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# Modeling volatility with Range-based Heterogeneous Autoregressive Conditional Heteroskedasticity model


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**Abstract**

In this paper a new ARCH-type volatility model is proposed. The Range-based Heterogeneous Autoregressive Conditional Heteroskedasticity (RHARCH) model draws inspiration from Heterogeneous Autoregressive Conditional Heteroskedasticity presented by Muller et al. (1995), but employs more efficient, range-based volatility estimators instead of simple squared returns in conditional variance equation. In the first part of this research range-based volatility estimators (such as Parkinson, or Garman-Klass estimators) are reviewed, followed by derivation of the RHARCH model. In the second part of this research the RHARCH model is compared with selected ARCH-type models with particular emphasis on forecasting accuracy. All models are estimated using data containing EURPLN spot rate quotation. Results show that RHARCH model often outperforms return-based models in terms of predictive abilities in both in-sample and out-of-sample periods. Also properties of standardized residuals are very encouraging in case of the RHARCH model.



**Keywords:**
volatility modelling, volatility forecasting, ARCH, range-based volatility estimators, heterogeneity of volatility

**JEL:**
C13, C22, C53

## Introduction

Since famous Engle article (1982), ARCH class models have become commonly used for modeling and forecasting volatility of financial assets returns. They owe its popularity to their flexible framework, and relatively straightforward estimation. Over the past 30 years, many ARCH class models have been proposed. Most of them differ from each other mainly in the conditional variance equation. First of all, GARCH model presented by Bollerslev (1986), and EGARCH model proposed by Nelson (1991) should be recalled. The aforementioned models, like many others, are very dissimilar in terms of their structural forms, but they use the same kind of data: returns, or squared returns, varying only in time horizon (e.g. daily returns, weekly returns, etc.). In 1997 Muller, Dacorogna, Dave, Olsen, Pictet and von Weizsacker undermined the time coherence of data used in ARCH model. They proposed Heterogeneous Autoregressive Conditional Heteroskedasticity (HARCH) model, which takes into account several time-dependent components of volatility. The authors assume that different market participants take different time horizons into consideration, so they contribute to the overall level of volatility in their specific way. All of these ways should be separately treated in the model.

The second important deviation from the canonical approach in ARCH models is using a less obvious volatility approximation. Since Parkinson paper (1980), it is well known that there exist more efficient volatility estimators than squared returns. Those estimators are based on widely available data such as high, low, close, and open prices, yet they contain much more information than simple close-to-close returns. Having such approximations of volatility, it is possible to rearrange most of ARCH class models and replace squared returns in their conditional volatility equations with these approximations.

In this paper the Range-based Heterogeneous Autoregressive Conditional Heteroskedasticity model is proposed. The RHARCH model is a new approach to volatility modeling, the aim of this model is to incorporate range-based volatility proxies into HARCH-like framework. The RHARCH model is compared not only with well established ARCH class model such as GARCH and EGARCH models, but also with HARCH and RGARCH (Range-based GARCH) models from which it takes inspiration.

The rest of paper is organized as follow. Section 2 reviews volatility estimators based on high, low, open and close prices. Section 3 describes the data set and reviews reference models, in this section the RHARCH model is derived. Section 4 presents empirical results of models comparison, especially in terms of forecasting performance. Section 5 concludes.

## Volatility Estimators

Supposing that mean of returns equals zero, the mean of squared returns is unbiased variance estimator, so if intraday prices are easily available, computing daily variance estimator is quite straightforward. The problem arises when intraday prices are unavailable, then aforementioned estimator is simply a squared daily return. This estimator is still unbiased, but it can be shown that there exist much more efficient volatility estimators based exclusively on daily data including high, low, open and close prices (also known as range-based variance estimators).

