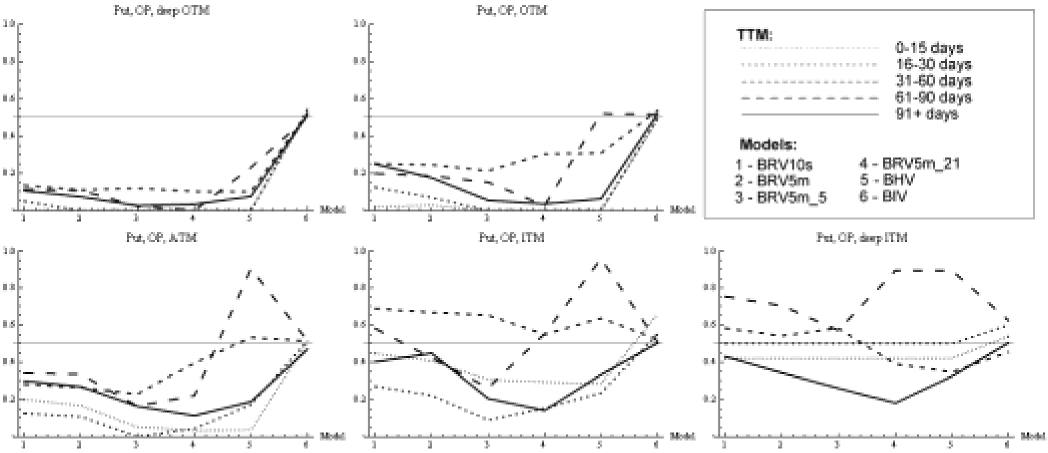
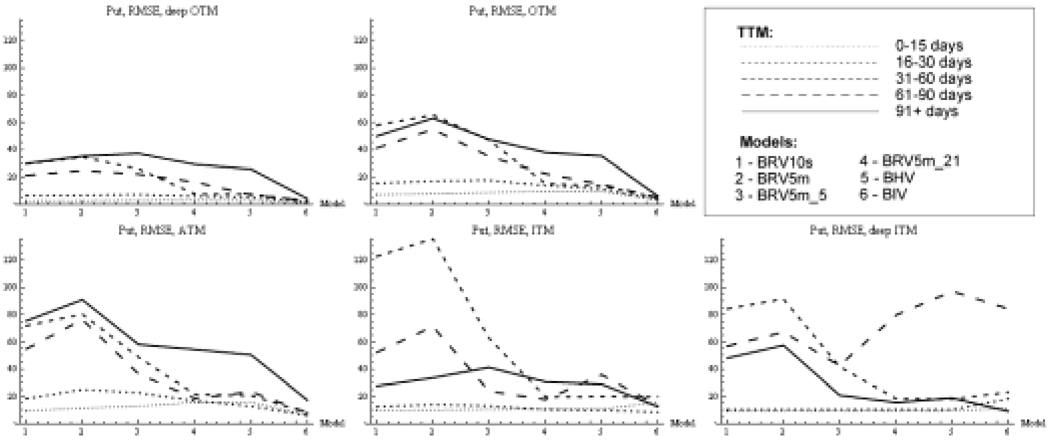


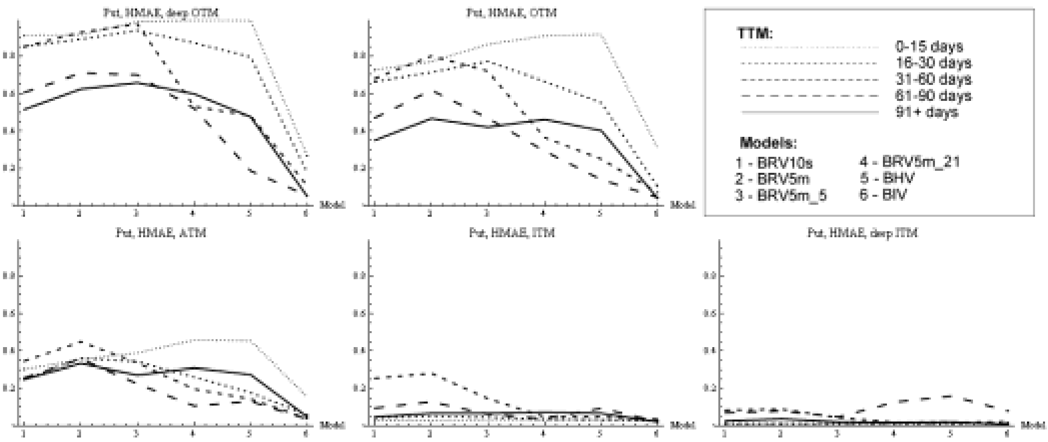
Figure 5.5 presents RMSE statistics for put options and shows almost the same results as those for call options. The errors gradually decrease from the left-hand side to the right-hand side with practically identical error values for all models with TTM equal to 0-30 days. The only exception we observe here are high values of error statistics when MR is deepITM and TTM equals 61-90 days for the BRV5_21, the BHV and the BIV models. The reason for so untypical observation could be a very low number of transaction for deepITM put options with high TTM values. Nevertheless, these results confirm the ranking of models (model 6 dominates model 1).

Figure 5.5. RMSE statistics for **put** options for all MR with respect to different pricing models and TTM.



