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Emerging versus developed volatility indices. The comparison of VIW20 and VIX indices

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Abstract

Modeling of financial markets volatility is one of the most significant issues of contemporary finance, especially with regard to analyzing high-frequency data. Accurate quantification and forecast of volatility are of immense importance in risk management (VaR models, stress testing and worst-case scenario), models of capital market and options valuation techniques. What we show in this paper is the methodology for calculating volatility index for Polish capital market (VIW20 – index anticipating expected volatility of WIG20 index). The methods presented are based on VIX index (VIX White Paper, 2003) and enriched with necessary modifications corresponding to the character of Polish options market. Quoted on CBOE, VIX index is currently known as the best measure of capital investment risk perfectly illustrating the level of fear and emotions of market participants. The conception of volatility index is based on the combination of realized volatility and implied volatility which, using methodology of Derman et al. (1999) and reconstructing volatility surface, reflects both volatility smile as well as its term structure. The research is carried out using high-frequency data (i.e. tick data) for index options on WIG20 index for the period November 2003 - May 2007, in other words, starting with the introduction of options by Warsaw Stock Exchange. All additional simulations are carried out using data gathered in years 1998-2008. Having analyzed VIW20 index in detail, we observed its characteristic behavior during the periods of strong market turmoils. What we also present is the analysis of the influence of VIW20 and VIX index-based instruments both on construction of minimum risk portfolio and on the quality of derivatives portfolio management in which volatility risk and liquidity risk play a key role. The main objective of this paper is to provide foundations for introducing appropriate volatility indices and volatility-based derivatives. This is done with paying attention to crucial methodology changes, necessary if one considers strong markets inefficiencies in emerging countries. As the introduction of appropriate instruments will enable active management of risks that are unhedgable nowadays it will significantly contribute to the development of the given markets in the course of time. In the summary we additionally point to the benefits Warsaw Stock Exchange might obtain from, being one of the few emerging markets possessing appropriately quantified investment risk as well as derivatives to manage it.

Keywords:

financial market volatility, high-frequency data, realized volatility, realized range, implied and historical volatility, volatility forecasting, option pricing models, investment strategies, portfolio optimization

JEL:

G11, G14, G15, G24

The content of this material reflects personal opinions and views of the authors.
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1. Introduction

Nearly every aspect of financial market activity is connected with risk that is usually represented by volatility of rate of return. The conception of volatility and particularly forecasting its future levels is crucial for a number of reasons which we describe below. The appropriate estimation of volatility parameter is essential in classical portfolio analysis for optimizing the relation between return and risk as well as in capital market models (CAPM APT, multifactor models). In all VaR models estimating the most probable portfolio loss with a given level of significance and an assumed type of distribution, volatility once more constitutes the most important parameter. While analyzing options valuation techniques, volatility parameter again influences the theoretical option value in the most significant way. One should also not forget about implied volatility, calculated from option valuation models, in practice traded in options market. Finally, considering management of derivatives portfolio (futures, options and swaps), one should underline that what is of key importance in the process of risk management is not only the forecast of price change direction but also, and most of all, correct forecast of future volatility levels.

Extensive literature concerning volatility presents a few fundamental approaches to volatility risk quantification and estimators used for calculation of its levels. Within the concept of market volatility one can distinguish historical volatility, implied volatility and realized volatility as well as volatility indices based on methodology presented by Derman (Derman et al., 1999). All the concepts mentioned along with the latest researches on volatility are presented in detail in the second and third part of the paper.

Paying attention to the issue of risk management in institutions participating in capital market, we decided to implement the methodology of Derman, applied in 2003 to reconstruct the formula for calculating VIX index, in order to calculate similar measure in Polish market, i.e. VIW20[1] volatility index which quantifies expected volatility over 91 consecutive calendar days and is calculated on the basis of the quotations of options on WIG20 index. The construction of VIW20 index reflects current market volatility regardless of historical variations and, what is more, historical data are not required to calculate this measure[2]. Furthermore, VIW20 shows a strong negative correlation with WIG20 as well as futures based on it (FW20). This fact allows not only for the use of VIW20 in risk management models, where Vega hedging is the key problem, but it also enables the implementation of instruments based on VIW20, particularly of derivatives, into the problem of investment portfolio optimization. Our research makes it possible to thoroughly analyze the entire history of volatility since the moment of introducing options trading in Warsaw Stock Exchange. Moreover, it lays the foundations for volatility quantifications and future forecasting. Our further step is the verification of effectiveness of volatility index in Polish Stock Exchange environment, where volatility indices and derivatives based on them were not available.
For a deeper understanding of the methodology underlying the presented index, we also decided to compare calculated VIW20 index with other volatility estimators, which had been examined in the earlier article (Ślepaczuk and Zakrzewski, 2008), as well as with VIX index. Showing the most significant differences between the volatility measures, we comment on the possibility of basing derivatives on volatility of one of the estimators/indices mentioned.

Theoretical and empirical investigations presented in this article enable expressing the attitudes towards the following hypotheses and research questions:

- there is a high negative correlation between VIW20 and WIG20 index as well as contracts

[1] VIW20 – volatility index of WIG20 index.
[2] The value of VIW20 index for the specified time unit bases strictly on data from the given time interval.

based on them; there is a positive correlation between corresponding volatility indices calculated for the American market and Polish equity market; i.e. VIX index and VIW20 index;

- negative correlation does not disappear in moments of extreme market behavior; unlike the standard instruments (e.g. stocks), where an initial strong negative correlation in the moments of market turmoils turns out to be seeming, it becomes more intensive;
- the use of VIW20 index while optimizing portfolio allows to increase the rate of return at a stable level of risk; in case of lack of appropriate instruments based on the volatility of Polish equity market for the strategies mentioned one can use VIX index;
- the long memory effect in volatility indices is much stronger when we base our calculations on the longer period of time.

In the article presented we additionally state a hypothesis that placing standardized volatility-based derivatives on the market (Warsaw Stock Exchange) would contribute to faster development of derivatives market. Arguments supporting this hypothesis are presented in the final part of the article. It would make it possible for a larger number of market participants to apply advanced strategies build on derivatives (options, futures, swaps), where the portfolio is exposed to one risk and is hedged against all the other risks. This situation is not possible at the moment as there are no appropriate instruments to hedge volatility, regardless of price movement.

The rest of the article is organized as follows. In Section 2 we describe different approaches to volatility estimation. In Section 3 we present financial data used in research as well as stylized facts about volatility issue. Section 4 provides detailed analysis of methodology applied to calculate VIW20 index with all the necessary corrections for the base methodology. Next, we characterize distribution of a new volatility index. In Section 6 we define, enriched with volatility index, simple investment strategies whose rate of return increases at a simultaneous stable level of risk. We also present forecasting properties of VIW20 index which result from strong dependence between behavior of indices and their volatility. Section 7 presents strategy based on VIX and futures contracts on WIG20, as an alternative to the lack of appropriate volatility indices on emerging markets. In Section 8 we concentrate on the rationale of introducing volatility indices and volatility-based derivatives on emerging markets. The last section concludes the article and presents value added to this paper.

2. Historical, implied vs. realized volatility

Since 70s of XX century, when the Black-Scholes (Black and Scholes, 1973) option pricing formula was published and foundations of derivatives development were formed, the issue of volatility has been paid a lot of attention from financial theorists. The standard and most common approach is estimating volatility by means of standard deviation of rate of return, using both daily and intraday data. However, the approach to modeling volatility has evolved significantly over the past few decades, developing at least three separate ways of its estimation:

- calculating volatility estimators on *ex post* data,
- parametric econometric models (GARCH and their modifications, stochastic volatility models etc.),
- calculating implied volatility directly from the market price of the options, basing on the option pricing models (e.g. Black-Scholes model, Heston model).

When discussing the first approach, one can observe a large number of estimators that try to approximate 'true volatility' and at the same time increase effectiveness in comparison to classical measure of the standard deviation of rate of return, calculated on the base of daily data. Volatility estimators calculated on high frequency data additionally undertake a task of taking into account a bias that results from microstructure bias and the selection of the most appropriate time interval for their estimation[3]. What needs to be analyzed in order to choose appropriate parametric model is the distribution that volality undergoes. Then, in order to forecast volatility, we have to estimate the parameters of the model on the basis of historical data. Since the parameters estimation may expose the process of volatility modeling to serious errors, we try to calculate volatility using nonparametric methods. The example of that is the methodology used for VIW20 index, presented in the other part of the paper. In order to estimate the volatility using the third way one has to choose an appropriate option pricing model, which is the basic disadvantage of this approach. Unfortunately, not meeting the assumptions of the model mentioned leads to significant errors in the above estimation. This approach served as a base for methodology used for evaluating VIX index until 2003. We discuss this issue in Section 4 as well as in appendix.

The third approach is directly connected with the methodology of volatility indices, which constitutes the fundamental part of the given article. It is based on the paper of Derman et al. (1999), describing basis for valuation of volatility and variance swaps, which became a cornerstone for development of derivatives based on volatility. Presenting the way of swaps valuation, Derman et al. provided the basis for hedging of volatility-based derivatives, which was the necessary condition for their further development. The details concerning above methodology are presented in the next section. Below we demonstrate calculation formulas for the three most popular volatility estimators (i.e. SD, RV and RR), which in the other parts of the article are compared with volatility indices constituting the main part of the paper.

$$VAR_{\Delta}^{n} = \frac{1}{(N_{\Delta} * n) - 1} \sum_{t=1}^{n} \sum_{i=1}^{N_{\Delta}} (r_{i,t} - \bar{r})^2 \tag{1}$$

where :

VAR_{Δ}^{n} - variance of rate of return calculated from intraday data on the basis of quotations from last n days,

$r_{i,t}$ - logarithmic rate of return for i_{th} interval of length Δ on a day t, calculated as follows:

$$r_{i,t} = \log C_{i,t} - \log C_{i-1,t} \tag{2}$$

$C_{i,t}$ - close price for i_{th} interval of length Δ on a day t,

$\bar{r}$ - mean rate of return at the level of interval of length Δ from the last n days, calculated from the formula:

$$\bar{r} = \frac{1}{N_{\Delta} * n} \sum_{t=1}^{n} \sum_{i=1}^{N_{\Delta}} r_{i,t} \tag{3}$$

N_{Δ} - number of intervals of length Δ in trading session,

n - memory of the process (in days), used in calculations of appropriate estimators and mean values.

[3] Estimators most often appearing in volatility literature, which properties were tested in numerous papers, are:
1. *realized volatility* (Merton, 1980; Andersen et al., 1999a, 2000, 2001a, 2001b; Taylor and Xu, 1997),
2. *Parkinson's range* (Parkinson, 1980),
3. *Garman-Klass estimator* (Garman and Klass, 1980),
4. *Rogers-Satchell estimator* (Rogers i Satchell, 1991),
5. *Yang-Zhang estimator* (Yang and Zhang, 1991),
6. *scaled realized volatility* and *scaled realized range* (Martens and Dijk, 2007).

$$RV_{\Delta,t} = \sum_{i=1}^{N_\Delta} {r_{i,t}}^2 \tag{4}$$

$$RR_{\Delta,t} = \sum_{i=1}^{N_\Delta} \frac{(h_{i,t} - l_{i,t})^2}{4\log 2} \tag{5}$$

where:

$RV_{\Delta,t}$ - estimator of realized volatility, calculated on the basis of Δ interval on a day t,

$RR_{\Delta,t}$ - estimator of realized range, calculated on the basis of Δ interval on a day t,

$l_{i,t}$ - logarithm of a minimum price ($\log L_{i,t}$) for ith interval of length Δ on a day t,

$h_{i,t}$ - logarithm of a maximum price ($\log H_{i,t}$) for ith interval of length Δ on a day t,

Before comparison estimators were annualized using the given formula. Instead of calculating variance we calculated standard deviation:

$$_{annual\_std}SD_\Delta^n = \sqrt{252 * N_\Delta * VAR_\Delta^n} \tag{6}$$

$$_{annual\_std}RV_\Delta^n = \sqrt{\frac{252}{n}\sum_{t=1}^{n} RV_{\Delta,t}^n} \tag{7}$$

$$_{annual\_std}RR_\Delta^n = \sqrt{\frac{252}{n}\sum_{t=1}^{n} RR_{\Delta,t}^n} \tag{8}$$

where:

$_{annual\_std}SD_\Delta^n$ - annualized SD value,

$_{annual\_std}RV_\Delta^n$ - annualized RV value,

$_{annual\_std}RR_\Delta^n$ - annualized RR value.

