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PIRACY AS AN ETHICAL DECISION

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Abstract

We consider a monopolist producer of information goods that may be subject to unauthorized copying. The key feature of our model is that we allow consumers to have ethical concerns based on equity theory that may reduce their utility of such a copy. We derive the formulas describing demand for the product. We find that piracy reduces prices and producers' profit, an effect that can be limited by such measures as copyright enforcement (proxied by expected value of punishment for piracy) and anti-piracy campaigns. Welfare effects are also analyzed and generally turn out to be ambiguous.

1 Introduction

With the rise of computer- and Internet-based technology, information goods quickly grow in importance. This is reflected in both academic analysis and public debates concerning several of their specific aspects. One of them concerns the unauthorized sharing of information goods, notably computer software (including games), music and films, aka "digital piracy". Its impact on the production of content and on general welfare continues to be hotly debated, with the two sides often taking quite extreme views. The corporate world often emphasizes that if unauthorized copies are easily available, it is very hard to monetize creative work, leading to "underproduction". Of course, this claim is based on the presumption that pirated copies actually crowd out official sales, which is found in most (Rob and Waldfogel, 2004) but not all (Andersen and Frenz, 2010) empirical studies. On the other hand, digital piracy may undercut the copyright holders' monopolistic power, limiting "underutilization", thus possibly increasing welfare.

It cannot be expected that scientific analysis comes up with easy and ultimate answers to the key questions (especially concerning the optimal level of copyright protection), given that

- the industry is subject to rapid technological changes

- data on digital piracy (as is often true for other "gray zones") is highly imperfect and incomplete

- both benefits and (especially) costs of piracy tend to be diffuse, indirect and lagged

The theoretical advances on piracy behavior typically follow one of two rather distinct paths. The first one is rooted in sociology and criminology and seeks to establish antecedents of pirates' "deviant behavior" in terms of their intentions, beliefs, perceived social norms etc. These models nearly never take on a well-defined functional form that would i.a. allow making predictions concerning the changes in welfare resulting from piracy. They are typically verified using survey-based constructs, rather than any behavioral data obtained in the field.

An important issue that these models seek to explore concerns the role of ethical decision making in pirates' behavior. The term itself suggests a highly unethical activity and the industry often tries to argue that piracy is tantamount with theft. However, most authors agree that level of moral intensity of digital piracy remains quite low (Logsdon et al., 1994). Harrington (1989), among others, finds no impact of students' individual characteristics related to moral reasoning on (self-reported) software copying.

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Nevertheless, some authors propose that ethical judgments affect propensity to engage in piracy and have successfully used ethical notions, notably the equity theory, to explain pirates' behavior. In particular, Glass and Wood (1996) investigate reciprocal sharing of content between consumers. Kwong and Lee (2002) find that an equitable relationship between consumer and copyright holders reduces piracy.

On the other hand, there are several formal models of piracy. They typically all but neglect the psychological or ethical dimension of pirates' decision-making, basing on a simple "rational crime" paradigm.

To name a few examples, Novos and Waldman (1984) focus on the cost of obtaining an unauthorized copy. They show that when this is taken into account, copyright protection does not lead to increase in welfare cost associated with underutilization. Bae and Choi (2006) distinguish between two types of cost of piracy: reproduction vs. degradation, finding rather different implications.

Landes and Posner (1989) show how curbing piracy by copyright protection may lead to welfare losses due to increased cost of production and copying of creative works (rather than simply reduced access for consumers due to higher prices). A similar approach is pursued by Chen and Png (2003). Yoon (2002) in turn allows for private costs to consumers arising from copyright protection.

Gopal and Sanders (1997) assume that software is purchased by a "club" of individuals who subsequently share it. They show that preventive (front-end) measures to reduce such illicit sharing will negatively affect both producers' profits and welfare, while deterrent (back-end) measures may be beneficial. While these authors do consider the "ethical index" they do not include it in their formal model.

Substantial literature has also evolved on possible positive effects of piracy. A prominent example involves sampling – see Belleflamme and Peitz (2010) for a review. Reavis Conner and Rumelt (1991) show that copyright protection may backfire if the good is characterized by substantial network externality. Benefits from piracy under demand network externalities are also exposed by Takeyama (1994).

