University of Warsaw
Faculty of Economic Sciences

Working Papers
No. 27/2014 (144)

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Options delta hedging with no options at all

Warsaw 2014

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Abstract

The adjustment speed of delta hedged options exposure depends on the market realized and implied volatility. We observe that by consistently hedging long and short positions in options we can eventually end up with pure exposure to volatility without any options in the portfolio at all. The results of such arbitrage strategy is based only on speed of adjustment of delta hedged option positions. More specifically, they rely on interrelation between realized volatility levels calculated for various time intervals (from daily to intraday frequency). Theoretical intuition enables us to solve the puzzle of the optimal frequency of hedge adjustment and its influence on hedging efficiency. We present results of a simple hedge strategy based on the consistent hedging of a portfolio of options for various worldwide equity indices.

Keywords:
options hedging efficiency, optimal hedging frequency, realized and implied volatility, index futures, investment strategies

JEL:
G11, G14, G15, G23, C61, C22

Acknowledgments:
We gratefully acknowledge government financial support via grant no. UMO-2011/03/B/HS4/02298.
The views presented in this text are those of the authors and do not necessarily represent those of the National Bank of Poland or Union Investment TFI S.A.

1 Introduction

The puzzle of optimal hedging frequency is a well-known problem in quntitative finance and researchers try to solve it focusing on many different issues. Firstly, we can focus on various sepcification of option pricing models, starting from BSM model (Black and Scholes [1973], Merton [1973]) and going to many more advanced including stochastic volatility (Heston [1993], Hull and White [1987]), price jumps (Doman and Doman [2009]) or regime switching (autor, rok), etc. Secondly, we have to choose the frequency of adjustment of our delta parameter, which is naturally dependent on the chosen option pricing model but strongly affects the efficiency of our hedge through noise trading, transaction costs, etc. Thirdly, we have to differentiate periods when it is optimal to hedge our options exposure more often compared to periods when more optimal is to hedge less frequent (eg. based on the comarison of volatiltiy estimators calculated with various frequencies). These and many other issues will help us to answer the question concerning optimal frequency in option hedging and possibility to design profitable investment strstegy based on RV disequilibrium.

2 Options as volatility instruments

Exposure to volatility arises naturally in delta hedged options positions. To see this, recall some basic delta hedging arithmetic and consider a trader who writes a call option C on an underlying S with implied volatility Σ and hedges away the delta risk by going long $(\partial C/\partial S)$ units of the underlying instrument (typically a futures contract). Observe that

$$C\left(S+\Delta S,t+\Delta t\right)=C(S,t)+\frac{\partial C}{\partial S}\Delta S+\frac{\partial C}{\partial t}\Delta t+\frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\Delta S)^2+... \tag{1}$$

Hence

$$\Delta C\left(S,t\right)=\frac{\partial C}{\partial S}\Delta S+\frac{\partial C}{\partial t}\Delta t+\frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\Delta S)^2+... \tag{2}$$

And the profit-and-loss, $P\&L$, on the trader's delta hedged option position can be expressed as

$$P\&L=d\left(-C(S,t)+\frac{\partial C}{\partial S}S\right)\approx-\frac{\partial C}{\partial t}\Delta t-\frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\Delta S)^2, \tag{3}$$

where the approximation stems from the fact that we omit the higher order derivatives in the Taylor expansion. Since the option was written at Black-Scholes implied volatility Σ, it also satisfies the Black-Scholes differential equation, i.e.:

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\Sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC. \tag{4}$$

Assuming that the risk free rate r is close to zero (which does not seem extravagant given currently low interest rate environment globally), equation (4) reduces to

$$\frac{1}{2}\Sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} \approx -\frac{\partial C}{\partial t}, \tag{5}$$

or equivalently, using the conventional notation for option Greeks, $\frac{1}{2}\Sigma^2 S^2 \Gamma \approx -\Theta$. Plugging equation (5) into (3) we obtain that the trader's daily P&L on a delta hedged option position is given by

$$P\&L \approx \frac{1}{2}\Gamma S^2 \left(\Sigma^2 \Delta t - \left(\frac{\Delta S}{S}\right)^2\right). \tag{6}$$

Hence, the total profit and loss accummulated until expiration can be approximated by

$$Total\ P\&L \approx \sum_{t=0}^{T} \frac{1}{2}\Gamma_t S_t^2 \left(\Sigma^2 \Delta t - \left(\frac{\Delta S_t}{S_t}\right)^2\right). \tag{7}$$

The equations above suggest that the trader delta hedging an option is essentially exposed to the difference between the implied volatility at which the option was written, Σ, and the volatility (variance) actually realized in the process of delta hedging the position. Thus, if realized volatility is greater than implied volatility, our trader should lose money. Note, however, that the exposure to realized volatiliy is weighted by the product of option gamma Γ and stock price at t. This introduced path-dependence into the trader's position manifested in the fact that if implied volatility is higher than realized volatility throughout most of the period $[0, T]$ with low Γ, but realized volatility happens to spike exactly at a time when Γ is high (i.e. S is near the money), then the trader's P&L can suffer badly. A simple example to this effect is shown in Figure 1.

3 Replicating realized volatility

As we have seen above, delta hedging a single option results in a path dependent P&L, in which changes in the price of the underlying instrument affect the position's exposure to volatility (through the varying gamma). By equation (7), to earn pure realized volatility (net of some implied volatility strike Σ^2) the delta hedged contract would have to have a gamma equal exactly $1/S_t^2$. Since $\Gamma = \partial^2 C/\partial S^2$, simple integration shows that an option C satisfying this condition would have to have a log-payoff profile, as $d^2(\log(x))/dx^2 = -x^{-2}$. Although such a log contract is not traded in practice, Derman et al. have shown that it is synthetically equivalent to an infinite strip of options.

To see why adding more options (with different strikes) purifies the exposure to volatility consider the following example shown in Figure 2. Moving from only 3 options (with strikes 50, 100 and 150) to 10 and eventually 100 options (spanning strikes from 1 to 199), portfolio gamma clearly flattens.