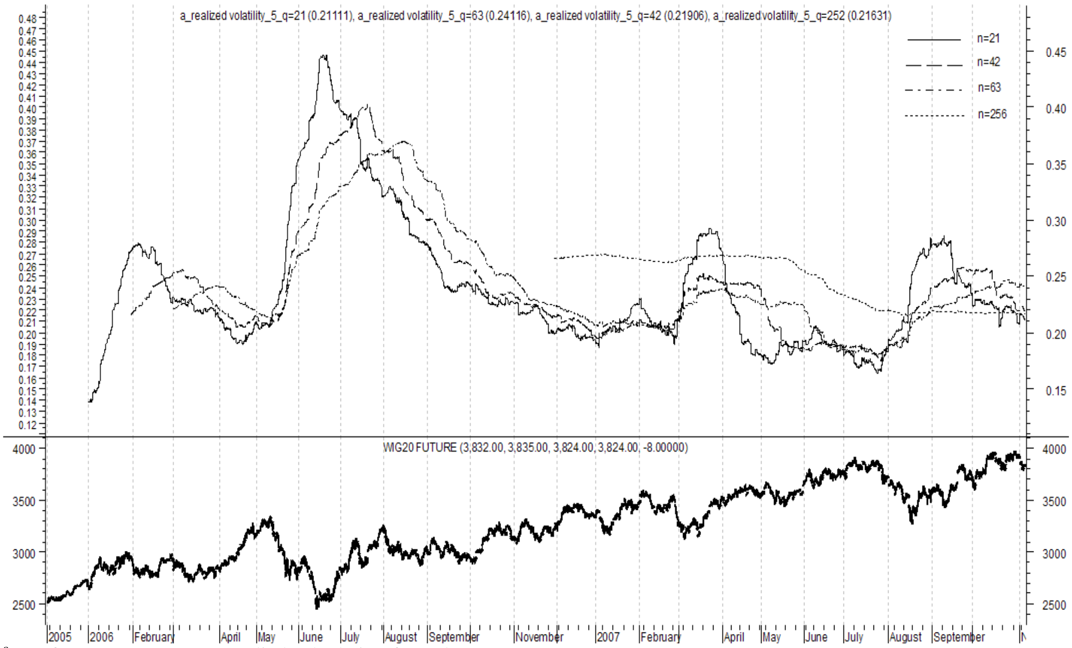
Basing on the results of analysis of effectiveness of volatility estimators[4] (Ślepaczuk and Zakrzewski, 2008), for the purpose of further comparisons we chose Δ=5 and n=63, and so: $_{annual\_std}RV_5^{63}$, $_{annual\_std}RR_5^{63}$, $_{annual\_std}SD_5^{63}$. Now let us discuss the consequences of choosing particular value of parameter *n* in the process of estimating volatility, on the example of RV estimator.

Figure 2.1 illustrates how important role in estimating current volatility as well as forecasting its future level is played by parameter n which determines memory on the basis of which volatility estimator is calculated. In the given parameter one can observe two opposing reactions. The higher the value n, the longer the price history included in the process of volatility estimation, which leads to data smoothing and limited strong volatility fluctuations (Figure 2.1 for n=252). Assuming that volatility undergoes a mean/minimum reverting process given choice would be reasonable from substantial point of view. On the other hand, with low value of parameter n we are able to

[4] In the given research we analyzed the effectiveness of estimators, using modified and relative coefficient of effectiveness. What we analyzed were estimators most often discussed in volatility literature. They were compared with standard deviation of rates of return, one of the basic variables in majority of financial models.

reflect all, often sudden, jumps in volatility which influence option value, hence influence the value of portfolio, as well as quality of risk management in short time period. Consequences arriving from the choice of parameter n are thoroughly described in Section 5.

Figure 2.1. The chart of ${}_{annual\_std}RV_{\Delta}^{n}$ calculated for Δ=5 min and n=21, 42, 63 and 252.[a]



[a] RV for FW20 contract. Applied calculation formula (7).

3. Data and stylized facts about volatility

In order to calculate VIW20 index we used high frequency data for options market (options on WIG20 index – all the series quoted on Warsaw Stock Exchange)[5]. The main problem was getting tick data, containing bid and ask offer for all the option types, strike prices and available expiration dates. All the above data for the analyzed period (October 2003-May 2007) we received directly from Warsaw Stock Exchange. Time series of the option data consisted of the following variables: *date, time, open, high, low, close, bid, ask, oi* and *volume.*

What was also needed to calculate VIW20 index was the value of risk-free interest rate. For our calculations we took quotations of the rate of WIBOR-3m, available on Stooq.pl data provider (www.akcje.net).

For comparing purposes we used VIX index, whose quotation can be accessed via official web page of CBOE (http://www.cboe.com/micro/vix/historical.aspx), as well as the data for future contracts on S&P500 index (FS&P500), available on Stooq.pl internet service (www.akcje.net).

The construction of investment strategies, further discussed in Section 6 and 7, is based on data for futures contracts on WIG20 index received also from Warsaw Stock Exchange. For the purpose of the research we used continuous index for futures contracts where expiring series was

[5] In the analyzed period the options quoted on Warsaw Stock Exchange were only those expiring in the next two months of a March cycle. In October 2007 options expiring in another two months were introduced.

substituted the moment the OI of the next series reached the higher value[6].

In simulations we used data for FW20 pertaining to period January 1998 - January 2008, in other words from the very beginning of their quoting on Warsaw Stock Exchange up to the present time.

VIW20 index was calculated during fixed quotations: 9:00 a.m. - 4:30 p.m. and for various intervals Δ (Δ={5, 10, 15, 30, 60, 120, 240, daily}). Results gathered in Section 5 and Section 6 are only based on Δ=5 as well as data aggregated at daily level.

All the calculations were done in SAS application. Additional charts were created in MetaStock and Excel.

Before focusing on the key part of the analysis that aims to measure volatility, let us look at the conclusions drawn from abundant researches carried by theorists and practitioners interested in the matter.

- Volatility series follow a mean reverting process. A very interesting character of distribution which in case of volatility series additionally takes form of minimum reverting process.
- Long memory process in the volatility series. Both positive and negative shocks in volatility series expire slowly (fractionally integrated time series, Baillie at al. 1996).
- Volatility clustering. One can observe distinct periods when volatility maintains at higher or lower level for a longer time. The effect mentioned is connected with long memory process described above.
- The leverage effect. Unsymmetrical reaction of volatility on shocks in base index, i.e. rapid growth of volatility in the moments of strong downward movements in contrast to moderate growth or lack of change in case of strong upward moves (Black, 1976; Ebens, 1999; Andersen et al., 2001a). Andersen et al. (2001a) additionally point out that the leverage effect is stronger at aggregated level (market indices) rather than for single stocks.
- Strong negative correlation between volatility and base index which gets even stronger in the moments of market shocks, contrary to standard instruments (e.g. stocks) where initially identified negative correlation may disappear in the moments of extreme market behavior (e.g. stock market crash).
- Characteristic features of volatility distribution (i.e. variation) are: high kurtosis, right skewness and nonnormality. However, logarithms of standard deviation (realized volatility) posses approximately normal distribution (Giot and Laurent, 2004; Andersen et al., 2001a and 2001b).
- Volatility-in-correlation effect; high positive correlation between single stock volatilities, i.e. one can observe a high/low correlation between separate stocks when their volatility is high/low and additionally when a correlation between remaining stocks is high/low (Andersen et al., 2001a).

In the further part of the paper we follow the methodology of calculating indices as well as necessary modifications allowing for its implementation on Polish market. In Sections 5-7 we aim to verify described conclusions on the example of volatility indices: VIW20 and VIX.

4. Theoretical framework

Significant problem that follows the introduction of the most frequently discussed volatility estimators is their calculation. The difficulty refers to the choice of appropriate number of days n on the basis of which given estimator is calculated. In other words, it concerns the choice of a shorter or longer price history for the purpose of appropriate estimation of 'true volatility'.

[6] The method applied is one of three ways of creating continuous charts for futures contracts where due to the presence of expiring series one does not possess quotations for long time horizons. For that reason continuous charts are created.

Although we make this attempt in the previous article (Ślepaczuk and Zakrzewski, 2008) it seems to us that the use of volatility estimator resistant to the given imperfection is a significantly better solution. For this purpose we turn to methodology of Derman et al. (1999), which constituted a base for improving the formula of calculation of VIX[7] index as well as volatility indices on European equity markets[8].

The new conception of volatility index (VIX) eliminated the imperfections, which were typical of the old formula (VXO[9]). It eradicated the following weakest assumptions:

- VXO formula was based only on at-the-money options, therefore the index did not reflect well documented phenomena of volatility smile and skew, and did not fully reflect the volatility surface.
- In the old conception implied volatility was derived from Black-Scholes formula, which directly presupposed fulfillment of assumptions which, as it is widely known, are not fulfilled.
- Moreover, VXO was calculated on the basis of S&P100 index options, which, contrary to S&P500 index serving as benchmark for majority of investment funds of American equity market, does not reflect the fluctuations of broad market

Above inaccuracies were eliminated with the following solutions in the new methodology for calculating VIX index:

- apart from at-the-money options, quotations of options reflecting a wide range of strike prices and volatility surface were used,
- calculation formula was reconstructed so as to abandon calculating volatility from Black-Scholes formula in order to derive volatility directly by averaging the weighted prices of at-the-money and out-of-the-money puts and calls. The result of the given modification was a model-free index, which was the most significant change.
- Moreover, what the new methodology used were options on S&P500 index, whose representativeness for reflecting broad market in comparison with S&P100 cannot be questioned.

Described changes enabled the introduction of volatility derivatives (futures contracts, options) which have been developing rapidly, both in respect of the volume of turnover and the number of open positions[10], since the moment of their introduction on CBOE market. The generalized formula used in the new VIX[11] index calculation looks as follows:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} * Q(K_i) - \frac{1}{T}[\frac{F}{K_0} - 1]^2 \tag{10}$$

where:

$$\sigma = \frac{VIX}{100} \Rightarrow VIX = \sigma * 100 \tag{11}$$

T - time to expiration (in years; with minutes accuracy),

F - forward price derived from option prices,

K_i - strike price of i_{th} out-of-the-money option; a call if K_i>F and a put if K_i<F,

[7] Methodology of calculating VIX index was changed in 2003. Until that moment given index had been calculated on the basis of averaged implied volatility derived from Black–Scholes formula and calculated only for at-the-money option. The improved formula has been used for calculation of volatility indices of the remaining important stock market indices (VXN – NASDAQ, VXD – DJIA, RVX – Russell 2000).

[8] VDAX-NEW – volatility index of DAX30 index, VSMI – volatility index of SMI index, VSTOXX – volatility index of Dow Jones EURO STOXX 50 index

[9] VXO – volatility index of American equity market, calculated on the basis of the old methodology until 2003

[10] In 2004 CBOE introduced VIX index futures contracts. Two years later VIX index options were launched.

[11] See: VIX White Paper, 2003.

ΔK_i - interval between strike prices - half of the distance between the higher and lower strike price in relation to K_i:

K_0 - first strike below the forward index level, (F)

R - risk-free interest rate,

$Q(K_i)$ - the midpoint spread (0.5*(bid+ask)) for each option with strike price K_i.

Unfortunately, the utilization of the original methodology made it impossible to calculate appropriate volatility index for Polish stock market due to the following imperfections of Polish options market:

- too low liquidity of the market, particularly for long-term options, which would be visible at the beginning and at the end of a trading session,
- too extensive participation of a single investor (market maker), in comparison to the number of bids and asks, who withdraws all his offers in the moments of sudden market turmoils. The situation made it impossible to estimate index value or caused its violent fluctuations resulting from insufficient number of offers in calculation process,
- insufficient number and spread of expiration terms on options market.

In other words, **insufficient number of market participants, which is a direct cause of low liquidity, and in addition, a key role of a market maker in providing a full order sheet essential for continuous calculation of VIW20** index**.**

Seeing that, we decided to modify the original VIX formula so as to provide the changes enabling the use of described methodology on Polish ground and preserve the fundamental logic behind it:

- We assume a process memory to be dependent on an average number of strike prices with active bid-ask quotations for a previous day. Memory activates when a number of strikes for a given interval is significantly different from the average value for a previous day. Given modification aims to eliminate unnatural jumps caused by withdrawal of orders by market maker.
- Deletion of outlier data. We noticed significant outliers in 5-min data, which we subsequently eliminated before calculating index.
- We calculate VIW20 index solely for period 9:00 a.m. - 4:00 p.m., as throughout the period of our research (October 2003 - October 2005) quotations always closed precisely in the given period.
- Change of base period of expected volatility from 30 to 91 calendar days caused by availability of options with only two expiry terms, what will be of crucial significance in the process of interpreting the properties of VIW20 index in comparison to VIX index,
- Lack of assumption of switching into option expiring on a later date if the time left to option expiration is less than 9 days. Such assumption was made in case of calculation of VIX index to minimize deviations caused by strong option fluctuation observed just before expiration. There are two reasons for which we decided not to make the assumption mentioned. Over the period of research the options analyzed had only two expiration terms so such a modification in the analyzed data was impossible. Secondly, the contribution of expiring option to the value of VIW20 index is only minimal in the given period.

Abridged description of methodology for calculating VIW20 index as well as necessary corrections of original methodology are thoroughly described in appendix A.1.

5. Properties of volatility index distribution and comparison with other volatility estimators.

Having calculated VIW20 (methodology presented in appendix), let us try to describe its properties as of separate volatility measure as well as in comparison with VIX index and other volatility estimators frequently discussed in the literature. In the table below we show descriptive statistics for 5-min data in various time periods.