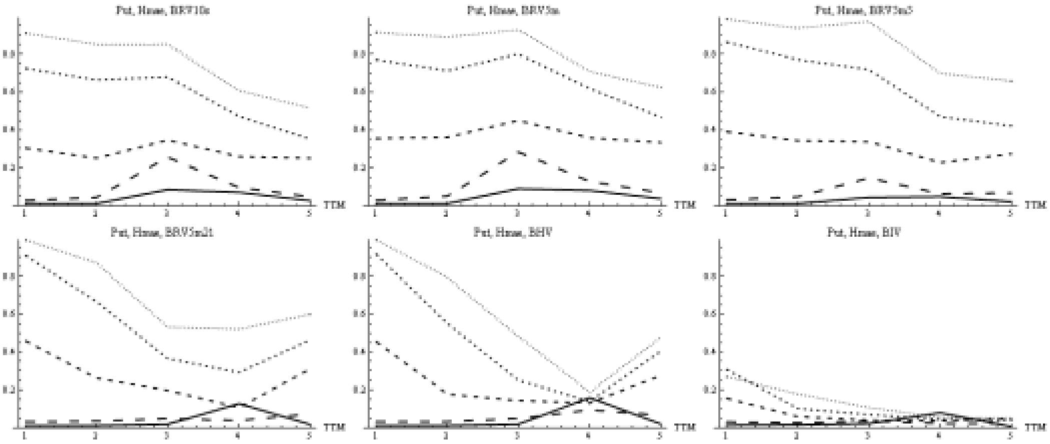
HMAE statistics are shown in Figure 5.6. They indicate that it does not matter whether we take the absolute or relative value statistics because the final outcomes do not differ: the error decreases from the left-hand side of the chart to its right-hand side, i.e. from the BRV model to the BIV one. Moreover, we observe that the value of the HMAE statistics is on average very high for deepOTM and OTM options when compared to its lower average values for ITM and deepITM put options. This stronger departure from theoretical prices in case of OTM and deepOTM options could be caused by stronger activity of individual investors in this segment of the option market in contrast with institutional investors being more active in the case of ITM and deepITM options. Our hypothesis can be supported by the percentage of trades with market makers participation which is significantly larger for ITM and deepITM options.

Figure 5.6. HMAE statistics for **put** options for all MR with respect to different pricing models and TTM.



Next figure (Figure 5.7) presents HMAE statistics for put options with a separate model in each box. Each box contains the results for a single model for five classes of TTM (TTM-1 = ‘0-15 days’, TTM-2 = ’16-30 days’, TTM-3 = ’31-60 days’, TTM-4 = ’61-90 days’, TTM-5 = ’91+days’) and five classes of MR ratio (from the dotted line indicating deepOTM through dashed lines indicating OTM, ATM and ITM to the solid line indicating deepITM). This way of presentation visualise the same pattern of valuation which has been revealed in the case of midquotes research. We observe the dependence of HMAE value on TTM and MR ratios. It gradually decreases with TTM going from '0-15 days' to '91+days' and MR going from deepOTM to deepITM. The similar pattern could be probably observed in the case of call options but because of the distortion of data with substantial outlier for the BIV model we are not able to show this.

Figure 5.7. HMAE statistics for **put** options for all pricing models with respect to different TTM and MR.



Final figures in this section present results for the BRV models only, with different averaging parameter for each model. Figure 5.8 with RMSE values indicate strong dependence of the error on the value of the averaging parameter. The lowest value of error we obtain is for the highest tested value of *n* equal to 21, what confirms our initial hypothesis that the non-averaged value of the RV estimator is not the best choice for an option pricing model. On the other hand, these results do not give us the definite answer to the question what is the optimal value of parameter *n*. Further research should address this issue.

Figure 5.8. RMSE for **call** options, BRV model, different averaging parameters and MR.

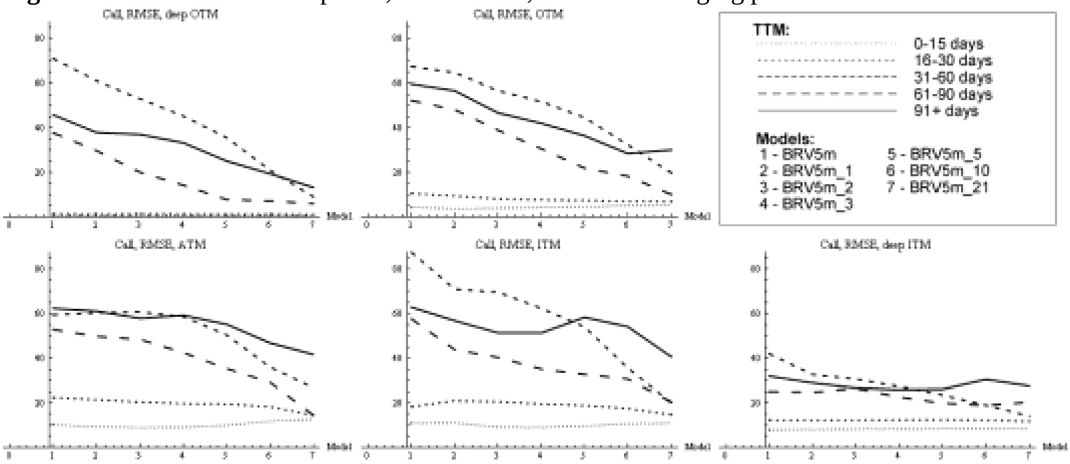
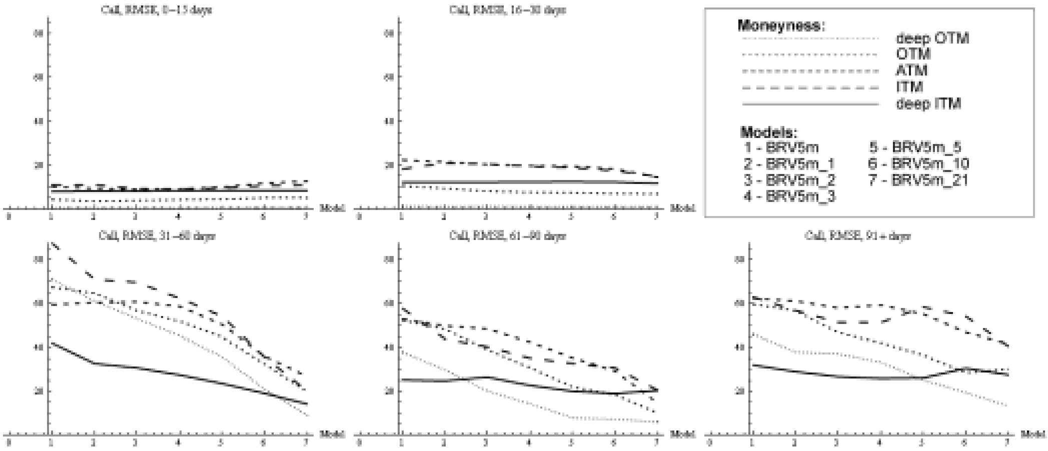


