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Volatility as a new class of assets? The advantages of using volatility index futures in investment strategies

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The advantages of using volatility index futures in investment strategies

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Abstract

This paper investigates the changes in the investment portfolio performance after including VIX. We apply different models for optimal portfolio selection (Markowitz and Black-Litterman) assuming both the possibility of short sale and the lack of it. We also use various assets, data frequencies, and investment horizons to get a comprehensive picture of our results' robustness. Investment strategies including VIX futures do not always deliver higher returns or higher Sharpe ratios for the period 2006-2013. Their performance is quite sensitive to changes in model parameters. However, including VIX significantly increases the returns in almost all cases during the recent financial crisis. This result clearly emphasizes potential gains of having such an asset in the portfolio in case of very high volatility in financial markets.

Keywords:
volatility, VIX futures, investment strategies, optimal portfolio selection, Markowitz model, Black-Litterman model

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1 Introduction

Uncertainty is inseparably linked with financial markets. Since the very beginning of their existence market participants faced risk connected with price changes. The nature of these changes has been intensively researched. Markets are characterized by volatility of asset prices, but what it really is? How can volatility be formally defined? How to measure this phenomenon? A natural and intuitively appealing measure could be the standard deviation of returns. However it is not always clear how to calculate it, in particular if one is interested in the expected or contemporaneous/current volatility. The standard deviation of past returns measures only historical volatility, which is already realized. Therefore there is no single obvious measure which could be simply used to numerically represent uncertainty in financial markets.

In 1993 CBOE started quotations of the first volatility index called VIX. Its aim is to measure the level of 30-day expected volatility. At the beginning S&P100 was used as the underlying index and VIX was directly based on the implied volatility of options. In September 2003 the methodology and the base index have changed and S&P500 is used since then and calculation of VIX is based on options.

VIX index appeared to be a very useful tool in assessing the situation on the stock market. Its idea spread out widely and currently there are many different volatility indices quoted on different exchanges.

Different volatility indices are strongly positively correlated. The same is observed for different stock indices. However, stock index and its implied volatility are strongly negatively correlated. This is a very appealing characteristic of the volatility index from the perspective on any investment strategy and has crucial importance in the analysis of potential benefits of investing in derivatives based on VIX or other volatility indices.

The aim of this paper is to analyse how including VIX in the investment portfolio changes investment strategy performance. We expect that considering VIX futures as one of the assets in the portfolio improves performance of the investment strategy measured by the average annualized return and Sharpe ratio. To verify our hypothesis we simulate a set of investment strategies on a relatively long sample period (2006–2013) using two different optimal portfolio selection models for different sets of assets on different data frequency with several additional strategy parameters which allows us to perform the sensitivity analysis. We also distinguish between strategies with and without the possibility of short sales.

The remaining part of the paper is organized as follows. In section 2 we discuss some earlier research on the topic, section 3 reviews theoretical background of the Markowitz model while section 4 refers to theoretical description of the Black-Littrerman model. Sections 5 and 6 describe the data used, discuss the assumptions of the simulation and empirical results for the Markowitz and Black-Litterman example respectively. Finally section 7 summarizes the conclusions.

2 Volatility as a traded asset

Volatility index itself is a valuable source of information about the market, but does not increase investment possibilities. Therefore introducing derivatives (futures and options) on VIX was in the ordinary course of events. CBOE introduced futures for VIX in 2004. Currently on each day there are quotations of eight different VIX contracts (with different maturities). Each month the expiration date is Wednesday that is the 30-th day before the third Friday of the next month. The value of a single contract is 1000$ (a point value) times the value of the VIX index.

In 2006 options on VIX were introduced. These are European type options, which means that CBOE Volatility Index options generally may be exercised only on the expiration date. And expiration dates are the same as for VIX futures.

The instruments shortly gathered huge interest of market participants. In the first four months of 2014 the average daily volume of VIX futures ranged up to 200 thousands of contracts and in the same time there were on average about 5 millions open positions of options (put and call together). They also created new investment possibilities and facilitated the application of hedging strategies, which were more difficult do apply and very costly before. Many investors consider the VIX Index to be the world's premier barometer of investor sentiment and market volatility, and VIX futures and options are very powerful risk management tools.

The introduction of the derivatives on volatility followed the introduction of the volatility index itself after a long time. The time was needed for the derivation of the theoretical models for pricing new instruments. The behaviour of volatility is very different from the behaviour of most financial time series – that is why pricing models commonly used before were not suitable and could not be directly used for new assets.

The emergence of futures contracts for volatility supplemented the financial market with a complete new class of investment strategies. Long position in futures or options on VIX can be a good example of hedging a position once an investor is afraid of severe decreases of prices of some other assets in their portfolio.

Although there are many empirical works devoted to volatility indices, there are few referring to investment possibilities introduced by derivatives on volatility.

Daigler and Rossi (2006) conducted a very simple analysis using the Markowitz model and just two assets: S&P500 index and VIX. The Authors show how including VIX changes the area of available and efficient portfolios. They indicate that the difference is significant and is an advantage of considering VIX as a traded asset.

Szado (2009) shows how investment in VIX might influence diversification of risk during the last financial crisis. The Author considers investments based on futures contracts and options using the Markowitz model. Several initial portfolios are constructed including stocks, bonds and other assets as real estate, commodities, investment funds, reflecting the weights of an average investor. The analyses are performed for several strategies based on options and futures on data since March 2006 until December 2008 and separately for the period August-December 2008. The most important conclusion from the research is that exposure on VIX is a permanent and important component of efficient portfolios. What follows – derivatives on VIX are indeed a useful investment tool.

Briere et al. (2010) show how including VIX influences investment possibilities of the long-term investor. They consider two investment strategies: long investment in VIX futures and the strategy of long investment in volatility risk premium. The premium is defined as the difference between the implied and realized volatility and in most cases is positive. Investment in such premium is possible through a variance swap. The Authors do not assume a standard deviation of returns to be a measure of risk (as did the Authors of the previously mentioned articles), but a modified Value-at-Risk is used. This measure is an approximation of the Value-at-Risk through the Cornish-Fisher expansion (Taylor expansion of a quantile of the distribution by the first four moments). As a result the maximized utility function depends not only on an average and variance of the distribution, but also on skewness and kurtosis, which seems to be a good choice considering the distribution of VIX and S&P500 returns. Comparing the possibilities of investing in stocks and bonds with the two above mentioned strategies the Authors show that both strategies separately significantly increase investment possibilities. They significantly move the efficient frontier to the left and the effect is stronger for the volatility risk premium strategy. The effect is intensified if one allows for investing in both strategies simultaneously.

With the development of financial markets and derivatives for volatility the number of works considering the attractiveness of aforementioned strategies will increase. The research done so far unequivocally corroborates that volatility is an important asset which may well diversify risk of investment portfolios.

3 Markowitz model – short theoretical review

Markowitz (1952) is an influential article, which became one of the turning points in the theory of finance. It presented totally new approach to investment and risk. There are two key ideas in the Markowitz model: expected (mean) return and variance of the portfolio. The aim of the market analysis is to find efficient portfolios, i.e. such as there does not exist a portfolio with lower variance and higher expected return in the same time (Harry H. Panjer, 1998). In the next step from the set of efficient portfolios one selects the portfolio which is optimal with respect to the selected criteria, e.g. Sharpe ratio or any other utility function.