Table 5.1. Descriptive statistics for VIW20 index – 5-min data.[a]

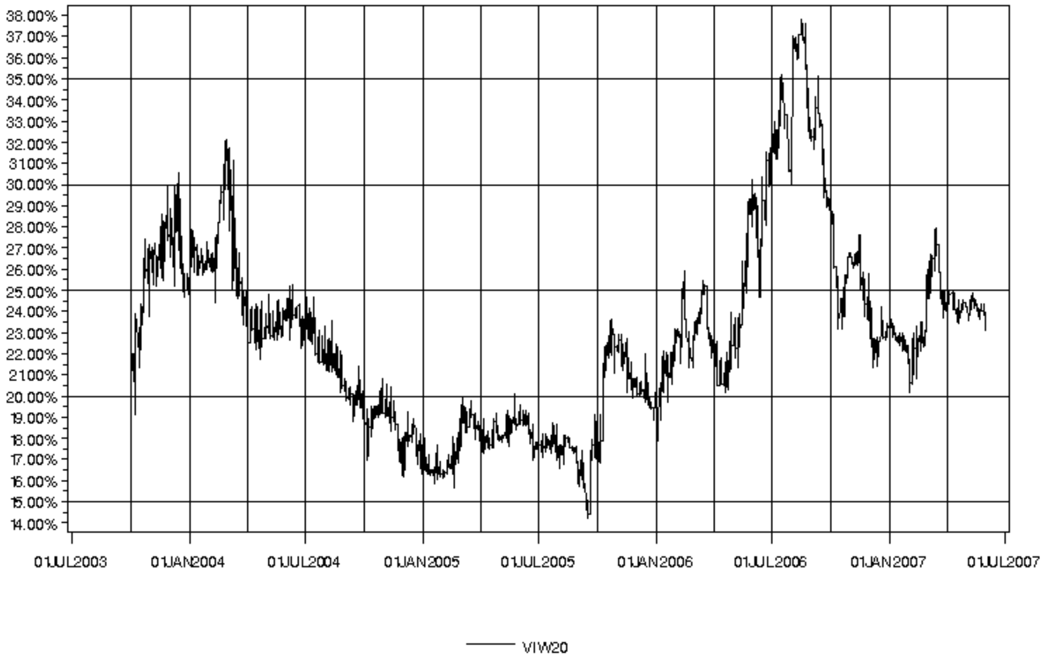
statistics		VIW20				
		1.10.03-31.05.07	1.10.03-31.08.04	1.09.04-31.07.05	1.08.05-30.06.06	1.07.06-31.05.07
N		78024	19233	19319	19778	19694
Mean		0.229	0.247	0.182	0.218	0.268
Median		0.227	0.239	0.181	0.218	0.248
Variance		0.0021	0.0007	0.0001	0.0013	0.0020
Std Dev		0.046	0.026	0.010	0.036	0.045
Minimum		0.142	0.191	0.157	0.142	0.202
Maximum		0.378	0.322	0.214	0.319	0.378
Kurtosis		0.441	-0.344	-0.614	0.284	-0.425
Skewness		0.780	0.561	-0.045	0.550	0.927
Test for Normality						
Kolmogorov-Smirnov	**Statistic**	0.066691	0.124396	0.039828	0.103987	0.188568
	p-value	<0.0100	<0.0100	<0.0100	<0.0100	<0.0100
Cramer-von Mises	**Statistic**	111.061	47.72453	8.182369	44.51618	206.4707
	p-value	<0.0050	<0.0050	<0.0050	<0.0050	<0.0050
Anderson-Darling	**Statistic**	866.9136	257.6142	63.71919	284.4272	1114.794
	p-value	<0.0050	<0.0050	<0.0050	<0.0050	<0.0050

[a] The table presents basic descriptive statistics for 5-min data: number of observations, mean, median, variance, standard deviation, minimum, maximum, kurtosis, skewness, test for normality for the whole analyzed period as well as for equal 11-month time periods.

Data shown in Table 5.1. and Figure 5.1. exhibit a big spread of volatility fluctuations over the analyzed period (illustrated by the distance between the minimum and the maximum as well as high standard deviation), lack of normality (tests for normality to be found in the Table 5.1.), slight kurtosis and positive skewness. The analysis of Table 5.1 divided into sub-periods suggests the existence of volatility clustering. Shape of Figure 5.1., even in a short 4-years period, makes us suppose that VIW20 index undergoes a mean-reverting process. Given observations do not differ significantly from those widely described in the literature and mentioned in the previous parts of the article.

For better understanding of the properties of VIW20 index, we provide a comparison of descriptive statistics for VIW20 index, VIX index and for three volatility estimators – RV, RR and SD calculated for futures contracts on WIG20 index (see Table 5.2).

Figure 5.1. VIW20 index for the analyzed period (5-min data).[a]



[a] Fluctuations of VIW20, October 2003 - May 2007.

The comparison of VIW20 index with other conceptions of volatility estimators calculated for the same time period does not reveal any significant differences in relation to mean value, spread, variance or lack of normality. However, we should remember that the expected volatility for subsequent 3 calendar months in VIW20 index is estimated in the given moment on the basis of data relating to 5-min. interval, whereas what we need to calculate compared volatility estimators are intraday quotations from the last three calendar months. This fact makes researchers aware of the superiority of Derman's method and induces to continue the studies on volatility indices so as to obtain a thorough understanding of their properties and use.

The comparison of volatility index of Polish and American equity markets confirms the supposition of a significantly higher volatility index and definitely higher variance of the volatility of an emerging market (mean values, minimums and maximums are at considerably higher levels for VIW20 index). As in the case of 5-min. data none of variables is normally distributed. We can also observe a slight kurtosis and a positive skewness. However, a definitely more interesting observation relates to corresponding percentiles of VIW20 and VIX index. The distance between P10 and MIN (0,012 for VIX and 0,03 for VIW20) in comparison with the distance between P90 and MAX (0,065 for VIX and 0,086 for VIW20) as well as significant difference between MEAN-MIN and MAX-MEAN (0,04/0,099 for VIX and 0,084/0,148 for VIW20 respectively) induces to suppose that the mean reversion, frequently observed in volatility distributions, takes a form of a minimum reverting process which contains an additional condition, that is the mean of the process is very close to its minimum value.

Table 5.2. Descriptive statistics for VIX index, VIW20 index and three volatility estimators – daily data.[a]

statistics		VIX	VIW20	${}_{annual\_std}RV_{\Delta}^{n}$	${}_{annual\_std}RR_{\Delta}^{n}$	${}_{annual\_std}SD_{\Delta}^{n}$
N		919	915	921	921	921
Mean		0.139	0.228	0,215	0,173	0,215
Median		0.133	0.226	0,208	0,167	0,208
Variance		0.0006	0.0021	0,0034	0,0024	0,0034
Std Dev		0.024	0.046	0,058	0,049	0,058
Minimum		0.099	0.144	0,137	0,111	0,137
Maximum		0.238	0.376	0,369	0,297	0,369
Kurtosis		-0.082	0.474	-0,084	-0,260	-0,090
Skewness		0.660	0.787	0,817	0,815	0,819
P1		0.102	0.160			
P5		0.107	0.168			
P10		0.111	0.175			
P90		0.173	0.290			
P95		0.183	0.320			
P99		0.203	0.366			
Test for Normality						
Kolmogorov-Smirnov	**Statistic**	0.068806	0.090791	0.105331	0.120639	0.108036
	p-value	<0.0100	<0.0100	<0.0100	<0.0100	<0.0100
Cramer-von Mises	**Statistic**	1.284063	1.864589	3.081353	4.03119	3.102534
	p-value	<0.0050	<0.0050	<0.0050	<0.0050	<0.0050
Anderson-Darling	**Statistic**	9.994147	11.20433	22.05543	26.93477	22.29116
	p-value	<0.0050	<0.0050	<0.0050	<0.0050	<0.0050

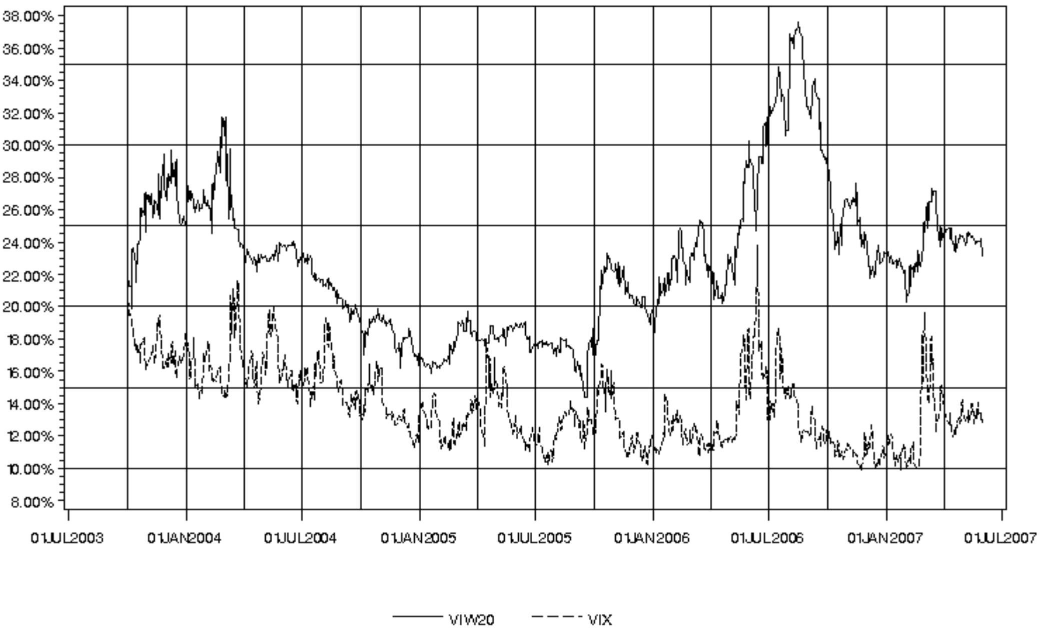
[a] The table presents basic descriptive statistics calculated on daily data: number of observations, mean, median, variance, standard deviation, minimum, maximum, percentiles, kurtosis, skewness, tests for normality for the full research period. ${}_{annual\_std}RV_{\Delta}^{n}$, ${}_{annual\_std}RR_{\Delta}^{n}$, ${}_{annual\_std}SD_{\Delta}^{n}$ – calculations on data aggregated at the daily level for n=63.

For better understanding of differences between VIX and VIW20 index let us take a look at their fluctuations (see Figure 5.2). The main difference that can be observed concerns different moments of occurrence of turning points. Apart from this, in VIW20 chart one can notice a substantially longer memory. All these differences are caused by periods for which we calculate expected volatility for Polish and American equity markets (VIW20 - 91, VIX - 30 calendar days).

The analysis of Figure 5.2, especially when one compares it with Figure 2.1, informs about strong correlation of changes as well as coexistence of extreme points for ${}_{annual\_std}RV_{5}^{21}$ with VIX and ${}_{annual\_std}RV_{5}^{63}$ with VIW20.

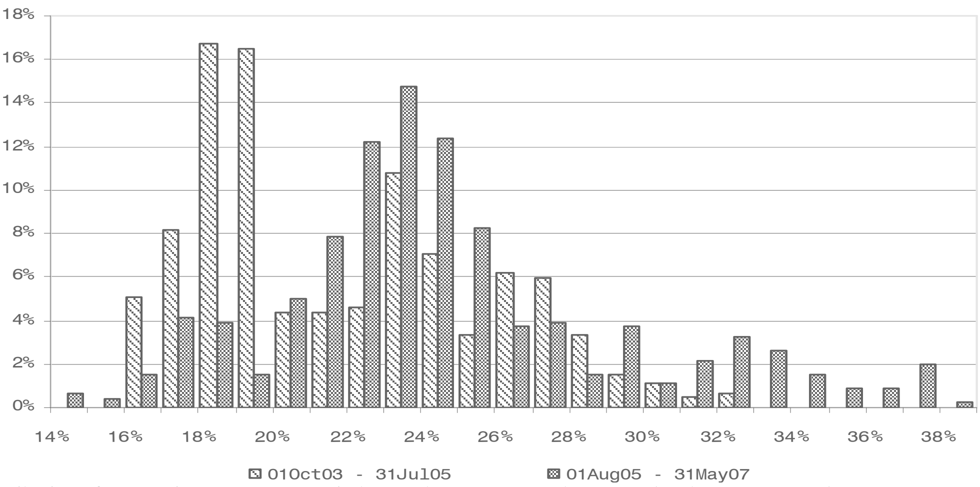
In spite of seeming differences, observed dependence suggests strong relation between volatility measures calculated for different markets (S&P500 and WIG20 index). It also underlines a significant influence of parameter *n*, and in case of volatility indices, of option time horizon (VIX – 30, VIW20 – 91 calendar days) in comparison to the calculation formula employed. Obviously, it does not eliminate the necessity of having significantly longer time series in case of volatility estimators in comparison to data necessary for the calculation of volatility indices.