The formalism behind this result is due to Carr and Madan who show that any twice-differentiable payoff function f of the terminal underlying price S_T can be written as:

$$f(S_T) = f(S_0) + f'(S_0)(S_T - S_0) + \int_0^{S_0} f''(K)(K - S_T)^+ dK + \int_{S_0}^{\infty} f''(K)(S_T - K)^+ dK \tag{8}$$

where S_0 is the strike threshold (e.g. ATM underlying price) separating call from puts and $(\cdot)^+$ is shorthand for $\max(0, \cdot)$. Applying (8) to $f(S_T) = \log(S_T)$ we obtain

$$\log\left(\frac{S_T}{S_0}\right) = \frac{S_T - S_0}{S_0} - \int_0^{S_0} \frac{(K - S_T)^+}{K^2} dK - \int_{S_0}^{\infty} \frac{(S_T - K)^+}{K^2} dK \tag{9}$$

Note that if the evolution of the underlying is modeled as

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dW_t \tag{10}$$

then for $\log(S_t)$

$$d\log(S_t) = \left(\mu_t - \frac{1}{2}\sigma_t^2\right) dt + \sigma_t dW_t \tag{11}$$

which implies

$$\frac{1}{2}\sigma_t^2 dt = \frac{dS_t}{S_t} - d\log(S_t) \tag{12}$$

Summing over the whole interval $[0, T]$ we finally get that the total realized variance RV is given by

$$RV = \int_0^T \sigma_t^2 dt = 2\int_0^T \frac{dS_t}{S_t} - 2\int_0^T d\log(S_t) = 2\int_0^T \frac{dS_t}{S_t} - 2\log\left(\frac{S_T}{S_0}\right) \tag{13}$$

To recapitulate: the results (9) and (13) taken together imply that pure exposure to realized variance can be synthetically engineered through a dynamic position is $(1/S_t)$ shares of the underlying instrument and a static short position in a log payoff. In turn, the log payoff can be created synthetically out of a long position in $(1/K^2)$ call options struck at K for all strikes $K > S_0$ and a long position in $(1/K^2)$ put options struck at K for all strikes $K < S_0$.

4 Frequency volatility arbitrage strategy

The theoretical intuitions presented above suggest that to engineer pure exposure to realized volatility requires:

1. a static position in a strip of options;
2. a dynamic position in $(1/S)$ futures on the underlying instrument.

If we observe that realized volatility calculated at different frequencies (e.g. weekly vs. monthly, daily vs. weekly, hourly vs. daily etc.) exhibits a persistent pattern (e.g. hourly RV > daily RV > ...) we can construct a trading strategy that aims to take advantage of this. Specifically, the strategy should consist of:

1. long (short) static position in an option strip plus a dynamic position in $(1/S)$ futures on the underlying instrument adjusted daily
2. short (long) static position in an option strip plus a short dynamic position in $(1/S)$ futures on the underlying instrument adjusted weekly

The option strip nets out (hence we do not have to worry about the fact that in practice only a finite number of strikes is available as opposed to an infinite number required to replicate log payoff) and we are left with a combination of a long and short position in a futures contract that is adjusted at different frequencies. Based on the comparison of RV calculated on short vs. long-term we decide which futures position is adjusted daily and which is adjusted weekly.

5 Main hypothesis and research questions

Based on this theoretical introduction we can formulate our research hypotheses and research questions. The main hypothesis of this research is that interrelation of RV calculated on various time intervals is the way to extract in the process of options portfolio hedging P/L coming from realized volatility disequilibrium. Additional research questions are as follows:

- how should we set the frequency of volatilities in the switch (intraday, daily, weekly, monthly)?
- what kind of RV estimator should we use?
- what kind of volatility conditions are preffered by momentum approach versus mean-reverting approach?

6 Data and research description

6.1 Data

The research will be based on 5-minutes and daily data for the following equity indices: WIG20, Bovespa, Bux, Kospi, Dax, Nikkei, FTSE, SPX. These data are characterized by various volatility and peristence. Below please find desciptive statistics for daily data and charts for these indices (Table 1 and Figure 3)

The data shows that distribution of returns are leptokurthic and slightly negatively skewed with highest range, min and max values for emerging markets indices in comparison to developed markets. The fluctuations of the price series for all indices can be seen on Figure 3.

6.2 Research description

6.2.1 Calculation of volatility measure

At the begining we have to choose volatility estimator and frequencies which will be taken into account in the option hedging process. We decided to use realized volatility (RV) as one of the most common volatility estimators:

$$Realized\ Volatility = \frac{1}{n * T} \sum_{t=0}^{n*T} (logS_t)^2 \tag{14}$$

where: S_t is 5-minutes close price of the underlying instrument and in reality can be taken with many different frequencies. n means the number of days in price history and T is the number of 5-minutes intervals per day. In the next step this estimator is annnualized before we use it for comparison pruposes. In this research we will use two different price data histories:

- LT - long term period for volatility estimator calculation (e.g. price data history cover only one week)
- ST - short-term period for volatility estimators calculation (e.g. price data history cover only one day)

in order to make the decision concerning optimal hedging frequency.

The detailed descriptive statistics for all our RV measures for all equity indices calculated based on daily and weekly history we can find in Table 2 and their fluctuations can be found on Figure2.

Table 2 shows several interesting features describing Realized Volaitlity. RV calculated based on longer prices history (e.g. weekly vs. daily) is much smoother, has lower range, higher mean and lower SD. These characteristics are responsible for specific interrelations between RVdaily and RVweekly.

6.2.2 Strategy description

Based on RV estimators obtained in the first step we make the decision concerning the direction of short-term adjustment:

- RV(LT) > RV(ST) ==> we increase our positions in long futures in the same direction as market changes
- RV(LT) < RV(ST) ==> we increase our positions in long futures in the oppposite direction as market changes

The direction of adujstment can change at the end of each LT period. Detailed description of our intuition behind the strategy definition can be found below.