Figure 5.9 presents results only for the BRV model, but in a different way - with each box showing error statistic for a single value of TTM. This modification does not change the interpretation. The gradual decrease of RMSE from model 1 to model 7 is still observed almost for all MR values.

Figure 5.9. RMSE statistics for **call** options, the BRV model, different averaging parameters and TTM.

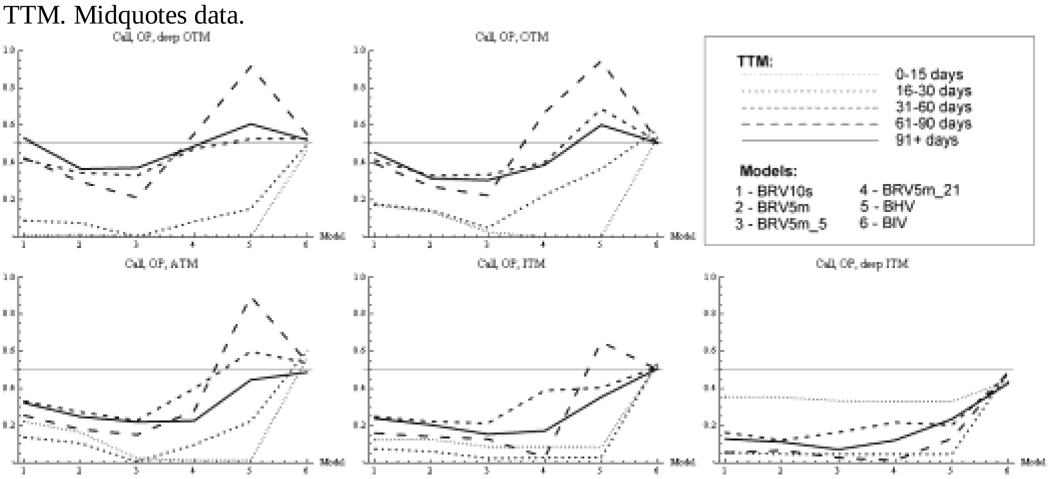


5.2. The comparison of results for midquotes and transactional data.

One of the main goals of this paper is to answer the question how firm are our conclusions concerning the option market that we have got using midquotes data. To check this we repeat the previous study of Kokoszczyński et al. 2010 using now transactional data. After discussion of results for the latter in section 5.1 the comparison of results for two different data sets will be presented in this section.

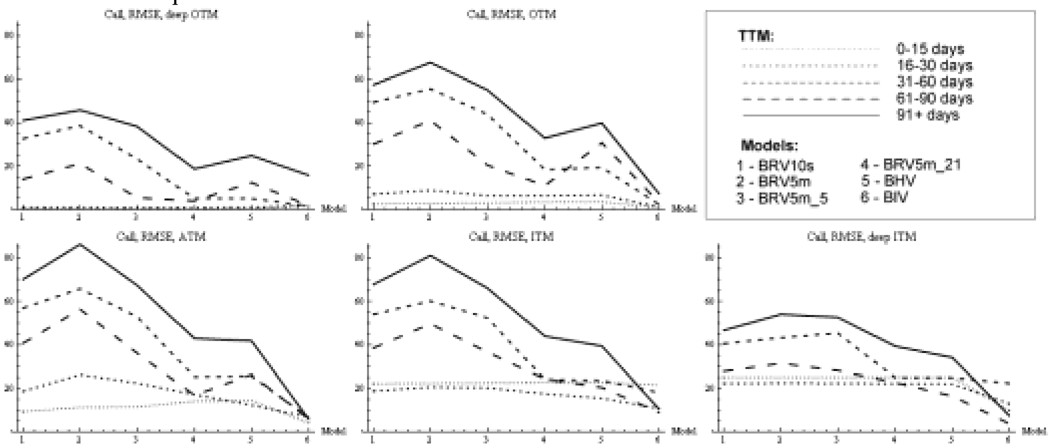
Figure 5.10 with OP statistics for call options does not reveal any significant differences when compared with figure 5.1, both with respect to the ranking of models and to errors dependence on TTM or MR values.

Figure 5.10. OP statistics for **call** options for all MR with respect to different pricing models and TTM. Midquotes data.