In the Markowitz model one selects an investment portfolio from N risky assets and looks for a vector of weights $w = (w_1, w_2, \ldots, w_N)$ (shares of each asset in the portfolio), such as $\sum_{i=1}^{N} w_i = 1$, which would be optimal with respect to risk. The calculation of weights is based on a variance-covariance matrix of risky assets and a vector of expected returns. Efficient portfolios can be defined as the solution to one of the following sets of the optimization criteria:

$$\begin{aligned} max\ w'\Sigma w \\ w'\bar{r} = r_e \\ w'e_N = 1 \\ w_i \geq 0\ \forall i = 1, \ldots, N \end{aligned} \tag{1}$$

or

$$\begin{aligned} min\ w'\Sigma w \\ w'\Sigma w = \sigma_e^2 \\ w'e_N = 1 \\ w_i \geq 0\ \forall i = 1, \ldots, N \end{aligned} \tag{2}$$

where

w – column vector of weights of the efficient portfolio,
Σ – variance-covariance matrix of returns,
$\bar{r}$ – column vector of expected returns,
r_e – target return of the selected portfolio,
σ_e^2 – target variance of the selected portfolio,
$e_N = (1, \ldots \ldots, 1)'$ – N-dimensional vector of ones.

In other words optimization means minimization of the variance (risk) of the portfolio for a selected target return or maximization of the expected return with respect to the selected target variance of the portfolio.

All weights must sum up to one, i.e. 100% (the third condition in both above variants) and weights should be non-negative. Allowing for negative weights may be interpreted as allowing for short sales. If negative weights are allowed for all assets there exists a closed form analytical solution of the first optimization problem (variance minimization – equation (1)):

$$\sigma_e^2\gamma - \frac{(r_e - \frac{\beta}{\gamma})^2\gamma^2}{(\alpha\gamma - \beta^2)} = 1 \tag{3}$$

where $\alpha = \bar{r}'\Sigma^{-1}\bar{r}$, $\beta = \bar{r}'\Sigma^{-1}e$, $\gamma = e'\Sigma^{-1}e$.

Problem (1) without the last condition (non-negativity of weights) can be reformulated and presented in another way. First, one can introduce a risk-free asset with a constant return, which allows for taking short and long positions. Let excess returns be defined as the difference between the expected return and the risk-free rate $\mu = \bar{r} - r_f$. Then one can ignore the restriction that the sum of all weights has to be equal to 1, assuming that the remaining part is invested in the risk free asset $w_f = 1 - w'e$. Next one can introduce the parameter λ, which will represent aversion to risk. This parameter reflects the expected (by the investor) relationship between the risk (variance) and the expected return. Then the first two conditions of the optimization problem (2) can be represented as (Mankert, 2010, p. 97–98):

$$max\ w'\mu - \frac{\lambda}{2}w'\Sigma w \tag{4}$$

which has the solution

$$w^* = (\lambda\Sigma)^{-1}\mu \tag{5}$$

And for the optimal portfolio the ratio of the expected return and the variance is equal to the risk aversion parameter λ:

$$\lambda = \frac{\mu *}{\sigma^2 *} \tag{6}$$

Such Markowitz problem representation is used when introducing its extension into Black-Litterman model.

It is worth mentioning the most important assumptions and limitations of the Markowitz model.

First, if estimation of the expected returns and the variance-covariance matrix is based on typical estimators using historical data, we base on the assumption that the returns have multivariate normal distribution, which in addition is constant in time. In case of most financial time series this assumption is not fulfilled.

One assumes that investors are rational and are risk-averse. What is more one also assumes that there are arbitrage opportunities in the market, that the market is efficient, there are no transaction costs and all assets are perfectly divisible. In case of a large amount invested and small number of assets considered, the latest assumption should not distort the analysis much.

The important disadvantage of the Markowitz model is often a result with relatively large concentration around a single asset. Another significant limitation of the Markowitz model is considering just two measures in portfolio selection (expected return and variance of returns). This method is static and does not allow for including subjective views and expectations of an investor.

In the classical approach one can only add them in a form of additional constraints used in the optimization process. It may however significantly restrict the optimization process as several additional constraints may almost unequivocally determine the solution.

This issue has found a solution in the Black-Litterman model – an extension of the Markowitz model where investor's views may be considered in the optimization process.

4 Black-Littermann model – short theoretical review

The model presented at the beginning of 1990. by Black and Litterman (1992) solves many problems of the classical Markowitz model.

Its construction allows for defining different investors' views (forecasts) on market behaviour and including them in the optimization process.

Similarly as in Markowitz model, N risky assets are considered: these can include stocks, bonds, commodities or any other assets, and the variance-covariance matrix of their returns is denoted as Σ. One assumes that the market is in equilibrium, i.e. all investors optimally select weights of their investment portfolios. All assets are therefore divided between investors in equal proportions, which directly relate to market capitalization of every asset. If k_i denotes market capitalization of an asset i, the weights in each portfolio are:

$$w_{mkt} = \frac{1}{\sum_{i=1}^{N} k_i}(k_1, k_2, \ldots, k_N)' \tag{7}$$

All investors are the same, so they also have the same coefficient of risk aversion. Therefore the optimal capital allocation results in the same selection of all investors (He and Litterman (1999, p. 3), see also equation (5)):

$$w^* = w_{mkt} = (\lambda\Sigma)^{-1}\Pi \Longleftrightarrow \Pi = \lambda\Sigma w_{mkt} \tag{8}$$

where Π are implied expected returns and λ is a coefficient of market risk aversion. Π defines the returns expected by investors in equilibrium. These are also initial expected returns in the Black-Litterman model and according to equation (8), w_{mkt} are the initial weights of the assets in the portfolio. In other words, capitalization, variance-covariance matrix and risk aversion coefficient derive the return expected by investors. In the literature the aforementioned scheme is called the reverse optimization.

If an investor does not formulate any views, the initial portfolio is defined by weights w_{mkt}. The next step is defining investor's expectations (views) on future price movements. Each view is defined as the portfolio and expected return. An investor may define any number of views (also higher that the number of assets), creating the following matrices:

$$P = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_K \end{bmatrix} = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,N} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ p_{K,1} & p_{K,2} & \cdots & p_{K,N} \end{bmatrix}; Q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_K \end{bmatrix} \tag{9}$$

where

p_i – weights of the portfolio being i-th view,
q_i – expected return of the i-th view.

In the above example there are K independent views formulated. To use the Black-Litterman formula one also needs $K \times K$ dimensional variance-covariance matrix of expectations Ω. One usually assumes that Ω is a diagonal matrix, i.e. random error for a specific view is independent of another view. As indicated by He and Litterman (1999, p. 4) this assumption can be omitted.

The Authors indicate one of the ways of constructing Ω matrix, in which the variance of the views is proportional to the variance of asset returns in each of the views: $\Omega = diag(P'(\tau\Sigma)P)$, where τ is the adjustment coefficient controlling for uncertainty of the created views. There are also other approaches used in the literature. Generally we will assume that:

$$\Omega = \begin{bmatrix} \omega_1 & 0 & \cdots & 0 \\ 0 & \omega_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_K \end{bmatrix} \tag{10}$$

So far the investor has the data on market equilibrium and some views defined. The most important result of the Black and Litterman model is a framework allowing for putting these information together. This was done with the use of the Bayesian methodology. The model assumes, that the initial *a priori* distribution of the returns has a form: $N(\Pi, \tau\Sigma)$ (where τ is a scalar). The distribution of a random variable matched with the views is treated as the *a posteriori* distribution and assumed to be normal: $N(\Pi, \Sigma)$. Therefore we have initial distribution $P(A)$ and the conditional distribution $P(B|A)$ ($N(\Pi, \Sigma)$). Based on these data and using the Bayes theorem one obtains $P(A|B)$, i.e. the distribution of returns conditional on the views.