1. If RV(LT) < RV(ST) (we assume that daily and weekly RV is anualized)

   (a) buy strip of T options, delta-hedge daily −> RVdaily,T

   (b) sell strip of T options, delta-hedge weekly −> RVweekly,T

      i. LC+SFdaily and SC+LFweekly -> when S goes up -> daily: we sell fut in order to adjust hedge to increasing value of LC; weekly: we buy fut in inrder to adjust hedge to increasing value of SC

      ii. LP+LFdaily and SP+SFweekly -> when S goes up -> daily: we sell fut in order to adjust hedge to decreasing value of LP; weekly: we buy fut in order to adjust hedge to decreasing value of SP

      iii. LC+LP+ZeroFdaily and SC+SP+ZeroFweekly -> when S goes up -> daily: we sell fut in order to adjust hedge to increasing value of LC and to decreasing value of LP; weekly: we buy fut in order to adjust hedge to increasing value of SC and to decreasing value of SP

   Therefore, we basically use mean reverting strategy with total exposure adjusted weekly.

2. if weekly hist vol > daily hist vol: (we assume that daily and weekly RV is anualized)

   (a) sell strip of T options, delta-hedge daily −> RVdaily,T

   (b) buy strip of T options, delta-hedge weekly −> RVweekly,T

      i. SC+LFdaily and LC+SFweekly -> when S goes up -> daily: we buy fut in order to adjust hedge to increasing value of SC; weekly: we sell fut in inrder to adjust hedge to increasing value of LC

ii. SP+SFdaily and LP+LFweekly -> when S goes up -> daily: we buy fut in order to adjust hedge to decreasing value of SP; weekly: we sell fut in order to adjust hedge to decreasing value of LP

iii. SC+SP+ZeroFdaily and LC+LP+ZeroFweekly -> when S goes up -> daily: we buy fut in order to adjust hedge to increasing value of SC and to decreasing value of LP; weekly: we sell fut in order to adjust hedge to increasing value of LC and to decreasing value of LP

Therefore, we basically use momentum strategy with exposure adjusted weekly.

Above mentioned conditions are summarised in the Table 3.

6.2.3 Calculation of P/L based on the speed of adjustment for SPX

We start from calculation of \Delta P/L_{LT-ST}^{i} during LT period:

$$\Delta P/L_{LT-ST}^{i} = N * \sum_{t=1}^{k} \left(\frac{1}{S_t} - \frac{1}{S_{t-1}}\right) * (S_t - S_{t-1}) \tag{15}$$

and then we end with the formula for the whole period:

$$\Delta P/L_{Total} = \sum_{i=1}^{n} \Delta P/L_{LT-ST}^{i} \tag{16}$$

6.2.4 Calculation of equity lines for SPX

We add each daily P/L and create equity line for the given equity index based on which we compute all required risk and return statistics:

- ARC - annualized compunded return,
- aSD - annualized standard deviation of returns,
- IR - information return for absolute return funds,
- MD - maximum drawdown,
- IR/MD ratio

based on the same formulas like in Slepaczuk et al. [2012]

6.2.5 Sensitivity analysis

It is importatn to add that in order to fully interpret our research it is important to perform thorough sensitivity analysis including all the below mentioned.

- to volatility estimator formula/type,
- to frequency of adjustments (daily vs weekly, weekly vs monthly, daily vs intraday),
- transaction costs, bid/ask spreads, etc.
- the research for other equity indices

Due to volume constraints we will not be able to present complete sensitivity anaysis in this research and we will focus only on a few points from mentioned above but the remaining points will be the scope of our future research.

7 Empirical research

7.1 S&P500 index - the most developed market

We have started our research from the most developed market and almost the most recognizable equity index, i.e. S&P500 index. Below please find Table 4with the results of all strategies for this index (LVD, HVD and B&H strategy). We can see that the results for LVD strategy are much better than BH strategy and HVD. LVD has lower risk (aSD, asemiSD and MD) and at the same time much higher risk/return statistics (IR, Sharpe, ARC). The results for HVD strategy are completely different and we can see that such approach on highly developed market can lead us to total bakruptcy.

Figure 5presents the fluctuations of LVD and HVD strategies in comparison to BH strategy. We can clearly see that equity line for LVD is very stable with low MD and seldom strong spikes. We can see that spikes occur in the time of very high RV jumps (specifically when RV jumps above IV). HVD strategy behaves just opposite to LVD strategy presenting very low results,

Taking all the above mentioned into account we can say that based on the initial theoretical intuition we can create profitable strategy when the decision concerning hedging frequency and its direction is based on the comparison of realized volatility calculated based on various price history. Based on this initial results we were curious if such stable results can be repeated for other developed and emerging markets.

7.2 The case for other developed markets (FTSE, NIKKEI225, DAX)

As we can see in Table 5the situtation is not so stragihtforward in case of other developed markets. We can not distinguish any clear pattern in case of FTSE, NIKKEI225 and DAX index. Based on IR criterion we can see that LVD is the best strategy for FTSE index but in casse of NIKKEI225 and DAX the best strategy is HVD. The detailed fluctuations of all strategies for NIKKEI225, DAX, FTSE100 are prsented on Figure 6, 7 and 8.

Figure 6 shows that results for NIKKEI225 are quite ambiguous nad patterns visible for S&P500 are totally different in case of NIKKEI225.

The same observation regards results or FTSE100. We can see two quite substantial jumps at the end of the crash of internet bubble and during 2008 crisis but the problem is that they cause the strategy move in two different directions.

The last strategies for developed markets do not change previous results. We can see that LVD and HVD produce results much worse than simple B&H strategy.

7.3 The case for emerging markets (WIG20, KOSPI, BOVESPA)

Based on statistics presente in Table 2we can see that RV for emerging markets indexes is on average much higher than for developed markets and this was the reason why we have decided to checck reults of our strategy for these markets as well. Table 6 presents results for WIG20, KOSPI and BOVESPA indices.

The comparison based on IR ratio shows that the results are not dependent on the higher level of average RV. Once again we can observe that there are no any siginficant results between three presented emerging market economies. What is more important, we do not observe any siginificant differences between LVD, HVD and BH on Figures 9, 10 and 11 presenting results for Kospi, WIG20 and Bovespa.

We can see that besides the case for WIG20, LVD strategy does not present any interesting result with regard to maimizing risk/return ratio.

8 Summary

Taking into account presented results the summary of them is very difiicult issue. Based on the theoretical intuition and detailed description presented in section 6.2.2 we can say that empirical calculation do not much initial presumptions concerning the behaviour of the market based on the theory.