Figure 5.2. VIX and VIW20 index for the analyzed period, daily data.[a]



[a] VIW20 - expected volatility for 91 consecutive calendar days, VIX – expected volatility for 30 consecutive calendar days.

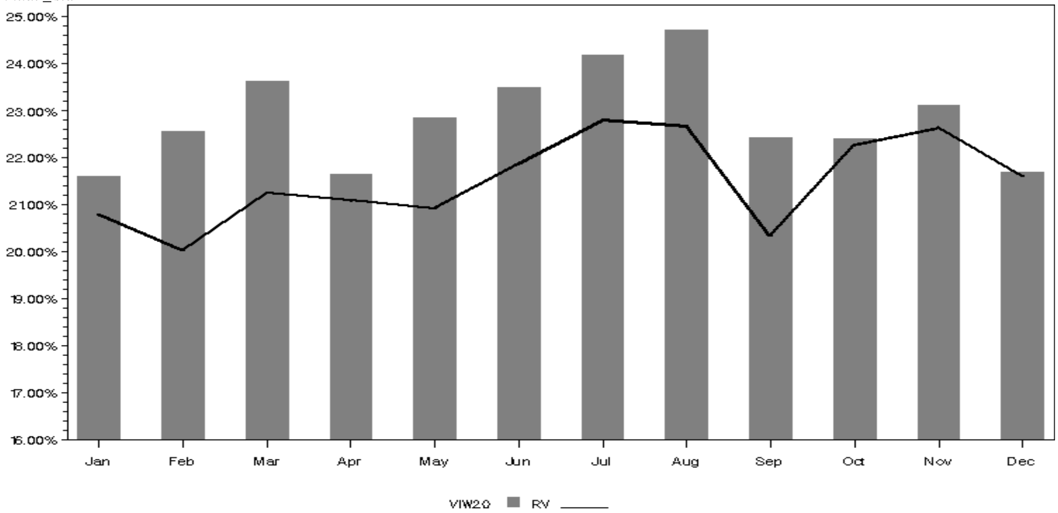
Figure 5.3. Frequency distribution for VIW20 index.[a]



[a] Distribution of VIW20 in two separate periods: October 1, 2003 – July 31, 2005 and August 1, 2005 – May 31, 2007.

Additional properties of volatility indices are presented in Figure 5.3 which reflects the volatility of VIX index over time. The figure shows a shift in volatility distribution between high and low periods, both in the sense of its mean value as well as its variance.

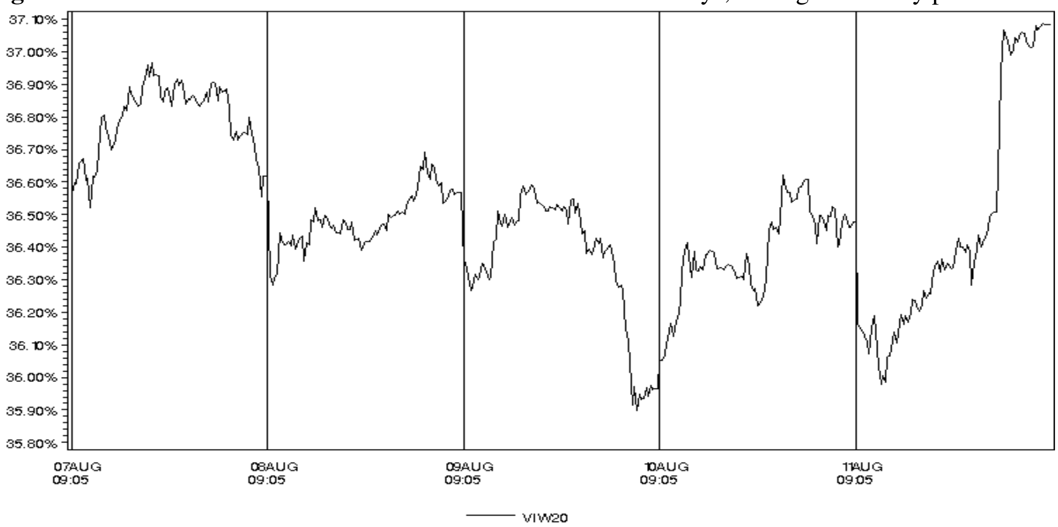
Figure 5.4. Mean monthly value of VIW20 index in comparison with mean monthly value of ${}_{annual\_std}RV_{\Delta}^{n}$ estimator.[a]



[a] Simulation for period: October 1, 2003 – May 31, 2007. Calculations for RV base on Δ = 5 min and n = 63.

The aim of the next Figure 5.4 was to reflect the monthly seasonality of the process, with the months of lower and higher than average volatility. Unfortunately, due to short test period influenced by external data, we do not focus on particular months but only suggest that given analysis is helpful in case of a longer history of quotations.

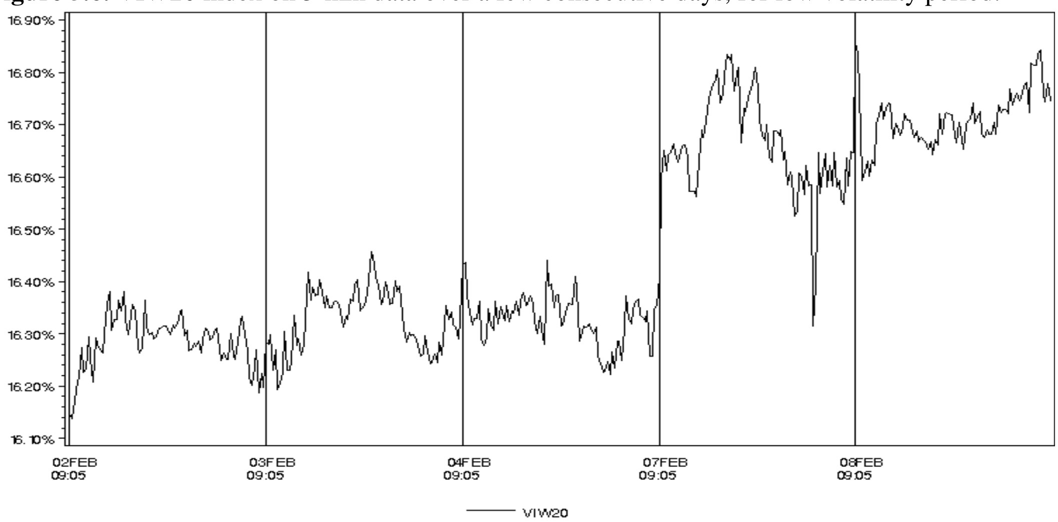
Figure 5.5. VIW20 index on 5-min data over a few consecutive days, for high volatility period.[a]



[a] Intraday volatility fluctuations in period August 7-11, 2006, reflecting a week of high volatility.

Summing up the properties of VIW20 volatility index, it can be noticed that it inherited all the features ascribed to realized volatility, described in Section 3, and, in addition, it has some specific advantages connected with the conception of calculation formula. Before we move to the next part, where by construction of appropriate investment strategies we try to illustrate another properties of VIW20 and VIX index (e.g. strong negative correlation with stock market indices, long memory and a mean reverting process), let us take a look at the example of volatility fluctuation during the trading session, on the example of a week characterized by high (Figure 5.5) and low volatility (Figure 5.6).

Figure 5.6. VIW20 index on 5-min data over a few consecutive days, for low volatility period.



[a] Intraday volatility fluctuations in period February 2 - 8, 2005, reflecting a week of low volatility.

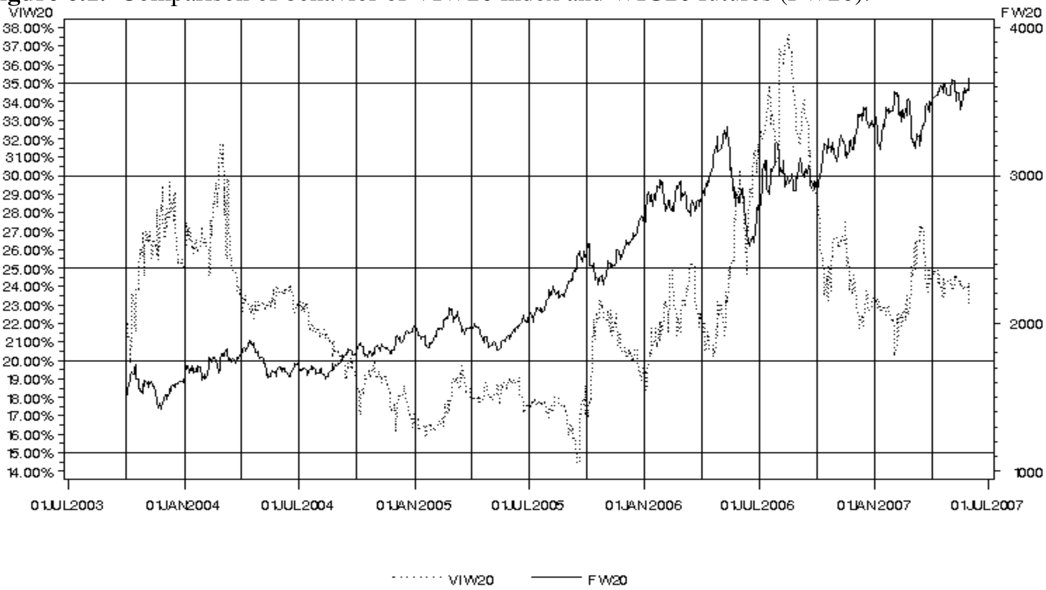
Despite the seeming difference (week of high and low volatility), figures share one feature, i.e. instability of volatility index over time, which is the key characteristic for risk management practitioners.

6. Index portfolio optimization based on VIW20 index

Distribution properties of presented volatility indices, thoroughly described in the previous section, inclined us to define a few simple investment strategies which use both volatility indices and future contracts on stock market indices, in order to discuss the properties of volatility measure (VIW20 index) not used on Polish capital market. The direct reason for creating such strategies was a visual analysis of figures 6.1-6.3, which similarly to tables 6.1-6.2 suggested a strong negative correlation between changes in volatility indices (VIW20 and VIX) and index futures (FW20 and FS&P500).

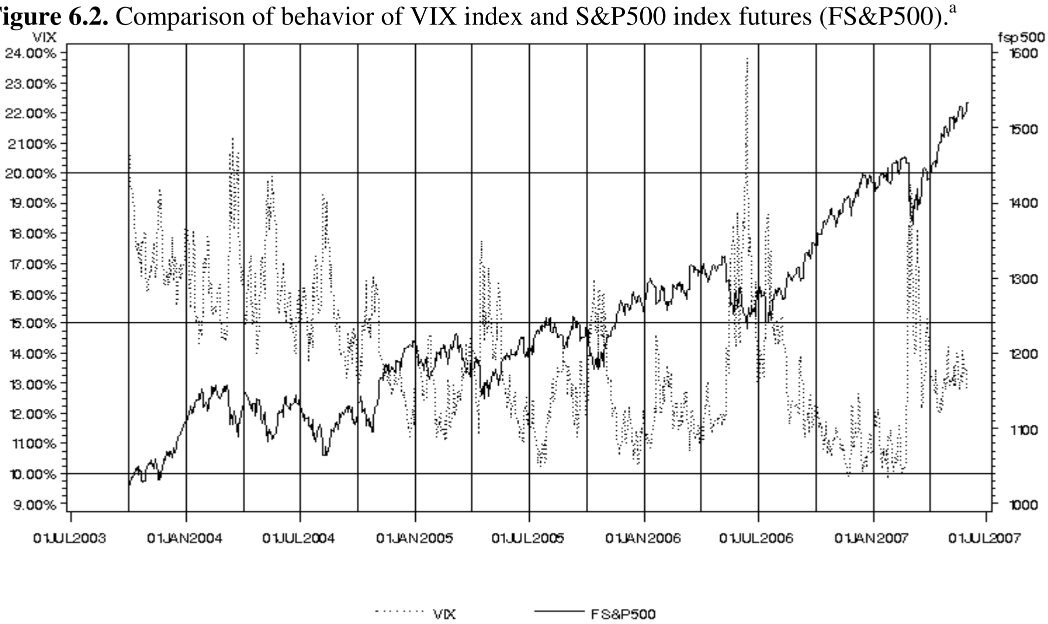
While analyzing figures 6.1-6.2, we also notice significant differences in behavior of VIW20 index and VIX index. To a large extent it is caused by the fact that VIX anticipates 30-day volatility, whereas its Polish equivalent expects volatility over the 91-day period. For this reason, basing on the phenomenon of long memory in volatility series, initiated volatility movement displayed in VIW20 figure lasts definitely longer than volatility movement in VIX figure. However, the very moment of volatility jump, if one takes into consideration their strong correlation, should be identical in both figures. Unfortunately, the above observation has serious consequences for the process of defining investment strategies described in this and next sections of the article. Therefore, they should be defined both while using VIX and the new VIW20 index.