Based on above observations we obtain the parameters of the new distribution which are the crux of the Black-Litterman model (Black and Litterman, 1992, p. 15):

$$\bar{\mu} = E(R) = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right] \tag{11}$$

where $\bar{\mu} = E(R)$ is a new vector of expected returns and τ is a scalar reflecting lack of confidence to the stated views (equivalent to trust in the*a priori) distribution.* The remaining components have the same meaning as before.

There is no consensus among researchers using Black-Litterman model which variance-covariance matrix to use. He and Litterman (1999, p. 4) suggest using "new" *a posteriori* covariance matrix:

$$\bar{\Sigma} = \Sigma + \bar{M}^{-1} \; where \; \bar{M}^{-1} = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1} \tag{12}$$

while other authors (e.g. Walters (2011)) use simply Σ in the next steps. For the purpose of our research the *a prori* covariance matrix was used, which should not affect the results significantly.

One gets that the returns conditioned on views have normal distribution $N(\bar{\mu}, \Sigma)$ (He and Litterman, 1999, p. 6). Therefore based on equation (5) one obtains:

$$w_{BL} = (\lambda\Sigma)^{-1}\bar{\mu} \tag{13}$$

Although there were a number of works which presented or extended the Black-Litterman model since it was presented 20 years ago, there are still many doubts concerning for example the λ and τ parameters. On the J. Walters website `www.blacklitterman.org` one can find a detailed comparison of methods used by different researchers. A good introduction to Black-Litterman model can be found in Walters (2011), Mankert (2010) or Idzorek (2005).

The assumptions of the Black-Litterman model differ significantly from those of the classical Markowitz model. In the Black-Litterman model the starting point is a market equilibrium. It results in obtaining portfolios which are generally quite well balanced without extreme short positions and without large concentration of the optimal portfolio around few assets.

In addition an investor/analyst has practically unlimited possibilities of creating views modifying an initial portfolio. A number of different forecasting approaches can be used within one estimation. Even contradictory forecasts can be formulated and used, which makes the Black-Litterman approach a very useful and universal scheme.

On the other hand versatility is also a strongest disadvantage of the Black-Litterman model. Many approaches of estimation of a risk aversion coefficient λ, τ coefficient or a matrix Ω makes its usage somehow inconvenient. These factors have crucial importance for the estimation results and arbitrary assumptions of their values may influence the results beyond control.

In this study we assumed that parameters τ and λ will take values: 0.05 and 2.5 respectively. These values are most common in the literature, where the calculations of the Black-Litterman models are shown.

5 Markowitz model – application

5.1 Data used

In the empirical analyses of the Markowitz model we decided to select diversified assets representing different classes of potential investments.

We decided to analyse assets of different types – S&P500 index represents stocks, GSCI is a general index of commodities, while the third asset is a general approximation of world's bond market.

The first asset – S&P500 index represents the most liquid stock market in the world (source of data: `stooq.com`). The second asset – Goldman Sachs Commodity Index (currently also referenced as S&P GSCI), is used as a benchmark for investment in the commodity markets (source: `stooq.com`). It also measures commodity performance over time. The index is a tradable asset, which is available to market participants on the Chicago Mercantile Exchange. The index was originally developed by Goldman Sachs, but since 2007 it is owned by Standard&Poors.

As the third asset we decided to use one of the global bonds indices – Citi World Government Bond Index (source: Bloomberg). The Citi World Government Bond Index provides a broad benchmark for the global sovereign fixed income market. It is a widely used benchmark that currently comprises sovereign debt from over 20 countries, denominated in a variety of currencies, and has more than 25 years of history available.

And as the last asset considered we selected a volatility index, which in this case meant futures contracts for VIX (CFE listed on CBOE, source: `http://cfe.cboe.com/Products/historicalVIX.aspx`).

We gathered close prices for VIX index futures and three other assets for each trading day between 2005-01-01 and 2013-12-31. The data for 2005 are used only for calculating returns for the purpose of average returns and variance-covariance matrix, as we assume to start our investment at the beginning of the year 2006.

The first step of the analysis is the correlation matrix of daily returns of above mentioned assets for the period of our potential investment, i.e. 2006-01-01 and 2013-12-31, which is presented in table 1 (all values are significant at 1% level).

Table 1: Correlation matrix of daily returns of all assets considered in the Markowitz model: SP500, GSCI, WGBI, VIX in 2006–2013

	SP500	GSCI	WGBI	VIX
SP500	1.000	0.394	-0.333	-0.751
GSCI	0.394	1.000	-0.261	-0.322
WGBI	-0.333	-0.261	1.000	0.267
VIX	-0.751	-0.322	0.267	1.000

Note: All values are statistically significant at 1% level.
Source: Own calculations.

The returns of the VIX futures are negatively correlated with the returns of the stock index S&P 500 and with the commodities index GSCI, which was expected. However, one can also observe a negative correlation of the returns of bond index with the stock index returns. The bond index returns are also positively correlated with VIX futures returns. Above observations mean that even without VIX included in the portfolio we may have an asset which returns are negatively correlated with the remaining two.

5.2 Simulation

Having selected the assets we create a simulation which mimics the behaviour of an investor assuming some predefined strategy. It is assumed that an investor starts to invest at the beginning of 2006 and invests until the end of 2013. The selection of portfolio weights is always done after Friday close.

When selecting the weights of the portfolio an investor considers historical data from the recent 60, 120, 180 or 240 working days (approximately 3, 6, 9 or 12 months) – which is a strategy parameter called *memory*. Based on simple daily returns for a selected period one calculates the average (expected) returns of each asset and their variance-covariance matrix. Using the results one searches for a tangency portfolio (i.e. portfolio with the best Sharpe Ratio over the analysed period) – it's weights are used to determine the investment portfolio for the next period.

The length of the investment period is assumed to be 1, 2, 4 or 8 weeks (another strategy parameter called *weeks.invest*). The whole investor's capital is invested in the selected portfolio and an investor keeps their portfolio until the end of investment horizon. After the period is over the vector of average returns and the variance-covariance matrix are recalculated and portfolio is rebalanced for the next period (the analysis is therefore performed on a rolling basis).

The procedure is applied with two additional parameters. The *short.sales* parameter defines whether short sales are allowed. If one allows for short sales the weight of an individual asset in the portfolio may range from -1 to 1, while if short sales are not allowed the weight of an individual asset in the portfolio is restricted to the interval from 0 to 1. These restrictions are necessary for the optimization algorithm to converge and to eliminate the problem of taking extreme positions.

The last parameter *include.VIX* defines whether an investor can consider VIX futures (CFE) among the available assets. If not, only the three remaining assets can be used: S&P500 index, GSCI and WGBI. This is a crucial parameter which will allow us to determine whether investment in VIX is indeed improving our strategy performance (investment possibilities).

For all combinations of above mentioned parameters and for each Friday between 2006-01-01 and 2013-12-31 the weights of the optimal (tangency) portfolio were determined. In each case the procedure of finding the optimal portfolio was the following. The range of potential target returns was set to cover the interval from 0 to twice the maximum of the average (expected) returns of all considered assets in historical data considered on particular Friday. When short sales was allowed the range was set from 0 to twice the maximum of the absolute value of average returns. We started from 0 as we wanted to consider only portfolios with non-negative target return (non-negative Sharpe ratio). The range was divided into intervals of the length of 0.01%. For each potential target return the efficient portfolio was found and among all efficient portfolios considered we selected the one with highest Sharpe ratio (if positive). Its weights were used to define a portfolio for the next investment period. If none of the considered efficient portfolios had positive Sharpe Ratio we assumed all weights equal to 0.

