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Option Pricing Models with HF Data – a Comparative Study. The Properties of Black Model with Different Volatility Measures*

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Abstract

This paper compares option pricing models, based on Black model notion (Black, 1976), especially focusing on the volatility models implied in the process of pricing. We calculated the Black model with historical (BHV), implied (BIV) and several different types of realized (BRV) volatility (additionally searching for the optimal interval Δ, and parameter n - the memory of the process). Our main intention was to find the best model, i.e. which predicts the actual market price with minimum error. We focused on the HF data and bid-ask quotes (instead of transactional data) in order to omit the problem of non-synchronous trading and additionally to increase the significance of our research through numerous observations. After calculation of several error statistics (RMSE, HMAE and HRMSE) and additionally the percent of price overpredictions, the results confirmed our initial intuition that that BIV is the best model, BHV being the second best, and BRV – the least efficient of them. The division of our database into different classes of moneyness ratio and TTM enabled us to observe the distinct differences between compared pricing models. Additionally, focusing on the same pricing model with different volatility processes results in the conclusion that point-estimate, not averaged process of RV is the main reason of high errors and instability of valuation in high volatility environment. Finally, we have been able to detect "spurious outliers" and explain their effect and the reason for them owing to the multi-dimensional comparison of the pricing error statistics.

Keywords:

option pricing models, financial market volatility, high-frequency financial data, realized volatility, implied volatility, microstructure bias, emerging markets

JEL:

G14, G15, C61, C22

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1. Introduction

Option trading dates back to the seventeenth century, when options were part of (and one of the reasons for) the South Sea bubble and Amsterdam tulip mania. However, only the 1970s brought the rapid growth in the options market. First, two famous papers by Black and Scholes (1973) and Merton (1973) introduced a formula for valuing European options (the BSM model). 1973 is also the year when the Chicago Board of Options Exchange is founded – it means the beginning of trading on standardized listed options (CBOE adopted the Black-Scholes-Merton model for option pricing in 1975).

The rapid growth of option markets - due to the combination of (seemingly) reliable pricing formula and a good exchange mechanism - brought a lot of data and stimulated intensive development of option pricing research. Quite soon, empirical studies have shown rather clearly that some theoretical assumptions of the BSM model are not fully supported by these data (cf. Bates 2003) and that the BS formula exhibits substantial pricing biases across both moneyness and maturity (Bakshi et al. 1997). Recent decades have witnessed a great number of new models, each of them relaxing some of the restrictive assumptions of the BSM model (Broadie and Detemple 2004, Garcia et al. 2010, Han 2008, Mitra 2009). There is also a growing literature devoted to comparisons of their various features, though even the best metric for comparison is a controversial issue (Bams et al. 2009). On the other hand, the BSM model is still widely used not only as some kind of benchmark in comparative studies mentioned earlier, but also among financial practitioners. Christoffersen and Jacobs 2004 show that much of its appeal is related to the treatment of volatility – the only parameter of the BSM model that is however not directly observed.

Detailed analysis of literature (An and Suo 2009, Andersen et al. 2007, Bates 2003, Brandt and Wu 2002, Ferreira et al. 2005, Mixon 2009, Raj and Thurston 1998) seems to suggest that the BSM model with implied volatility[1] calculated on the basis of the last observation performs quite well even when compared with many different pricing models (standard BSM model, BSM with realized volatility, GARCH option pricing models or various stochastic volatility models). This fact leads us to the following hypothesis: the Black model with implied volatility gives the lowest pricing error as it includes the most recent observation when estimating volatility.

Of course, there is more than one way of measuring and estimating volatility and a number of other studies (Ammann et al. 2010, Berkowitz 2010, Martens and Zein 2004) suggest plausibly that it makes a difference what kind of volatility – historical, implied or realized – has been applied. That observation has been one of major factors defining the scope of this study. Thus, in order to verify our initial hypothesis we have to consider a few additional questions:

- what kind of volatility process should be used in the Black model?
- what length of time period (parameter n – responsible for the memory of the process) should be used for averaging volatility in the estimation?
- what is the optimal interval (delta) for estimating volatility?
- do errors depend on option's time to maturity (TTM) and moneyness ratio (mr)?

The general objective of the research project this paper describes is thus to join the search for the best option pricing model in the sense of its outcomes being close to market prices[2]. We begin with the Warsaw Stock Exchange (WSE) as the Polish market is what we know best and what is more important, it is often used by foreign investors to reflect their opinion about Central-Eastern European markets[3]. However, it means that we face several barriers typical for emerging markets

[1] We use the Black model instead of the BS model for reasons briefly described in section 3.

[2] We assume that market prices are reflected by mid quotes (calculated on the basis of bid and ask quotes) and explain this later in this paper.

[3] The reason for this is its relative liquidity in comparison to the neighbouring markets. It was clearly seen during 2008 financial crisis on USDPLN and EURPLN markets.

(low liquidity, nonsynchronous trading etc.). They are the reason for using high frequency (HF) data (10-second data interval, based on tick data) for WIG20 index option quotes (bid and ask) in order to increase observed liquidity of the market and to remove nonsynchronous bias[4].

The choice of the market we study here has also another reason - the very limited knowledge we have on the option pricing in the Polish capital market. Most studies we know of exist only in Polish or in the form of unpublished papers, what makes them practically inaccessible to a wider audience. Moreover, they usually limit themselves to GARCH option pricing models (Osiewalski and Pipień 2003). This paper is intended to close, at least partially, this gap as the only text covering similar issues we study here is Fiszeder 2008, albeit he works only with daily data.

The remaining sections of this paper are organized as follows. Section 2 introduces option pricing methodology with special focus on various volatility measures and their estimators. Section 3 provides detailed description of data we use and of the volatility processes we study. HF data bring a significant number of specific technical issues that constrain to some extent the whole research – section 4 begins with some comments covering these issues before presenting the results of our study in detail, and section 5 presents implications these results have for other financial models and for further research on option pricing and concludes.

2. Option pricing methodology

The literature mentioned above allows us to assume that the type of volatility process included in the option pricing model is the most important issue when searching for the best model. Therefore we decided to base our research on the standard BSM model for futures pricing, ie. the Black model (it is called further BHV – the Black model with historical volatility). The formulas for Black model (Black, 1976) are presented below:

$$c = e^{-rT}[FN(d_1) - KN(d_2)] \tag{1}$$

$$p = e^{-rT}[KN(-d_2) - FN(-d_1)] \tag{2}$$

$$where:$$

$$d_1 = \frac{\ln(F/K) + \sigma^2 T/2}{\sigma\sqrt{T}} \tag{3}$$

$$d_1 = \frac{\ln(F/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{4}$$

where *c* and *p* are respectively valuation of a call and a put option, *T* is the expiration date, *r* is the risk-free rate, *F* – the futures price, *K* – underlying strike, and *N(.)* is the cumulative standard normal distribution.

We assume that we can price a European style option on WIG20 index applying the Black model for futures contract, where WIG20 index futures contract is the basis instrument. This is possible because of two reasons:

- WIG20 index futures mature exactly the same day as WIG20 index options, and the expiration prices are set exactly in the same way,
- WIG20 index options are European-style options, so there is no need to worry about early expiration as there is in the case of American options[5].

Our use of the Black model instead of the BSM model has been motivated by two following facts:

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[4] The WIG20 is the index of twenty largest companies on the Warsaw Stock Exchange (further detailed information may be found at www.gpw.pl).

[5] Early expiration of American-style option could result in the significant error in the case of such pricing, because of the difference in prices of index futures and of WIG20 index before the expiration date (the basis risk).

- We have to calculate the dividend ratio for the index, which might influence the final risk of our estimation – the Black model simplifies the process,
- We can use the data from the period between 9.00 and 9.30 a.m. each day, though index quotation starts only at 9.30 a.m., which gives us longer trading day.

We check the properties of the Black model with three different types of volatility estimators: historical volatility, realized volatility and implied volatility. Formulas for all three estimators are presented below.

Historical volatility (HV) estimator (standard deviation for log returns based on the daily interval) is directly derived from

$$VAR_{\Delta}^{n} = \frac{1}{(N_{\Delta} * n) - 1} \sum_{t=1}^{n} \sum_{i=1}^{N_{\Delta}} (r_{i,t} - \bar{r})^2 \tag{5}$$

where:

VAR_{Δ}^{n} – variance of log returns calculated on high frequency data on the basis of last *n* days,

$r_{i,t}$ - log return for *i*-th interval[6] on day t with sampling frequency equal to Δ, which is calculated in the following way:

$$r_{i,t} = \log C_{i,t} - \log C_{i-1,t} \tag{6}$$

$C_{i,t}$ – close price for *i*-th interval on day *t* with sampling frequency equal Δ,

N_{Δ} – number of Δ intervals during the stock market session,

n – memory of the process measured in days, used in the calculation of respective estimators and average measures.

$\bar{r}$ – average log return for *i*-th interval on the basis of last *n* days with sampling frequency Δ, which is calculated in the following way:

$$\bar{r} = \frac{1}{N_{\Delta} * n} \sum_{t=1}^{n} \sum_{i=1}^{N_{\Delta}} r_{i,t} \tag{7}$$

Realized volatility (RV) estimator is based on the following formula:

$$RV_{\Delta,t} = \sum_{i=1}^{N_{\Delta}} r_{i,t}^{\ 2} \tag{8}$$

Implied volatility (IV) estimator is based on the most recent observation (mid-quotes), therefore sigma has been derived from the Black formula with the assumption that other parameters and the valuation results are given. We calculate the implied volatility for the previous observation separately for each TTM and moneyness classes (i.e. for 50 different classes).[7] The details of our option classification are presented in Section 4. This estimator is then treated as an input variable for volatility parameter in calculation of the theoretical value for the Black model with implied volatility (BIV) for the next observation.

In the next step, historical volatility is annualized and transformed into standard deviation because this is the parameter used in the Black model[8]:

$$HV = {}^{annual\_std}SD_{\Delta}^{n} = \sqrt{252 * N_{\Delta} * VAR_{\Delta}^{n}} \tag{9}$$

Contrary to historical volatility which is based on the information from many periods (n>1), realized volatility estimator requires information only from the single period (interval Δ).

Therefore, the procedure of averaging and annualizing realized volatility estimator is slightly different from that presented in formula (9)[9]:

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[6] In the case of historical volatility estimator i=1 and $N_{\Delta}=1$ for every $r_{i,t}$ (daily log returns) and $C_{i,t}$ in formulas (5), (6) and (7). Moreover, we use constant value of parameter n=21, because we want to reflect the historical volatility from the last trading month.

[7] The concept of division into TTM and moneyness classes is presented in detail in the results section.

[8] We assume that we have 252 trading days in one calendar year.

$$ {}^{annual\_std}[RV]_{\Delta}^{n} = \sqrt{252}\sqrt{\frac{1}{n}\sum_{t=1}^{n}[RV]_{\Delta,t}} \qquad (10) $$

Having these volatility estimators we study several types of option pricing models:

- The Black model with historical volatility (sigma as standard deviation, n=21) – BHV,
- The Black model with realized volatility (realized volatility as an estimate of sigma; RV calculated on the basis of observations with different interval Δ and different parameter *n* in the process of averaging) – BRV,[10]
- The Black model with implied volatility (implied volatility as an estimate of sigma; IV calculated for the previous observation, separately for each TTM and moneyness classes - 50 different groups) – BIV.

Finally, we calculate the following error statistics for all these models in order to verify our research hypothesis.

- Root Mean Squared Error (RMSE):

$$ RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Black_i - MID_i)^2} \qquad (11) $$

where:

MID_i - means the market price (midquote in our research),
$Black_i$ - means the Black model price (BHV, BRV or BIV),

- Heteroscedastic Mean Absolute Error (HMAE):

$$ HMAE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{Black_i - MID_i}{MID_i}\right| \qquad (12) $$

- Heteroscedastic RMSE (HRMSE):

$$ HRMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{Black_i - MID_i}{MID_i}\right)^2} \qquad (13) $$

- Percentage of overprediction (OP):

$$ OP = \frac{1}{n}\sum_{i=1}^{n}OP_i, where: OP_i = \begin{cases} 1 & if \quad Black_i > MID_i \\ 0 & if \quad Black_i < MID_i \end{cases} \qquad (14) $$

3. The data and the description of volatility processes

3.1. Data

The empirical analysis is based on high-frequency financial data for WIG20 index options and WIG20 futures[11], supplied by Information Products Section of the WSE. These data cover the

[9] In this study realized volatility is calculated for Δ = 10s, 1m, 5m, 15m. However, the procedure of averaging has been done only for 5-minute interval and n days, where n=1, 2, 3, 5, 10, and 21. It is widely accepted in financial literature that interval between 5 to 15 minutes makes the consensus between the nonsynchronous bias and other microstructure biases.