In order to summary obtained results we have to focus on several issues.Firstly we can observe very convincing results for the most developed economy i.e. S&P500 index where LVD strategy presents very promising results in comparison to BH strategy. Secondly, below mentioned results are not confirmed in case of other developed and emergin markets. Thirdly, we can not find the reason for this in differennces between realized volatiltiy fluctuations nor in the diffierences in distributions of returns of analyzed indices.

In our opinion the reason for such ambiguous results can be attributed to the highest development of USA markets and that based on many previous results it is the most efficient markets among all analysed in this paper what cause that our theoretical assumptions works only in case of this market. Further research should focus on a few important issues starting from the utilisation of other volatility estimators for switch purposes, other price histories and the inclusion of many various costs associated with trading.

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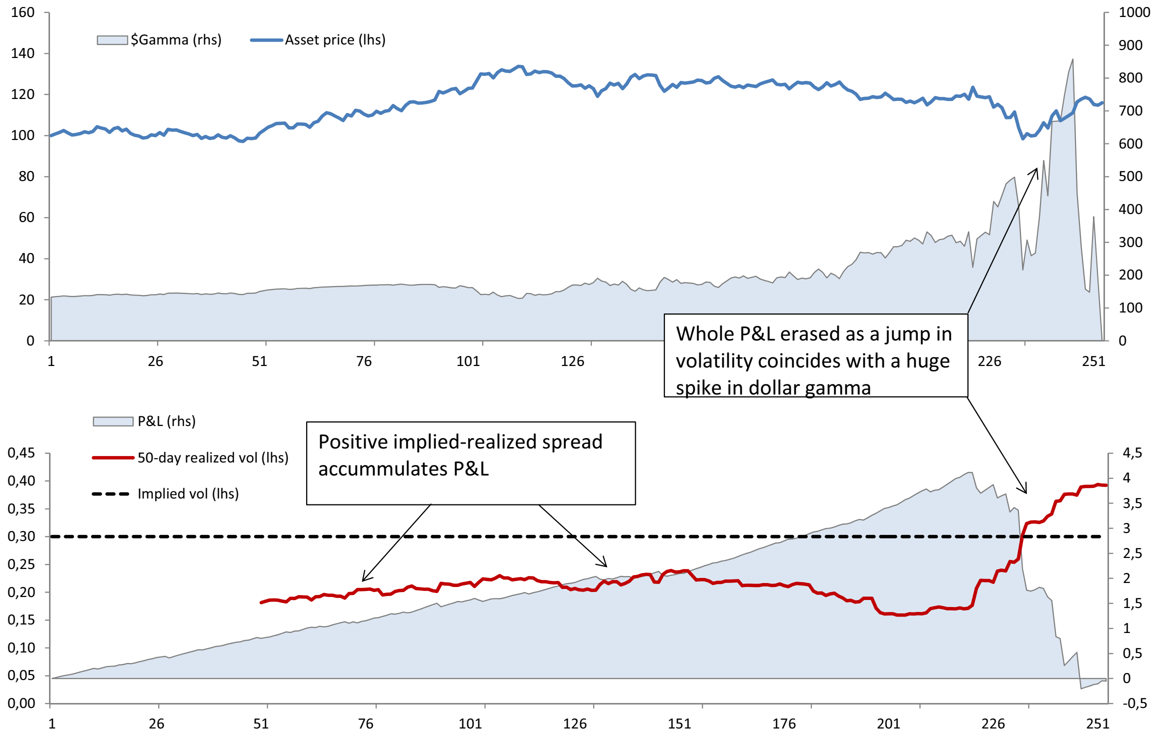
Figure 1: Path dependence of a delta hedged option position



A trader writes a 1Y call option at implied volatility $\Sigma = 30\%$, strike 110 and delta hedges daily. The underlying is assumed to be lognormally distributed with drift $\mu = 0.12$ and ("true") volatility that changes starting from 20% and jumping to 40% towards year end.

Figure 2: Variance exposure (dollar gamma) of three different option portfolios against the total portfolio gamma (dashed line).