All calculations were done in R CRAN software.

5.3 Empirical results

For each variant of the strategy the annualized average return, annualized standard deviation and annualized SR was calculated (based on daily returns of the strategy).

The summary of obtained results is presented in the following tables – gross annualized return for the whole investment period in table 2 and gross annualized Sharpe Ratio in table 3. The results of the strategies are compared with the basic long only equal weights portfolio which assumes equal weight for each asset along the whole investment period and keeping only long positions.

The conclusions from the above mentioned tables are mixed – they do not always match our expectations:

- allowing for short sales does not improve the strategy results if VIX is not included among the considered assets,
- allowing for short sales in case of portfolios with VIX does improve the strategy results only in case of memory = 120 days,
- sometimes including VIX worsens the results of the strategy (gives lower average returns and lower SRs) – in particular for a longer memory of the variance-covariance matrix (240 days = 1 year),
- including VIX into the portfolio does not necessarily improve the results if short sales are not possible (memory = 60 and 180 days),
- including VIX helps more often for shorter memory of the variance-covariance matrix and average returns (being a basis for portfolio weights calculation),
- strongest improvement of adding VIX to the portfolio is observed for half-yearly memory of the variance-covariance matrix (120 days = 6 months), both in case of

Table 2: Summary of gross annualized average return for the portfolio: S&P500, GSCI, WGBI, VIX

		short.sales=0		**short.sales=1**	
memory	**weeks invest**	**include VIX=0**	**include VIX=1**	**include VIX=0**	**include VIX=1**
60	1	5.53%	4.82%	4.06%	10.45%
	2	4.89%	3.55%	2.93%	9.75%
	4	2.90%	2.84%	3.12%	5.63%
	8	5.93%	3.85%	5.49%	6.82%
120	1	2.91%	4.51%	2.97%	12.83%
	2	1.75%	9.56%	1.50%	16.22%
	4	1.31%	1.41%	1.02%	5.80%
	8	-0.95%	-0.54%	-0.36%	8.35%
180	1	2.85%	4.57%	2.16%	7.30%
	2	3.45%	4.63%	1.76%	9.31%
	4	3.44%	5.08%	1.82%	6.68%
	8	3.56%	8.17%	1.94%	7.40%
240	1	3.74%	2.89%	4.31%	0.42%
	2	4.33%	3.60%	4.65%	1.10%
	4	3.75%	3.03%	3.69%	-0.20%
	8	3.80%	2.90%	3.96%	1.60%
equal weights		5.91%	-6.77%		

Source: Own calculations.

short sales possible and not possible,

- including VIX when short sales are allowed improves the results in most cases (except memory = 240), but generally for the frequency of portfolio rebalancing not lower than every 4 weeks,
- for all variants the annualized average returns and annualized SR are relatively low,
- the equal weights portfolio without VIX seems to be close to best of the strategy variants in terms of the annualized return (5.91% against 5.93% for memory equal to 60 and investment for 8 weeks), but once corrected for risk is rather mediocre,
- in case of portfolios with VIX assuming equal weights does not seem to be a reasonable choice here – its results are worst among all.

So to sum up these conclusions – if short sales are not allowed portfolios with VIX are

Table 3: Summary of gross annualized Sharpe Ratio for the portfolio: S&P500, GSCI, WGBI, VIX

		short.sales=0		**short.sales=1**	
memory	**weeks invest**	**include VIX=0**	**include VIX=1**	**include VIX=0**	**include VIX=1**
60	1	0.6532	0.5741	0.3565	0.4990
	2	0.5471	0.3906	0.2646	0.4789
	4	0.3230	0.3409	0.2893	0.3025
	8	0.6246	0.4436	0.4844	0.2802
120	1	0.3596	0.4594	0.2927	0.5898
	2	0.2105	0.7678	0.1476	0.6547
	4	0.1542	0.1657	0.0986	0.2426
	8	-0.0966	-0.0524	-0.0324	0.3136
180	1	0.4851	0.3888	0.2794	0.2861
	2	0.5924	0.3460	0.2340	0.3374
	4	0.5624	0.3227	0.2224	0.2253
	8	0.6115	0.4352	0.2794	0.2235
240	1	0.8475	0.6338	0.7575	0.0261
	2	0.9662	0.7904	0.7985	0.0677
	4	0.8379	0.6650	0.6674	-0.0122
	8	0.7965	0.6114	0.6826	0.0954
equal weights		0.4656	-0.4661		

Source: Own calculations.

generally not better than those without VIX. This may result from the characteristics of selected assets mentioned before – investment in bonds is on average negatively correlated with stocks and commodities, which may result in possibilities of hedging the portfolio even without VIX futures. In such a case including VIX may not extend the hedging possibilities much. However, in case of short sales allowed strategies with VIX are better in all variants when an investor uses relatively recent data for calculation of the variance-covariance matrix and the expected returns (60, 120 or 180 days) and their investment horizon is not longer than 4 weeks. In all those cases both the Sharpe ratios and annualized average returns are higher. Using longer memory (240 days) for deriving weights of the portfolio may result in less accurate assessment of the current behaviour of individual assets and their relationships. Investing for 8 weeks may have similar effect – relationships (correlations) between the assets may change during such a long period and that is why updating them more regularly improves the performance of the strategy.

In the next step we wanted to check how the strategies compare in subsequent years and if including VIX is more helpful in some sub-periods. The following tables 4 and 5 show the annualized Sharpe ratios for all variants of the strategy and for all individual years of the investment period. In each row of both tables we bolded the yearly values of the Sharpe ratio which are higher than the overall value for all years.

Most strategies were performing better in recent years – 2013, 2012 and 2010, the ones with shorter memory (60, 210) also in 2011. Few strategies performed better than average in 2006. The majority of the strategies which performed better than average in 2007 and 2008 included VIX.

To analyse the contents of aforementioned tables more formally we performed a set of multifactor ANOVAs. In each of the analyses we assumed that the Sharpe Ratio (overall or in particular year) is a dependent variable and the strategy parameters (*memory*, *weeks.invest*, *short.sales* and *include.VIX* are independent variables (factors) – only main effects were analysed. The results indicated that memory of the strategy was jointly significant at 5% level in all cases except for Sharpe Ratio in 2009. The possibility of short sales was in turn significant in five out of nine cases. As far as inclusion of VIX was concerned, it was significant in four years (2007, 2008, 2009 and 2012) and the largest improvement of the Sharpe ratio due to VIX was observed in 2008 – it increased in average by 0.65 *caeteris paribus* (the detailed results of the analyses are available upon request).

The next table 6 summarizes the increase of the annualized Sharpe ratio resulting from the inclusion of VIX as compared to the situation when just three assets are considered.

The calculations in table 6 clearly indicate that having assets negatively correlated with each other is very helpful during the financial crisis. It allows for reduction of losses or even obtaining profits during that period. This is in particular observed in 2007 and 2008 when for almost all variants the strategy with VIX is more profitable than without the volatility index with the only exception for memory of the strategy equal to 240 days. As noted before, since 2010 most strategies were profitable and noted relatively high Sharpe ratios, however including VIX did not in general improve their profitability.