[10] Initially, we calculated BRV model with different Δ: 10s, 1m, 5m, and 15m. We have checked the properties of average RVs with different values of parameter n in option pricing models. Therefore, we calculate BRV models based on 5m interval with different values of averaging parameter (n=1, 2, 3, 5, 10, and 21). As a result, we obtain the following seven BRV models: BRV5m, BRV5m_1, BRV5m_2, BRV5m_3, BRV5m_5, BRV5m_10, and BRV5m_21. In section 5 we present all these models in two set of comparisons:

1. BRV10s, BRV5m, BRV5m_5, BRV5m_21, BHV, and BIV – in order to choose the best model,
2. BRV5m, BRV5m_1, BRV5m_2, BRV5m_3, BRV5m_5, BRV5m_10, BRV5m_21 – in order to reveal the properties of averaging.

period from January 2, 2008 to June 20, 2008. Tick data have been aggregated - because of well-known statistical problems - to 10-second quotes.

The number of 10-second bid-ask quotes for a trading day depends on the trading hours for option and futures contracts. The trading takes place from 9:00 a.m. to 4:30 p.m. for the time period we study[12]. We take into account only those quotes for which we had both bid and ask quotes simultaneously, so we are able to calculate the mid quotes[13]. They are later treated as the market consensus of option investors and are used for comparison with theoretical prices obtained from the option pricing models. We do not make any corrections for outliers, because we want to show fully properties of models we test here even for options with low prices and short time to maturity (TTM), which are usually excluded from similar studies.

Additionally, we use WIBOR interest rate (converted into 10-second intervals) as the interest free rate in option pricing models and we calculate TTM in seconds.

As a result we obtain complete data for 128 index options (65 call and 63 put options expiring in March, June and September). Thus, we had 318 718 10-second observations (midquotes, Wibor rates, TTM and strike prices for each options) in our sample period (118 trading days with 2701 observations for each day). These data are then used in the process of calculation of volatility parameters (HV, RV and IV) and later on for theoretical option valuation (BHV, BRV and BIV model).

3.2. The descriptive statistics for WIG20 futures time series.

Table 3.1 summarizes the descriptive statistics for 10-second interval data for continuous futures contract (with and without the opening jump effect – described respectively as R_f and $R_{f'}$) in order to show the distribution for the basis instrument. It is shown in order to reference to the crucial assumption of option pricing models tested in this paper, e.g. the normality of returns[14]. The statistics we present below seem to confirm our belief that the distribution of HF data is not exactly normal.[15]

Table 3.1. The descriptive statistics for index futures returns (with and without opening jump effect).

	$\mathbf{R_f}$ [a]	$\mathbf{R_{f'}}$ [b]
N	318717	318482
Mean	-0.000000787	-0.000000624
Median	0	0
Std Deviation	0.0003985	0.0003358
Range	0.07847	0.02551
Minimum	-0.047473855	-0.010453057
Maximum	0.030991753	0.015059446
Kurtosis	1369.388606	79.746338
Skewness	-7.31722	1.781351

[11] The study is based on the separate time series for futures contracts (F_1 – the expiration date is March 21, 2009, F_2 - the expiration date is June 20, 2009, and F_3 – the expiration date is September 19, 2009) where the choice of specific futures contract depends on the expiration date (the same date as for the options).

[12] In practice, the continuous trading stops at 4:10 p.m., then the close price is set between 4:10 p.m. and 4:20 p.m., and next investors can trade until 4:30 p.m. only on the basis of close price.

[13] Mid quote = (bid+ask)/2.

[14] The continuous time series for futures contracts was created based on the notion that the expiring futures contract was replaced by the next series.

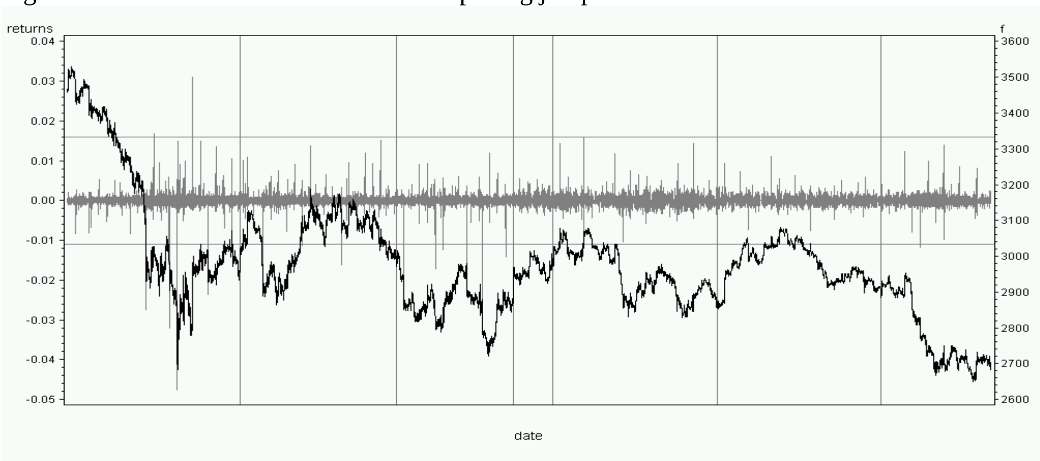
[15]Analyzing both return series we can see high kurtosis, enormous Jarque-Berra statistics and high negative (in case of R_f) or high positive (in case of $R_{f'}$) skewness. Mean returns are small and are not significantly different from zero. Distributions of both time series are leptokurtic, i.e. they have fat tails and a substantial peak at zero.

Kolmogorov-Smirnov	**Statistic**	0.3683	0.3684
	p-value	<0.01	<0.01
Jarque-Berra	**Statistic**	24 904 800 000	84 556 553.7
	p-value	<0.00001	<0.00001

[a] the original data. [b] the modified data, without opening jump effect.

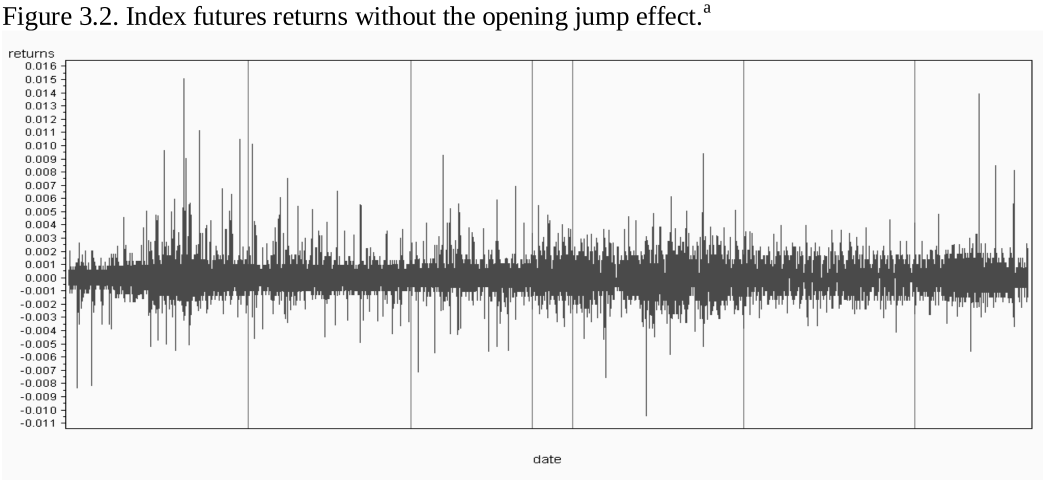
Figures 3.1 and 3.2 show that the opening jump effect is responsible for the large fraction of the departure from the normality.

Figure 3.1. Index futures returns with the opening jump effect. [a]



[a] The returns cover the data span between January 2, 2008 to June 19, 2008.

Figure 3.2. Index futures returns without the opening jump effect.[a]



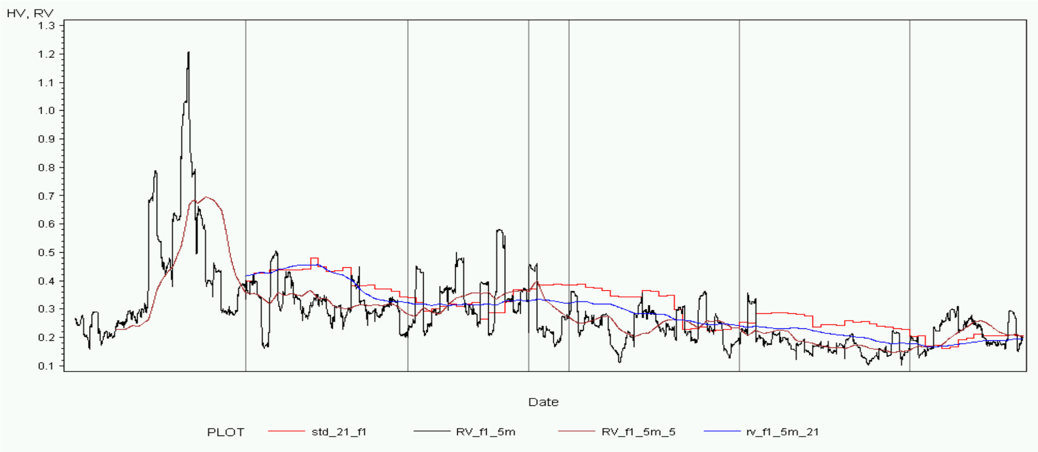
[a] The 10-second returns between the closing price from each day and the opening price from the next day have been excluded. The returns cover the data span from January 2, 2008 to June 19, 2008.

Formally, this non-normality means that we should not use the standard BSM model to price an option on such a basis instrument. Therefore, we have decided to transform the standard BSM model through the inclusion of non-standard volatility parameters.

3.3. The description of volatility processes

Finally, before we come to the main section of results, we present some properties of volatility parameters distributions (presented as Figures 3.3, 3.4 and 3.5). We believe that they are the main reason of differences between option pricing models we compare.

Figure 3.3. Historical and realized volatility (5m, 5m_5, 5m_21). [a]



[a] The volatility time series cover the data period between January 2, 2008 and June 19, 2008.

First of all, realized volatility time series which is not averaged (RV_f1_5m) exhibits substantial volatility of volatility (parameter kappa in stochastic volatility models), especially in comparison with averaged RV, which then strongly influences the results for BRV model. This feature of RV is responsible for the high errors of these models, especially in high volatility environment.

Table 3.2 additionally confirms the observation based on Figure 3.3 concerning the effect of averaging RV estimator on its efficiency (decreasing standard deviation and narrower range of fluctuations of RV while parametr *n* increases). On the other hand we can see that the mean value of RV is robust to the process of averaging, what informs us, that higher volatility for estimator with lower *n*, is responsible for rather symmetrical departure from the mean value.

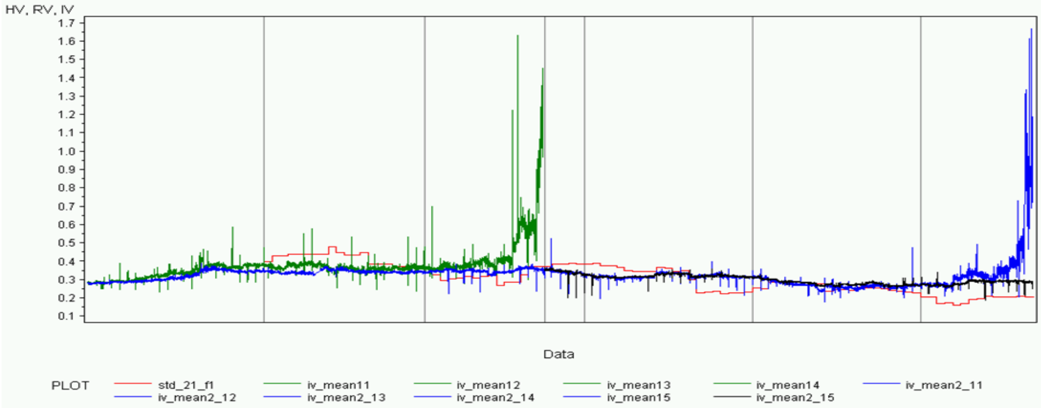
Table 3.2. The descriptive statistics for realized volatility estimators.[a]

	RV5m	RV5m_2	RV5m_5	RV5m_10	RV5m_21
N	316 017	310 615	302 512	289 007	259 296
Mean	0,286	0,287	0,289	0,291	0,286
Std Deviation	0,137	0,123	0,112	0,102	0,082
Range	1,107	0,778	0,550	0,428	0,290
Minimum	0,101	0,124	0,146	0,156	0,166
Maximum	1,208	0,902	0,695	0,584	0,457

[a] The different sample size is the result of different number of intervals which are necessary to compute the first value of averaged RV. The latter depends on parameter *n*.