Figure 6.1. Comparison of behavior of VIW20 index and WIG20 futures (FW20).[a]



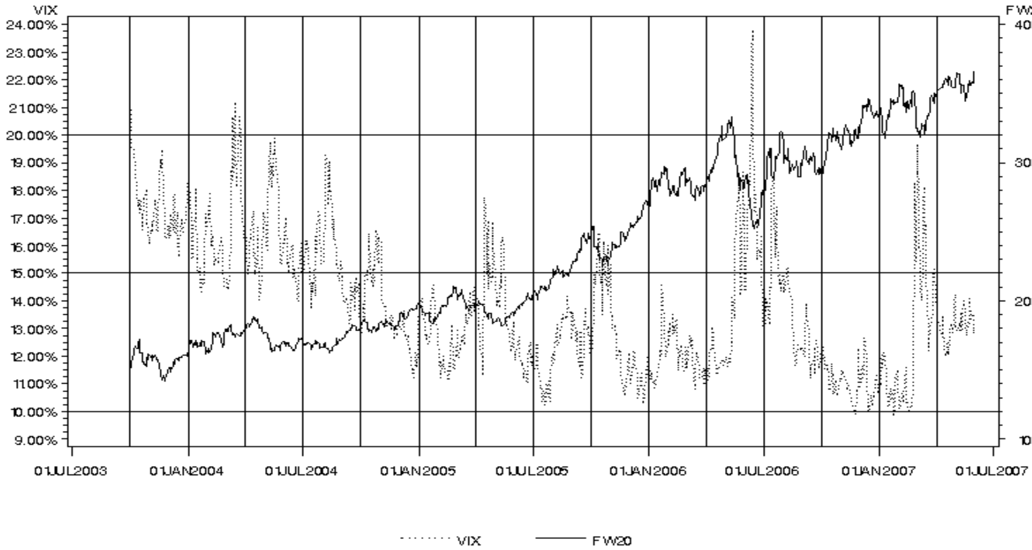
[a] VIW20 index and FW20 in period October 2003 – May 2007, daily data.

Figure 6.2. Comparison of behavior of VIX index and S&P500 index futures (FS&P500).[a]



[a] VIX index and FS&P500 in period October 2003 – May 2007, daily data.

Figure 6.3. Comparison of behavior of VIX index and WIG20 futures in analyzed period - daily data.[a]



[a] VIX and FW20 in period October 2003 – May 2007, daily data.

Our next step is to prove the visual observations statistically. Tables 6.1 – 6.2 present daily and averaged (5-period moving average) correlations of rates of return of volatility indices and futures on stock market indices[12] for the following pairs: VIW20 – FW20, VIX – FS&P500 and VIX – FW20.

Table 6.1. Correlation between daily rates of return from volatility indices: VIW20, VIX, and daily rates of return from stock market indices: WIG20, S&P500.[a]

period	Corr_ VIW20_ FW20	p-value	N	Corr_ VIX_ FS&P500	p-value	N	Corr_ VIX_ FW20	p-value	N
01.10.03-31.08.04	-0.114	0.088	225	-0.808	<0.001	231	-0.148	0.0277	225
01.09.04-31.07.05	-0.201	0.0022	229	-0.753	<0.001	230	-0.232	0.0005	224
01.08.05-30.06.06	-0.228	0.0005	231	-0.849	<0.001	232	-0.111	0.0966	225
01.07.06-31.05.07	-0.405	<0.001	229	-0.230	0.0005	225	-0.102	0.1306	229
01.10.03-31.05.07	-0.229	<0.001	914	-0.648	<0.001	918	-0.135	<0.001	903

[a] Calculations on daily data for consecutive annual study periods and a full research period.

[12] In described strategies we use futures contracts on WIG20, instead of stock market index, The reason is a strong correlation of these two instruments and the fact that transaction costs connected with futures contract are significantly lower.

Table 6.1 presents correlations between daily rates of return of volatility indices and futures contracts on stock market indices. The comparison of corresponding correlation coefficients between VIW20 and FW20 shows a significant correlation over the full period, which is additionally increasing over the consecutive 11-month periods, suggesting unstable character of volatility over time and increasingly negative correlation between VIW20 and FW20. Changing correlation coefficients observed for Polish indices can also be noticed for American indices, where one can see a significantly stronger negative correlation, statistically significant even at the 1% significance level. After the analysis of Figure 6.3, we present the correlation between VIX index and FW20 (see Table 6.1) the analysis of which inclines us to define appropriate investment strategies for these two instruments.

Table 6.2. Correlation between averaged (5-period moving average) returns from volatility indices: VIW20, VIX and averaged returns from stock market indices: WIG20 and S&P500.[a]

period	**Corr_ mov5- VIW20_ mov5- FW20**	**p-value**	**N**	**Corr_ mov5- VIX_ mov5- FS&P500**	**p-value**	**N**	**Corr_ mov5- VIX_ mov5- FW20**	**p-value**	**N**
01.10.03-31.08.04	-0.150	0.0257	221	-0.805	<0.001	227	-0.355	<0.001	221
01.09.04-31.07.05	-0.176	0.0075	229	-0.718	<0.001	230	-0.406	<0.001	224
01.08.05-30.06.06	-0.368	<0.001	231	-0.830	<0.001	232	-0.372	<0.001	225
01.07.06-31.05.07	-0.488	<0.001	229	-0.664	<0.001	225	-0.332	<0.001	229
01.10.03-31.05.07	-0.318	<0.001	910	-0.741	<0.001	914	-0.355	<0.001	899

[a] Calculations on daily data for consecutive annual study periods and a full research period.

Assuming a different correlation for averaged and daily data, we show the correlation between 5-day moving average returns (see Table 6.2). In this presentation we can observe a substantially stronger negative correlation. A significant change is observed particularly in case of pairs: VIW20 – FW20 and VIX-FW20, which confirms our suppositions about the direction and strength of this relation as well as the assumption of a negative correlation between both VIW20-FW20 and VIX-FW20. The above relation indirectly implies a strong positive correlation between volatility indices (VIX and VIW20), despite of their calculation done for different markets (emerging and developed market) and dissimilar time horizons (30 and 91 calendar days). We will develop this observation in subsequent section as well as in final conclusions.

Observed strong negative correlation, both between daily and averaged returns, inclined us to exemplify this property with a simple investment strategy. The objective of strategy A is to invest 50% of portfolio value into volatility index and 50% of portfolio value into FW20 under the condition that VIW20 is below 1st, 5th or 10th percentile. The investment in volatility index is held over predetermined period of n days, for n ={21, 42, 63, 126}, the result of which it to depict a mean reverting process. The behavior of the strategy is analyzed for period October 2003 – May 2007 (for results see Table 6.3).

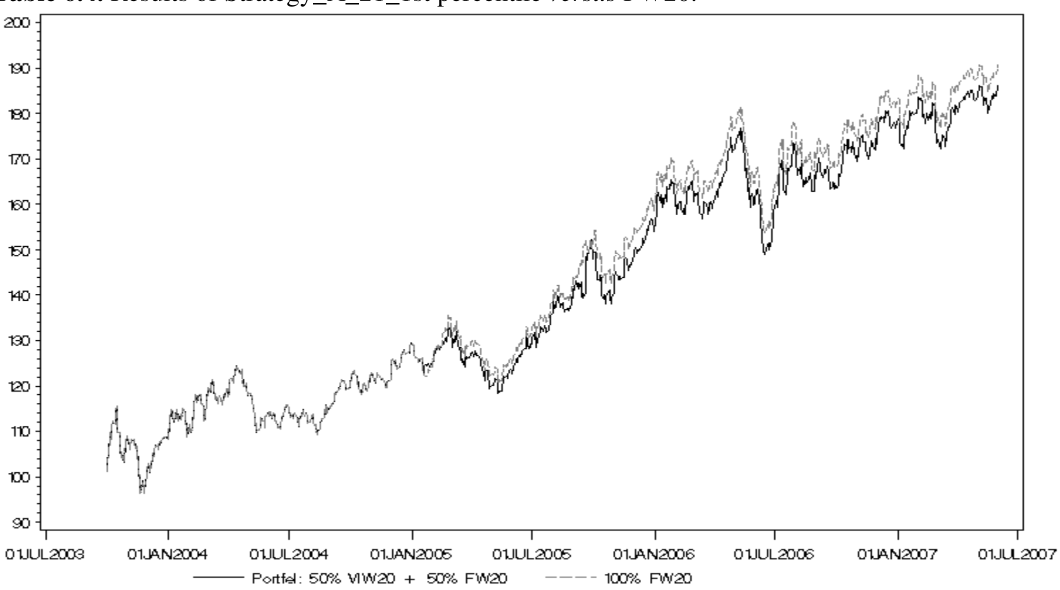
Table 6.3. Results of Strategy_A_n_k_{th} percentile for n = {21, 42, 63, 126} for the full period.[a]

stat	instr.	n = 21	n = 42	n = 63	n = 126
		VIW20 < 1 percentyl			
r	FW20	0.0009827	0.0009827	0.0009827	0.0009827
	Strategia_A: 50/50	0.0009326	0.00123	0.00111	0.00105
Corr_VIW20_FW20		-0.22204	-0.22204	-0.22204	-0.22204
	p-value	<.0001	<.0001	<.0001	<.0001
sd	FW20	0.01450	0.01450	0.01450	0.01450
	Strategia_A: 50/50	0.01479	0.01493	0.01497	0.01478

[a] The table shows average daily returns and standard deviations for investment in strategy A or in FW20 in period: October 2003 – May 2007.

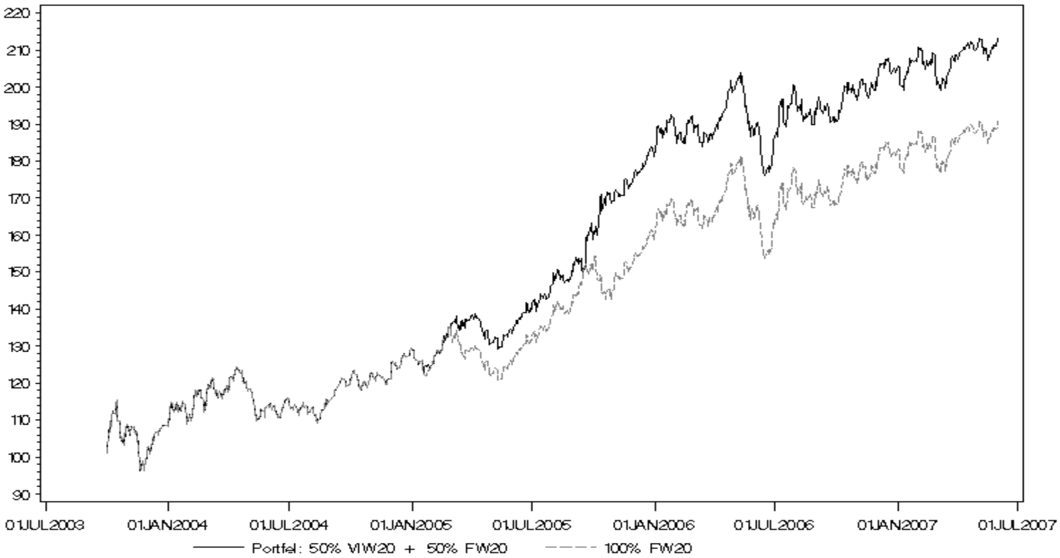
Results shown in table 6.3 confirm a significant negative correlation between volatility indices and stock index futures contracts on given stock indices, which can be used in various investment strategies as well as advanced risk management models. The results mentioned show the possibility of a significant increase of return at a simultaneous stable initial level of standard deviation.

Table 6.4. Results of Strategy_A_21_1st percentile *versus* FW20.[a]



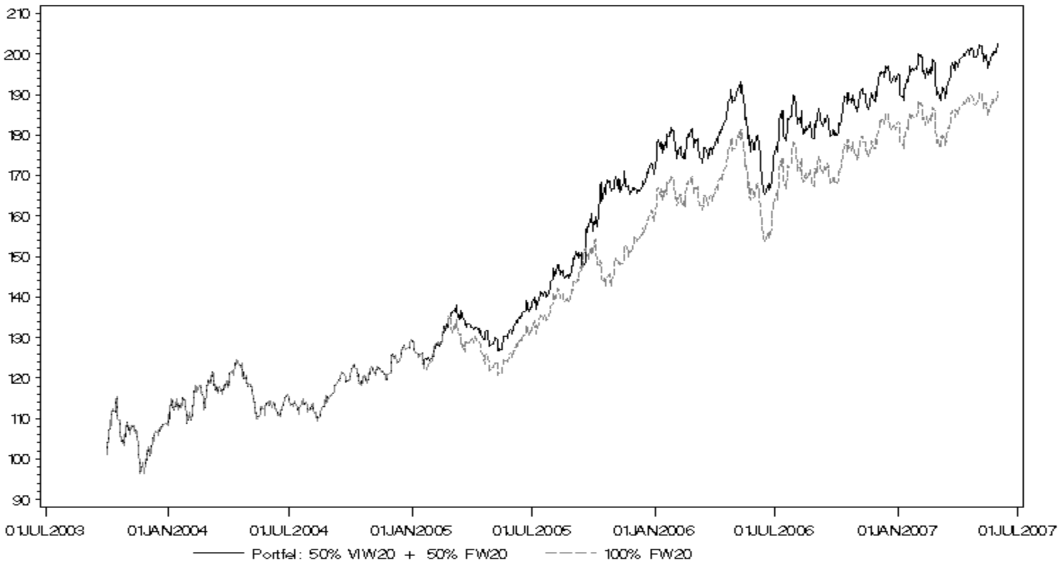
[a] Strategy A initiates when VIW20 is below 1st percentile. Investment period in VIW20 is 21 days. Strategy behavior is compared with 100% capital investment in FW20. The figure shows accumulated rate of return with the assumed initial investment of 100.