Taking into account all above mentioned results one can conclude that investing in VIX futures may be a valuable tool of risk diversification, especially in periods of extremely high volatility. However, investors should use it with caution and properly select their investment algorithm as the performance of portfolios with VIX is very sensitive to the selection of strategy parameters.

We should clearly emphasise that all results presented above are calculated in gross terms, i.e. without extraction of transaction costs, which to some extent overestimated results of presented strategies. However, these costs should not be very high from the perspective of a large investor.

Table 4: Summary of yearly gross annualized Sharpe Ratio for the portfolio: SP500, GSCI, WGBI, VIX – memories 60 and 120

memory	weeks.invest	short.sales	include.VIX	all	2006	2007	2008	2009	2010	2011	2012	2013
60	1	0	0	0.65	0.54	0.00	-0.11	**1.06**	**1.52**	**1.45**	**1.32**	**0.71**
			1	0.57	0.22	**1.25**	-0.34	**0.82**	**1.33**	**1.71**	**0.91**	**0.72**
		1	0	0.36	0.28	-0.23	-0.61	0.27	**1.37**	**1.84**	**1.01**	**0.44**
			1	0.50	**0.73**	-0.33	0.24	**0.51**	**1.51**	**1.82**	0.39	-0.41
	2	0	0	0.55	**0.67**	-0.15	0.24	0.49	**1.24**	**1.41**	**1.30**	**0.63**
			1	0.39	**0.54**	**1.16**	-0.21	0.04	**1.05**	**1.43**	**0.85**	0.31
		1	0	0.26	**0.52**	-0.43	-0.25	-0.49	**1.35**	**1.80**	**1.20**	**0.41**
			1	0.48	**0.74**	-0.44	**0.86**	0.02	**1.18**	**1.70**	**1.56**	-0.20
	4	0	0	0.32	**0.74**	-0.31	0.06	0.09	0.00	**1.62**	**1.23**	**0.36**
			1	0.34	**0.89**	**0.96**	0.33	-0.66	-0.23	**1.54**	**1.02**	-0.22
		1	0	0.29	**0.84**	-0.48	-0.03	-0.62	**0.32**	**1.84**	**0.61**	0.27
			1	0.30	0.05	-0.32	**1.26**	-0.13	**1.50**	**1.46**	**0.50**	-0.88
	8	0	0	0.62	0.11	0.52	0.60	**1.28**	0.55	**1.40**	**1.85**	-0.76
			1	0.44	-0.35	**1.21**	**0.75**	-0.22	**0.56**	**1.37**	**1.67**	-0.65
		1	0	0.48	-0.32	0.37	0.19	**0.78**	**0.90**	**1.46**	**1.10**	-0.05
			1	0.28	-0.55	-0.08	**1.62**	0.00	**1.90**	**1.28**	**0.81**	0.02
120	1	0	0	0.36	-0.16	**1.40**	-0.87	**0.38**	**1.21**	**0.89**	**1.77**	**1.86**
			1	0.46	-0.73	**1.70**	**0.50**	-0.37	**1.88**	**1.14**	**1.37**	**1.70**
		1	0	0.29	-0.87	**1.18**	-0.35	0.25	**1.19**	**1.14**	**1.52**	**1.09**
			1	0.59	-0.50	**0.99**	**1.47**	**0.96**	**1.65**	**0.68**	**1.14**	**0.82**
	2	0	0	0.21	-0.42	**1.20**	-0.85	-0.12	**1.95**	**0.51**	**1.97**	**1.64**
			1	0.77	-0.62	**1.45**	**1.80**	-0.42	**1.32**	0.75	**1.79**	**1.47**
		1	0	0.15	-0.98	**1.21**	-0.49	0.04	**1.08**	**0.66**	**2.05**	**0.67**
			1	0.65	-0.88	0.40	**2.25**	**0.84**	**1.50**	0.38	**1.09**	0.20
	4	0	0	0.15	-0.53	**1.30**	-1.08	-0.10	**1.06**	**0.76**	**1.73**	**1.70**
			1	0.17	-0.27	**1.75**	-0.79	-0.41	**0.90**	**0.75**	**1.52**	**1.53**
		1	0	0.10	-0.70	**1.31**	-0.84	-0.48	**1.19**	**0.65**	**1.98**	**0.61**
			1	0.24	-0.34	**1.02**	**0.41**	**0.45**	**1.39**	**0.87**	**0.76**	**0.41**
	8	0	0	-0.10	-0.51	**1.34**	-1.70	**0.96**	**1.10**	**0.86**	**2.68**	**1.34**
			1	-0.05	-0.69	**1.52**	-1.34	**0.91**	**1.00**	**0.82**	**1.81**	**1.73**
		1	0	-0.03	-0.68	**1.28**	-1.28	**0.49**	**1.11**	**0.66**	**1.60**	**0.34**
			1	0.31	-0.62	**0.60**	**0.97**	**1.51**	**1.55**	**0.63**	**0.93**	**0.68**

Note: Values above the average for all years are bolded.

Source: Own calculations.

Table 5: Summary of yearly gross annualized Sharpe Ratio for the portfolio: SP500, GSCI, WGBI, VIX – memories 180 and 240

memory	weeks.invest	short.sales	include.VIX	all	2006	2007	2008	2009	2010	2011	2012	2013
180	1	0	0	0.49	-0.41	0.36	0.43	0.46	**0.66**	0.21	**3.13**	**1.61**
			1	0.39	-0.49	**0.82**	**0.59**	0.08	**1.93**	0.32	**1.46**	**1.89**
		1	0	0.28	-0.13	-0.09	-0.20	**0.36**	**2.49**	0.00	**1.95**	**1.40**
			1	0.29	-0.35	0.26	**0.74**	**0.47**	**1.60**	**0.74**	**1.51**	**1.56**
	2	0	0	0.59	0.00	0.52	-0.09	**1.27**	**1.48**	0.10	**3.40**	**1.53**
			1	0.35	-0.04	**0.88**	**0.50**	0.21	**1.41**	0.13	**1.40**	**1.78**
		1	0	0.23	0.02	0.05	-0.60	**0.64**	**1.40**	0.12	**2.12**	**1.25**
			1	0.34	-0.25	**0.63**	**1.02**	**0.52**	**1.63**	0.25	**1.69**	**1.43**
	4	0	0	0.56	-0.14	0.45	0.54	**1.15**	**1.47**	0.09	**3.33**	**1.34**
			1	0.32	-0.17	**1.13**	**0.50**	0.28	**1.09**	0.05	**2.00**	**1.67**
		1	0	0.22	0.00	-0.08	-0.35	**0.56**	**1.48**	0.20	**2.11**	**1.17**
			1	0.23	-0.35	**0.73**	**0.93**	**0.45**	**1.19**	-0.38	**2.02**	**1.40**
	8	0	0	0.61	0.41	0.60	0.32	**0.92**	**1.08**	0.12	**2.32**	**1.48**
			1	0.44	0.41	**1.08**	**0.81**	0.22	**0.94**	0.09	**1.93**	**1.66**
		1	0	0.28	**0.48**	0.21	-0.66	**0.47**	**1.04**	-0.03	**1.83**	**1.27**
			1	0.22	0.08	**0.66**	**0.67**	**0.43**	**1.63**	-0.88	**1.70**	**1.60**
240	1	0	0	0.85	0.02	**1.14**	**0.92**	0.82	**0.95**	0.11	**2.19**	**2.38**
			1	0.63	0.02	**1.03**	**1.04**	-0.34	**1.84**	-0.05	0.28	**2.37**
		1	0	0.76	0.13	**1.01**	**1.10**	0.60	**1.83**	-0.29	**2.37**	**1.99**
			1	0.03	-0.43	**0.47**	**0.68**	-0.18	**1.85**	-0.46	**1.22**	**1.83**
	2	0	0	0.97	0.52	**1.21**	0.69	0.72	**2.12**	0.08	**1.84**	**2.21**
			1	0.79	0.52	**1.46**	0.72	-0.10	**1.57**	0.31	**0.84**	**2.27**
		1	0	0.80	0.35	**1.07**	**1.25**	0.48	**1.86**	0.17	**1.98**	**1.75**
			1	0.07	-0.26	**0.55**	**0.75**	-0.24	**1.73**	-0.49	**1.36**	**1.68**
	4	0	0	0.84	0.30	**1.11**	0.66	0.54	**1.92**	0.34	**2.03**	**1.76**
			1	0.66	0.30	**1.24**	0.60	-0.32	**1.63**	0.61	0.53	**2.01**
		1	0	0.67	-0.10	**1.00**	**0.99**	0.49	**1.59**	0.39	**2.66**	**1.56**
			1	-0.01	-0.35	**0.35**	**0.30**	-0.43	**1.42**	-0.34	**0.93**	**1.58**
	8	0	0	0.80	0.09	**1.20**	**0.81**	0.70	**2.36**	0.39	**1.00**	**1.63**
			1	0.61	0.09	**1.51**	**0.62**	-0.13	**1.70**	0.28	**0.63**	**1.82**
		1	0	0.68	0.02	**1.15**	0.66	0.47	**1.82**	0.30	**2.74**	**1.79**
			1	0.10	0.05	**0.49**	-0.34	-0.26	**1.69**	-0.80	**0.77**	**1.96**