In 1980 Parkinson presented daily volatility estimator based on price range defined as a difference between natural logarithms of highest and lowest daily prices. The Parkinson estimator is asymptotically unbiased under the assumption that a geometric Brownian motion without drift can describe the path of the asset price changes, it can be expressed by the following formula:

$$\sigma^2_{Parkinson} = \frac{1}{4\ln 2} \times (\ln(H_t / L_t))^2 \tag{1}$$

where $H_t$ and $L_t$ are respectively: highest and lowest daily price. In the same year Garman and Klass (1980) proposed even more efficient volatility estimator defined in the following way:

$$\hat{\sigma}^2_{GK,t} = 0.511(\ln(H_t / L_t))^2 - 0.19[\ln(C_t / O_t)(\ln H_t + \ln L_t - 2\ln O_t) \\ - 2(\ln(H_t / O_t)\ln(L_t / O_t)] - 0.383(\ln(C_t / O_t))^2 \tag{2}$$

In the Garman-Klass estimator beside highest and lowest prices, also close ($C_t$ ) and open ($O_t$ ) daily prices are used. Practitioners tend to use simpler form of Garman-Klass estimator:

$$\hat{\sigma}^2_{GK,t} = 0.5(\ln(H_t / L_t))^2 - (2\ln 2 - 1)(\ln(C_t / O_t))^2 \tag{3}$$

The Garman-Klass estimator is asymptotically unbiased under the assumption of no drift in geometric Brownian motion process. Rogers and Satchell (1991) repealed this assumption and derived estimator that is asymptotically unbiased even in the presence of drift in DGP:

$$\hat{\sigma}^2_{RS,N,t} = \frac{1}{N}\sum_{n=t-N}^{t} \ln(H_n / O_n)(\ln(H_n / O_n) - \ln(C_n / O_n)) \\ + \ln(L_n / O_n)(\ln(L_n / O_n) - \ln(C_n / O_n)) \tag{4}$$

Where $N$ is a length of time horizon on which Rogers-Satchell estimator is computed.

There are several other variance estimators based on high, low, open and close prices. Especially those proposed by Kunimoto (1992) or Yang and Zhang (2000) should be mentioned. The common feature of range-based variance estimators is their high relative efficiency. Parkinson reported that his estimator is 2.5 to 5 times more efficient than simple close-to-close variance estimator. Numerical experiments show that more complex estimators can achieve even higher theoretical values.

## Data and models

In this paper EURPLN spot rate quotation is examined. The data set is obtained from financial website stooq.pl, it covers period from 30 September 2007 to 30 September 2013 and contains high, low, open and close daily price. On the basis of these data other variables are calculated such as: daily, weekly, monthly and quarterly logarithmic returns, daily range, Garman-Klass daily variance estimator (using simplified formula) and three Rogers-Satchell daily variance estimators, each one computed with different time horizon (one week, one month and one quarter).

Five ARCH-class models are used as a reference to the RHARCH model. Three of them are well established: GARCH(1,1) (Bollerslev, 1986), EGARCH(1,1) (Nelson, 1991) and GJR-GARCH(1,1) (Glosten, Jaganathan, Runkle, 1993). Functional forms of those three models are similar in case of conditional mean equation, yet they differ substantially in conditional variance specification. Their conditional variance equations are given by formulas (5)-(7) respectively.

$$h_t = \omega + \alpha_1 \varepsilon^2_{t-1} + \beta h_{t-1} \tag{5}$$

$$\ln h_t = \omega + \alpha_1 (abs(\varepsilon_{t-1} / \sqrt{h_{t-1}}) - \sqrt{2/\pi}) + \beta \ln h_{t-1} + \delta(\varepsilon_{t-1} / \sqrt{h_{t-1}}) \tag{6}$$

$$h_t = \omega + \alpha_1 \varepsilon^2_{t-1} + \alpha_2 I_{t-1}\varepsilon^2_{t-1} + \beta h_{t-1} \\ I_t = sign(\min(0, \varepsilon_t)) \tag{7}$$

In the recent years several range-based volatility models have been proposed. Many of them, like CARR model by Chou (2005) or very similar model presented by Mapa (2003), focus on modeling the range itself, thus they omit impact of the distribution of returns. However there are researches that employ classical ARCH framework and simply replace squared returns (squared innovations) with more efficient volatility proxies such as range-based estimators. One of those researches is Molnar paper (2012) where RGARCH(1,1) (Range-based GARCH) model is presented. Conditional variance equation in this model takes the following form:

$$h_t = \omega + \alpha_1 \hat{\sigma}^2_{Parkinson,t-1} + \beta h_{t-1} \tag{8}$$

In above equation instead of squared innovations, as it is in standard GARCH model, the Parkinson estimator is used, but it is obvious that other volatility approximations can be used as well, like in this paper, where the Garman-Klass estimator was chosen.