Secondly, the process of averaging RV estimator drives it closely to the classical volatility estimator, especially in cases with the same value of parameter *n* (responsible for the memory of the process).

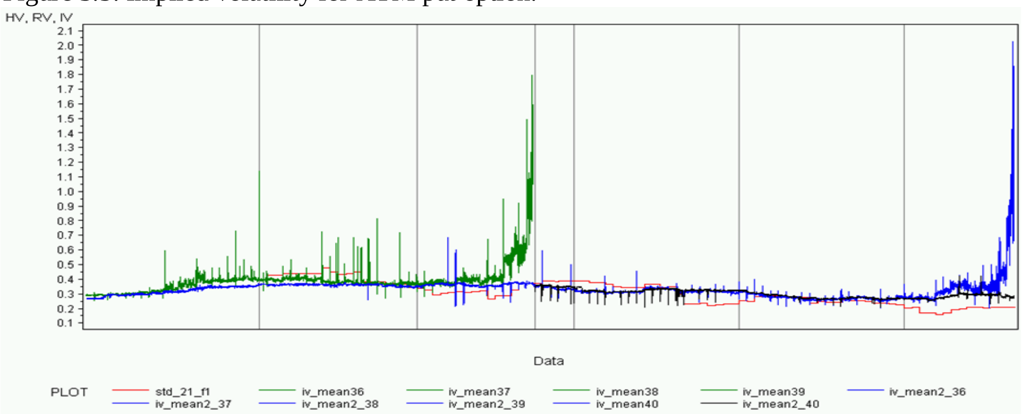
Figure 3.4. Implied volatility for ATM call option. [a]



[a] The volatility time series cover the data period between January 2, 2008 and June 19, 2008.

Thirdly, Figure 3.4 and Figure 3.5 reveal that implied volatility explodes for the very short TTM (less than 5 days) and very low-priced options (i.e. deep OTM), whether they are call or put options. Probably, this was the reason for excluding options with short TTM and market premium lower than 5 or 10 in most of research studies comparing different volatility models. However, we have decided to conduct our research on non-modified initial database in order to deeply investigate these properties of volatility estimators and additionally to answer the question what observations and when should we treate as outliers.

Figure 3.5. Implied volatility for ATM put option. [a]



[a] The volatility time series cover the data period between January 2, 2008 and June 19, 2008.

4. Results

4.1. Option classification

After calculating theoretical prices for each model we obtain more than 21 millions theoretical premiums. Basing on these premiums and the mid-quotes we calculate error statistics for 6 pricing models (BHV, BRV10s, BRV5m, BRV5m_5, BRV5m_21, and BIV)[16]. We order them according to:

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[16] First set of comparison was prepared for: BHV, BRV10s, BRV1m, BRV5m, BRV15m, and BIV) but the results were very similarto these presented in this paper, with BRV models as clearly the worst (the detailed results are available upon

- 2 types of options (call and put),
- 5 classes of moneyness ratio[17]*:* deep OTM (0 – 0.85), OTM (0.85 – 0.95), ATM (0.95 – 1.05), ITM (1.05 – 1.15) and deep ITM (> 1.15) for call options and in the opposite order for put options,
- 5 classes for time to maturity: (0-15 days], [16-30 days], [31-60 days], [61-90 days], [91+ days),

This classification allows for multidimensional comparison of pricing models we have used in this study. Table 4.1 and Figures 4.1 and 4.2 present the sample size of each class for BRV models for call and put options separately.

Table 4.1. Number of theoretical premiums for different classes of moneyness and TTM for BRV model.[a]

Option	moneyness ratio	0-15 days	16-30 days	31-60 days	61-90 days	91+ days	Total
CALL	**Deep OTM**	339 797	359 055	948 267	685 768	1 910 423	4 243 310
CALL	**OTM**	180 354	213 726	421 413	331 134	1063 066	2 209 693
CALL	**ATM**	171 757	169 101	337 286	261 326	898 348	1 837 818
CALL	**ITM**	127 573	131 510	267 731	201 209	703 620	1 431 643
CALL	**Deep ITM**	61 428	126 065	264 199	122 424	426 429	1 000 545
	Total CALL	**880 909**	**999 457**	**2 238 896**	**1 601 861**	**5 001 886**	**10 723 009**
PUT	**Deep OTM**	68 013	130 617	271855	128 425	537 867	1 136 777
PUT	**OTM**	134 076	133 480	275763	211 347	716 996	1 471 662
PUT	**ATM**	172 384	169 165	337752	260 727	898 810	1 838 838
PUT	**ITM**	197 227	209 591	404305	315 727	954 303	2 081 153
PUT	**Deep ITM**	521 267	322 806	812245	520 952	1 773 912	3 951 182
	Total PUT	1 092 967	965 659	2 101 920	1 437178	4 881 888	10 479 612
	Total CALL and PUT	**1 973 876**	**1 965 116**	**4 340 816**	**3 039 039**	**9 883 774**	**21 202 621**

[a] BHV model 17 million because the first value of HV we had for February 1, 2008; BIV model 21 million observations.

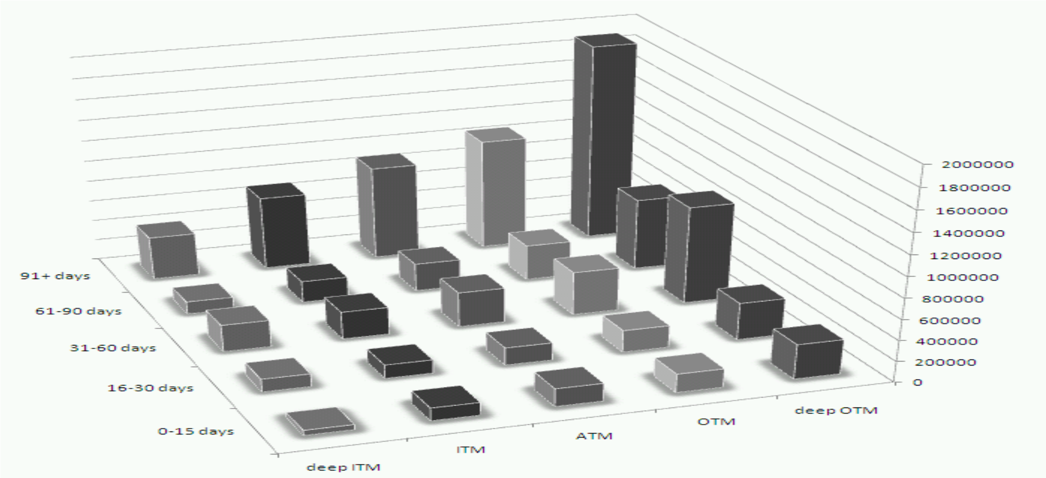
The numbers presented in the above table and figures below inform us that the activity of market participants, within the research period, was focused on deep ITM and ITM put options and deep OTM and OTM call options. However, it was not real emergin market characteristic of liquidity but only the result of a sharp downward movement of WIG20 prices in the period we study and of the procedure of introducing new strike prices by the stock exchange.

request). Then, we decided to additionally present the results for models with averaged value of RV estimator. Moreover, in the final part of the result section we present the comparison only for the BRV models with RV5m with different values for parameter *n*. They are presented to show properties of averaging the volatility parameter in the process of option pricing.

[17] Moneyness ratio is calculated according to the following formula, which was only adjusted for the use of futures contract as the basis instrument:

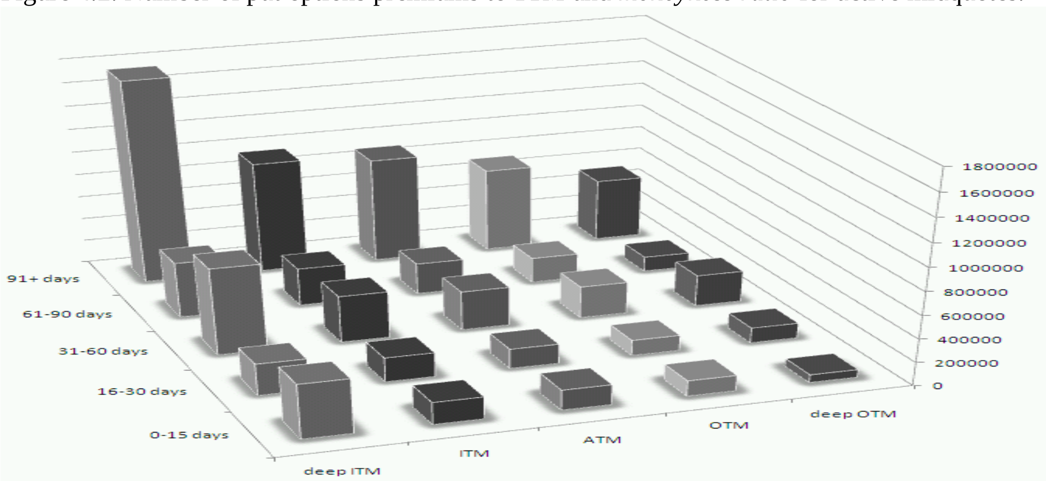
$$\text{moneyness ratio} = \frac{S}{K / e^{rT}} = \frac{F}{K} \qquad (15)$$

Figure 4.1. Number of call options premiums to *TTM* and *moneyness ratio* for active midquotes. [a]



[a] active mid-quotes mean options that were quoted in the sample period.

Figure 4.2. Number of put options premiums to *TTM* and *moneyness ratio* for active midquotes. [a]

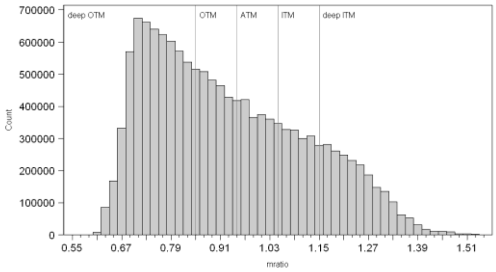


[a] active mid-quotes mean options that were quoted in the sample period.

Additional information about available strike prices and active mid quotes is presented in Figures 4.3, 4.4, 4.5, and 4.6, separately for call and put options[18]. These histograms confirm observation revealed by previous figures that we had a great number of ITM and deep ITM put options and OTM and deep OTM call option available on WSE in the sample period. However, only part of the available strike prices were quoted by the market participants. That is an additional confirmation that the Polish equity option market is not fully mature in terms of liquidity.

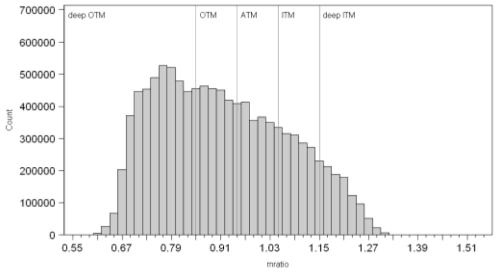
[18] Taking into account that this research is based on bid-ask quotes, instead of transactional prices, we wanted to have some reference to liquidity through the presentation of the fraction of quoted options. Available strike prices mean the span of strike prices which were available to trade for market participants, whether they were quoted or not. Active mid-quotes stand for options with bid-ask quotes that were actually quoted.

Figure 4.3. *Moneyness ratio* histogram for call options and available strike prices.[a]



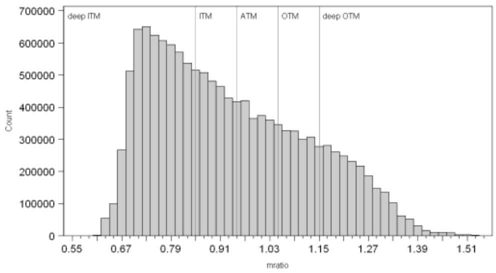
[a] available strike prices mean options which were introduced by the WSE on the market.

Figure 4.4. *Moneyness ratio* histogram for call options and active mid-quotes. [a]



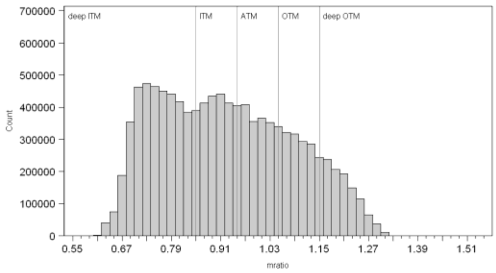
[a] active mid-quotes mean options which were quoted in the sample period.

Figure 4.5. *Moneyness ratio* histogram for put options and available strike prices.



[a] available strike prices mean options which were introduced by the WSE on the market.

Figure 4.6. *Moneyness ratio* histogram for put options and active mid-quotes.



[a] active mid-quotes mean options which were quoted in the sample period.