The primary goal for this paper is to allow for ethical preference for authorized sources in a formal model of piracy. We seek to establish how such 'non-standard' preference affects welfare analysis, especially with regard to such policy measures as penalization of digital piracy and social campaigns discouraging the use of unauthorized content.

In our model of an ethical consumer we choose the teleological rather than deontological perspective – we focus on the perceived consequence of piracy rather than the act itself. There is some evidence in the literature (Thong

and Yap, 1998) that this perspective may be more fruitful, mostly because there is little evidence that piracy is universally seen as unethical per se. The teleological perspective also helps to bring out the role of circumstances of the act such as price of the pirated good in ethical assessment. More specifically, we consider equity-oriented customers who suffer utility loss when they obtain valuable, reasonably priced content from an illicit source, thus failing to reciprocate the developer's contribution.

A paper that may be closest to ours is (Kanniainen and Pääkkönen, 2007) which considers social norms decreasing the utility of the illegal version. However, it differs substantially in the specification of the non-material cost of piracy which in their model stems for social disapproval, a form of network externality – it weakens as more people begin to pirate. It also considers a different market structure, assuming central distribution of the illegal product by a profit-maximizing "producer".

2   The benchmark model

An individual can choose among three alternatives: he[1] can purchase the product, obtain it for free from an unauthorized source (an act of "piracy") or not consume at all. Denoting the price charged for the product by P and the direct use value resulting from its features and functionalities by V, we assume the consumer seeks to maximize the following utility function

$$U = \begin{cases} V - P & \text{the consumer buys} \\ 0 & \text{no consumption} \\ V - \alpha f(V, P) & \text{the consumer pirates} \end{cases}$$

where $f(V, P)$ represents the "moral cost of piracy" and α stands for sensitivity to such a cost. Concerning the former, we propose it is natural to assume that the feeling that piracy is inappropriate grows as the value of the product V increases relative to its price. Indeed, as mentioned before, equity theory proposes that individuals feel they should reciprocate (i.e. pay) when an attractive product is offered for a reasonable price. It seems less of a problem to pirate if the price is outrageous. On the other hand, if the product is offered at a very low price, the consumer can reason that it was cheap to develop (and possibly also that the customer base will be large, enabling the producer to recoup her expenses anyway) and thus there is little moral weight attached to the act of piracy. Indeed, most people feel quite OK about

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[1]We shall refer to the consumer as "he" and the monopolist as "she", obviously an arbitrary convention.

occasionally "borrowing" a pencil. It appears thus justified to assume that for fixed P, $F(V, P)$ increases in V and that it initially increases and then decreases in P for a fixed V.

Another way to obtain the same relationship is to employ Gneezy's 2002 suggestion that suffering from unethical behavior corresponds to the harm inflicted upon the other party in the process. In our case, the primary harm involved in the act of piracy is the lost revenue. We can thus assume

$$f(P, V) = \Pr(\text{Purchase})P,$$

Where $\Pr(\text{purchase})$ is consumer's perceived likelihood of ever purchasing the product in the future. This expected loss of revenue is highest for intermediate price – if the price is too high, the consumer reasons he would never consider purchasing the product anyway. For concreteness we assume that the probability of purchase is $\Pr(\text{Purchase}) = \frac{1}{1+e^{P-V}}$ and thus the moral cost of pirating is $f(P, V) = \frac{P}{1+e^{P-V}}$.

It is natural to assume that individuals in the society differ with respect to their moral qualities, which in this model is expressed by the weight attached to the moral cost, namely α. Furthermore, specific individuals value the product differently. Let us assume that V, α are independent and are uniformly distributed, that is, $V \sim U[0, V_{max}], \alpha \sim U[0, \alpha_{max}]$ in a population of consumers with a mass of one. Then, individuals' optimal choices given the price can be summarized as follows (for expositional clarity we divide the space into three subsets depending on the value of α):

Lemma 1   1. *Individuals with $\alpha \in [0, 1+e^{-V_{max}})$ will not buy the product. Those who belong to $[0, V_0)$, where V_0 fulfills $V_0(1 + e^{P-V_0}) = \alpha P$, will not consume the product at all; those who belong to $(V_0, V_{max}]$ will pirate it.*

   2. *Individuals with $\alpha \in [1 + e^{-V_{max}}, 2)$ fall into three categories. Those whose V belongs to $[0, V_0)$ will not consume the product; those in $(V_0, P - \ln(\alpha - 1))$ will pirate it; those in $[P - \ln(\alpha - 1), V_{max}]$ will buy it.*