Table 6.5. Results of Strategy_A_42_1st percentile *versus* FW20.[a]



[a] Strategy A initiates when VIW20 is below 1st percentile. Investment period in VIW20 is 42 days. Strategy behavior is compared with 100% capital investment in FW20. The figure shows accumulated rate of return with the assumed initial investment of 100.

Table 6.6. Results of Strategy_A_63_1st percentile *versus* FW20.[a]

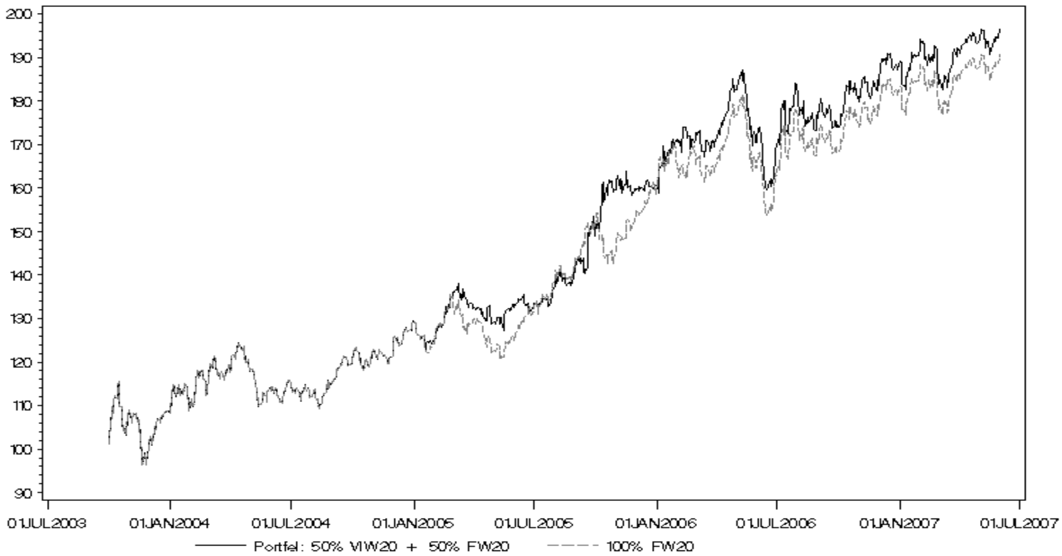


[a] Strategy A initiates when VIW20 is below 1st percentile. Investment period in VIW20 is 63 days. Strategy behavior is compared with 100% capital investment in FW20. The figure shows accumulated rate of return with the assumed initial investment of 100.

Comparing the results from Table 6.3 as well as Figures 6.4 - 6.7 presenting accumulated rate of return for the strategy A and control investment: 100% of capital value in FW20, we draw our attention to significant influence of parameter n, which was to reflect the mean length of mean/minimum reverting process. Differences in rates of return, standard deviations as well as accumulated rates of return presented in the figure suggest that appropriate estimation of the length of the process plays a crucial role in the use of volatility indices properties. As far as our strategy is concerned, the best results were obtained for n = 42 days, however, we do realize that the analyzed time interval is not long enough to draw far-reaching conclusions.

It should also be noticed that the strategy described above represents the least complicated *buy&hold* technique, the aim of which is to draw one's attention only to the impact of a new element, which is volatility index, quite apart from sophisticated *timing* techniques which can be used in order to correct the final rate of return. Moreover, due to the short history of Polish options market we were not able to show how the strategy works over a long time period, where its effectiveness should have been significantly more noticeable. Similar strategies can be built for American (VIX and FS&P500) or German equity market (VDAX and FDAX), where volatility indices and volatility derivatives are both published and traded.

Table 6.7. Results of Strategy_A_126_1st percentile *versus* FW20.[a]



[a] Strategy A initiates when VIW20 is below 1st percentile. Investment period in VIW20 is 126 days. Strategy behavior is compared with 100% capital investment in FW20. The figure shows accumulated rate of return with the assumed initial investment of 100.

To summarize this section, we would like to underline a few significant elements concerning VIW20 index. Firstly, we focused on the use of negative correlation between VIW20 index and stock index futures. Particularly interesting feature of the given correlation is that, unlike standard assets, it does not disappear in the moments of market turmoils but additionally becomes more intensive while facing a stock index downturn. Secondly, we highlight the significance of the period for which the investment in volatility index is done, by connecting its length with the mean length of mean reverting process. Thirdly, the results obtained indicate the significance of long memory process and the importance of time horizon, for which the volatility index is calculated. In the last part of the paper the issue described above will be subjected to further discussion in relation to its weight for derivative development in emerging markets.

7. VIX *versus* FW20, investment strategy for WIG20 index futures (starting from I.1998).

Results obtained in the previous section inclined us to develop one more simulation including hypothetical possibility of hedging FW20 futures by means of derivatives based on volatility index for American equity market (VIX index) over the period from the beginning of FW20 trading on Warsaw Stock Exchange (I.1998) to January 2008. We call this possibility hypothetical due to unavailability of derivatives on VIX index in the analyzed time period. However, VIX derivatives has noticeably developed (options and futures quoted on CBOE) since then, making our analysis relevant from the point of view of valuation of the possible use of given instruments in Polish market, where there are neither corresponding indices nor index derivatives.

Table 7.1. Descriptive statistics for VIX index for period 1998-2008 – daily data.[a]

statistics		**20.01.1998 – 22.01.2008**	**20.01.1998 – 16.04.2003**	**16.04.2003 – 22.01.2008**
N		2 439	1 274	1 165
Mean		0,207	0,255	0,154
Median		0,204	0,243	0,146
Variance		0,0049	0,0030	0,0015
Std Dev		0,070	0,055	0,039
Minimum		0,099	0,162	0,099
Maximum		0,457	0,457	0,311
Kurtosis		0,188	0,999	0,903
Skewness		0,649	1,070	1,048
P1		0,103	0,174	0,102
P5		0,113	0,187	0,108
P10		0,120	0,197	0,112
P90		0,299	0,332	0,208
P95		0,335	0,367	0,233
P99		0,406	0,426	0,272
Test for Normality				
Kolmogorov-Smirnov	**Statistic**	0.061738	0.101001	0.096253
	p-value	<0.0100	<0.0100	<0.0100
Cramer-von Mises	**Statistic**	2.102419	4.171041	3.868214
	p-value	<0.0050	<0.0050	<0.0050
Anderson-Darling	**Statistic**	18.75417	25.29076	24.29215
		<0.0050	<0.0050	<0.0050

[a] The table presents basic descriptive statistics calculated on daily data: observations number, mean, median, variance, standard deviation, minimum, maximum, percentiles, kurtosis, skewness, tests for normality for the full research period. VIX – calculations on data aggregated at the daily level.

In order to show the features of a hedging strategy extensivelly, we present descriptive statistics for VIX index in period January 1998 – January 2008 (see Table 7.1.) and VIX fluctuations (see Figure 7.2) for the same period in order to picture the properties of that volatility measure and accentuate two significant volatility sub-periods in the period analyzed (volatility clustering). Table 7.1 contains statistics for the full research period as well as statistics divided into two sub-periods: of high and volatile volatility as well as low and significantly stable volatility.

The above descriptive statistics show a definitely broader range of volatility fluctuations of VIX index in the period 1998-2008 (signalized by the distance between minimum and maximum as well as high standard deviation) in comparison with data from the period 2003-2008 (see Table 5.2). They also demonstrate lack of normality (tests for normality), slight kurtosis and a positive skewness. Considerably more interesting conclusions can be drawn on the basis of sub-periods analysis (see Table 7.1). Not only does it suggest the existence of volatility clustering, but it also points at two characteristic periods in the analyzed 10-year history of VIX. The first period (January 20, 1998 – April 16, 2003) connected with a high volatility and volatility instability (high value of StD, variance and spread of fluctuations) can be easily explained if one considers numerous market turmoils observed in the analyzed period: Russian and LTCM crisis (1998), the end of internet boom (2000) and setting minimums following the end of a bear market (2002-2003). In the second period we can observe lower volatility and its definitely smaller fluctuations. This fact corresponds with American bull market driven by the fall of economic uncertainty and decreasing volatility.

Figure 7.1 confirms the data presented in the Table 7.1, and additionally reflects the processes discussed (i.e. periods of high and low volatility). What is more, a thorough analysis of the figure suggests a probable another change resulting from the beginning of a new high volatility period with the end of 2007. The analysis of Figure 7.1 and Table 7.1 depicts volatility clustering effect and mean reverting process.

Figure 7.1. VIX index for period January 20, 1998 – January 22, 2008; calculated on daily data.[a]

[a] VIX – volatility expected over 30 consecutive calendar days, daily data

Above-mentioned conclusions and data presented in last columns of Tables 6.1 and 6.2 (comparison of VIX and FW20, strong negative correlation between VIX and FW20 especially visible on smoothed data) inclined us to define strategy B based on the investment in FW20 and VIX index. With the purpose of presenting value added resulting from employment of volatility-based instrument we decided to simulate the results of strategy B. We compared average daily returns from WIG20 futures and average daily returns from synthetic portfolio which was composed of WIG20 futures and VIX index (shares x% and (1-x)%). Assumed investment period was January 1998 (first FW20 quotation on Warsaw Stock Exchange) – January 2008 (end of tests). At first the portfolio was based only on FW20 (100%). VIX was added when its level was falling below 1st percentile (strategies: B0, B1, B2, B3, B4, B5, see Table 7.2) or below 5th percentile (strategies: B0', B1', B2', B3', B4', B5', see Table 7.3.). Percentiles were calculated daily for the last 250 quotations (1-year equivalent). Investment in VIX lasted for n days (n=10, 21, 42, 63, 126). Results showing correlation coefficient, rates of return, standard deviations as well as investment period for analyzed strategies are presented in Tables 7.2 and 7.3.

Table 7.2. Results of investment in FW20 contract vs. complex portfolio (FW20 and VIX); based on 1st percentile.[a]

instrument	statistics	n = 10 [b)]	n = 21	n = 42	n = 63	n = 126
		VIX < 1th percentile [c)]				
FW20	**r**	0.0002849	0.0002849	0.0002849	0.0002849	0.0002849
	sd	0.01850	0.01850	0.01850	0.01850	0.01850
Strategy_B1: 60/40 [d)]	**r**	0.0006396	0.0004974	0.0006277	0.0007824	0.0005593
	sd	0.02102	0.02217	0.02466	0.02489	0.02793
Strategy _B1: 50/50	**r**	0.0005804	0.0004620	0.0005705	**0.0006995**	0.0005135
	sd	0.02011	0.02086	0.02249	**0.02244**	0.02453
Strategy _B3: 40/60	**r**	0.0005213	0.0004266	0.0005134	0.000616	0.0004678
	sd	0.01937	0.01979	0.02065	0.02039	0.02157
Strategy _B4: 30/70	**r**	0.0004622	0.0003911	0.0004563	0.0005336	0.0004221
	sd	0.01882	0.01898	0.01925	0.01885	0.01927
Strategy _B5: 20/80	**r**	*0.0003557*	*0.0003557*	*0.0003991*	*0.0004507*	*0.0003763*
	sd	*0.01849*	*0.01849*	*0.01838*	*0.01798*	*0.01788*
Strategy _B6: 10/90	**r**	*0.0003440*	*0.0003203*	*0.0003420*	*0.0003678*	*0.0003306*
	sd	*0.01838*	*0.01832*	*0.01813*	*0.01785*	*0.01760*
Corr_VIX_FW20		-0.26	-0.26	-0.26	-0.26	-0.26
p-value		<0.0001	<0.0001	<0.0001	<0.0001	<0.0001

[a] Table presents average daily rate of return and standard deviation on investment in WIG20 futures (FW20) and separately in portfolio (consisted of FW20 and VIX). [b] period of holding VIX in portfolio, [c] condition suggesting when to purchase VIX, [d] describes portfolio mix, e.g. 60/40 indicates respectively 60% VIX and 40% FW20, [e] an italic type font indicates portfolios of higher return at lower risk level.

Results in Tables 7.2 and 7.3 indicate a significantly higher rate of return on portfolio (even above 100%) than on individual investment in FW20. Portfolio investment risk expressed by standard deviation only slightly exceeds FW20 risk (several to a dozen or so percent). We can also identify strategies which provide higher return (about 30-50%) at a lower or the same level of risk (strategies: B4, B4', B5 and B5' for all n days).