4.2. Technical and statistical issues when dealing with HF data.

Before we go to the main section of results we want to describe briefly the main problems and obstacles we encounter in the process of this research. The first problem we face is how to present our results. Taking into account that we want to show detailed results (4 error statistics) for six models (call and put separately) divided into 5 class of TTM and 5 class of moneyness ratio we obtain 1200 values of error statistics. Presenting them in a table or a number of tables does not seem practical. Therefore, we decided to use 3-D figures with boxes scaled with global and local minima and maxima in order to show in a transparent manner differences among our models along various dimensions. The detailed description of the way of the presentation is described in the results section.

The comparison of option pricing models is based on three types of error statistics: relative (HMAE and HMRSE), absolute (RMSE), and additionally OP. However, we believe that RMSE (the type of absolute statistics), that is most often used in that kind of research, is not appropriate in some situations. It can lead to wrong conclusions (Figure 4.8 or 4.12), especially when we try to find some patterns comparing models in the same TTM or moneyness ratio class. Relative statistics are much better suited for evaluation in this case. We describe this phenomenon in detail in the next subsection.

Analyzing results we have discovered several untypical observations. We investigate the cause thereof and we try to answer the question whether we could treat them as outliers. These observations happen for the following models and error statistics:

- HMAE and HRMSE, Call, TTM=3 and mr=1 for BRV10s, BRV5m, BRV5m_5;
- HMAE and HRMSE, Call, TTM=1 and mr=1 for BIV;
- RMSE, Put, TTM=1, 2, 3 and mr=5 for all models;

- HMAE and HRMSE, Put, TTM=1 mr=2,3 for BIV models;

4.3. Results

Before we present our results, some comments about the Black model with realized volatility are in order. Actually, we have considered several different BRV models. They were tested using different values of the Δ parameter: 10 seconds, 1 minute, 5 minutes and 15 minutes. We also examined a number of averaging parameters: 1 day, 2 days, 3 days, 5 days, 10 days and 21 days. Eventually, we have decided to include in our comparison the BRV models with the Δ parameters only of 10 seconds and 5 minutes and additionally, for the latter one the averaging parameter of 5 days and 21 days. Therefore, when we investigate the impact of averaging parameter we focus on the realized volatility computed in the interval of 5 minutes that is averaged across different periods.

The very great number of values of 4 error statistics we use force us to present those results in a smart way in order to spot any patterns emerging from them. Thus, we present our results in three different ways.

In the first approach (Figures 4.7 - 4.14) we present values of 4 error statistics: OP, RMSE, HMAE, HRMSE. They have been separately calculated for six different models. Each figure contains five boxes presented for five moneyness classes. Vertical axes in each box show values of a given statistic for six models (first horizontal axis) and five TTM classes (second horizontal axis). The models are always presented in the same order: 1. BRV10s, 2. BRV5m, 3. BRV5m_5, 4. BRV5m_21, 5. BHV, 6. BIV. Within each figure, each box has the same scale on vertical axis. Aditionally, local minima are marked with blue color and maxima with red color.

The second approach is to look at the pricing errors in a somewhat different way. Figure 4.15 presents HMAE statistics for PUT options with respect to time to maturity and moneyness ratio for six models separately. This allows us to identify the effect of TTM and mr on pricing error and how this effect differs for different models.

The third way is to compare the BRV models only. Figures 4.16 - 4.17 present RMSE statistics for call options with respect to given model and TTM or mr. Such an approach makes it easier to observe the effect of averaging realized volatility across different time horizons and the way it affects pricing errors.

Figure 4.7. **OP** statistics for **call** options, all pricing models, TTM and mr

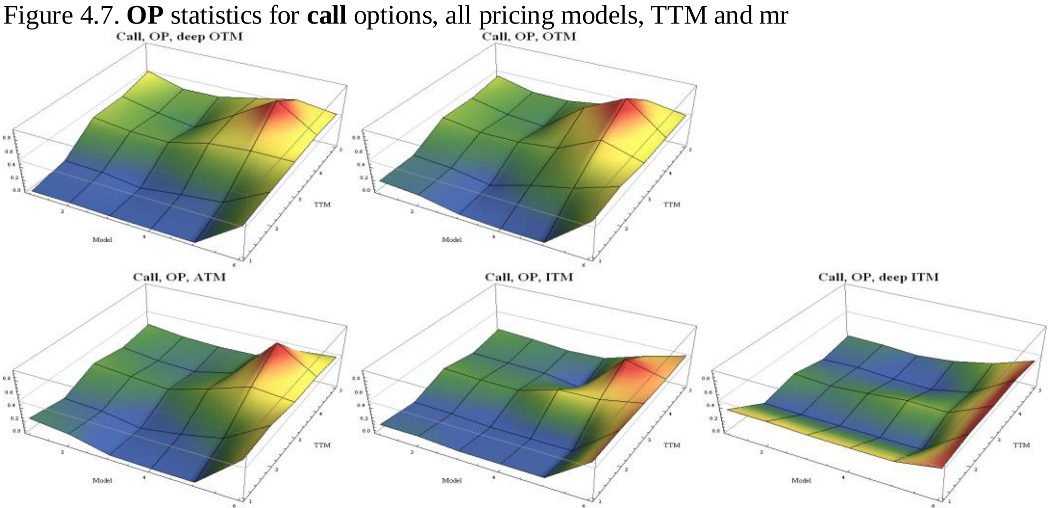
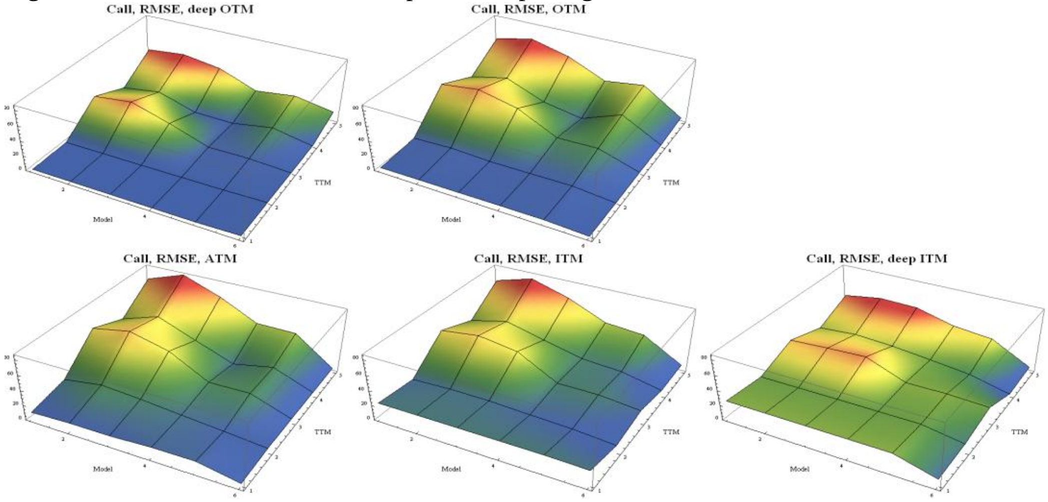


Figure 4.7 presents values of the overprediction ratio for call options. We can spot no significant differences for models with realized volatility: BRV10s, BRV5m, BRV5m_5. The

most expected value of OP (approx. 0.5, the same fraction of over- and underprediction) has the BIV model. BHV model and BRV5m_21 model are slightly worse. Most models show underpredicted premia for time to maturity between 0-15 days and 16-30 days. Exceptions are the BIV model and all models for deep ITM class.

Figure 4.8. **RMSE** statistics for call options, all pricing models, TTM and mr



RMSE statistics for call options presented on Figure 4.8 confirm lack of significant differences for models with realized volatility, e.g. BRV10s, BRV5m and BRV5m_5. Figures indicate lowest values of RMSE for the BIV model and slightly higher for the BHV and the BRV5m_21 models. Moreover, we observe that the pricing error increases with time to maturity. However, this is not confirmed later on by HMAE and HRMSE statistics. Thus, such an effect may arise only due to higher option values with relatively long time to maturity. Hence, we argue that the RMSE statistics used for comparing pricing error for options with different TTM is a misleading metric.

Figure 4.9. **HMAE** statistics for **call** options, all pricing models, TTM and mr

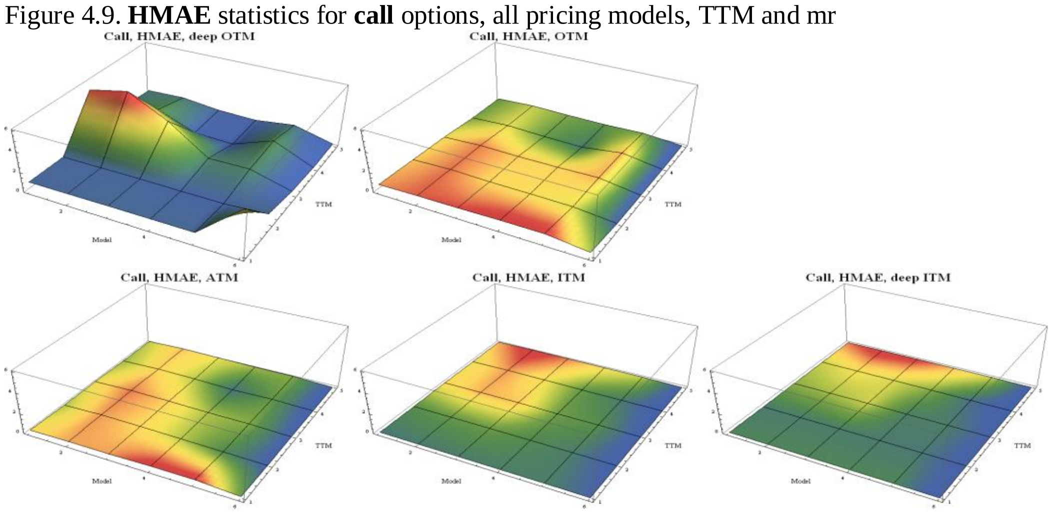


Figure 4.9 contains values for the first relative statistic, HMAE. Again, no significant differences for models with realized volatility (BRV10s, BRV5m, BRV5m_5) can be found. We observe best results for the BIV model, and then for the BHV and the BRV5m_21 models.

Very high HMAE values for the BRV models for deep OTM class seem to suggest the existence of two kinds of outliers in initial data. We intentionally left them at this stage although they distort other results[19]. These outliers are the outcome of point estimates for volatility. That is why we call them „spurious outliers" as their cause is the specific nature of the BRV model and not the data values themselves.

The first kind of "spurious effect" we observe for the BRV10s, BRV5m, BRV5m_5 models and for time to maturity between 31 and 60 days. We notice that for these classes, average pricing error values were up to 600%. The main reason for this error is that the BRV model is based on sigma parameters (RV estimator) computed for the previous day (only one day – not averaged). During the periods of high market volatility differences between values of sigma parameter for two consecutive days are up to 50% and occasionally even higher. Enormous mispricings emerging as a result (even up to 40 000% in sepcific cases) what obviously influences the average value of pricing error. One can ask why so high errors appear when time to maturity is between 31-60 days and only for deep OTM options. The reason is that during high market volatility periods (in mid January, 2008) options that mature in March were actually classified with TTM=3 status. Secondly, deep OTM options have highest value of the relative Vega parameter and hence their price is very sensitive with respect to changes of volatility of the futures prices of the market index.

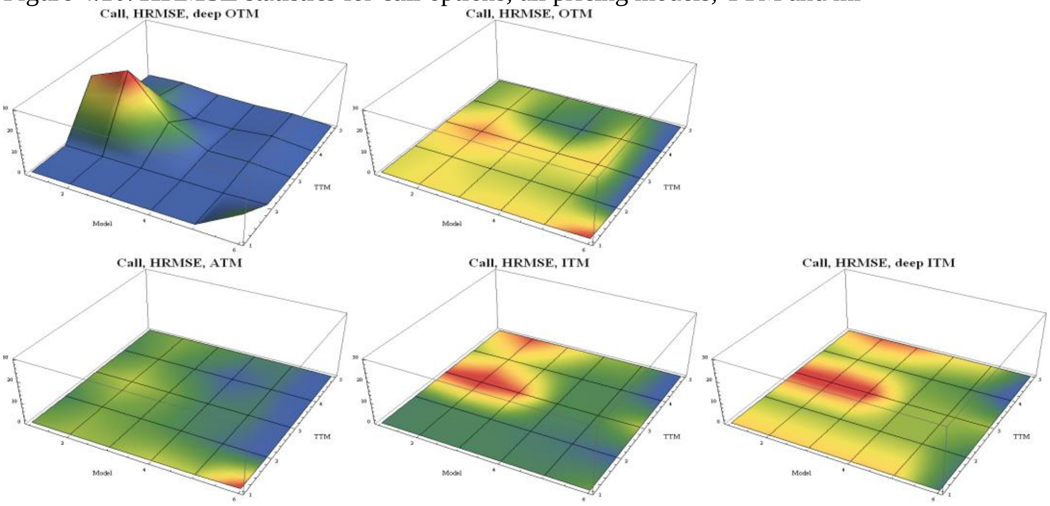
The second kind of „spurious effect" we can observe for the BIV model and for time to maturity between 0 and 15 days. However, this effect is not visible for RMSE statistic. Hence, it concerns only low-priced options. Secondly, it is also not visible for the OP statistics. That means that the fraction of price underpredictions equals the fraction of price overpredictions. The pricing error is due to very high differences between the BIV model valuation and mid-quote when the former is lower than the latter. Such a situation appears mostly when the market-maker withdraws his bid offers. The new mid-quote is then calculated on the basis of an old ask offer and a new bid offer (mostly significantly lower). As a result, mid-quote often changes by dozens of percent points.