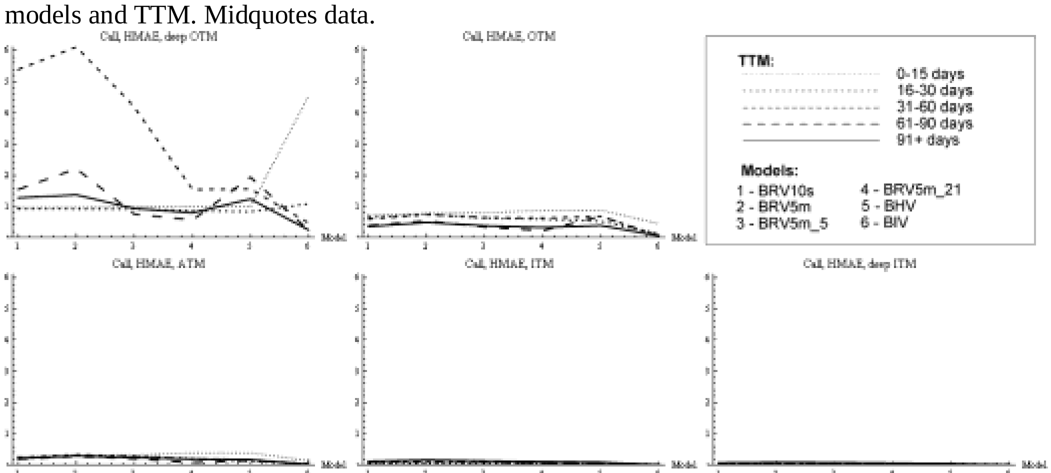
The same conclusion can be drawn from comparing Figure 5.11 (midquotes data) and Figure 5.2 (transactional data), where even the size of maximum error is the same for both datasets.

Figure 5.11. RMSE statistics for call options for all MR with respect to different pricing models and TTM. Midquotes data.



Figures 5.12 and 5.3 enable us to have a closer look at the problem of outlier identification and their influence on the final results. These two figures are distorted due to the presence of outliers for 3 models: BIV, BRV10s and BRV5m with MR equal to deepOTM. However, the magnitude of this error varies significantly for our two data sets. The error is significantly larger for transactional data where outlier for the BIV model is almost five times higher than for midquotes data. The reason for this can be the much lower number of observations for transactional data what means that some extraordinary deviations from the normal market behaviour stronger affects the final results.

Figure 5.12. HMAE statistics for **call** options for all MR with respect to different pricing models and TTM. Midquotes data.



The comparison of the results for put options (Figures 5.13, 5.14, 5.15 for midquotes data vs. Figures 5.4, 5.5 and 5.6 for transactional data) does not add any new insights to the conclusions based on the results for call options. OP, RMSE and HMAE error statistics for transactional data present the same picture as for midquotes data, with the only exception of outliers or rather spurious outliers which distort results in a different manner.

Figure 5.13. OP statistics for **put** options for all MR with respect to different pricing models and TTM. Midquotes data.

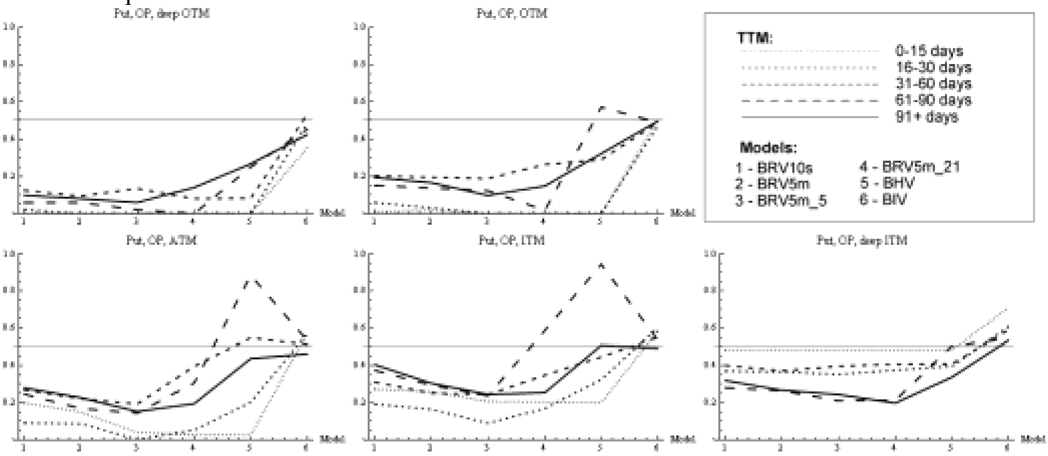


Figure 5.14. RMSE statistics for **put** options for all MR with respect to different pricing models and TTM. Midquotes data.