   3. *Individuals with $\alpha \geq 2$ will purchase the product if and only if their V exceeds P. Remaining consumers will either pirate it or not consume at all. Those who belong to $[0, V_0)$, where V_0 fulfills $V_0(1 + e^{P-V_0}) = \alpha P$, will not consume the product at all; those who belong to $(V_0, V_{max}]$ will pirate it.*

First of all, Lemma 1 states that individuals who do not attach much weight to disutility from pirating will not buy the product. If their value of

the product is low, they will abstain from consumption. If it is sufficiently high (higher than V_0) they will pirate the product. Secondly, if the individuals are highly moral ($\alpha \geq 2$) they will buy the product provided it is worth its price. Only when the net value of the product is lower than zero they will consider pirating it. Thirdly, the option of pirating introduces the most significant change for individuals with the intermediate sense of morality ($\alpha \in [1 + e^{-V_{max}}, 2)$). Even if they value the product more than its price they may still decide to pirate the product.

We are now in a position to determine the demand functions for each of our categories.

Lemma 2
The demand among individuals with an given value of α is

$$D_2(P) = \begin{cases} \frac{V_{max} - P}{V_{max}} & \alpha \geq 2 \\ \frac{V_{max} - P + \ln(\alpha - 1)}{V_{max}} & 1 + e^{-V_{max}} \leq \alpha < 2 \\ 0 & 0 \leq \alpha < 1 + e^{-V_{max}} \end{cases}$$

The second demand curve is the first demand curve shifted by the constant $\ln(\alpha - 1)$. This constant represents the loss in consumer demand due to the option of pirating (in the "no piracy" scenario the demand is simply $D_1(P) = (V_{max} - P)/V_{max}$ no matter the value of α).

To set stage for further analysis we assume that the product is produced by a monopolist with zero marginal cost. It seems a reasonable assumption for non-material, intellectual goods.

Let us first consider the world without piracy. Producer's revenue will be given by:

$$R_1(P) = Pr(V \geq P)P = \frac{V_{max} - P}{V_{max}} P$$

Remark 1 *The optimal price in the "no piracy" scenario is $P_1 = \frac{V_{max}}{2}$. Monopoly's revenue is then $R_1 = \frac{V_{max}}{4}$.*[2]

In the "piracy" scenario the monopolist faces three distinct groups of consumers; those who are the most sensitive to moral cost ($\alpha \geq 2$); those who are

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[2]Here, we do not need to assume uniform distribution. Let F_1, F_2 denote, respectively, the cdf of V and α, which are independent. For P_1 to be the unique maximum , it is sufficient to assume that the so called reliability function $1 - F_1$ is log-concave. Then $\frac{F_1'}{1 - F_1}$ $((\log F_1)')$ is decreasing. There are numerous applications of log-concave probabilities in economics: Laffont and Tirole (1988) in game theory; Caplin and Nalebuff (1988) in spatial economics; Bagnoli and Khanna (1992) in real estate markets; Flinn and Heckman (1983) in job search; Bergstrom and Bagnoli (1993) in the marriage market.

moderately sensitive to moral cost ($[1+e^{-V_{max}}, 2)$) and those who do not care much about moral cost ($[0, 1 + e^{-V_{max}})$). The members of the last group do not buy the product. The monopoly has two options depending on whether she knows the morality of the consumer. If she knows the "moral type" of the consumer, she can perfectly price discriminate and charge different prices to different consumers.[3]

Lemma 3
If the monopolist could perfectly price discriminate between more and less moral consumers he would charge the price $P_2^h = \frac{V_{max}}{2}$ to the more morally sensitive consumers ($\alpha \geq 2$) and prices $P_2^\alpha = \frac{V_{max} + \ln(\alpha - 1)}{2}$ tailored to any α in the less morally sensitive range ($\alpha \in [1 + e^{-V_{max}}, 2)$). Obviously $P_2^h > P_2^\alpha$ for any ($\alpha \in [1 + e^{-V_{max}}, 2)$). Moreover, resulting revenue R_2 is lower than R_1, that is, monopoly's revenue is lower when there is the option of pirating.

If the monopolist does not know α she cannot perfectly price discriminate and maximizes her revenue knowing only the distribution of α.