However, there is also an alternative explanation for the second kind of "spurious effect", even more appropriate when we discuss the properties of BIV model. It may be partly explained by the characteristic path of the implied volatility when time to maturity is less than 10 or 15 days. In such periods the implied volatility simply explodes. As a result, for the low-priced deep OTM options the pricing error related to the option price is so high that it could significantly alter an average value of HMAE or HRMSE statistics, even when the number of observations with extremely high volatility is relatively low.

Values of the second relative statistic, HRMSE, are presented in Figure 4.10. The results confirm conclusions derived from HMAE statistics. Fistly, we observe "spurious outliers". Secondly, there are no differences for BRV10s, BRV5m and BRV5m_5. And finally, again we can acknowledge the BIV model as the most efficient one.

[19] These outliers can additionally be the reason that the patterns of pricing presented for put options on Figure 4.15 are not revealed for call options.

Figure 4.10. **HRMSE** statistics for **call** options, all pricing models, TTM and mr



The next four figures present four statistics for the put options in the same order.

Figure 4.11. **OP** statistics for **put** options, all pricing models, TTM and moneyness classes. [a]

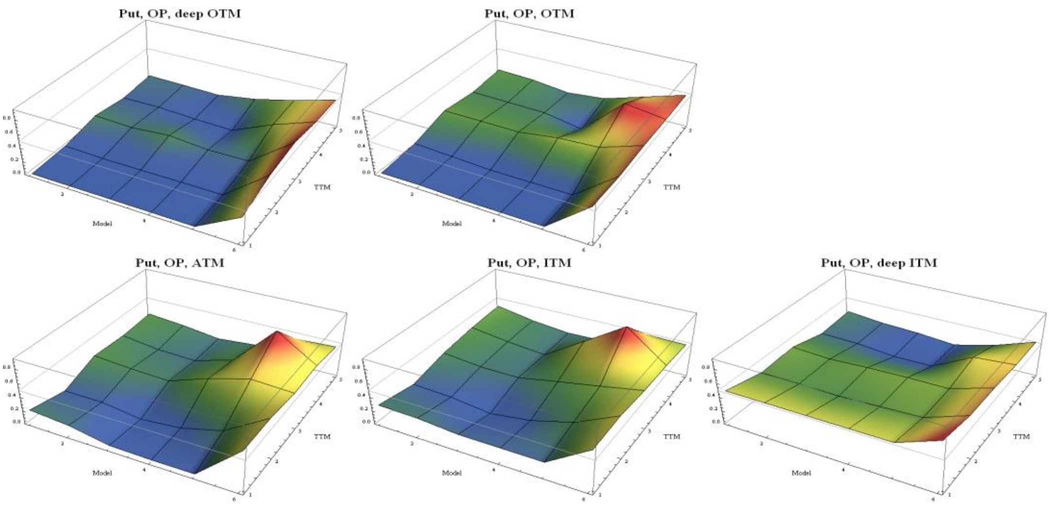
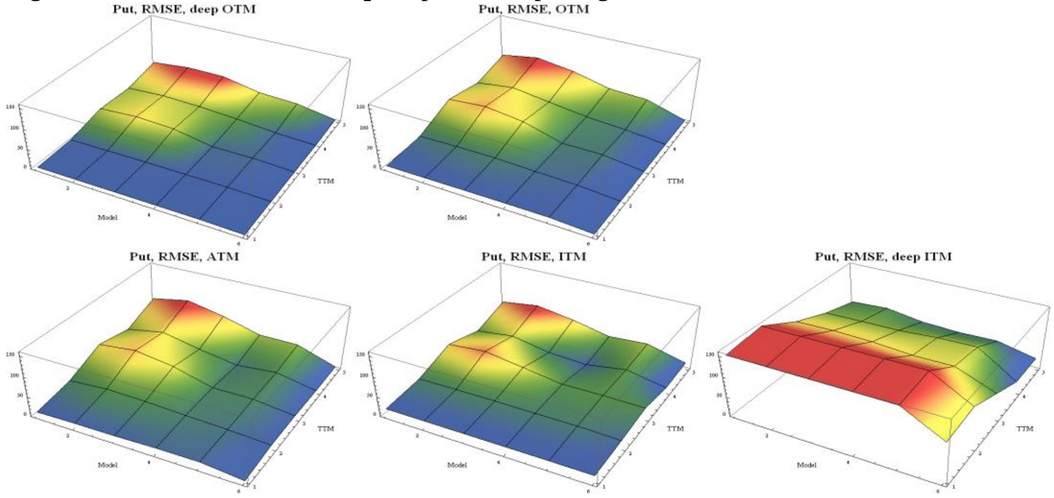


Figure 4.11 does not show any significant differences between the OP statistics for BRV models. Their theoretical premiums are on average underestimated when compared with the actual prices. Best results are observed for the BIV model, and then for the BHV and the BRV5m_21 models. On average, all models underestimate market prices (exception are options within the deep ITM class).

The reason for the high values of the OP statistics for the BHV model and time to maturity between 61 and 90 days is that prices of the BHV model are affected by the long-memory effect typical for the historical volatility estimator. Opposed to that, market participants adjust to the new market volaitlity levels much more rapidly, what is reflected in mid-quotes. This is actually related to one of our reasearch questions: what is the optimal level of *n* parameter, representing long-memory effect of the volatility process.

Figure 4.12. **RMSE** statistics for **put** options, all pricing models, TTM and mr



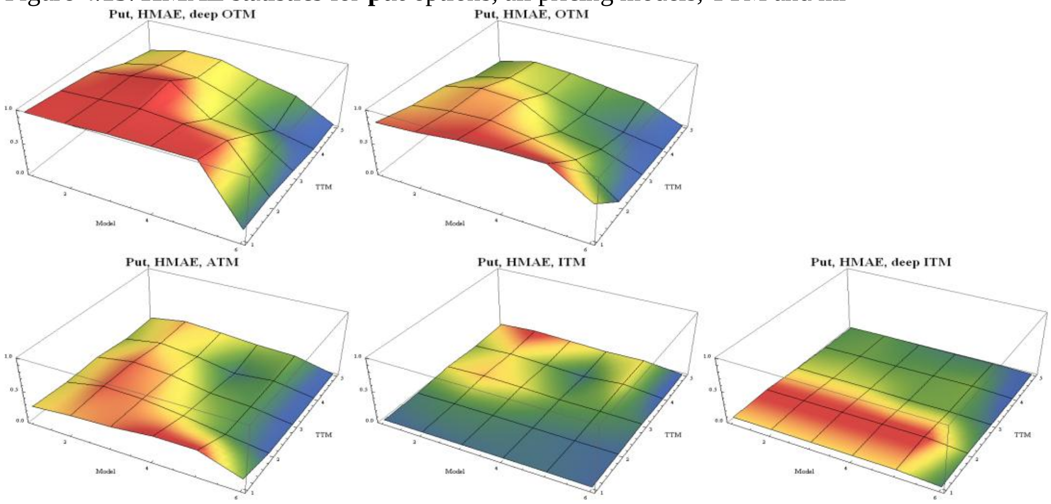
Analysing RMSE values on Figure 4.12 we again observe no significant differences between the BRV10s, then BRV5m and the BRV5m_5 models. The lowest values of RMSE statistics have been obtained for the BIV model, slightly higher for the BHV model and even more higher for the BRV5m_21 model. Similarly to the results for the call options, the pricing error seems to increase with higher time to maturity (exception are options within the deep ITM class). Again, the reason for that could be much more higher option prices when time to maturity is relatively long. For that reason, we argue that the far better way to compare pricing errors for options with different TTM are relative statistics, like HMAE or HRMSE.

Last "spurious effect" is revealed through very high RMSE values for the BRV and the BHV models for the deep ITM class with time to maturity between 0 and 15 days. Again, this may be due to possible "spurious outliers" present in the data. This effect, however, is not visible for the HMAE and HRMSE statistics which means that the effect concerns only highly priced options, where the pricing error related to the mid-quote is not as high as it is when computed in absolute values. Therefore, the possible reason for that is the situation when the market-maker withdraws his ask offers and hence the mid-quotes can suddenly increase by a high amount. This effect is present not only for the time to maturity between 0 and 30 days, but also for other TTM classes. On the other hand, this effect is visible only for the deep ITM options because they have highest prices and hence their pricing errors in absolute values have the greatest effect on the RMSE values.

Two relative statistics once again confirm previous findings that there are no significant differences between the BRV10s, the BRV5m and the BRV5m_5 models (Figure 4.13). Similarly to the previous findings the lowest values have been obtained for the BIV model, then for BHV and BRV5m_21 models.

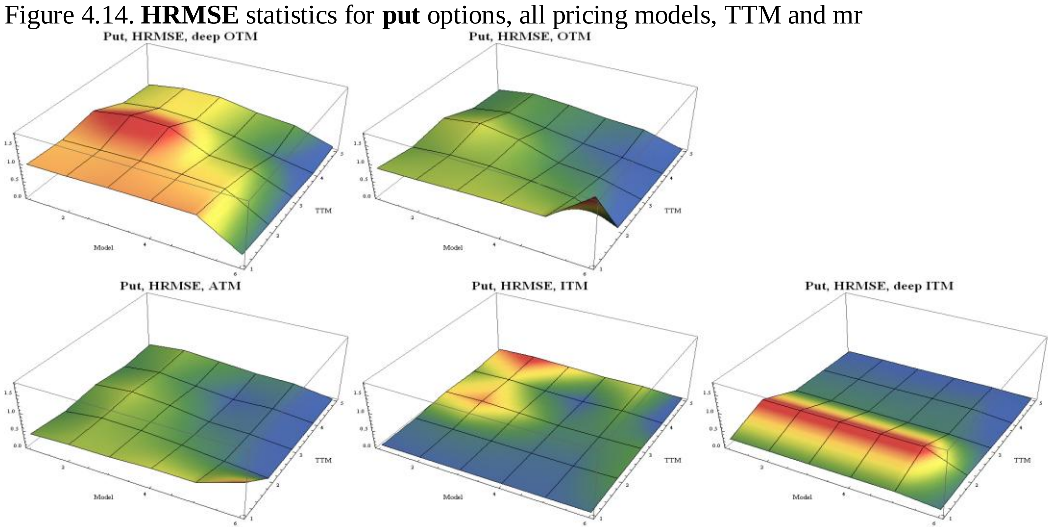
This time, we obtain very high values for the deep OTM and OTM classes with time to maturity between 0 and 60 days (exception are values for the BIV model). For these classes we observe low OP values and low RMSE values. This means that model pricings were significantly smaller when compared with market prices. On the other hand, for the ITM and the deep ITM classes we observe values close to zero.

Figure 4.13. **HMAE** statistics for **put** options, all pricing models, TTM and mr



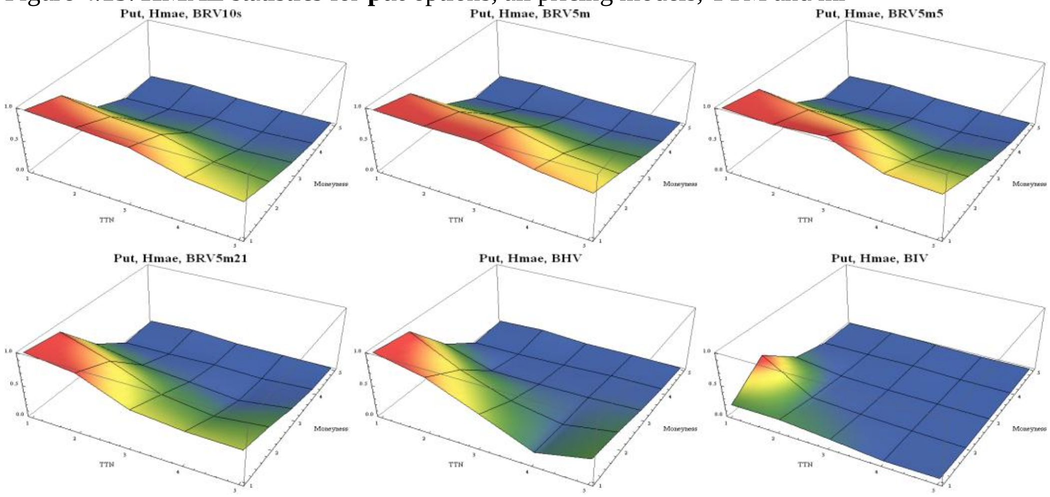
Analysing HRMSE statistics (Figure 4.14) we come to the similar conclusions as those we have from HMAE statistics. High values for the OTM/ATM options when time to maturity is less than 15 days for the BIV model could be explained in the same way as the first “spurious outlier” on the Figure 4.9.