The RHARCH model draws inspiration from HARCH model. The structural form of HARCH model is expressed by formula (9). In the original Muller et al. article, intraday data were used, so the model encompassed seven components of volatility ($n$=7), while the parameter $j$, which itself describes the length of each component time horizon, took values from 30 minutes to about one quarter.

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &\sim N(0, h_t^2) \\ h_t &= \omega + \sum_{j=1}^{n} \alpha_j \Big(\sum_{i=1}^{j} r_{t-i}\Big)^2 \end{aligned} \tag{9}$$

It should be noticed that conditional variance equation in HARCH model does not include GARCH-like term, so there is no path-dependency problem. Since in this paper data with daily frequency are employed, it is necessary to reformulate the original model. In the modified HARCH model there are four volatility components, each can be associated with a different time horizon (one day, one week, one month and one quarter). The exact form of modified model is given by the following formula:

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &\sim N(0, h_t^2) \\ h_t &= \omega + \alpha_1 r^2_{1,t-1} + \alpha_2 r^2_{5,t-1} + \alpha_3 r^2_{21,t-1} + \alpha_4 r^2_{63,t-1} \\ r_{j,t-1} &= \sum_{i=1}^{j} r_{t-i} \end{aligned} \tag{10}$$

The RHARCH model combines features of range-based GARCH and HARCH models. It preserves time dependent form of HARCH model, yet it employs more efficient volatility proxies than squared returns. Daily variance in horizon of one day is approximated with the Garman-Klass estimator, in case of longer time horizons (one week, one month, one quarter) the Rogers-Satchell estimator is used. The structural form of RHARCH model is expressed in the following way:

$$\begin{aligned} r_t &= \mu + \varepsilon_t \\ \varepsilon_t &\sim N(0, h_t^2) \\ h_t &= \omega + \alpha_1 \hat{\sigma}^2_{GK,t-1} + \alpha_2 \hat{\sigma}^2_{RS,5,t-1} + \alpha_3 \hat{\sigma}^2_{RS,21,t-1} + \alpha_4 \hat{\sigma}^2_{RS,63,t-1} \end{aligned} \tag{11}$$

Just like in case of HARCH model, there is no GARCH-like term in conditional variance equation of RHARCH model, so path-dependency problem does not occur.

## Results

All analyzed models have been estimated using Maximum Likelihood Estimation method with the assumption of conditional normality of returns. In the first part of the research, models have been estimated on whole data set. Table (1) contains models parameters estimates along with their p-values. Parameters significant at the confidence level of 0.05 are bolded. For each model predicted values of conditional variance have been calculated. On the basis of these values standardized residuals have been computed.

**Table 1:** Models parameters estimates.

| | *GARCH* | *EGARCH* | *GJR-GARCH* | *RGARCH* | *HARCH* | *RHARCH* |
|---|---|---|---|---|---|---|
| $\mu$ | *-0.0152* | *0.0054* | *0.0033* | *-0.0036* | - | *-0.0054* |
| | *0.2732* | *0.7044* | *0.8161* | *0.7965* | | *0.6975* |
| $\omega$ | ***0.0059*** | ***-0.0134*** | ***0.0061*** | *0.0083* | ***0.1638*** | *0.0141* |
| | *0.0035* | *0.0143* | *0.0009* | *0.0786* | *0.0000* | *0.4014* |
| $\alpha_1$ | ***0.0943*** | ***0.1550*** | ***0.1310*** | ***0.2647*** | ***0.1305*** | ***0.2449*** |
| | *0.0000* | *0.0000* | *0.0000* | *0.0000* | *0.0000* | *0.0000* |
| $\alpha_2$ | - | - | ***-0.1050*** | **-** | ***0.0310*** | ***0.5558*** |
| | | | *0.0000* | | *0.0000* | *0.0000* |
| $\alpha_3$ | - | - | - | - | ***0.0140*** | *0.0000* |
| | | | | | *0.0000* | *0.9999* |
| $\alpha_4$ | - | - | - | - | ***0.0031*** | ***0.3116*** |
| | | | | | *0.0000* | *0.0000* |
| $\beta$ | ***0.8949*** | ***0.9835*** | ***0.9083*** | ***0.7533*** | - | - |
| | *0.0000* | *0.0000* | *0.0000* | *0.0000* | | |
| $\delta$ | - | ***0.0787*** | - | - | - | - |
| | | *0.0000* | | | | |