Lemma 4
If the monopolist cannot perfectly price discriminate, then there exists the unique price P_3 that maximizes her revenue. Moreover, $P_3 < P_1$ and monopoly's revenue is lower compared to the "no piracy" scenario.

Although the monopoly's revenue is lower comparing to the "no piracy" scenario, lower price of the product increases consumer surplus, therefore the overall social welfare effect of piracy is a priori not known. In particular, we are interested whether it is possible that piracy brings about higher social welfare.

Lemma 5 *There exists V_{max}^* and α_{max}^* such that for all $V_{max} > V_{max}^*$ and $\alpha_{max} > \alpha_{max}^*$ social welfare under "piracy" scenario is higher than under "no piracy" scenario. Increasing α makes social welfare under "piracy" and "no piracy" scenario converge.*

Lemma 5 states that one can find parameters which reflect product evaluations and moral sensitivity such that piracy results in higher social welfare. For $\alpha*_{max} = \infty$ social welfare in both settings is equal, that is, the moral costs of piracy is so large that piracy scenario is treated as if there was no option of piracy at all. Lemma 5, however, states that we can find values

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[3]For example, research shows that young people tend to be less sensitive to ethical constraints. Offering software to students at discount prices may thus be an attractive option not only because of their low incomes.

of α_{max} lower than infinity and social welfare in the piracy world will be unambiguously higher than in the no piracy world. This is primarily because of two effects: first, prices are lower and second, some individuals with low α and high V can now obtain the product for free and with low moral cost. Within this parameter range these effects outweigh the negative social welfare effect of some purchasers switching to piracy – the resulting moral cost is pure welfare loss. Please note that Lemma 5 does not establish what the overall relationship between α_{max} and social welfare is. One could think that relationship should be negative, since increasing α_{max} means that there are proportionally more individuals who care more about the moral consequences of the piracy, social welfare under piracy should go down with higher values of α_{max}. Yet, this omits a second effect that is at play here, namely, the fact that the price P_3 depends on α_{max} too. Since the price increases with α_{max} [4], this lowers social welfare under piracy. It follows from Lemma 5 that we can find such values of α_{max} for which the former effect unambiguously dominates.

3 Policy implications

3.1 A policy that punishes piracy

We now introduce the cost of piracy c, which is a fine that a pirate must pay when caught (which happens with probability p) expressed as a percentage of the product price. Further, let $h := pc$. We now have the following utility function

$$\begin{cases} \text{purchase} & V - P = \max(V - P, 0, V - \alpha\frac{P}{1+e^{P-V}} - hP) \\ \text{no consumption} & 0 = \max(V - P, 0, V - \alpha\frac{P}{1+e^{P-V}} - hP) \\ \text{piracy} & V - \alpha\frac{P}{1+e^{P-V}} = \max(V - P, 0, V - \alpha\frac{P}{1+e^{P-V}} - hP) \end{cases}$$

Lemma 6 *1. Individuals with $\alpha \in [0, (1-h)(1+e^{-V_{max}})]$ will buy the product. Let $V_0^h = \alpha\frac{P}{1+e^{P-V}} + hP$. Those with V in $[0, V_0^h]$ will not consume the product, and within $[V_0^h, V_{max}]$ will pirate it.*

2. If $\alpha \in [(1-h)(1+e^{-V_{max}}), 2(1-h)]$ then we have three distinct types of consumers: buyers, non-buyers and pirates.

3. If $\alpha > 2(1-h)$ then all those who value the product more than its price will buy the product.

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[4]Please look at the proof of Lemma 5.

Let $\alpha_1 = (1 + e^{-V_{max}}), \alpha_2 = 2$ denote the critical values of α when there was no policy. Further, let $\alpha_1^h = (1-h)(1+e^{-V_{max}}), \alpha_2^h = 2(1-h)$ denote the critical values of α when policy is implemented. We obtain the following results.