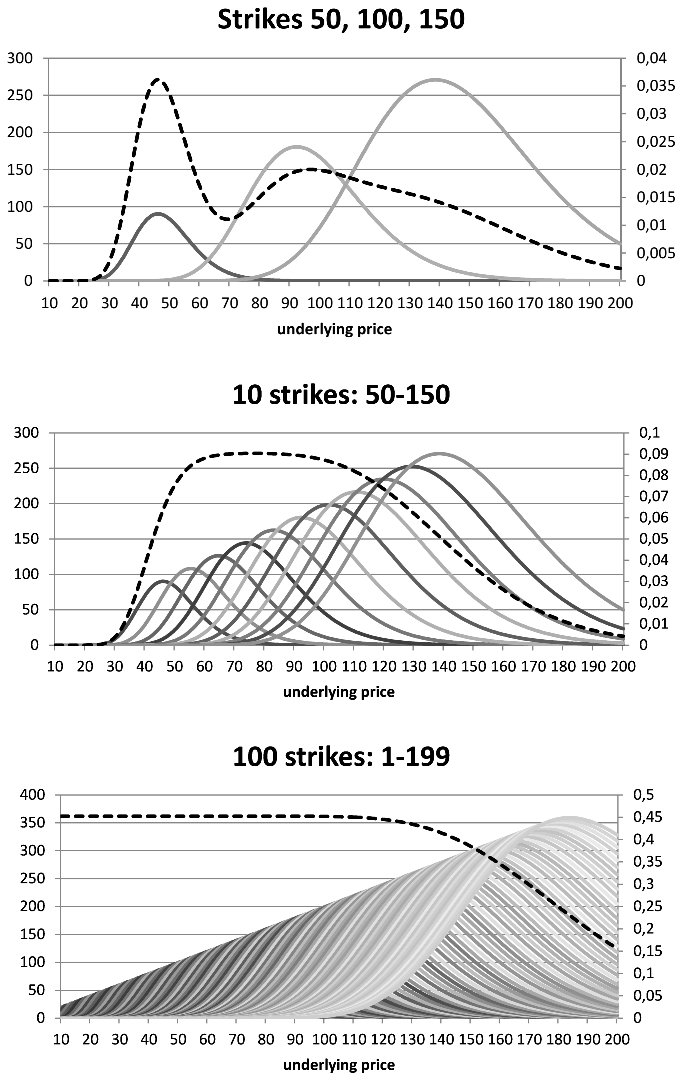
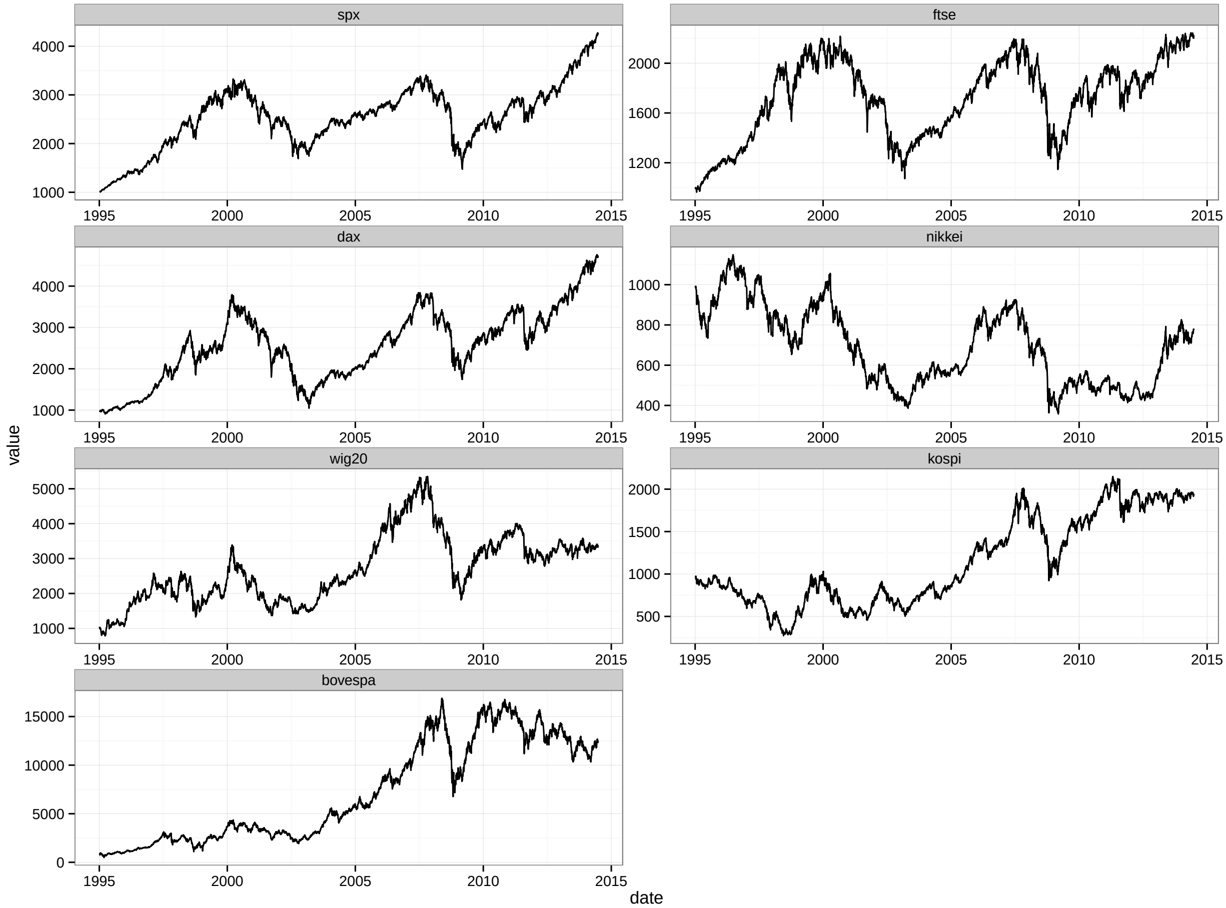


Table 1: Descriptive statistics for equity indices returns (WIG20, Bovespa, Dax, Kospi, Nikkei, FTSE100, S&P500)

	SPX	NIKKEI	FTSE	DAX	WIG20	KOSPI	BOVESPA
obs	4908	4929	4941	4794	4876	4976	4824
mean	0.00	0.00	0.00	-0.00	0.00	0.00	0.00
sd	0.01	0.01	0.02	0.02	0.02	0.02	0.02
median	0.00	0.00	0.00	0.00	0.00	0.00	0.00
min	-0.09	-0.09	-0.09	-0.12	-0.14	-0.13	-0.17
max	0.11	0.09	0.11	0.13	0.14	0.11	0.29
range	0.20	0.19	0.20	0.25	0.28	0.24	0.46
skewness	-0.25	-0.16	-0.10	-0.30	-0.15	-0.21	0.47
kurtosis	8.01	5.93	4.26	5.38	3.92	4.50	13.42

Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014

Figure 3: Equity indices for developed and emerging markets (WIG20, Bovespa, Bux, Kospi, Nikkei225, FTSE100, S&P500)