Note: Values above the average for all years are bolded.

Source: Own calculations.

Table 6: Summary of an increase of gross annualized Sharpe Ratio due to addition of VIX futures to the portfolio

memory	weeks.invest	short.sales	all	2006	2007	2008	2009	2010	2011	2012	2013
60	**1**	0	-0.08	-0.32	1.25	-0.23	-0.24	-0.18	0.26	-0.41	0.01
		1	0.14	0.45	-0.10	0.85	0.24	0.14	-0.02	-0.62	-0.85
	2	0	-0.16	-0.13	1.31	-0.46	-0.45	-0.18	0.02	-0.45	-0.32
		1	0.21	0.22	-0.01	1.11	0.51	-0.17	-0.09	0.36	-0.61
	4	0	0.02	0.15	1.27	0.26	-0.75	-0.23	-0.08	-0.21	-0.58
		1	0.01	-0.79	0.16	1.28	0.49	1.18	-0.38	-0.11	-1.15
	8	0	-0.18	-0.46	0.69	0.15	-1.50	0.00	-0.03	-0.18	0.11
		1	-0.20	-0.23	-0.45	1.43	-0.78	0.99	-0.19	-0.29	0.08
120	**1**	0	0.10	-0.57	0.30	1.37	-0.76	0.67	0.26	-0.40	-0.15
		1	0.30	0.37	-0.19	1.82	0.72	0.45	-0.46	-0.38	-0.27
	2	0	0.56	-0.20	0.25	2.65	-0.30	-0.63	0.24	-0.18	-0.17
		1	0.51	0.11	-0.81	2.74	0.80	0.42	-0.28	-0.96	-0.47
	4	0	0.01	0.26	0.45	0.29	-0.31	-0.17	0.00	-0.22	-0.17
		1	0.14	0.35	-0.29	1.25	0.93	0.20	0.22	-1.21	-0.20
	8	0	0.04	-0.18	0.18	0.36	-0.06	-0.10	-0.04	-0.87	0.39
		1	0.35	0.06	-0.69	2.25	1.01	0.44	-0.03	-0.67	0.34
180	**1**	0	-0.10	-0.09	0.46	0.16	-0.38	1.27	0.10	-1.68	0.28
		1	0.01	-0.21	0.35	0.94	0.11	-0.89	0.74	-0.44	0.16
	2	0	-0.25	-0.04	0.36	0.58	-1.06	-0.07	0.02	-2.00	0.25
		1	0.10	-0.26	0.58	1.62	-0.12	0.23	0.14	-0.42	0.18
	4	0	-0.24	-0.03	0.68	-0.04	-0.87	-0.38	-0.04	-1.34	0.32
		1	0.00	-0.36	0.81	1.27	-0.11	-0.30	-0.59	-0.09	0.23
	8	0	-0.18	0.00	0.48	0.49	-0.70	-0.14	-0.03	-0.39	0.18
		1	-0.06	-0.41	0.45	1.33	-0.05	0.59	-0.85	-0.14	0.34
240	**1**	0	-0.21	0.00	-0.11	0.12	-1.16	0.89	-0.16	-1.91	-0.01
		1	-0.73	-0.56	-0.54	-0.42	-0.77	0.02	-0.17	-1.14	-0.16
	2	0	-0.18	0.00	0.25	0.03	-0.82	-0.55	0.23	-1.00	0.06
		1	-0.73	-0.62	-0.52	-0.50	-0.72	-0.13	-0.67	-0.63	-0.07
	4	0	-0.17	0.00	0.13	-0.05	-0.87	-0.29	0.28	-1.50	0.24
		1	-0.68	-0.25	-0.65	-0.68	-0.92	-0.17	-0.73	-1.73	0.03
	8	0	-0.19	0.00	0.32	-0.19	-0.83	-0.67	-0.11	-0.37	0.19
		1	-0.59	0.03	-0.66	-1.00	-0.73	-0.13	-1.09	-1.97	0.17

Source: Own calculations.

The above analyses to some extent highlighted the well known issues of the classical Markowitz model. The most important problem is estimation method of the expected returns and their covariances. In case of using historical data long sample gives more stable results, but almost completely ignores current market situation.

The Black-Litterman model presented in the next section allows to get rid of some of the disadvantages of the Markowitz model. Including views of the investor allows for more a flexible selection of initial parameters. In addition we will check how VIX may improve investment possibilities on another market.

6 Black-Litterman model – application

6.1 Data used

To verify our findings concerning the potential advantages of using VIX in investment portfolios we performed the analysis of the Black-Litterman model on a different set of assets, changing also the frequency of the data. For each trading day between 2005-01-01 and 2013-12-31 we gather close prices for VIX index futures and ten stock indices for the developed and emerging financial markets. Our research is based on the following data indices: S&P500, FTSE100, DAX, CAC40, IBEX35, Nikkei 225, Bovespa, Hang Seng, NIFTY, ISE30 and in addition the VIX index futures (all data took from `stooq.com` with the exception for VIX futures, which can be found at `http://cfe.cboe.com/Products/historicalVIX.aspx`).

All indices are significantly positively correlated with each other, most of then strongly – the highest correlation of weekly returns is found for the three European indices: FTSE100, DAX and CAC40, which are also very strongly correlated with the returns of S&P500 and IBEX35. Bovespa is an index from a developing financial market with the strongest correlation with the above mentioned developed indices. All ten indices face strong negative correlation with VIX futures in terms of weekly returns – for details see table 7.

6.2 Simulation

Having selected the assets we create a simulation which mimics the behaviour of an investor assuming a predefined strategy. It is assumed that an investor starts to invest at the beginning of 2006 and invests until the end of 2013. The selection of portfolio weights is done on a weekly basis, always after Friday's close.