Figure 4.14. **HRMSE** statistics for **put** options, all pricing models, TTM and mr



Additionaly, we analyse pricing errors looking at them along some other dimensions. Figure 4.15 presents HMAE statistics calculated for put options with respect to TTM and moneyness ratio, for six models separately. Here again we find the BIV model as the best one. The BHV model has slightly higher HMAE values. We can also see no significant differences among first three BRV models.

Figure 4.15. **HMAE** statistics for **put** options, all pricing models, TTM and mr



Moreover, there is striking pattern visible. The best model pricings are obtained for high TTM and moneyness ratio while the highest error values are calculated for low TTM and moneyness ratio classes.

Figure 4.16. **RMSE** for **call** options, BRV model, different averaging parameters and mr

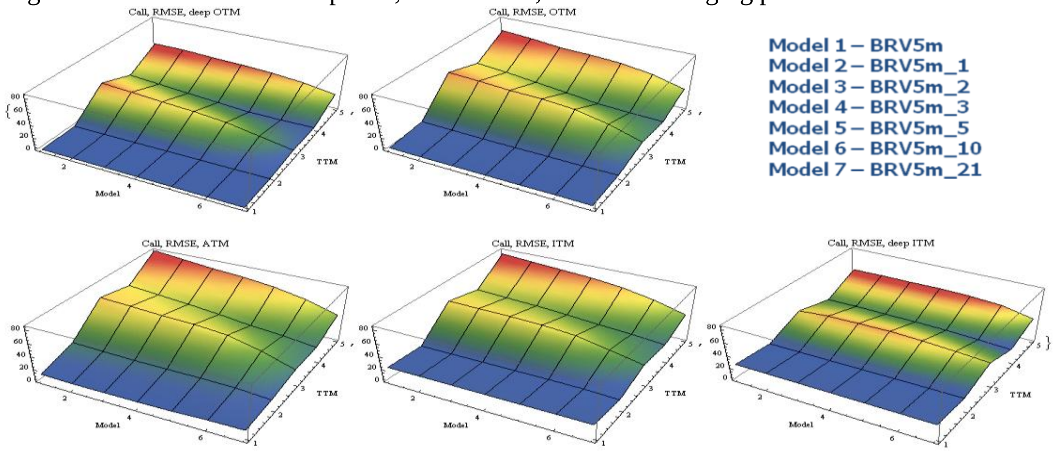


Figure 4.16 presents a comparison for different BRV models. They differ by the averaging parameter *n* and are presented in the following order: 1. BRV5m, 2. BRV5m_1, 3. BRV5m_2, 4. BRV5m_3, 5. BRV5m_5, 6. BRV5m_10, 7. BRV5m_21. We observe that the pricing error decreases as the averaging parameter increases. We get the smallest pricing error for the BRV5m_21 model, ie. BRV model with 5-minute realized volatility averaged across the last 21 days. On the other hand, error values increase with the higher time to maturity, but not in a stable way. Moreover, we can see that moneyness ratio does not influence model quality.

Figure 4.17. **RMSE** statistics for call options, BRV model, different averaging parameters and TTM

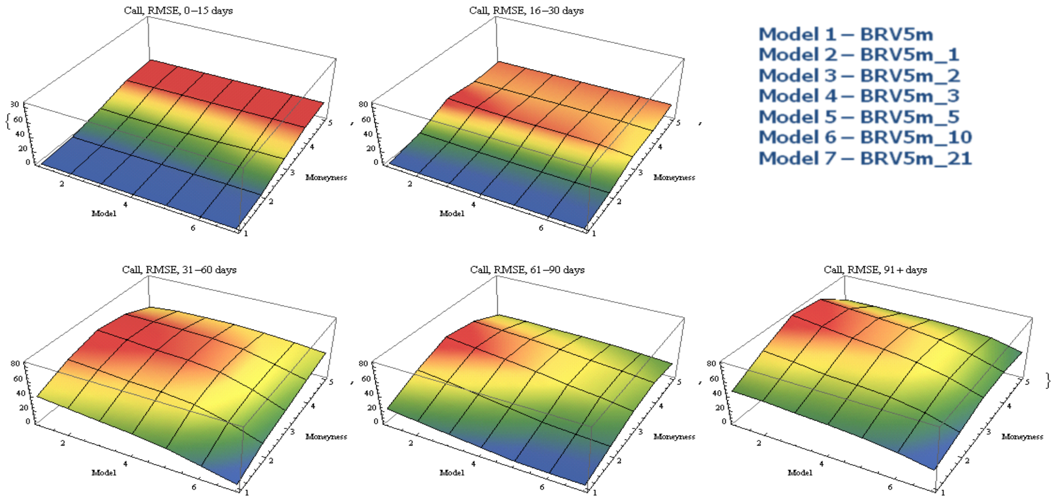


Figure 4.17 presents another comparison for the BRV models. Each box contains values of the RMSE statistics for call options calculated for different models and moneyness ratio classes, and separately for five TTM classes. Models are shown on the first vertical axis in the same order as in Figure 4.16. For time to maturity over 30 days pricing error values seem to decline as the averaging parameter increases. When time to maturity is shorter than 30 days we observe no differences among the BRV models.

To sum up, we can conclude that, on average, the smallest pricing errors have been obtained for the BIV model. It has been the best model in most of comparisons, regardless of the TTM and moneyness ratio class and the type of option. The second place belongs to the Black model with historical volatility and the third place to the Black model with realized volatility averaged across the period of 21 days. The pricing errors for the latter one were only slightly worse than for the BHV model. The highest pricing error values have been obtained for the Black model with non-averaged realized volatility. Additionally, we observe no significant differences between the Black models with non-averaged realized volatilities computed for 10-second and 5-minutes intervals. It can mean that there is no point in calculating the RV for higher-frequencies because the accuracy of volatility estimates can be offset by revealing microstructure biases.

We have also observed that the averaging parameter used in calculation of realized volatility has an important effect on pricing error. When realized volatility is averaged across the period of 21 days the error statistics are very similar to those obtained for the Black model with the historical volatility.

In addition, when we investigate values of error statistics with respect to different TTM and moneyness ratio class we can observe that the pricing error is much smaller when time to maturity is relatively long and the option belongs to the ITM and the deep ITM class.

We also detected some „spurious outliers”, i.e. observations with model valuations extraordinarily distant from other theoretical prices and market mid-quotes within a given TTM and moneyness ratio class. We do not exclude them, although, when present, they make the whole analysis much more difficult. Nevertheless, we have been able to detect them and explain their effect and the reason for thanks to the multi-dimensional comparison of the pricing error statistics.

5. Conclusions and further research.

Applying high-frequency (10-seconds) data for WIG20 index options we have verified the efficiency of different option pricing models. We applied various volatility processes (historical, realized, and implied) for the Black model in order to check our research hypothesis. Additionally, we calculated these models for different interval Δ and parameter *n* in order to discuss the influence of these parameters on final valuation and stability of volatility estimators. Moreover, we analyze results for 5 classes of moneyness ratio and TTM in order to reveal some patterns in valuation and explain the behavior of models we use for options with different time to maturity and whole span of strike prices. Finally, we discuss the possibility of and the reason for outlier exclusion.

Our findings support our initial hypothesis that the BIV model gives the best results, the BHV model is slightly worse, and BRV models give clearly the worst results (but the results for different concepts of BRV models significantly differ). These results are robust to changing TTM and moneyness ratio and are confirmed by four different types of error statistics. We believe that the reason for poor outcomes for the BRV model is the way how RV estimator is calculated (this estimator is characterised by very high volatility). RV can be described as point estimate in comparison with historical volatility which is rather range estimate, and we observe that this characteristic of RV is responsible for the worst results. Focusing on BRV models, we obtain best results for averaged models with the largest parameter *n* we test for (n=21). This value of *n* makes the result for the BRV5m_21 closer to the BHV model but does not clearly answer the question what is the best value for the parameter describing the process memory in volatility estimation. This issue requires further detailed studies.

Presenting results for different classes of TTM and moneyness ratio reveals some patterns of valuation in the case of put options. There is a clear relation between model error and TTM, and model error and moneyness ratio, which can be described briefly as follows: high error values for low TTM and moneyness ratio, and best fit for high TTM and moneyness ratio. The incidence of this pattern only for put options does not mean that call options are not characterized by similar behaviour. However, the possible existence of outliers made it impossible to reveal analogous pattern for call options.

Multidimensional presentation of raw data allows us to indicate some spurious outliers that actually are no true outliers at all. They result from the model misspecification (e.g. not appropriate volatility estimator) and can change the final evaluation of the specific model efficiency when excluded from further calculations. We provide the detailed explanation and the reason for them in the result section.

Results we present here and significant lack of papers testing various option pricing models for emerging CEE markets data suggest several enhancements which can be made. First of all, we should test other option pricing models like GARCH (based on the methodology presented in Duan (1995, 1999)) and SV models (Heston (1993), Hull and White (1987)). Secondly, there is a space for models with different assumptions of volatility distributions taking into account not only the rigorous definitions of parameters and delta interval in realized volatility (Slepaczuk and Zakrzewski (2009)) or the way of implied volatility estimator calculations but the latest results concerning high-frequency and model-free volatility indexes (based on VIX index methodology) as well. Thirdly, more effort should be put to outlier identification because as it is explained in detail in the previous section it is not clear whether we can exclude these observations for which TTM is lower than specific number of days (e.g. 5 or 10 days), mr is not limited by the fixed interval (e.g. 0.8 to 1.2), and the market premium is lower than some established value (e.g. 5, 10 or 15 pts), as it is assumed in many other research papers without any further investigations. Fourthly, the results for bid-ask quotes tested in this paper should be compared with the transactional prices. It will significantly decrease the number of observations but will enable us to verify the results for the real market behaviour not only the potential one. Time needed for calculations and disk space limitation made it impossible to conduct this study for the whole period of index options quotations on WSE (starting from September, 2003), but we are aware of the fact that option pricing models can behave differently in high or low volatility environment or for upward or downward market trends. This issue requires further investigations. Results could significantly differ for the markets with various degree of efficiency, so conducting the similar study for other markets in different countries (emerging and developed with different depth in terms of liquidity) should be the subject of further analyses. Moreover, results we have obtained in terms of the efficiency of implied volatility estimator in comparison with HV and RV could be additionally verified by simple econometric regression of future RV (as dependent variable) on HV, RV or IV (as explanatory variables). The last issue, which can be developed further, is the verification of statistical significance of our results which can be done in several ways:

- estimating econometric models where we try to explain the magnitude of error statistics (RMSE, OP, HMAE or HRMSE) with respect to TTM, mr, and dummy variables for each model separately,
- analysis of variance for each error statistics for call and put models separately, in order to additionally confirm the differences between models and patterns in valuation presented in figures and in tables in section 4 and in Appendix.

Nevertheless, our paper is one of the first to compare the option pricing models on high-frequency data for Eastern European emerging markets, we are aware of the fact that further research is needed.

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Appendix

The detailed tables for the outcomes presented in the results section (Figures 4.7 to 4.17).