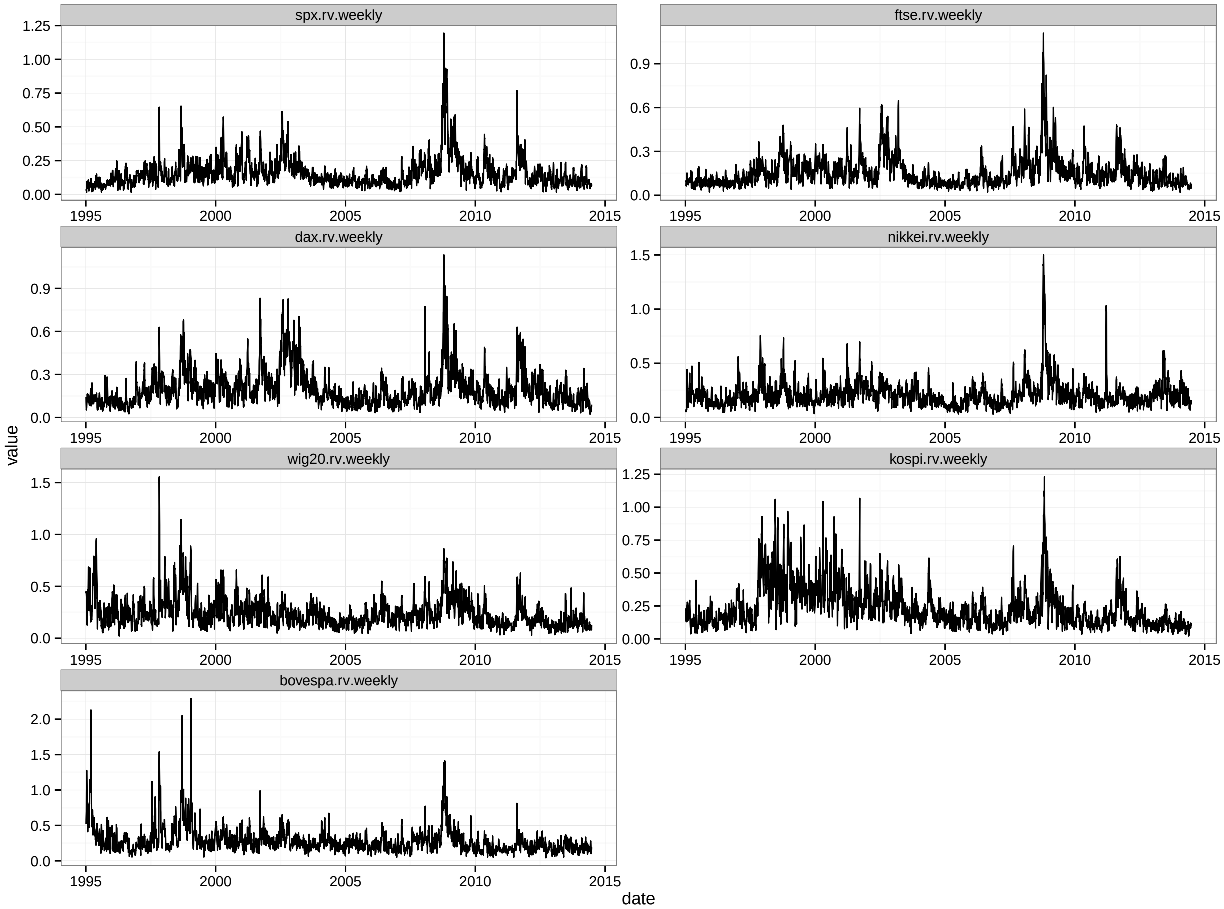
Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014

Table 2: Descriptive statistics for 5-minutes RV measures calculated independently for each equity indices.

	SPX	FTSE	DAX	NIKKEI	WIG20	KOSPI	BOVESPA
				daily RV			
mean	0.13	0.13	0.17	0.18	0.20	0.20	0.24
sd	0.14	0.13	0.17	0.17	0.20	0.21	0.25
median	0.09	0.09	0.12	0.13	0.15	0.13	0.18
min	0.00	0.00	0.00	0.00	0.00	0.00	0.00
max	1.74	1.49	1.71	2.10	2.25	2.03	4.58
range	1.74	1.49	1.71	2.10	2.25	2.03	4.58
skewness	3.10	2.75	2.37	2.67	2.29	2.39	3.96
kurtosis	17.29	13.46	9.19	14.74	9.46	8.66	35.69
				weekly RV			
mean	0.16	0.16	0.21	0.22	0.25	0.24	0.29
sd	0.11	0.10	0.13	0.12	0.14	0.16	0.20
median	0.14	0.13	0.17	0.20	0.22	0.20	0.25
min	0.01	0.02	0.02	0.03	0.02	0.02	0.04
max	1.19	1.11	1.13	1.50	1.56	1.23	2.35
range	1.18	1.09	1.11	1.47	1.53	1.21	2.31
skewness	2.78	2.47	1.93	3.17	2.08	1.75	3.79
kurtosis	12.59	10.33	5.51	20.25	8.37	3.94	24.25

Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014. RV estimator is annualized in this table. dailyRV means realized volatility calculated with daily price hisotory. weeklyRV means realized volatility calculated with weekly price history.

Figure 4: The fluctuations of 5-minutes RV measures (annualized) calculated independently for each equity index based on weekly price history.



Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014.

Table 3: The detailed description of LVD and HVD strategies.

	HighVolatilityDrive style strategy (HVD)	LowVolatilityDrive style strategy (LVD)
RV(ST) > RV(LT)	if RV(ST) > RV(LT) ==> ShortOptions+LongFutures adjusted daily ==> daily mean-reverting style	if RV(ST) > RV(LT) ==> LongOptions+ShortFutures adjusted daily ==> daily momentum style
RV(ST) < RV(LT)	if RV(ST) < RV(LT) ==> LongOptions+ShortFutures adjusted daily ==> daily momentum style	if RV(ST) < RV(LT) ==> ShortOptions+LongFutures adjusted daily ==> daily mean-reverting style

The description of these strategies is prepared based on the theoretical intuition presented in previous sections of this article.

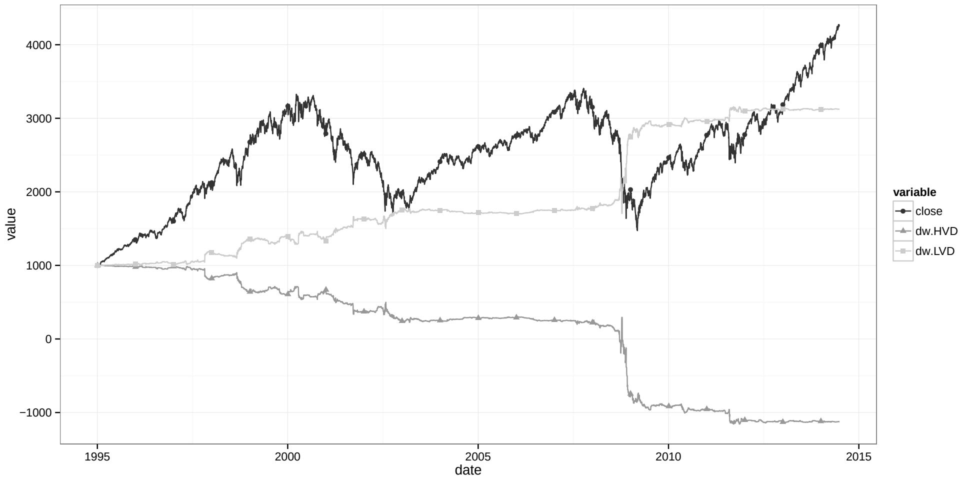
Table 4: The results of LVD and HVD investment strategy compared with BH strategy for S&P500 equity index