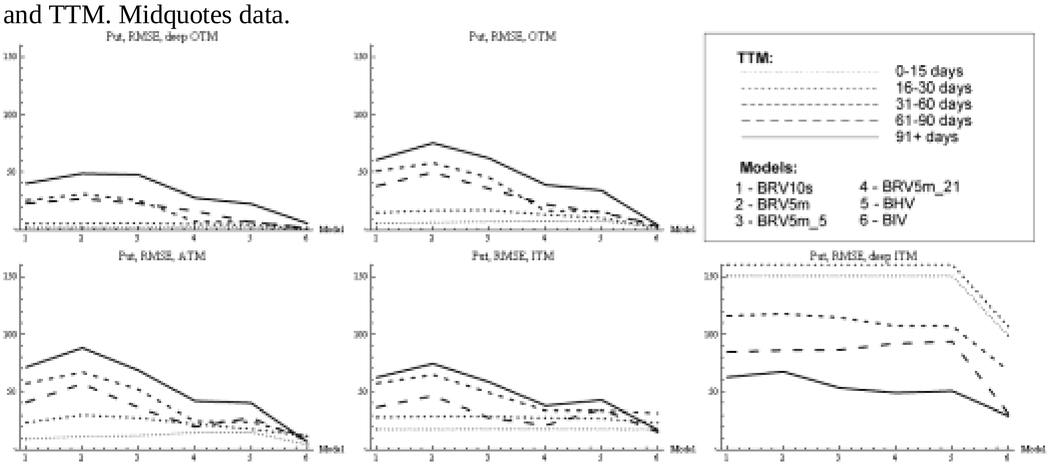
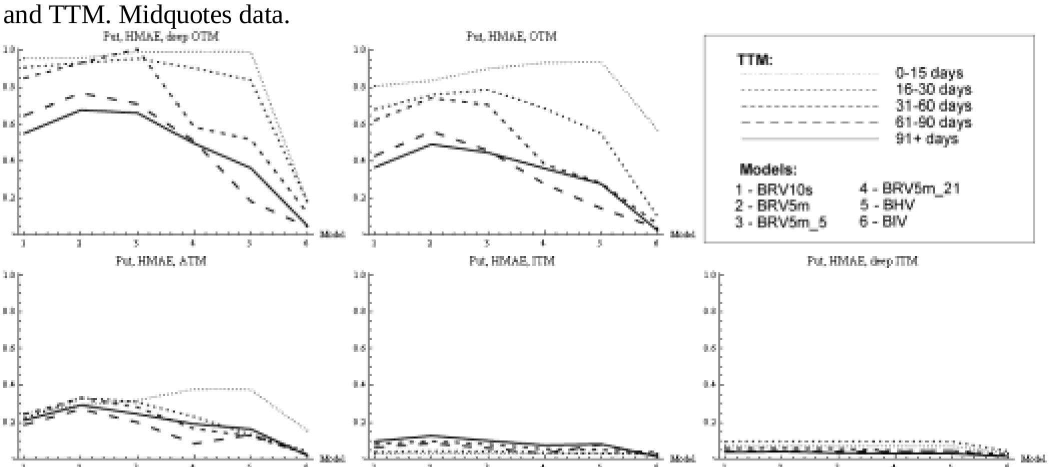
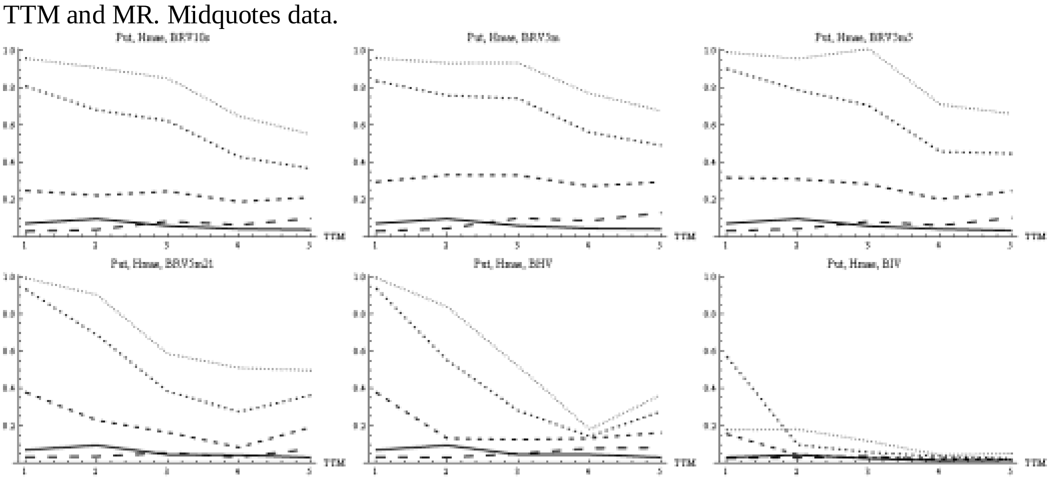


Figure 5.15. HMAE statistics for **put** options for all MR with respect to different pricing models and TTM. Midquotes data.



When we compare Figures 5.16 and 5.17, with 5.7 and 5.8 for transactional data, both presenting HMAE statistics for put options, we observe that the same pattern of valuation (the dependence of HMAE statistics on TTM and MR) is revealed on transactional and miquotes data, as well.

Figure 5.16. HMAE statistics for **put** options for all pricing models with respect to different TTM and MR. Midquotes data.



Final two figures (5.17 and 5.18) do not change the picture of results. We observe the same behaviour of RMSE statistics for midquotes and transactional data, no matter which way of presentation we choose. The best model is obtained when parameter n is set equal to the highest tested value of 21 days.

Figure 5.17. RMSE for **call** options, BRV model, different averaging parameters and MR for midquotes data.