Table A.1. **OP** statistics for **call** options, all pricing models, TTM and moneyness classes.

mr_status	ttm_status	BRV10s	BRV5m	BRV5m_5	BRV5m_21	BHV	BIV
DeepOtm	0-15 days	0,010	0,012	0,000	0,000	0,000	0,467
DeepOtm	16-30 days	0,087	0,075	0,001	0,083	0,151	0,498
DeepOtm	31-60 days	0,418	0,346	0,332	0,475	0,526	0,526
DeepOtm	61-90 days	0,425	0,297	0,212	0,553	0,916	0,550
DeepOtm	91+ days	0,530	0,367	0,372	0,489	0,606	0,521
Otm	0-15 days	0,170	0,135	0,026	0,000	0,000	0,535
Otm	16-30 days	0,177	0,143	0,048	0,227	0,368	0,578
Otm	31-60 days	0,414	0,330	0,336	0,403	0,688	0,512
Otm	61-90 days	0,396	0,272	0,223	0,669	0,947	0,511
Otm	91+ days	0,454	0,315	0,306	0,388	0,601	0,505
Atm	0-15 days	0,223	0,163	0,024	0,013	0,013	0,565
Atm	16-30 days	0,138	0,105	0,006	0,097	0,224	0,603
Atm	31-60 days	0,333	0,273	0,229	0,401	0,597	0,535
Atm	61-90 days	0,255	0,180	0,151	0,281	0,895	0,539
Atm	91+ days	0,323	0,248	0,220	0,227	0,447	0,489
Itm	0-15 days	0,123	0,125	0,086	0,086	0,085	0,535
Itm	16-30 days	0,075	0,061	0,024	0,026	0,028	0,539
Itm	31-60 days	0,249	0,222	0,214	0,390	0,405	0,517
Itm	61-90 days	0,160	0,141	0,128	0,030	0,650	0,500
Itm	91+ days	0,239	0,204	0,154	0,172	0,355	0,508
DeepItm	0-15 days	0,355	0,353	0,334	0,331	0,331	0,439
DeepItm	16-30 days	0,056	0,047	0,046	0,046	0,047	0,486
DeepItm	31-60 days	0,163	0,122	0,164	0,218	0,203	0,479
DeepItm	61-90 days	0,055	0,067	0,027	0,013	0,130	0,490
DeepItm	91+ days	0,129	0,112	0,072	0,122	0,235	0,429

Table A.2. **RMSE** statistics for call options, all pricing models, TTM and moneyness classes.

mr_status	ttm_status	BRV10s	BRV5m	BRV5m_5	BRV5m_21	BHV	BIV
DeepOtm	0-15 days	0,225	0,237	0,265	0,265	0,266	1,835
DeepOtm	16-30 days	0,890	0,890	0,704	0,775	0,884	1,755
DeepOtm	31-60 days	32,636	38,635	23,274	5,030	5,010	1,633
DeepOtm	61-90 days	13,950	21,118	5,345	3,898	12,377	1,835
DeepOtm	91+ days	41,063	45,836	38,297	18,696	24,799	15,776
Otm	0-15 days	2,476	2,837	2,978	3,330	3,401	0,949
Otm	16-30 days	6,991	8,782	6,268	6,242	6,574	1,184
Otm	31-60 days	49,415	55,560	43,885	18,314	19,316	2,353
Otm	61-90 days	30,157	41,068	20,406	10,964	30,710	3,166
Otm	91+ days	57,474	67,593	55,089	32,891	39,840	7,331
Atm	0-15 days	9,352	11,246	11,446	14,055	14,368	4,145
Atm	16-30 days	18,642	26,230	22,350	17,175	12,278	6,895
Atm	31-60 days	56,919	65,816	52,894	25,273	25,568	6,649
Atm	61-90 days	40,706	56,308	36,187	16,801	26,305	6,023
Atm	91+ days	70,160	86,339	67,424	42,899	41,831	5,922
Itm	0-15 days	22,157	22,295	22,444	22,600	22,633	21,491
Itm	16-30 days	18,770	20,540	20,218	17,507	15,434	10,821
Itm	31-60 days	54,009	60,203	52,370	24,095	23,599	18,075

Itm	61-90 days	38,515	49,627	36,652	24,308	20,371	8,972
Itm	91+ days	67,808	81,160	66,089	43,957	39,421	10,538
DeepItm	0-15 days	25,020	25,015	25,036	25,033	25,034	9,982
DeepItm	16-30 days	22,281	22,331	22,310	22,113	21,958	12,952
DeepItm	31-60 days	40,540	43,265	45,351	24,890	24,846	22,306
DeepItm	61-90 days	27,825	31,403	28,186	22,605	16,470	3,778
DeepItm	91+ days	46,658	53,971	52,613	39,324	34,125	7,505

Table A.3. **HMAE** statistics for **call** options, all pricing models, TTM and moneyness classes.

mr_status	**ttm_status**	**BRV10s**	**BRV5m**	**BRV5m_5**	**BRV5m_21**	**BHV**	**BIV**
DeepOtm	0-15 days	0,963	0,972	0,994	0,997	0,997	4,541
DeepOtm	16-30 days	0,929	0,922	0,921	0,868	0,834	1,087
DeepOtm	31-60 days	5,390	6,115	4,198	1,547	1,566	0,473
DeepOtm	61-90 days	1,552	2,216	0,757	0,566	1,944	0,245
DeepOtm	91+ days	1,280	1,374	0,938	0,800	1,242	0,251
Otm	0-15 days	0,716	0,776	0,801	0,871	0,883	0,448
Otm	16-30 days	0,649	0,753	0,629	0,591	0,526	0,126
Otm	31-60 days	0,596	0,740	0,646	0,630	0,680	0,061
Otm	61-90 days	0,389	0,536	0,344	0,228	0,682	0,055
Otm	91+ days	0,360	0,488	0,385	0,326	0,392	0,059
Atm	0-15 days	0,260	0,309	0,312	0,391	0,390	0,145
Atm	16-30 days	0,205	0,315	0,275	0,206	0,126	0,045
Atm	31-60 days	0,246	0,330	0,283	0,198	0,175	0,036
Atm	61-90 days	0,192	0,276	0,208	0,082	0,150	0,027
Atm	91+ days	0,213	0,294	0,238	0,191	0,175	0,021
Itm	0-15 days	0,038	0,040	0,043	0,045	0,045	0,026
Itm	16-30 days	0,057	0,064	0,065	0,054	0,043	0,015
Itm	31-60 days	0,108	0,125	0,120	0,067	0,058	0,015
Itm	61-90 days	0,100	0,131	0,105	0,064	0,041	0,012
Itm	91+ days	0,126	0,162	0,137	0,109	0,089	0,011
DeepItm	0-15 days	0,026	0,026	0,026	0,026	0,026	0,006
DeepItm	16-30 days	0,022	0,022	0,023	0,022	0,021	0,008
DeepItm	31-60 days	0,040	0,043	0,048	0,024	0,023	0,011
DeepItm	61-90 days	0,048	0,055	0,051	0,039	0,017	0,005
DeepItm	91+ days	0,072	0,086	0,081	0,065	0,048	0,005

Table A.4. **HRMSE** statistics for **call** options, all pricing models, TTM and moneyness classes.

mr_status	**ttm_status**	**BRV10s**	**BRV5m**	**BRV5m_5**	**BRV5m_21**	**BHV**	**BIV**
DeepOtm	0-15 days	0,973	0,979	0,995	0,997	0,997	12,600
DeepOtm	16-30 days	1,037	0,980	0,936	0,915	0,894	1,690
DeepOtm	31-60 days	23,569	30,290	10,812	2,107	2,327	0,636
DeepOtm	61-90 days	4,204	7,584	1,068	0,721	2,384	0,330
DeepOtm	91+ days	3,364	3,881	2,051	1,401	1,852	0,526
Otm	0-15 days	0,783	0,859	0,844	0,895	0,905	1,320
Otm	16-30 days	0,763	0,824	0,724	0,692	0,626	0,173
Otm	31-60 days	0,958	1,099	0,955	0,843	0,934	0,089
Otm	61-90 days	0,580	0,780	0,396	0,283	0,776	0,073
Otm	91+ days	0,554	0,674	0,549	0,426	0,493	0,092
Atm	0-15 days	0,389	0,440	0,426	0,496	0,491	0,905

Atm	16-30 days	0,252	0,381	0,335	0,264	0,176	0,079
Atm	31-60 days	0,397	0,477	0,384	0,222	0,224	0,057
Atm	61-90 days	0,265	0,366	0,232	0,110	0,183	0,041
Atm	91+ days	0,317	0,400	0,315	0,230	0,215	0,033
Itm	0-15 days	0,079	0,080	0,080	0,081	0,081	0,106
Itm	16-30 days	0,075	0,083	0,081	0,071	0,063	0,047
Itm	31-60 days	0,222	0,239	0,214	0,088	0,087	0,135
Itm	61-90 days	0,127	0,162	0,121	0,086	0,083	0,035
Itm	91+ days	0,183	0,220	0,178	0,125	0,112	0,027
DeepItm	0-15 days	0,090	0,090	0,090	0,090	0,090	0,029
DeepItm	16-30 days	0,060	0,060	0,060	0,060	0,060	0,034
DeepItm	31-60 days	0,118	0,123	0,119	0,060	0,060	0,063
DeepItm	61-90 days	0,059	0,066	0,060	0,050	0,042	0,008
DeepItm	91+ days	0,095	0,108	0,105	0,083	0,076	0,024

Table A.5. **OP** statistics for **put** options, all pricing models, TTM and moneyness classes.

mr_status	ttm_status	BRV10s	BRV5m	BRV5m_5	BRV5m_21	BHV	BIV
DeepOtm	0-15 days	0,000	0,000	0,000	0,000	0,000	0,352
DeepOtm	16-30 days	0,021	0,000	0,000	0,000	0,000	0,485
DeepOtm	31-60 days	0,125	0,094	0,135	0,080	0,083	0,534
DeepOtm	61-90 days	0,059	0,059	0,021	0,000	0,251	0,454
DeepOtm	91+ days	0,098	0,080	0,060	0,141	0,268	0,425
Otm	0-15 days	0,011	0,017	0,000	0,000	0,000	0,502
Otm	16-30 days	0,059	0,031	0,000	0,000	0,000	0,471
Otm	31-60 days	0,203	0,196	0,188	0,265	0,291	0,490
Otm	61-90 days	0,152	0,138	0,123	0,018	0,571	0,490
Otm	91+ days	0,194	0,167	0,098	0,149	0,323	0,498
Atm	0-15 days	0,199	0,149	0,040	0,026	0,026	0,570
Atm	16-30 days	0,090	0,086	0,002	0,052	0,202	0,571
Atm	31-60 days	0,268	0,222	0,192	0,395	0,548	0,513
Atm	61-90 days	0,245	0,169	0,143	0,307	0,883	0,533
Atm	91+ days	0,280	0,227	0,152	0,195	0,437	0,461
Itm	0-15 days	0,273	0,258	0,206	0,204	0,200	0,583
Itm	16-30 days	0,192	0,164	0,090	0,168	0,325	0,602
Itm	31-60 days	0,310	0,252	0,243	0,348	0,446	0,551
Itm	61-90 days	0,373	0,294	0,238	0,586	0,943	0,536
Itm	91+ days	0,404	0,304	0,244	0,254	0,508	0,492
DeepItm	0-15 days	0,482	0,482	0,481	0,481	0,481	0,708
DeepItm	16-30 days	0,369	0,365	0,352	0,375	0,395	0,607
DeepItm	31-60 days	0,401	0,371	0,396	0,407	0,409	0,589
DeepItm	61-90 days	0,281	0,264	0,211	0,213	0,504	0,544
DeepItm	91+ days	0,319	0,267	0,244	0,198	0,337	0,534

Table A.6. **RMSE** statistics for **put** options, all pricing models, TTM and moneyness classes.

mr_status	ttm_status	BRV10s	BRV5m	BRV5m_5	BRV5m_21	BHV	BIV
DeepOtm	0-15 days	2,255	2,255	2,533	2,537	2,545	0,533
DeepOtm	16-30 days	5,575	5,777	6,125	5,169	4,515	1,236
DeepOtm	31-60 days	25,787	31,167	25,833	7,247	7,157	1,764
DeepOtm	61-90 days	23,236	27,179	23,262	16,437	6,837	1,813

DeepOtm	91+ days	40,131	48,765	47,656	27,982	22,787	5,817
Otm	0-15 days	5,891	6,438	7,270	7,797	7,920	2,248
Otm	16-30 days	15,021	17,049	17,256	13,327	10,409	2,555
Otm	31-60 days	50,931	58,002	45,265	16,705	16,119	3,123
Otm	61-90 days	37,949	49,710	35,647	22,462	15,053	4,380
Otm	91+ days	60,683	75,418	62,158	38,952	34,108	4,284
Atm	0-15 days	9,254	11,437	12,136	14,919	15,181	4,297
Atm	16-30 days	23,572	30,140	27,551	22,248	17,954	11,841
Atm	31-60 days	57,285	67,016	52,025	24,494	24,200	11,362
Atm	61-90 days	40,920	56,913	36,456	19,804	27,793	7,763
Atm	91+ days	71,472	88,318	68,701	42,044	40,324	6,919
Itm	0-15 days	17,656	17,749	17,856	17,938	17,956	17,480
Itm	16-30 days	28,179	28,880	28,779	27,579	27,045	23,297
Itm	31-60 days	57,443	64,875	49,287	33,783	33,802	31,529
Itm	61-90 days	36,596	46,526	27,660	21,172	35,141	15,381
Itm	91+ days	62,553	74,478	58,631	37,997	42,881	17,366
DeepItm	0-15 days	150,608	150,608	150,608	150,608	150,608	99,021
DeepItm	16-30 days	160,558	160,560	160,560	160,562	160,563	107,063
DeepItm	31-60 days	116,317	117,999	114,850	107,377	107,356	68,656
DeepItm	61-90 days	84,750	86,141	86,587	92,159	93,739	31,371
DeepItm	91+ days	62,805	67,561	53,420	49,204	50,747	29,379