Remark 2 *For $h > 1$ we have that $V - P > V - \alpha\frac{P}{1+e^{P-V}} - hP$, so expected fine is higher than the price, and no one pirates the product. Hence, from now on we consider $h \in [0, 1]$.*

Lemma 7

1. *The policy reduces the set of individuals who never buy the product by reducing the set of pirates (comparing to the set of non-consumers).*

2. *The policy reduces the set of less morally sensitive consumers.*

3. *The policy expands the set of buyers among those who value the product more than its price.*

Lemma 8
The demand function for the product is

$$D^h(P) = \begin{cases} V_{max} - P & \alpha \geq \alpha_2^h \\ V_{max} - P + \ln(\frac{\alpha}{1-h} - 1) & \alpha_1^h \leq \alpha < \alpha_2^h \\ 0 & 0 \leq \alpha < \alpha_1^h \end{cases}$$

Remark 3 *For less morally sensitive consumers, the policy results in higher demand for the product. We have that $\frac{dD^h(P)}{d\alpha} > 0$ unless $\alpha > 1 - h$, which is always true for $\alpha_1^h \leq \alpha < \alpha_2^h$.*

Lemma 9
If the monopolist cannot perfectly price discriminate, then there exists the unique price P_4 that maximizes his revenue. Moreover, $P_4 > P_3$.

As expected, the punishing policy results in higher monopoly's revenue and lower surplus of both consumers and pirates. With the punishing as additional cost, monopoly can now charge higher price for its product.

3.2 A policy that impacts on moral qualities: social campaign

The government or producers may want to discourage people from piracy by running social campaign which shows that piracy is morally unacceptable or at least inferior to actually purchasing the product. One such example is the Music Matters campaign in the UK. We model such policy by changing the distribution of α, that is, the distribution that results following the social campaign is stochastically first-order dominates the pre-campaign distribution. In other words, for a given level of α (weight attached to individual's own payoff) there is lower percentage of individuals in pre-campaign distribution than in post-campaign who have given α or lower. The utility function remains unchanged.

We consider the pre-policy and post-policy distributions, which are, respectively, $U[0, \alpha_{max}^1]$, $U[0, \alpha_{max}^2]$, where $\alpha_{max}^2 > \alpha_{max}^1$.

Lemma 10 *Under no price discrimination, the policy that impacts on moral qualities increases demand for the product and the revenue of the monopolist.*

4 Conclusions

Our formal model gives basic insights into the role that ethical concerns may be playing for the phenomenon of piracy. First, we establish that moral reservations do not save firms from piracy-related profit losses (which are rather typical for piracy models with no network effects and no possibility of indirect rent appropriation) although these losses diminish as ethical concerns rise. Second, we see that ethical aversion to piracy leads to higher prices. In the short run, even the most honest customers benefit from their peers' lack of scruples, unless the monopolist is able to price discriminate. Third, we analyze two possible measures of affecting pirates' behavior. We show that both stronger copyright enforcement and (effective) anti-piracy campaigns lead to higher prices and higher revenue of the producer.

Our model is obviously highly simplified. In particular, the market structure is exogenous. Assuming some specific level of fixed costs, piracy may drive the monopolist out of the market and thus reduce welfare. Allowing for product differentiation is another natural extension – piracy may reduce incentives for firms to enter the market thus reducing product variety. If quality of the product may be freely chosen (and is costly to raise), piracy may distort producers' incentives. These effects will be considered in subsequent drafts of the paper.

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Proofs

Lemma 1
Proof.

The set of combinations of (V, P) that make the individual indifferent between purchasing and pirating is given by $V - P = V - \alpha \frac{P}{1+e^{P-V}}$, that is, $V = P + \ln(\alpha - 1)$ – individual will not buy if his value is below that and he will not pirate if his value is above (in either case he may still abstain). If $V < \ln(\alpha - 1)$, individual will not purchase the product no matter how low its price may be. Thus individuals with low sensitivity to moral cost, $\alpha < 1 + e^{-V_{max}}$ will never purchase the product, no matter how high value they attach to it and how cheap it is, which proves 1a).

The value V_0 that fulfills $V = \alpha \frac{P}{1+e^{P-V}}$ makes the individual indifferent between abstaining and pirating – for higher values of V, they will not abstain and for lower values they will not pirate which completes the proof of 1b) and 1c)

Proof of point 2 follows directly by verifying that for $\alpha \in [1 + e^{-V_{max}}, 2)$ we have $V_0 < P - \ln(\alpha - 1)$ Thus those with V below V_0 will neither consume nor pirate, i.e. they will abstain. Those in $(V_0, P - \ln(\alpha - 1))$ will prefer to pirate and those above $P - \ln(\alpha - 1)$ will purchase.

As for point 3, for $\alpha \geq 2$, $V > P$ implies $V - P > V - \alpha \frac{P}{1+e^{P-V}}$ which completes the proof.