It is worth to notice that sum of *"alfas"* and $\beta$ parameters estimates in RGARCH and RHARCH models are greater than one. In case of standard return-based GARCH model, such an observation would indicate nonstationarity of variance. However one should remember that range-based variance estimators are only asymptotically unbiased. Empirical results show that due to discrete nature of intraday price changes, as well as market microstructure effects (e.g. bid-ask spread), observed range is lesser than true range, thus range-based estimators are typically downward biased. That fact explains why sum of parameters estimates in conditional variance equations in range-based ARCH-type models (excluding constant) may be greater than one.

There are several diagnostic tests for volatility models, among them the most important are those for autocorrelation of standardized squared residuals and normality. The first one checks whether the model capture volatility-clustering phenomenon, the second one tests how good the model copes with leptokurtosis of returns. In this paper the Ljung-Box test was chosen as an autocorrelation test, while normality of standardized returns is checked with the Jarque-Bera test. Results of both tests are presented in tables (2) and (3) respectively.

**Table 2:** Ljung-Box test for autocorrelation of standardized squared residuals (number of lags = 5).

| | *GARCH* | *EGARCH* | *GJR-GARCH* | *RGARCH* | *HARCH* | *RHARCH* |
|---|---|---|---|---|---|---|
| $\chi^2(5)$ | *4.2510* | *4.6467* | *2.1878* | *3.7544* | *15.4050* | *1.9714* |
| *p-value* | *0.5139* | *0.4605* | *0.8226* | *0.5853* | *0.0088* | *0.8531* |

Surprisingly, standardized squared residuals from HARCH model are still autocorrelated, the null hypothesis of no autocorrelation within standardized squared residuals must be rejected at any reasonable significance level. Other models seem to perform very well in terms of volatility-clustering capture.

**Table 3:** Jarque-Bera test for normality of standardized returns.

| | *GARCH* | *EGARCH* | *GJR-GARCH* | *RGARCH* | *HARCH* | *RHARCH* |
|---|---|---|---|---|---|---|
| $\chi^2(2)$ | *24.5036* | *10.3647* | *10.8817* | *11.3483* | *74.3357* | *7.8292* |
| *p-value* | *0.0000* | *0.0056* | *0.0043* | *0.0034* | *0.0000* | *0.0200* |

It is evident that distributions of standardized returns from all estimated models are far from Gaussian. However at significance level of 0.02 the null hypothesis would not be rejected in case of RHARCH model. Results indicate that RHARCH model is the best performer among all analyzed models in terms of standardized residuals properties.

Measuring volatility forecast errors rely on volatility proxy. Since volatility is a latent variable, it is not possible to indicate the most accurate approximation. Patton (2010) argues that due to the presence of noise in volatility proxy, comparing predictive abilities of models demands careful choice of loss function. It is possible that inference based on values of loss function may be misleading because optimal forecast may depend on the form of approximation used. In his paper, Patton mentions that two widely used loss function are more robust to noise in proxy than others. Those loss functions are: Mean Squared Error and QLIKE loss function given by the formula (12). It should be underlined that QLIKE is an asymmetric loss function and it tends to favor models that overestimate rather than underestimate true volatility.

$$L(\sigma^2, h) = \ln(h) + \sigma^2 / h \tag{12}$$

In this paper two different daily volatility approximations are employed: the first one is simply a squared daily return (close-to-close estimator), the second one is obtained by Garman-Klass formula. Table (4) contains loss functions values computed for each model with respect to used volatility proxy.