Table A.7. **HMAE** statistics for **put** options, all pricing models, TTM and moneyness classes.

mr_status	**ttm_status**	**BRV10s**	**BRV5m**	**BRV5m_5**	**BRV5m_21**	**BHV**	**BIV**
DeepOtm	0-15 days	0,957	0,960	0,991	0,994	0,995	0,179
DeepOtm	16-30 days	0,908	0,932	0,956	0,904	0,839	0,181
DeepOtm	31-60 days	0,850	0,932	1,009	0,586	0,519	0,118
DeepOtm	61-90 days	0,649	0,768	0,711	0,510	0,182	0,043
DeepOtm	91+ days	0,552	0,679	0,663	0,496	0,363	0,050
Otm	0-15 days	0,806	0,836	0,901	0,933	0,938	0,569
Otm	16-30 days	0,682	0,758	0,787	0,687	0,551	0,099
Otm	31-60 days	0,623	0,741	0,705	0,383	0,282	0,057
Otm	61-90 days	0,429	0,562	0,458	0,276	0,144	0,036
Otm	91+ days	0,367	0,492	0,447	0,361	0,277	0,023
Atm	0-15 days	0,248	0,295	0,317	0,377	0,376	0,156
Atm	16-30 days	0,222	0,331	0,310	0,231	0,133	0,044
Atm	31-60 days	0,244	0,330	0,283	0,165	0,126	0,027
Atm	61-90 days	0,187	0,272	0,201	0,084	0,133	0,024
Atm	91+ days	0,211	0,294	0,245	0,191	0,163	0,020
Itm	0-15 days	0,028	0,029	0,031	0,032	0,032	0,027
Itm	16-30 days	0,038	0,044	0,042	0,035	0,030	0,028
Itm	31-60 days	0,082	0,098	0,081	0,055	0,050	0,037
Itm	61-90 days	0,064	0,085	0,060	0,031	0,079	0,014
Itm	91+ days	0,098	0,127	0,100	0,076	0,082	0,014
DeepItm	0-15 days	0,071	0,071	0,071	0,071	0,071	0,031
DeepItm	16-30 days	0,096	0,096	0,096	0,096	0,096	0,043
DeepItm	31-60 days	0,058	0,059	0,057	0,045	0,045	0,024
DeepItm	61-90 days	0,041	0,044	0,041	0,042	0,046	0,013
DeepItm	91+ days	0,038	0,041	0,033	0,030	0,030	0,015

Table A.8. **HRMSE** statistics for **put** options, all pricing models, TTM and moneyness classes.

mr_status	ttm_status	BRV10s	BRV5m	BRV5m_5	BRV5m_21	BHV	BIV
DeepOtm	0-15 days	0,962	0,964	0,991	0,994	0,995	0,300
DeepOtm	16-30 days	0,922	0,942	0,958	0,912	0,854	0,214
DeepOtm	31-60 days	1,087	1,233	1,201	0,691	0,610	0,155
DeepOtm	61-90 days	0,693	0,805	0,741	0,532	0,269	0,055
DeepOtm	91+ days	0,650	0,782	0,781	0,581	0,473	0,117
Otm	0-15 days	0,847	0,870	0,914	0,941	0,945	1,834
Otm	16-30 days	0,717	0,804	0,810	0,730	0,608	0,119
Otm	31-60 days	0,873	1,009	0,827	0,471	0,373	0,078
Otm	61-90 days	0,528	0,677	0,497	0,319	0,207	0,051
Otm	91+ days	0,483	0,616	0,538	0,412	0,344	0,040
Atm	0-15 days	0,384	0,429	0,440	0,495	0,491	0,891
Atm	16-30 days	0,277	0,407	0,377	0,302	0,200	0,090
Atm	31-60 days	0,365	0,453	0,356	0,189	0,165	0,063
Atm	61-90 days	0,253	0,354	0,230	0,119	0,157	0,043
Atm	91+ days	0,315	0,401	0,317	0,228	0,203	0,036
Itm	0-15 days	0,057	0,058	0,058	0,059	0,059	0,065
Itm	16-30 days	0,067	0,071	0,069	0,064	0,063	0,088
Itm	31-60 days	0,145	0,167	0,124	0,077	0,077	0,090
Itm	61-90 days	0,103	0,132	0,077	0,054	0,096	0,041
Itm	91+ days	0,165	0,194	0,153	0,109	0,121	0,052
DeepItm	0-15 days	0,237	0,237	0,237	0,237	0,237	0,147
DeepItm	16-30 days	0,624	0,624	0,624	0,624	0,624	0,170
DeepItm	31-60 days	0,179	0,181	0,177	0,166	0,166	0,108
DeepItm	61-90 days	0,135	0,137	0,138	0,148	0,151	0,048
DeepItm	91+ days	0,092	0,095	0,082	0,085	0,089	0,051

Table A.9. **HMAE** statistics for **put** options, all pricing models, TTM and moneyness classes.

model	mr_status	0-15 days	16-30 days	31-60 days	61-90 days	91+ days
BRV10s	DeepOtm	0,957	0,908	0,850	0,649	0,552
BRV10s	Otm	0,806	0,682	0,623	0,429	0,367
BRV10s	Atm	0,248	0,222	0,244	0,187	0,211
BRV10s	Itm	0,028	0,038	0,082	0,064	0,098
BRV10s	DeepItm	0,071	0,096	0,058	0,041	0,038
BRV5m	DeepOtm	0,960	0,932	0,932	0,768	0,679
BRV5m	Otm	0,836	0,758	0,741	0,562	0,492
BRV5m	Atm	0,295	0,331	0,330	0,272	0,294
BRV5m	Itm	0,029	0,044	0,098	0,085	0,127
BRV5m	DeepItm	0,071	0,096	0,059	0,044	0,041
BRV5m5	DeepOtm	0,991	0,956	1,009	0,711	0,663
BRV5m5	Otm	0,901	0,787	0,705	0,458	0,447
BRV5m5	Atm	0,317	0,310	0,283	0,201	0,245
BRV5m5	Itm	0,031	0,042	0,081	0,060	0,100
BRV5m5	DeepItm	0,071	0,096	0,057	0,041	0,033
BRV5m21	DeepOtm	0,994	0,904	0,586	0,510	0,496
BRV5m21	Otm	0,933	0,687	0,383	0,276	0,361
BRV5m21	Atm	0,377	0,231	0,165	0,084	0,191
BRV5m21	Itm	0,032	0,035	0,055	0,031	0,076

BRV5m21	DeepItm	0,071	0,096	0,045	0,042	0,030
BHV	DeepOtm	0,995	0,839	0,519	0,182	0,363
BHV	Otm	0,938	0,551	0,282	0,144	0,277
BHV	Atm	0,376	0,133	0,126	0,133	0,163
BHV	Itm	0,032	0,030	0,050	0,079	0,082
BHV	DeepItm	0,071	0,096	0,045	0,046	0,030
BIV	DeepOtm	0,179	0,181	0,118	0,043	0,050
BIV	Otm	0,569	0,099	0,057	0,036	0,023
BIV	Atm	0,156	0,044	0,027	0,024	0,020
BIV	Itm	0,027	0,028	0,037	0,014	0,014
BIV	DeepItm	0,031	0,043	0,024	0,013	0,015

Table A.10. **RMSE** for **call** options, BRV model, different averaging parameters and moneyness ratio.

mr_status	ttm_status	BRV5m	BRV5m_1	BRV5m_2	BRV5m_3	BRV5m_5	BRV5m_10	BRV5m_21
DeepOtm	0-15 days	0,237	0,246	0,256	0,260	0,265	0,269	0,265
DeepOtm	16-30 days	0,890	0,831	0,753	0,716	0,704	0,671	0,775
DeepOtm	31-60 days	38,635	35,186	31,460	28,260	23,274	14,393	5,430
DeepOtm	61-90 days	21,118	16,998	12,893	8,967	5,106	4,448	4,532
DeepOtm	91+ days	45,836	45,412	42,786	40,951	37,855	33,114	25,174
Otm	0-15 days	2,837	2,506	2,706	2,822	2,978	3,409	3,330
Otm	16-30 days	8,782	7,808	6,874	6,488	6,268	5,989	6,242
Otm	31-60 days	55,560	53,102	49,711	48,075	43,885	33,429	18,628
Otm	61-90 days	41,068	36,654	31,128	25,569	19,296	16,575	14,432
Otm	91+ days	67,593	64,189	59,013	57,075	53,857	46,424	34,065
Atm	0-15 days	11,246	10,068	10,365	10,640	11,446	13,380	14,055
Atm	16-30 days	26,230	24,491	23,050	22,643	22,350	21,303	17,175
Atm	31-60 days	65,816	62,982	59,636	57,633	52,894	40,818	24,857
Atm	61-90 days	56,308	51,621	46,977	41,586	35,078	30,521	24,062
Atm	91+ days	86,339	81,170	76,020	72,140	66,348	57,040	43,002
Itm	0-15 days	22,295	22,302	22,363	22,376	22,444	22,623	22,600
Itm	16-30 days	20,540	20,468	20,336	20,311	20,218	19,612	17,507
Itm	31-60 days	60,203	59,337	58,257	56,502	52,370	42,402	32,107
Itm	61-90 days	49,627	46,744	44,388	40,786	36,464	32,905	28,186
Itm	91+ days	81,160	77,586	74,320	70,931	65,460	56,265	43,482
DeepItm	0-15 days	25,015	25,033	25,034	25,033	25,036	25,036	25,033
DeepItm	16-30 days	22,331	22,294	22,280	22,292	22,310	22,277	22,113
DeepItm	31-60 days	43,265	46,730	47,259	47,438	45,351	42,130	38,050
DeepItm	61-90 days	31,403	31,539	30,260	29,604	28,186	25,617	22,575
DeepItm	91+ days	53,971	54,312	53,895	54,040	52,613	49,234	39,015

Table A.11. **RMSE** statistics for call options, BRV model, different averaging parameters ***and*** *TTM.* [a]

ttm_status	mr_status	BRV5m	BRV5m_1	BRV5m_2	BRV5m_3	BRV5m_5	BRV5m_10	BRV5m_21
0-15 days	deep OTM	0,237	0,246	0,256	0,260	0,265	0,269	0,265
0-15 days	OTM	2,837	2,506	2,706	2,822	2,978	3,409	3,330
0-15 days	ATM	11,246	10,068	10,365	10,640	11,446	13,380	14,055
0-15 days	ITM	22,295	22,302	22,363	22,376	22,444	22,623	22,600

0-15 days	deep ITM	25,015	25,033	25,034	25,033	25,036	25,036	25,033
16-30 days	deep OTM	0,890	0,831	0,753	0,716	0,704	0,671	0,775
16-30 days	OTM	8,782	7,808	6,874	6,488	6,268	5,989	6,242
16-30 days	ATM	26,230	24,491	23,050	22,643	22,350	21,303	17,175
16-30 days	ITM	20,540	20,468	20,336	20,311	20,218	19,612	17,507
16-30 days	deep ITM	22,331	22,294	22,280	22,292	22,310	22,277	22,113
31-60 days	deep OTM	38,635	35,186	31,460	28,260	23,274	14,393	5,430
31-60 days	OTM	55,560	53,102	49,711	48,075	43,885	33,429	18,628
31-60 days	ATM	65,816	62,982	59,636	57,633	52,894	40,818	24,857
31-60 days	ITM	60,203	59,337	58,257	56,502	52,370	42,402	32,107
31-60 days	deep ITM	43,265	46,730	47,259	47,438	45,351	42,130	38,050
61-90 days	deep OTM	21,118	16,998	12,893	8,967	5,106	4,448	4,532
61-90 days	OTM	41,068	36,654	31,128	25,569	19,296	16,575	14,432
61-90 days	ATM	56,308	51,621	46,977	41,586	35,078	30,521	24,062
61-90 days	ITM	49,627	46,744	44,388	40,786	36,464	32,905	28,186
61-90 days	deep ITM	31,403	31,539	30,260	29,604	28,186	25,617	22,575
91+ days	deep OTM	45,836	45,412	42,786	40,951	37,855	33,114	25,174
91+ days	OTM	67,593	64,189	59,013	57,075	53,857	46,424	34,065
91+ days	ATM	86,339	81,170	76,020	72,140	66,348	57,040	43,002
91+ days	ITM	81,160	77,586	74,320	70,931	65,460	56,265	43,482
91+ days	deep ITM	53,971	54,312	53,895	54,040	52,613	49,234	39,015

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