	SPX LVD	SPX HVD	SPX BH
min	-0.12	-54.13	-0.09
max	0.17	2.88	0.12
ARC(%)	6.02		7.73
aSD(%)	9.53	1238.82	19.51
aSemiSD(%)	5.24	1232.30	14.01
IR	0.63		0.40
Sharpe	0.30		0.23
Treynor	0.58		0.05
beta	0.05	6.17	1.00
MaxD(%)	22.19	215.23	56.78
ARC/MaxD	0.27		0.14
IR/MaxD	0.03		0.01

Source: Bloomberg data.

dw.LVD stands for low volatility drive strategy with switch based on RV calculated on daily and weekly price history. dw.HVD stands for high volatility drive strategy with switch based on RV calculated on daily and weekly price history. dw.BH stands for buy&hold strategy. The details of each strategy are presented in Table 3

Figure 5: The results of LVD and HVD investment strategy compared with BH strategy for S&P500 equity index



Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014

Table 5: The results of LVD and HVD investment strategy compared with BH strategy for other developed markets index (NIKKEI225, DAX, FTSE100)

	FTSE			NIKKEI			DAX		
	LVD	HVD	BH	LVD	HVD	BH	LVD	HVD	BH
min	-0.23	-0.49	-0.09	-0.49	-0.29	-0.11	-0.31	-0.30	-0.08
max	0.19	0.74	0.10	0.64	0.95	0.14	0.25	0.48	0.11
ARC(%)	0.23	-0.25	4.11	-0.45	0.41	-1.37	-2.62	1.75	8.17
aSD(%)	13.66	27.63	18.77	32.92	39.11	24.43	28.42	26.39	23.92
aSemiSD(%)	10.37	14.97	13.43	22.07	17.40	17.52	20.42	15.65	17.12
IR	0.02	-0.01	0.22	-0.01	0.01	-0.06	-0.09	0.07	0.34
Sharpe	-0.22	-0.12	0.05	-0.11	-0.07	-0.19	-0.20	-0.05	0.21
Treynor	-0.65	0.18	0.01	-0.61	0.13	-0.05	-0.59	0.16	0.05
beta	0.05	-0.19	1.00	0.06	-0.21	1.00	0.10	-0.09	1.00
MaxD(%)	51.60	76.18	52.57	81.70	88.52	68.88	78.96	70.08	72.68
ARC/MaxD	0.00	-0.00	0.08	-0.01	0.00	-0.02	-0.03	0.03	0.11
IR/MaxD	0.00	-0.00	0.00	-0.00	0.00	-0.00	-0.00	0.00	0.00

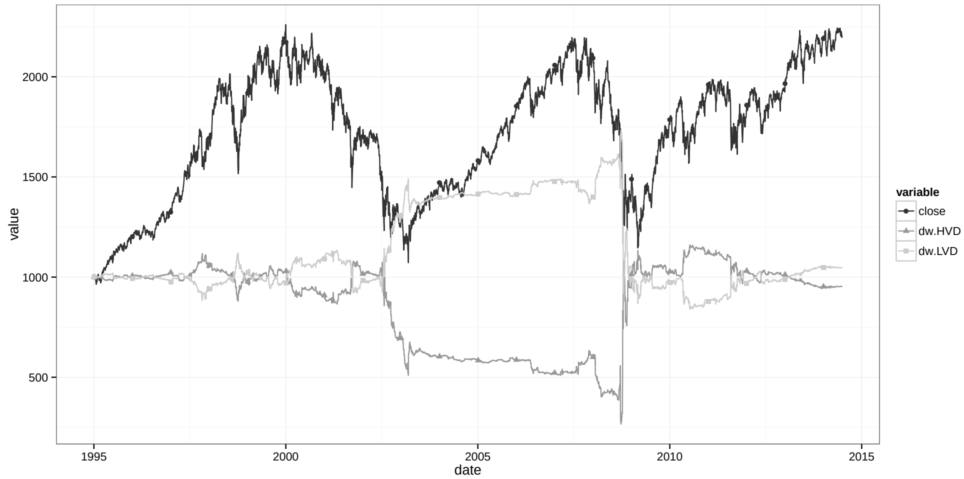
Source: Bloomberg data.

dw.LVD stands for low volatility drive strategy with switch based on RV calculated on daily and weekly price history. dw.HVD stands for high volatility drive strategy with switch based on RV calculated on daily and weekly price history. dw.BH stands for buy&hold strategy. The details of each strategy are presented in Table 3

Figure 6: The results of LVD and HVD investment strategy compared with BH strategy for Nikkei225 equity index

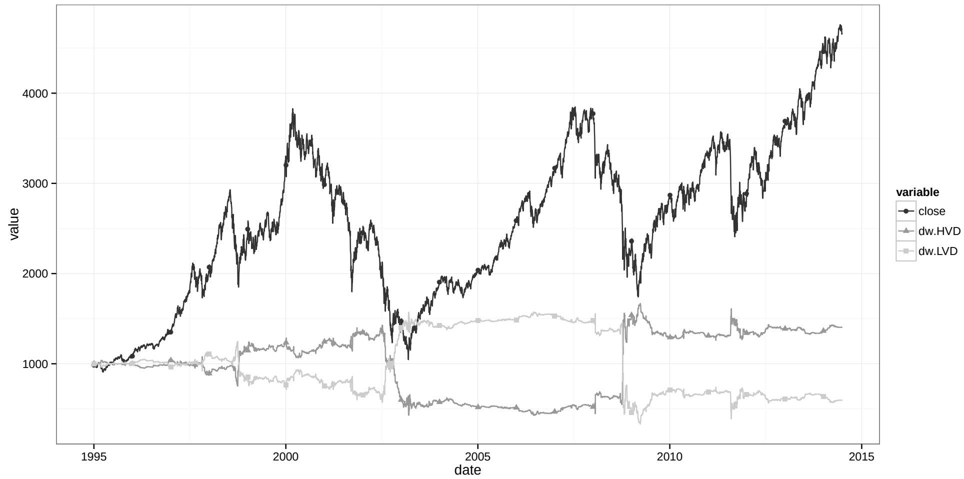
Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014