**Table 4:** Values of loss functions, in-sample period.

| | *using squared returns* | | *using G-K estimator* | |
|---|---|---|---|---|
| | *MSE* | *QLIKE* | *MSE* | *QLIKE* |
| *GARCH* | *1.3848* | *0.0311* | *0.4729* | *-0.1012* |
| *EGARCH* | ***1.3517*** | *0.0203* | *0.4536* | *-0.0999* |
| *GJR-GARCH* | *1.3645* | *0.0144* | *0.4646* | *-0.1080* |
| *RGARCH* | *1.4257* | *0.0073* | *0.4646* | *-0.1151* |
| *HARCH* | *1.4931* | *0.1011* | *0.5036* | *-0.0504* |
| *RHARCH* | *1.3923* | ***0.0042*** | ***0.4176*** | ***-0.1237*** |

In three of four cases, the RHARCH model takes the lowest values of loss function. Only in case of Mean Squared Error computed using squared returns as volatility proxy, RHARCH model is inferior to return-based GARCH-type models. At least two general conclusions can be drawn: the first one is that the HARCH model seems to be the worst performer, the second one is that range-based models clearly dominate in case of QLIKE loss function regardless of volatility approximation used.

Information criteria are often employed as simple, yet useful model selection tools. In table (5) Akaike Information Criterion (AIC) as well as Schwarz's Bayesian Information Criterion (BIC) are reported for each model. In both cases the RHARCH model is chosen on a basis of minimal value rule, while the RGARCH model seems to be the second best. It is worth to notice that a difference between those two models is larger in case of AIC. This should not be surprising as the BIC penalises the number of parameters stronger than the AIC.

**Table 5:** Values of Information Criteria.

| | *GARCH* | *EGARCH* | *GJR-GARCH* | *RGARCH* | *HARCH* | *RHARCH* |
|---|---|---|---|---|---|---|
| *AIC* | *2760.68* | *2745.37* | *2736.57* | *2724.06* | *2858.69* | ***2711.10*** |
| *BIC* | *2781.86* | *2771.84* | *2763.04* | *2745.23* | *2885.16* | ***2742.87*** |

In the second part of this research the out-of-sample forecasting performance of models are evaluated. All models have been estimated on a rolling window of 500 observations from analyzed period (30 September 2007 to 30 September 2013). At each step all models are estimated on most recent 500 observations and one-day-ahead forecasts are obtained. Then the window is moved up by one day, and the procedure is repeated. On the basis of these forecasts, values of the same four loss functions are calculated. The results are presented in table (6). Conclusions are pretty similar: the HARCH model still lags behind its competitors, while RHARCH model again takes the lowest values in three of four cases. It is worth to notice that in terms of MSE four models (EGARCH, GJR-GARCH, RGARCH and RHARCH) have very comparable results, but if one takes into account QLIKE function, range-based models still perform better.

**Table 6:** Values of loss functions, out-of-sample period.

| | *using squared returns* | | *using G-K estimator* | |
|---|---|---|---|---|
| | *MSE* | *QLIKE* | *MSE* | *QLIKE* |
| *GARCH* | *0.4624* | *-0.2174* | *0.1057* | *-0.3576* |
| *EGARCH* | ***0.4333*** | *-0.2378* | *0.0868* | *-0.3578* |
| *GJR-GARCH* | *0.4371* | *-0.2379* | *0.0895* | *-0.3649* |
| *RGARCH* | *0.4335* | *-0.2429* | *0.0872* | *-0.3731* |
| *HARCH* | *0.4404* | *-0.1886* | *0.0969* | *-0.3120* |
| *RHARCH* | *0.4351* | ***-0.2496*** | ***0.0854*** | ***-0.3763*** |

To examine statistical significance of differences between values of loss function predictive ability tests are used. In this paper Diebold-Mariano (1995) test is employed. The null hypothesis of this test is that models have the same level of forecasting accuracy, which is measured by chosen loss function. Tables (7) and (8) present pairwise comparison between analyzed models, p-values of Diebold-Mariano test are reported. In both cases Garman-Klass daily variance estimator is used an approximation of true volatility, tables differ in adopted loss function. The alternative hypothesis is that model specified in row has greater forecasting accuracy than model in column, values lesser than 0.05 are bolded.