Table 7: Correlation matrix of weekly returns of all assets considered in the Black-Litterman model in 2006–2013

	SP500	FTSE100	DAX	CAC40	IBEX35	Nikkei225
SP500	1.000	0.874	0.861	0.863	0.753	0.663
FTSE100	0.874	1.000	0.925	0.897	0.780	0.672
DAX	0.861	0.925	1.000	0.935	0.866	0.677
CAC40	0.863	0.897	0.935	1.000	0.814	0.675
IBEX35	0.753	0.780	0.866	0.814	1.000	0.569
Nikkei225	0.663	0.672	0.677	0.675	0.569	1.000
Bovespa	0.757	0.773	0.749	0.742	0.655	0.579
HSI	0.626	0.667	0.650	0.641	0.554	0.683
NIFTY	0.532	0.568	0.558	0.577	0.508	0.533
ISE30	0.533	0.558	0.568	0.579	0.509	0.488
VIX	-0.734	-0.689	-0.707	-0.672	-0.609	-0.542

	Bovespa	HSI	NIFTY	ISE30	VIX
SP500	0.757	0.626	0.532	0.533	-0.734
FTSE100	0.773	0.667	0.568	0.558	-0.689
DAX	0.749	0.650	0.558	0.568	-0.707
CAC40	0.742	0.641	0.577	0.579	-0.672
IBEX35	0.655	0.554	0.508	0.509	-0.609
Nikkei225	0.579	0.683	0.533	0.488	-0.542
Bovespa	1.000	0.689	0.591	0.577	-0.567
HSI	0.689	1.000	0.670	0.529	-0.504
NIFTY	0.591	0.670	1.000	0.486	-0.448
ISE30	0.577	0.529	0.486	1.000	-0.456
VIX	-0.567	-0.504	-0.448	-0.456	1.000

Source: Own calculations.

When selecting the weights of the portfolio an investor considers historical data from the recent 13, 26 or 52 weeks (approximately 3, 6 or 12 months) – which is a strategy parameter called *memory*. Based on simple weekly returns for historical data one calculates the average (expected) returns and their variance-covariance matrix. Next using equation (8) the implied expected returns Π are calculated.

Then one defines the views, which modify the expected returns. In our research we use a very simple naive forecasts. For each of the analysed assets we calculate the return in last week before setting new portfolio weights and assume that the expected return in the next week will be the same. Therefore the P matrix of portfolios used for forecasts is the identity matrix (each portfolio consists of just one asset, different in each view). Following Idzorek (2005) we assume that the Ω matrix is diagonal with the values equal to variances of each portfolio. In our case these are variances of weekly returns (the diagonal of the variance-covariance matrix).

Therefore the formula of calculating the expected returns simplifies to the following form:

$$\bar{\mu} = E(R) = \left[(\tau\Sigma)^{-1} + \Omega^{-1}\right]^{-1} \left[(\tau\Sigma)^{-1}\Pi + \Omega^{-1}Q\right] \tag{14}$$

where $\Omega = diag(\Sigma)$ and Q are asset returns from last week. We assume the correcting coefficient τ equal to 0.05.

The obtained value $\bar{\mu}$ and the matrix Σ calculated before are treated as inputs for portfolio optimization. Based on the obtained results an investor searches for a tangency portfolio (i.e. portfolio with the best Sharpe Ratio over the analysed period) – its weights are used to determine the investment portfolio for the next week.

The length of the investment period is always one week in this case. The whole investor's capital is invested in the selected portfolio and an investor keeps their portfolio until the end of next week. On the next Friday new views are formulated, the vector of average returns and the variance-covariance matrix are recalculated and portfolio is rebalanced for the next week.

The procedure is applied with two additional parameters. The *short.sales* parameter defines whether short sales are allowed. If one allows for short sales the weight of an individual asset in the portfolio may range from -1 to 1, while if short sales are not allowed the weight of an individual asset in the portfolio is restricted to the interval from 0 to 1.

The last parameter *include.VIX* defines whether an investor can consider VIX futures (CFE) among the available assets. If not, only ten stock indices are used. This is again a crucial parameter which will allow us to determine whether investment in VIX is indeed improving our strategy performance.

For all combinations of above mentioned parameters and for each Friday between 2006-01-01 and 2013-12-31 the weights of the optimal (tangency) portfolio were determined.

All calculations were done in R CRAN software using the `BLCOP` package.

6.3 Empirical results

Again for each variant of the strategy the annualized average return, annualized standard deviation and annualized SR was calculated (based on weekly returns of the strategy).

The summary of obtained results is presented in the following tables – gross annualized return for the whole investment period in table 8 and gross annualized Sharpe Ratio in table 9. The results of the strategies are again compared with the basic long only equal weights portfolio which assumes equal weight for each asset along the whole investment period and keeping only long positions.

Table 8: Summary of gross annualized average returns for the strategy based on Black-Litterman model

		short.sales=0		**short.sales=1**	
memory	**weeks invest**	**include VIX=0**	**include VIX=1**	**include VIX=0**	**include VIX=1**
13	1	5.94%	-19.05%	6.82%	-10.87%
26	1	3.25%	16.74%	10.57%	7.85%
52	1	2.55%	2.84%	5.20%	-5.68%
equal weights		6.89%	1.61%		

Source: Own calculations.

Table 9: Summary of gross annualized Sharpe Ratio for the strategy based on Black-Litterman model

		short.sales=0		**short.sales=1**	
memory	**weeks invest**	**include VIX=0**	**include VIX=1**	**include VIX=0**	**include VIX=1**
13	1	0.2563	-0.5200	0.2685	-0.3812
26	1	0.1333	0.3551	0.4214	0.2891
52	1	0.1064	0.0607	0.2332	-0.1694
equal weights		0.3393	0.1109		

Source: Own calculations.

The conclusions from the above mentioned tables again do not always match our expectations:

- if short sales are allowed, there is no advantage of considering VIX futures among investment assets – including VIX decreases the profitability of all variants of the strategy in such case,
- if short sales are not allowed, there is some advantage of investing in VIX – for two variants of longer memory the strategy with VIX included generates higher average returns, however, just in one case (memory = 26 weeks) it also gives a higher Sharpe ratio,
- for portfolios without VIX allowing for short sales consistently improves strategy performance – it suggests that short sales may be a good substitute for adding VIX

to long only portfolios,

- the equal weights portfolio without VIX seems to be better than any of the strategy variants both in terms of the annualized return and Sharpe ratio – this indicates that if all considered assets are (strongly) positively correlated and only long positions are available, assuming equal weights for all assets may be a good alternative to regular portfolio rebalancing, which also generates additional transaction costs,
- also in case of portfolios with VIX assuming equal weights is not the worst choice, which at the first sight seems to be in contrast to results obtained for the Markowitz model discussed earlier. However, using the long equal weights portfolio for eleven assets (as in our Black-Litterman example) assigns much lower weight for volatility than in the case of just four assets (as in our Markowitz model analysis). In our Markowitz case VIX returns have much stronger impact on the strategy results and therefore may more significantly distort the profitability of the equal weights portfolio.
- in case of the shortest memory (13 weeks) and lack of short sales the results of including VIX to the portfolio are the worst.