The results presented in Tables 7.2 and 7.3 confirm a significant negative correlation between VIX index and WIG20 futures. Additionally, they picture possibilities of use of given instruments in various investment strategies as well as advanced risk management models, where volatility plays a key role. Given results point at the possibility of increasing return at a simultaneous stable initial level of standard deviation. Moreover, they show how to significantly increase return at slightly increased investment risk.

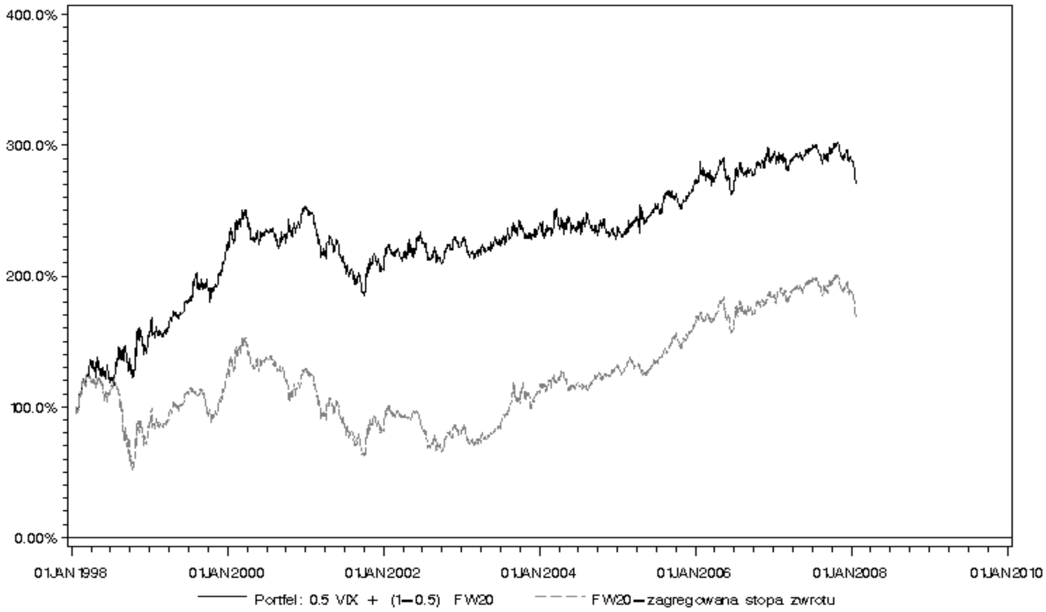
Table 7.3. Results of investment in FW20 contract vs. complex portfolio (FW20 and VIX); based on 5th percentile.[a]

Instrument	**statystics**	**n = 10**[b)]	**n = 21**	**n = 42**	**n = 63**	**n = 126**
		VIX < 5 th percentyl [c)]				
FW20	**r**	0.0002849	0.0002849	0.0002849	0.0002849	0.0002849
	sd	0.01850	0.01850	0.01850	0.01850	0.01850
Strategy_B1':60/40 [d)]	**r**	0.0007046	0.0006298	0.0007589	0.0005861	0.0004742
	sd	0.02366	0.02491	0.02663	0.02787	0.02960
Strategy _B2':50/50	**r**	0.0006346	0.0005723	**0.0006799**	0.0005359	0.0004427
	sd	0.02184	0.02266	**0.02372**	0.02449	0.02571
Strategy _B3': 40/60	**r**	0.0005647	0.0005148	0.0006009	0.0004857	0.0004111
	sd	0.02031	0.02076	0.02122	0.02156	0.02229
Strategy _B4': 30/70	**r**	0.0004947	0.0004573	0.0005219	0.0004355	0.0003795
	sd	0.01916	0.01930	0.01929	0.01927	0.01956
Strategy _B5': 20/80	**r**	*0.0004248*	*0.0003999*	*0.0004429*	*0.0003853*	*0.0003480*
	sd	*0.01844*	*0.01839*	*0.01811*	*0.01789*	*0.01786*
Strategy _B5': 10/90	**r**	*0.0003548*	*0.0003424*	*0.0003639*	*0.0003351*	*0.0003164*
	sd	*0.01822*	*0.01811*	*0.01784*	*0.01761*	*0.01748*
Corr_VIX_FW20		-0.26	-0.26	-0.26	-0.26	-0.26
p-value		<0.0001	<0.0001	<0.0001	<0.0001	<0.0001

[a] Table presents average daily rate of return and standard deviation on investment in WIG20 futures (FW20) and separately in portfolio (consisted of FW20 and VIX). [b] period of holding VIX in portfolio, [c] condition suggesting when to purchase VIX, [d] describes portfolio mix, e.g. 60/40 indicates respectively 60% VIX and 40% FW20, [e] an italic type font indicates portfolios of higher return at lower risk level.

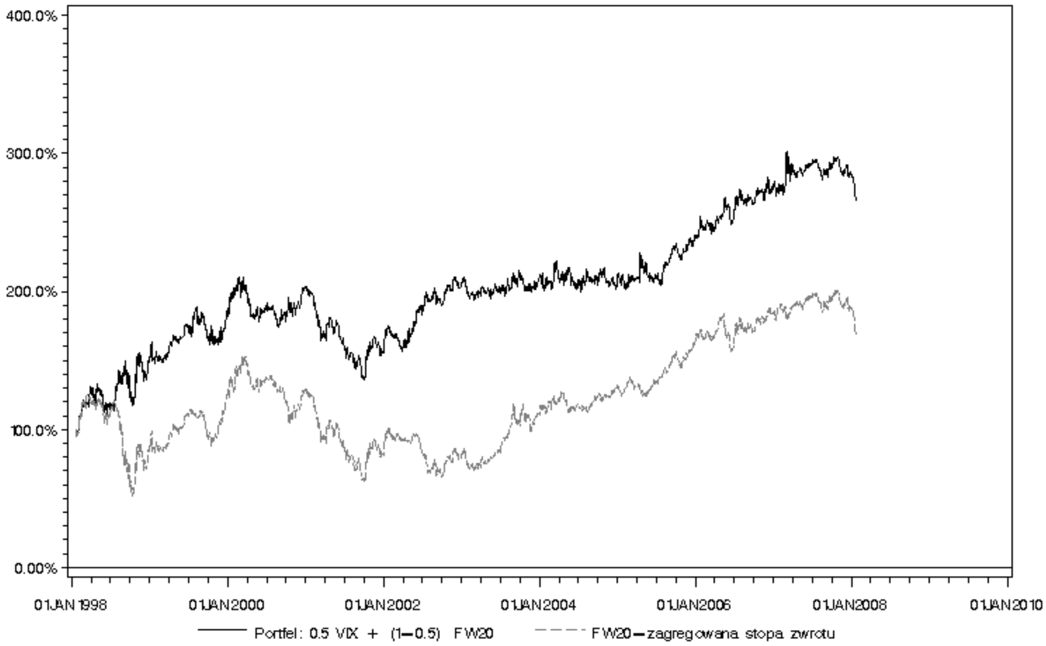
Analyzing the results from Tables 7.2 and 7.3 as well as Figures 7.2 - 7.3 one more time we draw our attention to the influence of parameter n, which was to reflect the mean length of mean/minimum reversion process. Differences respectively in rates of return, standard deviations as well as accumulated rates of return presented in the figures suggest that appropriate estimation of the length of the process plays an important role in the use of properties of volatility indices. On the other hand, the value of parameter n does not play a key role in the light of our study and conclusions. As far as simulation for 10-year period is concerned, the best results are obtained for n = 63 days. It should also be noticed that uncomplicated construction of the strategy analyzed was to draw one's attention only to the impact of a new element, which is volatility index (VIX index), quite apart from *timing* techniques which one can use to correct the final rate of return. The use of VIX instead of VIW20 as well as simulation comprising years 1998-2008 enabled to show how the strategy works over a longer time period, where its effectiveness was significantly more noticeable.

Figure 7.2. Results of Strategy_B1_63_1st percentile *versus* FW20.[a]



[a] Strategy B1 initiates when VIX is below 1st percentile. Investment period in VIX is 63 days. Strategy behavior is compared with 100% capital investment in FW20. The figure shows accumulated rate of return with the assumed initial investment of 100.

Table 7.3. Results of Strategy_B1'_42_5th percentile *versus* FW20.[a]



[a] Strategy B1' initiates when VIX is below 5th percentile. Investment period in VIX is 42 days. Strategy behavior is compared with 100% capital investment in FW20. The figure shows accumulated rate of return with the assumed initial investment of 100.

To summarize this section, we underline the possibility of use of VIX index (volatility index for developed market) in construction of investment strategies based on derivatives on WIG20 (stock market index for emerging market). Successfulness of strategy B and B' bases on strong negative correllation between VIX and FW20 (especially visible on smoothed data), long memory phenomenon as well as mean reverting process observed in volatility series.

In the further part of the paper, we describe the consequences of our results for derivatives market development in emerging markets. We also discuss possibilities of investment strategy construction during the periods of strong turmoils in worldwide capital markets.

8. Do emerging markets really need to develop advanced volatility risk measures?

Conclusions drawn in empirical part of the paper incline us to consider the need for introducing advanced volatility risk measures in order to ensure a stable development of emerging markets, as it plays the crucial role especially in the time of market turmoils.

The results of the study show that volatility index (VIW20) serves as a perfect barometer of WIG20 behavior and at the same time measures market expectations of the amplitude of WIG20 fluctuations. The main advantage of WIW20 index is the possibility of hedging volatility risk regardless of future price movement direction. What we also observe is the fact that negative correlation between volatility and stock index does not disappear in the moments of extreme market behavior, as it happens in standard instruments, but additionally gets stronger. This property makes it possible to hedge both volatility risk and the risk of price change in investment strategies, where the costs of standard hedging strategies increase significantly in moments of high volatility. In reality, market risk, in the sense of price change risk, can be hedged with many kinds of derivatives. Nevertheless, the strategy of hedging works effectively only up to the moment of strong growth of market volatility, when even the most advanced strategies do not bring expected results due to the following consequences: increased base risk, change in basic correlations between assets, sharp decrease in liquidity. All of these have a significant impact on investment cost and result in inadequacy of standard risk management approaches in periods of extreme market behavior.

To sum up the results of empirical analysis, we do think that there is a serious need to introduce VIW20 index or a similar volatility measure on Warsaw Stock Exchange in order to enable further development of derivatives market, *inter alia* by offering a broader investment palette. Such step would additionally encourage and enable professional investors and risk managers to actively reallocate a portfolio of derivative instruments, consequently leading to further intense development of structured products. What is of key importance in the analysis of potential growth of derivatives market after the introduction of volatility index and volatility-based derivatives is developing advanced investment strategies for both futures and options alike. Moreover, availability of volatility-based derivatives would facilitate the process of risk management and Warsaw Stock Exchange would be given a chance to become the first European emerging market enabling large number of investors to hedge volatility risk. What we also notice is the necessity of introducing options with shorter and longer expiration terms (weekly, monthly, 2-year period, 3-year period) which would make it possible to: reflect volatility surface, calculate volatility indices of varied time horizons of anticipated volatility and manage volatility risk using volatility-based derivatives.

Obviously, increasing globalization of financial markets provides alternative solutions, i.e. hedging WIG20 portfolio with volatility index from American equity market (VIX index), which has already been discussed in Section 7. To a certain extend, it is only a substitute solution for investors operating in Polish market, on the other hand, it will remain the only way out if Warsaw Stock Exchange does not introduce appropriate indices and volatility-based derivatives in the nearest future.

9. Summary. Value added to the paper. Directions for future investigations.

In this section we summarize the conclusions drawn from theoretical and practical investigations. In addition, we underline value added to the paper as well as the importance of the analyzed subject for development of Polish derivative market. In this place we also want to determine directions for future investigations.

The aim of collected theoretical and empirical material was to illustrate the possibility of calculating volatility index on intraday data from Polish capital market and during a trading session. Another objective was to focus the attention of Polish market participants on the necessity of introducing volatility-based derivatives. Value added to the paper can be summarized as follows:

- the first volatility index for European emerging market, enabling to quantify volatility risk on Polish capital market
- basing on high frequency data, the index calculates volatility during a trading session, when the information of its current level is of key importance for market participants
- the first instrument enabling to quantify risk and providing foundations for building volatility-based derivatives, therefore applicable to institutions operating on capital market: investment funds, Open-Ended Pension Funds, Asset Management as well as to individual investors

- theoretical framework – presentation of corrections in the methodology, which enable to apply the VIX formula on Polish equity market, to be more precise on Polish options market, which being in initial phase of development shows some imperfections not present on developed markets.

Modification of VIX methodology enabled us to calculate VIW20, i.e. volatility index for Polish equity market, the first synthetic volatility measure calculated on high-frequency data, enabling the assessment of stock market risk in a given time moment. Detailed analysis of VIW20 in comparison with VIX index and stock market indices, constituting the base for calculated volatility indices (WIG20, S&P500) made it possible to define a few significant properties of VIW20:

- strong and statistically significant negative correlation between volatility indices and given stock market indices, particularly high in points of local minimums and maximums of stock market indices (WIG20, S&P500) but dependent on assumed period for which volatility is anticipated.
- possibility of using VIW20 properties in investment strategies. VIW20 index enables greater rates of return at a stable level of risk even in case of simple *buy&hold* strategy for index portfolio.
- distribution analysis of VIW20 index additionally confirmed some properties of volatility series which had already been observed in relation to realized volatility: volatility clustering, mean and minimum reverting process, long memory process.
- An additional property is the fact that volatility index estimates volatility on the basis of data from one interval and does not require any additional information.