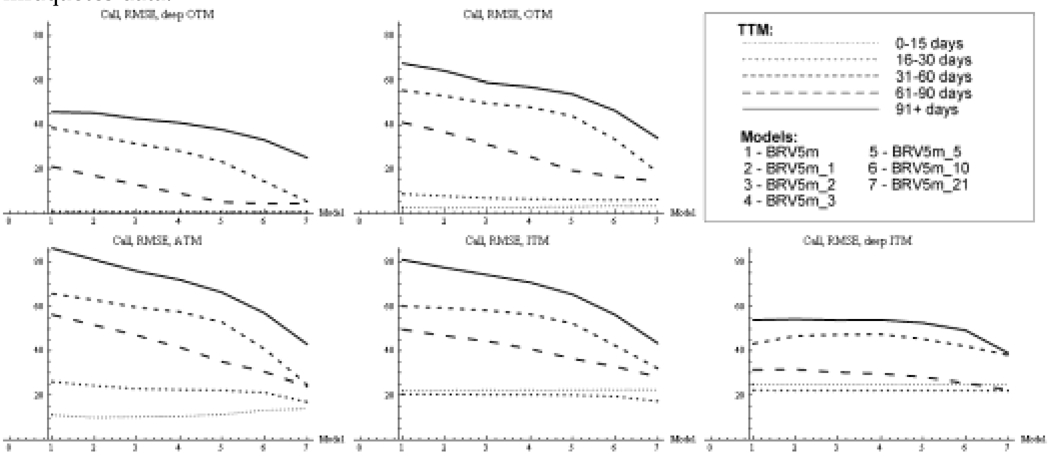
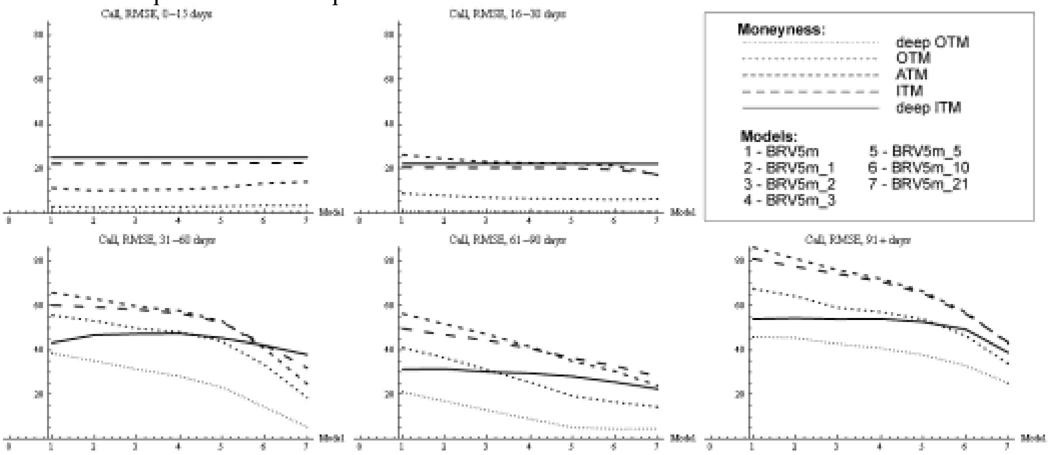


Figure 5.18. RMSE statistics for call options, BRV model, different averaging parameters and TTM for midquotes data. Midquotes data.



This brief comparison informs us that we do not observe any important differences between the results for midquotes and transactional data. Therefore, we can use the former in our research for countries where liquidity issue (which is usually the characteristic of emerging country) plays an important role. Two sets of data may give different outcomes with respect to outliers which can distort data in a different manner because the number of observations for midquotes and transactional data is usually not the same. This issue of outliers is the main subject of the next subsection.

5.3. The influence of spurious outliers exclusion on identification of various patterns in option pricing. The case of transactional data.

The issue of outlier identification and the reason for their exclusion require special attention, because in many cases outliers totally distort results. First of all, we would like to clarify the reasons why they occur in our data sets. Secondly, we attempt to identify cases when we can treat them as ordinary outliers and exclude them from the data set before the final intepretation of results. Finally, we try to separate cases when they appear to be outliers but in reality they result from the specific form of pricing model and its assumptions (spurious outliers).

Below we describe some cases where outliers (or sometimes spurious outliers) distort our results and we try to answer the question whether they can be excluded from our data:

a. Figure 5.1 and 5.10, Call, OP, BHV, TTM=4, MR=1,2,3,4.

Results we see here appear to be the outcome of an atypical observation but they are due to the fixed value of parameter n which in the case of the BHV model is responsible for large deviations of model values from transactional and midquotes data. Taking into account widely used definition of outlier[14] this observation is rather the deviation resulting from the specific shape of our model and should not be treated as an outlier.

b. Figure 5.2 and 5.11, Call, RMSE, BRV10s and BRV5m,TTM=3, MR=1,2,3,4.

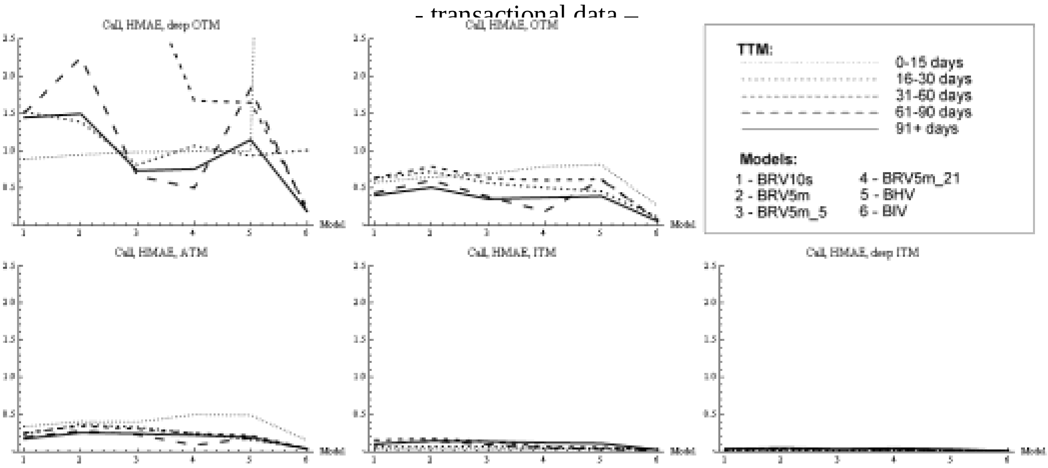
Very high values of RMSE occur for non-averaged value of the RV measure (the BRV10s model and the BRV5m model), so once again what we have here is not an outlier because it results from model specification and not from the nature of data.

c. Figure 5.3 or 5.12, Call, HMAE, model=1,2,3, TTM=3, deepOTM and Call, HMAE, model=6 TTM=1, deepOTM (in particular for transactional data).