**Table 7:** P-values of Diebold-Mariano test, MSE used as a loss function.

| | *GARCH* | *EGARCH* | *GJR-GARCH* | *RGARCH* | *HARCH* | *RHARCH* |
|---|---|---|---|---|---|---|
| *GARCH* | - | *0.9952* | *0.9931* | *0.9908* | *0.7674* | *0.9853* |
| *EGARCH* | ***0.0048*** | - | *0.1558* | *0.4645* | *0.0857* | *0.5967* |
| *GJR-GARCH* | ***0.0069*** | *0.8442* | - | *0.7006* | *0.1712* | *0.7203* |
| *RGARCH* | ***0.0092*** | *0.5355* | *0.2994* | - | *0.1301* | *0.6924* |
| *HARCH* | *0.2326* | *0.9143* | *0.8288* | *0.8699* | - | *0.8683* |
| *RHARCH* | ***0.0147*** | *0.4033* | *0.2797* | *0.3076* | *0.1317* | - |

**Table 8:** P-values of Diebold-Mariano test, QLIKE used as a loss function.

| | *GARCH* | *EGARCH* | *GJR-GARCH* | *RGARCH* | *HARCH* | *RHARCH* |
|---|---|---|---|---|---|---|
| *GARCH* | - | *0.3735* | *0.9712* | *0.9991* | ***0.0000*** | *1.0000* |
| *EGARCH* | *0.6265* | - | *1.0000* | *0.9995* | ***0.0000*** | *1.0000* |
| *GJR-GARCH* | ***0.0288*** | ***0.0000*** | - | *0.9552* | ***0.0000*** | *0.9990* |
| *RGARCH* | ***0.0009*** | ***0.0005*** | ***0.0448*** | - | ***0.0000*** | *0.9999* |
| *HARCH* | *1.0000* | *1.0000* | *1.0000* | *1.0000* | - | *1.0000* |
| *RHARCH* | ***0.0000*** | ***0.0000*** | ***0.0010*** | ***0.0001*** | ***0.0000*** | - |

Results of Diebold-Mariano tests confirm earlier observations. Definitely the HARCH model, at least in the form employed in this research, performs very poor, especially in terms of QLIKE loss function, which means it underestimates volatility more often than the rest of analyzed models. On the other side, range-based models cope with volatility underestimation problem significantly better than return-based models.

## Conclusions

In this paper Range-based Heterogeneous Autoregressive Conditional Heteroskedasticity (RHARCH) model is proposed. The RHARCH model draws inspiration from the HARCH model, presented by Muller et al. (1997), which takes into account several time-dependent components of volatility. The main difference between RHARCH and HARCH models is that the first one uses range-based estimators (Garman-Klass and Rogers-Satchell) rather than squared returns as a volatility approximation in conditional variance equation.

The RHARCH model is compared with five other ARCH-type models, all of them are estimated using MLE method with assumption of normal distribution of returns. The comparison is conducted on a set of EURPLN spot rate quotations. General properties of models and forecasting abilities are examined. In both cases the RHARCH models perform very well. Standardized residuals does not show any statistically significant autocorrelation, moreover their empirical distribution is closest to Gaussian among all analyzed models. Moreover, Information Criteria indicate the RHARCH model as the best one. Predictive ability of the RHARCH model is also very encouraging. Depending on loss function and volatility approximation used, the RHARCH model performs good or very good – in most cases it takes the lowest values of loss function. Results of Diebold-Mariano tests shows that range-based models (RHARCH and RGARCH) have significantly better forecasting accuracy measured by QLIKE loss function than return-based models. It is also worth to notice that RHARCH model does not have GARCH-like term in its conditional variance equations, thus it does not suffer path-dependency problem. That means estimation of regime-switching form of RHARCH model is quite straightforward and is feasible using MLE method.

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