The analysis of the properties of volatility indices as well as volatility-based derivatives enables to list a few key functions they perform on capital markets:

- informative – enables the objective assessment of equity market volatility; constitutes an excellent point of reference for designing investment strategies and forecasts based on econometrical models. Much as it enables inexperienced investors to assess the risk of current investments by means of comparison of a current volatility level with its historical data, this function is not thoroughly discussed in our article
- investing – as a speculative instrument for active, volatility risk-exposed investors, making use of advanced strategies based on index futures and options, who require volatility based instrument, as the component of their portfolio what results in increasing liquidity of index based derivatives,
- diversifying – for investors optimizing their equity portfolio in order to increase its rate of return while minimizing the risk,
- ensuring development of additional instruments – enables to build new and more advanced structured products based on volatility derivatives.

Exhaustive investigation of the subject suggests directions for future explorations of broadly understood concept of volatility, ways of its modeling and prediction as well as implication of theoretical models for practical purposes. Undoubtedly, the paper does not provide extensive analysis of prognostic value of the analyzed concept. Nevertheless, the main aim of the study was to provide the detailed description of described for the first time volatility index. In future investigations we will focus on the relation between assumed period, for which volatility is anticipated in volatility index (30-day period – VIX, 91-day period – VIW20), and the actual volatility of market calculated with appropriate estimator (RV – realized volatility) after the given number of days from now. In our next study we would also like to determine appropriate indices anticipating volatility for the period of 30, 91, 182 and 365 calendar days both for Polish and American equity market. On their basis we would like to analyze volatility term structure. What also needs a deeper analysis is the relation between volatility indices of emerging and developed markets. In other words, we want to answer the question of how quickly volatility shocks are transferred between markets. Another area that falls within the scope of our interest is the study of advanced investment strategies which combine volatility indices and market indices mainly to lower investment risk.

Finally, focusing on practical conclusions and referring to criticism about introducing volatility derivatives in the light of an early phase of development of Polish derivatives market, we ask the question: *'Are result and cause really put in the right order?'* By suggesting the introduction of given instruments, we claim that such a step, being connected with a broader investment and hedge palette, would contribute to further development of the market and strengthen the position of Warsaw Stock Exchange.

Appendix. Detailed methodology for calculating VIW20.

In this part we present the abridged description of methodology for calculating VIW20 index as well as necessary corrections in comparison to original methodology[13]. The generalized formula for calculation of VIW20 index looks as follows:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} * Q(K_i) - \frac{1}{T}[\frac{F}{K_0} - 1]^2 \qquad (A.1)$$

where:

$$\sigma = \frac{VIW20}{100} \Rightarrow VIW20 = \sigma * 100 \qquad (A.2)$$

T - time to expiration (in years; with minutes accuracy),

F - forward price, derived from option prices,

K_i - strike price of ith out-of-the-money option; a call if K_i>F and a put if K_i<F,

ΔK_i - interval between strike prices - half of the distance between the higher and lower strike price in relation to K_i:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2} \qquad (A.3)$$

for the lowest strike price this variable is the difference between the two lowest strike prices; likewise, for the highest strike price this variable is the difference between the two highest strike prices,

K_0 - first strike price below the forward index level, (F)

R - risk-free interest rate

$Q(K_i)$ - the midpoint spread (0.5*(bid+ask)) for each option with strike price K_i.

The VIW20 calculation formula uses call and put options with the two nearest expiration terms to cover the assumed 91-day calendar period, for which VIW20 is expected to estimate volatility.

Let us assume that we calculate the VIW20 at 10:00 a.m. and that the time to expiration of options is 44- day (series A) and 135-day period (series B). The time to expiration (T) is calculated in minutes rather than days in order to assure the precision. The time to expiration is calculated in the following way:

$$T = \{M_{\text{current day}} + M_{\text{expiration day}} + M_{\text{other days}}\}/T_{365} \qquad (A.4)$$

[13] see: VIX White Paper, 2003.

where:

$M_{current\ day}$ - number of minutes remaining until midnight of the current day,
$M_{expiration\ day}$ - number of minutes from midnight until 4:30 p.m. on expiration day,
$M_{other\ days}$ - number of minutes between current day and expiration day,
T_{365} - number of minutes in 365 days,

Time to expiration for series A and series B is:

$$T_A = \{840 + 990 + 61920\} / 525600 = 0.121289954$$
$$T_B = \{840 + 990 + 192960\} / 525600 = 0.370606022$$

The risk-free interest rate is assumed to be WIBOR-3M which for the given data is 5.11%. For the sake of simplicity, the same number of options is used for both series. In practice, however, one should remember that such a situation does not have to take place.

STEP 1. We select the options to be used in the VIW20 calculation.

We determine the Forward index level (F) on the basis of at-the-money option price. Chosen at-the-money strike price should minimize the difference between call and put prices. Table A.1. shows the process of choosing appropriate strike price for calculation of Forward index level, both for series A and B, for a hipothethical trading session.

Table A.1. Selection of appropriate strike price for calculation of Forward index level for series A and B.[a]

Seria A				Seria B			
Strike Price	Call	Put	|Difference|	Strike Price	Call	Put	|Difference|
3500	349.00	66.9	282.1	3500	445.5	156.3	289.2
3600	270.50	92.2	178.3	3600	372.7	189.3	183.4
3700	200.50	124.5	76	3700	310.1	226.9	83.2
3800	143.25	164.6	21.35	3800	254.7	269.1	14.4
3900	97.75	213.3	115.55	3900	206.5	328.0	121.5
4000	74.4	270.8	196.4	4000	169.8	372.8	203
4100	44.3	341.6	297.3	4100	139.3	434	294.7

[a] Calculations for a hypothetical trading session in October 2007, 10:00 a.m., WIG20 = 3806

In order to calculate the Forward index level we use the following formula:

$$F = K + e^{RT} * (c - p) \qquad (A.5)$$

where:

K - chosen strike price,
c - call price,
p - put price,

Using the data from the Table A.1, we calculate the Forward index price for series A and B:

$$F_A = 3800 + e^{(0.051*0.121289954)} * \left|143.25 - 164.4\right| = 3821.48$$

$$F_B = 3800 + e^{(0.051*0.370606022)} * |254.7 - 269.1| = 3814.67$$

Next, we determine K_0 – the strike price immediately below the Forward index level (F). In our case, K_0 = 3800 for both expiration terms.

Further, we sort the options in ascending order by strike price. We select call options that have strike prices greater than K_0 and a non-zero bid price. We add to the table consecutive options until we encounter two consecutive options with a bid price of zero. The same rules apply to selecting put options. First, we select put options that have strike prices less than K_0 and a non-zero bid price. We add to the table consecutive puts until we encounter two consecutive options with a bid price of zero. What we put in the table is the averaged price of selected option type; in case of options of a strike price of K_0 we put half the price between call and put options. Table A.2. presents the above-mentioned method of selecting options.

Table A.2. Options used in determining VIW20 index.[a]

Series A			Series B		
Strike Price	Option Type	Averaged Price	Strike Price	Option Type	Averaged Price
3500	Put	66.9	3500	Put	156.3
3600	Put	92.2	3600	Put	189.3
3700	Put	124.5	3700	Put	226.9
3800	Call i Put	153.9	3800	Call i Put	261.9
3900	Call	97.75	3900	Call	206.5
4000	Call	74.4	4000	Call	169.8
4100	Call	44.3	4100	Call	139.3

[a] Calculations for a hypothetical trading session in October 2007, 10:00 a.m., WIG20 = 3806

STEP2. Calculating volatility for options with shorter and longer time to expiration.

In order to calculate volatility for the shorter and longer time to expiration, i.e. T_A (series A) and T_B (series B), we apply the formula (9).

$$\sigma_A{}^2 = \frac{2}{T_A}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT_A} * Q(K_i) - \frac{1}{T_A}[\frac{F_A}{K_0} - 1]^2$$

$$\sigma_B{}^2 = \frac{2}{T_B}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT_B} * Q(K_i) - \frac{1}{T_B}[\frac{F_B}{K_0} - 1]^2$$

The VIW20 index bases on the information reflected in quotations of all the options used in calculation process. The contribution of a single option to the index level is proportional to the price of that option and inversely proportional to its strike price, e.g. the contribution of the option of the strike price 3900 to volatility for the shorter term is:

$$\frac{\Delta K_{3900\_Call}}{K^2_{3900\_Call}} e^{R*T_A} * Q(3900\_Call)$$ [14]

[14] ΔK_i is half the distance between the strike price on either side of K_i, with the exception of two extreme strike prices

The result of the calculation is the contribution of the above option of the given strike price to the final volatility value for the given expiration term:

$$\frac{\Delta K_{3900\_Call}}{K^2_{3900\_Call}} e^{R*T_A} * Q(3900\_Call) = \frac{100}{3900^2} e^{(0.051*0.121289954)} * 97.75 = 0.000647$$

Similar calculations are performed for all the options from Table A.2. Then, we sum the values for series A and multiply them by 2/T. Analogical calculation is performed for series B. Our next step is to determine the value: $\frac{1}{T}\left[\frac{F}{K_0}-1\right]^2$ for both the longer and shorter expiration term. Table A.3. presents the results of the above calculations as well as calculations of σ_A^2 and σ_B^2.

Table A.3. Method for determination of σ_A^2 and σ_B^2.[a]

Series A				Series B			
Strike Price	Option Type	Mid-quote Price	Contribution by strike	Strike Price	Option Type	Mid-quote Price	Contribution by strike
3500	Put	66.9	0.000550	3500	Put	156.3	0.001300
3600	Put	92.2	0.000716	3600	Put	189.3	0.001489
3700	Put	124.5	0.000915	3700	Put	226.9	0.001689
3800	Call i Put	153.9	0.001072	3800	Call i Put	261.9	0.001848
3900	Call	97.75	0.000647	3900	Call	206.5	0.001384
4000	Call	74.4	0.000468	4000	Call	169.8	0.001081
4100	Call	44.3	0.000265	4100	Call	139.3	0.000844
$\frac{2}{T_A}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT_A} * Q(K_i)$			0.076346	$\frac{2}{T_B}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT_B} * Q(K_i)$			0.051996
$\frac{1}{T_A}\left[\frac{F_A}{K_0}-1\right]^2 = \frac{1}{0.121289954} * \left[\frac{3821.48}{3800}-1\right]^2$			0.000263	$\frac{1}{T_B}\left[\frac{F_B}{K_0}-1\right]^2 = \frac{1}{0.370606022} * \left[\frac{3814.67}{3800}-1\right]^2$			0.000040
$\sigma_A^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT_A} * Q(K_i) - \frac{1}{T_A}\left[\frac{F_A}{K_0}-1\right]^2$			0.076346 - 0.000263 = **0.076083**	$\sigma_B^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT_B} * Q(K_i) - \frac{1}{T_B}\left[\frac{F_B}{K_0}-1\right]^2$			0.051996 - 0.000040 = **0.051956**

[a] Calculations for a hypothetical trading session in October 2007, 10:00 a.m., WIG20 = 3806

STEP 3. We interpolate σ_A^2 and σ_B^2 in order to obtain a single value of a constant 91-day maturity (3 months). We take the square root of the value and multiply it by 100. What we obtain is the final value of VIW20.

considered in the calculation process. For maximum and mininimum of K, ΔK_i is simply the difference between K_i and the adjacent strike price, e.g. $\Delta K_{4100\ Call}$ = 4000-3900 = 100.

$$\sigma = \sqrt{\left\{ T_A \sigma_A^2 \left[\frac{N_{T_B} - N_{91}}{N_{T_B} - N_{T_A}} \right] + T_B \sigma_B^2 \left[\frac{N_{91} - N_{T_A}}{N_{T_B} - N_{T_A}} \right] \right\} * \frac{N_{365}}{N_{91}}} \qquad (A.6)$$

where:

N_{TA} - number of minutes to expiration of series A, shorter expiration term,
N_{TB} - number of minutes to expiration of series B, longer expiration term,
N_{91} - number of minutes in 91 days (91 * 1440 = 131 040),
N_{365} - number of minutes in 365 days (365 * 1440 = 525 600).

$$VIW20 = 100 * \sigma \qquad (A.7)$$

For our data the above calculation and a final value of σ as well as VIW20 are:

$$\sigma = \sqrt{\left\{ 0.121289954 * 0.076083 * \left[\frac{194790 - 131040}{194790 - 63750} \right] + 0.370606022 * 0.051996 * \left[\frac{131040 - 63750}{194790 - 63750} \right] \right\} * \frac{525600}{194790}}$$

$$\sigma = 0.197008$$

$$VIW20 = 100 * \sigma = 19.70$$

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