Analysing values of error statistic for our first three models (BRV10s, BRV5m and BRV5m_5) we come to the same conclusion as in the case of RMSE error, i.e. that error values are strongly affected by model specification. Moreover, they gradually decrease when we move to other types of BRV models, what informs us that there is no reason to treat them as outliers. On the other hand, in the case of the BIV model we observe substantial increase of HMAE error (especially for transactional data, 30 times higher than the average) only in the case of deepOTM and the shortest TTM. Such an observation fulfills the condition for being an outlier and can be excluded from our results before the final interpretation.

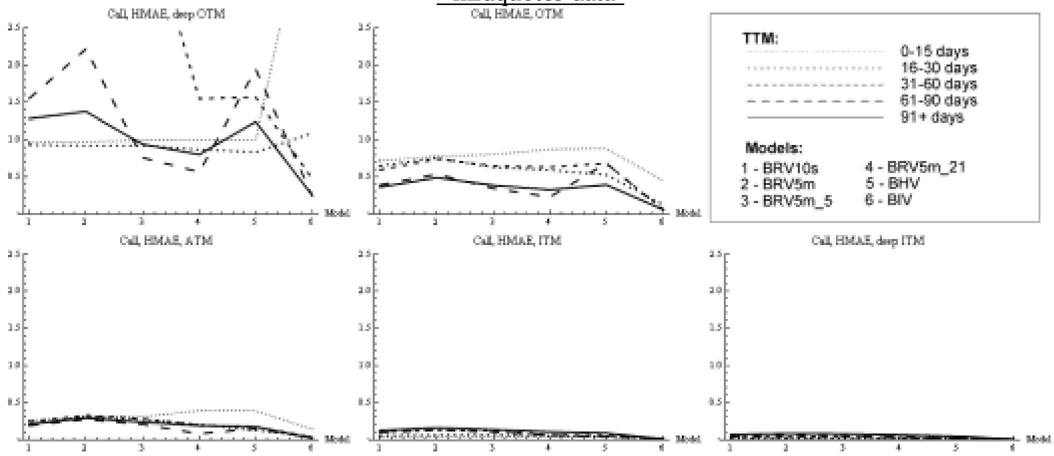
Figure 5.19. The effect of spurious outlier exclusion on **HMAE** statistics for **call** options, all pricing models, TTM and MR. Transactional and Midquotes data.[a]

- transactional data –



[14] In statistics, an outlier (Barnett and Lewis, 1994) is an observation that is numerically distant from the rest of data. On the other hand, Grubbs (Grubbs, 1969) defines an outlier as follows: “An outlying observation, or outlier, is one that appears to deviate markedly from other members of the sample in which it occurs”. Moreover, an outlier is an observation that lies outside the overall pattern of a distribution (Moore and McCabe, 1999). Usually, the presence of an outlier indicates some sort of problem. This can be a case which does not fit the model under study or an error in measurement.

- midquotes data-

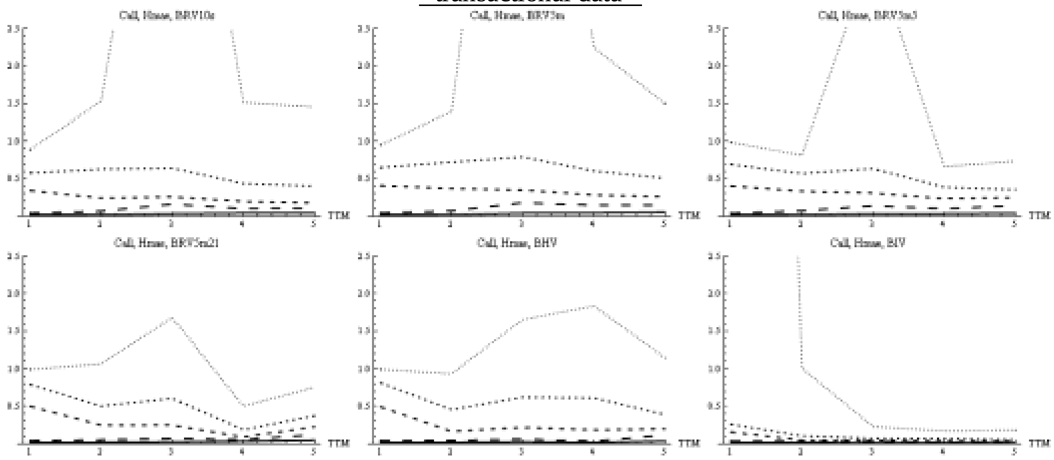


[a] The y-axis scale was transformed in order to exclude the effect of observation for model=1,2,3 when TTM=3 and MR=1, and additionally for model=6 when TTM=1 and MR=1.