In the latest issue mentioned above – in the strategy with VIX the weights of VIX futures in the portfolio are close to 100% quite often in 2008, 2010, 2011 and 2012. But it appears that in some crucial moments of significant market drops the weight of VIX was too low. For example on the first Friday of October 2008 (2008-10-03) the strategy with no short sales, but VIX included and memory of 13 weeks resulted in a weight for VIX for the next week equal to just 41%. In the next week all the indices considered dropped by 15-25%, while VIX futures noted a weekly return of 60%. The strategy was profitable over that week and gave 15% return. However, almost the same strategy, but with memory based on last 26 weeks put almost all (99.9%) in VIX and therefore resulted in 60% profit in the next week. Few more such differences between those two strategies resulted in a huge difference in the overall profitability (-19.05% vs 16.74% as the annualized average return is concerned and -0.52 vs 0.35 in terms of the Sharpe ratio).

This may be partly related to the mean reverting behaviour of financial time series. In the example mentioned above in the case of 26 weeks memory all assets except VIX futures had negative average returns calculated on historical data – therefore setting a high weight for VIX in lack of short sales was somehow natural in our strategy framework. In the case of 13 weeks memory however, two other stock indices (apart from VIX) noted slightly positive returns – therefore long position was assumed in more than one asset (not just VIX as above), which resulted in much smaller, but still positive return of the strategy in the next week. Calculations based on the longer memory to higher extent smooth the data, which is useful when short increasing and decreasing trends appear interchangeable. In case of 13 weeks memory our decision rule seemed to react to fast, based on potential trend reversals in some assets which were not continued in subsequent week.

The results indicate therefore strong sensitivity of the strategy performance to the change in parameters, which was also observed for the Markowitz model. Our intuition also tells us that the results are strongly dependent on the defined views and probably the naive forecasts used in our approach are not the best choice.

In the next step we wanted to check how the strategies compare in subsequent years and if including VIX is at least more helpful in some years. Below table 10 shows annualized Sharpe ratios for all considered variants of strategies and for all years of an investment.

Similarly as for the results of the strategies based on the Markowitz model, to analyse the contents of table more formally we performed a set of multifactor ANOVAs. In each of the analyses we assumed that the Sharpe Ratio (overall or in particular year) is a dependent variable and the strategy parameters (*memory*, *short.sales* and *include.VIX* are independent variables (factors) – only main effects were analysed. The results indicated that inclusion of VIX was statistically significant in all years apart from 2007 and 2013. However, only in 2008 and 2011 its impact was positive – on average adding VIX increased the Sharpe ratio in 2008 by 1.71 and in 2011 by 0.93 *caeteris paribus*. For the remaining significant years including VIX worsened strategy performance. As far as short sales is concerned, it appeared to be significant in four years (2006, 2009, 2010 and 2012) and on average increased the Sharpe Ratio in 2006 and 2010 (the detailed results of the analyses are available upon request).

Table 10: Summary of yearly gross annualized Sharpe Ratio for the strategy based on Black-Litterman model

memory	**weeks.invest**	**short.sales**	**include.VIX**	all	2006	2007	2008	2009	2010	2011	2012	2013
13	1	0	0	0.26	0.34	1.44	-1.73	2.30	0.06	-1.05	1.77	1.22
			1	-0.52	-1.02	1.15	-0.34	-0.06	-2.47	-0.20	-0.25	-0.66
		1	0	0.27	2.78	0.78	-2.48	1.15	1.06	-0.95	1.60	1.19
			1	-0.38	-0.61	0.89	-1.54	-0.53	-0.34	-0.72	-0.81	1.07
26	1	0	0	0.13	0.03	1.50	-1.96	1.63	0.48	-1.69	1.84	1.65
			1	0.36	-1.43	1.07	0.92	1.02	-1.24	1.03	-0.07	1.77
		1	0	0.42	1.39	0.71	-0.96	1.13	0.72	-0.52	1.48	0.89
			1	0.29	0.81	0.94	-0.54	-0.32	2.17	-0.65	0.01	1.73
52	1	0	0	0.11	0.78	0.74	-1.91	1.27	0.94	-0.73	1.28	1.01
			1	0.06	-0.36	0.01	1.22	-0.48	-2.27	0.62	-0.71	0.79
		1	0	0.23	0.84	1.30	-1.45	0.81	1.89	0.12	-0.96	0.93
			1	-0.17	0.01	0.61	0.03	-1.25	0.98	0.70	-3.02	-0.24

Source: Own calculations.

The next table 11 summarizes the increase of the annualized Sharpe ratio resulting from the inclusion of VIX as compared to the situation when just ten stock indices are considered.

Table 11: Summary of an increase in gross annualized SR due to inclusion of VIX futures in the portfolio for the strategy based on Black-Litterman model

memory	**short.sales**	all	2006	2007	2008	2009	2010	2011	2012	2013
13	0	-0.78	-1.36	-0.29	1.40	-2.35	-2.52	0.85	-2.02	-1.88
	1	-0.65	-3.39	0.11	0.94	-1.68	-1.39	0.23	-2.41	-0.12
26	0	0.22	-1.46	-0.43	2.89	-0.60	-1.71	2.71	-1.91	0.13
	1	-0.13	-0.59	0.23	0.41	-1.46	1.44	-0.13	-1.47	0.84
52	0	-0.05	-1.14	-0.73	3.13	-1.75	-3.20	1.34	-1.99	-0.22
	1	-0.40	-0.83	-0.69	1.48	-2.05	-0.91	0.58	-2.06	-1.17

Source: Own calculations.

The results presented in the table indicate that adding VIX consistently improves the strategy performance in 2008 for all variants and in 2011 for almost all variants (with only one exception), while almost consistently deteriorate the performance for all remaining years.

Conclusions from the yearly analysis of the Black-Litterman model for stock indices and VIX are therefore similar to those for the classical Markowitz model. Having VIX in the portfolio of assets is very helpful during the financial crisis. It reduces losses during that period.

7 Conclusions

Exposure on VIX was a permanent and important component of efficient portfolios in our analyses, which means that the derivatives on VIX are useful investment tools. But including VIX did not in general improve the performance of different strategies in many cases the performance of strategies with VIX was even worse than without VIX. What is more, the strategy performance was very sensitive to model parameters. However, in huge majority of the analysed cases for both models including VIX significantly increased the average returns and Sharpe ratios during recent financial crisis. This result perfectly emphasizes potential gains of having such an asset in the portfolio in case of very high volatility in financial markets. VIX is a very appealing asset due to its negative correlation with most of the others. In the same time it is also a specific asset which can not be easily added into investment portfolios and treated similarly to other assets in modern portfolio theory asset allocation models. Investing in VIX futures may be a valuable tool of risk diversification. However, investors should use it with caution and properly select their investment algorithm.

In the Markowitz model exercise we observed that if short sales were not allowed portfolios with VIX were generally not better than those without VIX included. However, this probably resulted from the fact that we had another asset (bonds), which was negatively correlated with stocks and commodities. In such case including VIX may not have extended the hedging possibilities much. On the other hand, when short sales were allowed the strategies with VIX were better in all variants when an investor used relatively recent historical data for the variance-covariance analyses and their investment horizon was not longer than 4 weeks. In all those cases both the annualized average returns and Sharpe ratios were higher. Using longer memory (240 days) for deriving weights of the portfolio may result in less accurate assessment of the current behaviour of individual assets and their covariances.

In turn, in case of the Black-Litterman model, where all assets apart from VIX were positively correlated with each other, including VIX gave some advantage in case of lack of short sales and did not improve the results when short sales were allowed. For portfolios without VIX allowing for short sales consistently improved strategy performance. This suggests that short sales may be a good substitute for adding VIX to long only portfolios.

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