Figure 7: The results of LVD and HVD investment strategy compared with BH strategy for FTSE100 equity index



Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014

Figure 8: The results of LVD and HVD investment strategy compared with BH strategy for DAX equity index



Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014

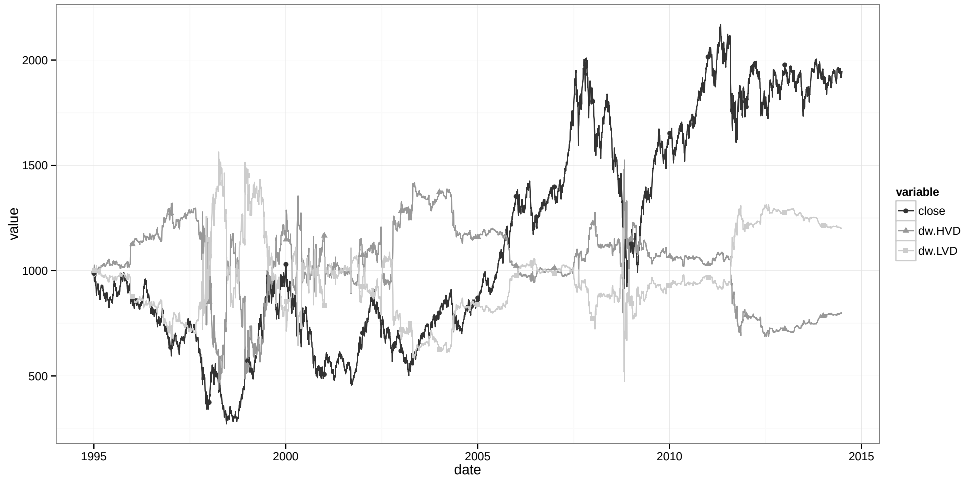
Table 6: The results of LVD and HVD investment strategy compared with BH strategy for emerging markets inices (WIG20, KOSPI, BOVESPA).

	WIG20			KOSPI			BOVESPA		
	LVD	HVD	BH	LVD	HVD	BH	LVD	HVD	BH
min	-0.24	-33.18	-0.13	-0.40	-0.38	-0.12	-4.35	-11.08	-0.16
max	1.03	34.14	0.15	0.33	0.32	0.12	7.03	45.00	0.33
ARC(%)	4.40		6.38	0.93	-1.12	3.44	6.34		13.96
aSD(%)	28.60	1648.08	28.35	36.52	36.78	28.58	272.13	1274.33	35.30
aSemiSD(%)	11.39	1172.12	19.96	24.13	24.14	20.36	150.78	306.21	24.27
IR	0.15		0.22	0.03	-0.03	0.12	0.02		0.40
Sharpe	0.04		0.11	-0.06	-0.12	0.01	0.01		0.30
Treynor	0.08		0.03	-0.11	0.28	0.00	-0.04		0.11
beta	0.15	1.78	1.00	0.21	-0.16	1.00	-0.85	0.04	1.00
MaxD(%)	39.44	199.42	66.11	69.63	66.97	54.54	165.57	145.29	59.96
ARC/MaxD	0.11		0.10	0.01	-0.02	0.06	0.04		0.23
IR/MaxD	0.00		0.00	0.00	-0.00	0.00	0.00		0.01

Source: Bloomberg data.

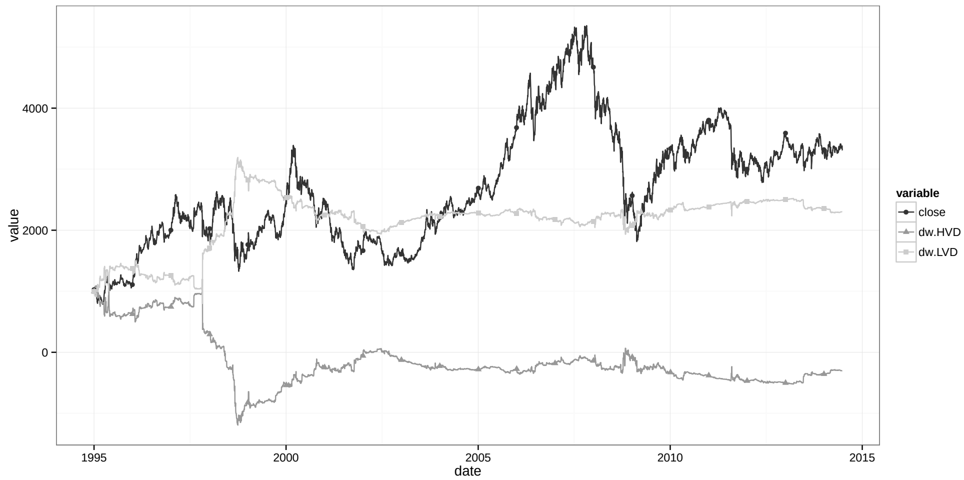
dw.LVD stands for low volatility drive strategy with switch based on RV calculated on daily and weekly price history. dw.HVD stands for high volatility drive strategy with switch based on RV calculated on daily and weekly price history. dw.BH stands for buy&hold strategy. The details of each strategy are presented in Table 3

Figure 9: The results of LVD and HVD investment strategy compared with BH strategy for Kospi equity index



Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014

Figure 10: The results of LVD and HVD investment strategy compared with BH strategy for WIG20 equity index



Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014

Figure 11: The results of LVD and HVD investment strategy compared with BH strategy for BOVESPA equity index

Source: Bloomberg data. The data used in this research have daily periodicity and cover the period between 1995-2014

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