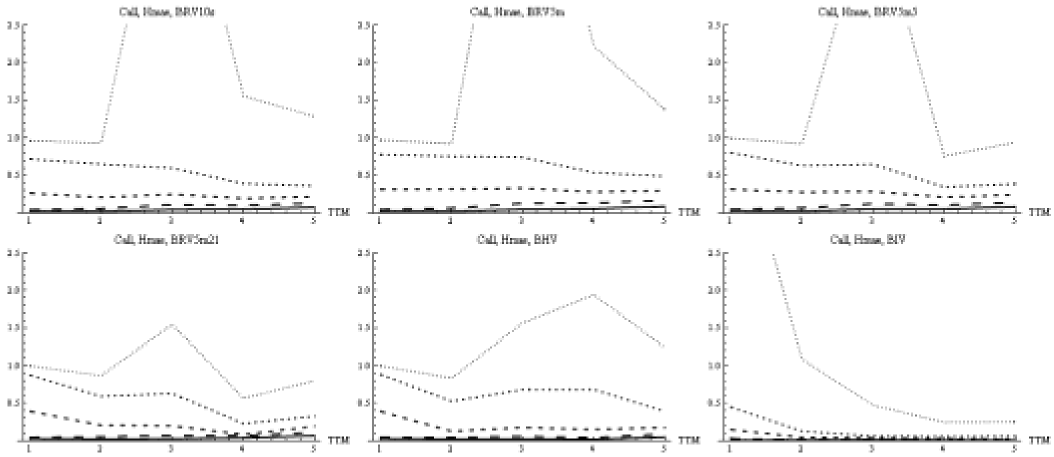
Finally, we decided to present in Figure 5.19 results from Figures 5.3 and 5.12 with different scale for deepOTM options. We do it in order to reveal the same patterns of distribution for call options which have been earlier presented for put options (Figures 5.6 and 5.15) revealing the dependence of HMAE error on TTM and MR ratios. Additionally, Figure 5.20 presents the same results for call options as those earlier presented for put options (Figures 5.7 and 5.16). The legend for Figures 5.19 and 5.20 is presented with the description of Figure 5.7.

Figure 5.20. HMAE statistics for **call** options for all pricing models with respect to different TTM and MR.

- transactional data –



- midquotes data-



d. Figure 5.4 or 5.13, Put, OP, BHV, TTM=4, MR=3,4,5 – the same case as for call options resulting from historically high value of volatility in this period.

e. Figure 5.5 or 5.14, Put, RMSE, model=1,2,3, TTM=3 and MR=2,3,4,5 – the same case as for call options resulting from a non-veraged value of RV measures in the BRV models.

f. Figure 5.5 or 5.14, Put, RMSE, model=4,5,6, TTM=4 and MR=5 **-** the reason for the deviation in this case is the very low number of observations.

Summing up the problem of outlier identification and the motivation for their exclusion we can formulate the following conclusions. Firstly, substantial deviations of error values for their average value are not always signalling a “true” outlier. In some cases the reason thereof is the nature of model used rather than data themselves. We call the former spurious outliers to differentiate them from “true” outliers. Secondly, excluding outliers (including these spurious ones), gives as a result similar patterns in error statistics for call options as those we managed to get for put options.

6. Conclusions and further research.

We decided to repeat and develop further the recent study by Kokoszczyński et al. 2010 based on WIG20 option index data for the first half of 2008 year and to check if those results were still valid not only for midquotes data but also for transactional data. Furthermore, we presented the analysis of liquidity for option market in order to better understand different behavior of option market within various classes of TTM and MR. Moreover, through the discussion of the possibility of outliers exclusions we revealed some additional patterns of distribution which were not visible in the original data set. Below, we try to summarize our conlusions regarding this research and we formulate some thoughts concering further research.

First of all, the results for transactional data do not differ significantly from the results based on midquotes data. The sequence of models, from the most efficient to the least one, is as follows: BIV, BHV, BRV5m_21, BRV5m_5, BRV5m, BRV10s. Moreover, the variability of observed values of analysed error statistics when we move from model 1 (BRV10s) to model 6 (BIV) become much lower what additionally confirms our previous results concerning the efficiency of these models. Focusing on parameter n and only on BRV models we observe that the lowest value of error is obtained for the highest tested n=21, what confirms our initial hypothesis that non-averaged value of RV estimator is not the best choice when we consider the efficiency of option pricing model. On the other hand these results do not give us the definite answer to the question what is the optimal value of parameter *n*? Further research should address this issue. Next, we observe the clear relation between model error and TTM, and model error and moneyness ratio (for call and put options): high error values for low TTM and moneyness ratios, and best fit for high TTM and moneyness ratios. All these outcomes confirm our initial hypothesis that midquotes are proper representation of market prices and can be used in similar studies, especially in case of low liquidity markets.

New interesting feature of transactional data when compared to midquotes data is that in many cases it escalates the problem of spurious outliers. Partly, it is due to substantially lower number of observations, which are responsible for averaging process of error statistics. On the other hand, it could inform us that actual transaction, especially made in more volatile markets when the spread between bid and ask increases, move away from the midquote resulting in an outlier which is not revealed on the basis of midquotes data. It does not change our conclusions concerning the efficiency of models or the variability of results but it should influence the treatment of outliers depending on which data set is used. Furthermore, we observe that highly valued option are relatively better priced because of larger participation of market makers and institutional investors in this part of the market, where psychological aspects of behavior should play significantly less important role.

Analysing liquidity issue we can observe several interesting feature of midquotes and especially transactional data. First of all, the volume of call and puts is focused on ATM, OTM and deepOTM options with hardly no any volume for deepITM and ITM options. What is more important the behaviour of this characteristic is robust for transactional data and depends on the actual market fluctuations for midquotes data. Secondly, the volume of turnover focuses around ATM options, indicating that when we consider the value of transactions the highest liquidity is observed for ATM options. Thirdly, we observe similar pattern for number of open positions as described for their volume. Fourthly, no matter which characteristic do we choose, the liquidity is significantly higher for put options. However, we are aware of the fact that the last conclusion could result from the sharp downward movement of the market in the time of research and high demand for put options for hedging purposes.

More generally, we seem to confirm that the nature of data used for studies of option models - midquotes or transactional ones – does not play the very important role in determining results one gets. Another observation, i.e. how important are liquidity issues for patterns we get comparing performance of various option pricing models, should be studied further. Our intention is thus to conduct a similar study for other markets, more advanced in this regard.

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