$\square$

Lemma 3
Proof. The demand curve for the more moral consumers is $D_2(P) = \frac{V_{max}-P}{V_{max}}$, the monopoly's revenue is then $R_2(P) = \frac{V_{max}-P}{V_{max}}P$, which is maximized for $P_2^1 = \frac{V_{max}}{2}$. Similarly we prove that the price for less moral consumers equals $P_2^2 = \frac{V_{max}+\ln(\alpha-1)}{2}$. Monopoly's revenue is now lower comparing to the "no piracy" scenario because then he could charge higher price (P_2^1) to all the consumers (no matter the α).

$\square$

Lemma 4
Proof. For a given price P the set of those who buy the product is given by $V \geq P$ and $\alpha \geq 1 + e^{P-V}$. If $P \geq V_{max}$ then this set is empty, hence we assume $P < V_{max}$. Also to consider all consumer types specified before, we assume $\alpha_{max} > 2$. The demand for the product is the following

$$D_3(P) = \begin{cases} \frac{1}{\alpha_{max}V_{max}} \int_P^{V_{max}} (\alpha_{max} - 1 - e^{P-V})dV & P \leq V_{max} \\ 0 & P > V_{max} \end{cases}$$

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and

$$D_3(P) = \begin{cases} -\dfrac{1-e^{P-V_{max}}+(1-\alpha_{max})(V_{max}-P)}{\alpha_{max}V_{max}} & P \leq V_{max} \\ 0 & P > V_{max} \end{cases}$$

The revenue is then

$$R_3(P) = \begin{cases} -P\dfrac{1-e^{P-V_{max}}+(1-\alpha_{max})(V_{max}-P)}{\alpha_{max}V_{max}} & P \leq V_{max} \\ 0 & P > V_{max} \end{cases}$$

This is a continuous function on the interval $[0, V_{max}]$ so it has a maximum. Furthermore, $R_3(P) = 0$ for $P = 0$ and $P = V_{max}$. In the neighborhood of $P = 0$ we have $R_3'(0) = \frac{e^{-V_{max}}-1+V_{max}(\alpha_{max}-1)}{V_{max}\alpha_{max}}$, which knowing that $\alpha_{max} > 2$ is positive for $V_{max} > 0$, so the maximum is attained in the interior.

We now need to prove that there is only one maximum. The third derivative of $R_3(P)$ is $R_3'''(P) = \frac{e^{P-V_{max}}(3+P)}{V_{max}\alpha_{max}} > 0$ for all P, hence the second derivative is a strictly increasing function. Therefore, we know that $R_3''(P) = 0$ has at most one solution, and in consequence $R_3'(P)$ has at most one local extremum. We have,

$$R_3'(P) = \frac{e^{P-V_{max}}(1+P) - 1 + (\alpha_{max}-1)(V_{max}-2P)}{V_{max}\alpha_{max}},$$

and Next,

$$R_3'(V_{max}) = -1 + \frac{2}{\alpha_{max}} < 0$$

for $\alpha_{max} > 2$ and as we already showed $R_3'(0) > 0$ for all $V_{max} > 0, \alpha_{max} > 2$. Therefore, we can conclude that $R_3'(P)$ attains zero at most one point, that is, $R_3(P)$ has at most one maximum.

We now need to prove that the price chosen by the monopolist who cannot discriminate is lower than the price in the "no piracy" scenario. We have that

$$R_3'(\frac{V_{max}}{2}) = \frac{e^{-\frac{V_{max}}{2}}(2 - 2e^{\frac{V_{max}}{2}} + V_{max})}{2V_{max}\alpha_{max}} < 0$$

for $V_{max} > 0$. Knowing that $R_3'(0) > 0, R_3'(\frac{V_{max}}{2}) < 0$ and that $R_3'(P)$ is a continuous function that attains zero at most one point, we have $P_3 < P_1$.

$\square$

Lemma 5
Proof. Please address the authors.

$\square$

Lemma 6
Proof. The proof follows along the lines of proof of Lemma 1.

$\square$

Lemma 7
Proof.

1. Clearly, $\alpha_1^h < \alpha_1$. In addition, V_0^h decreases with h.

2. Previously, we had $\alpha_2 - \alpha_1 = 1 - e^{V_{max}}$. Now we have $\alpha_2^h - \alpha_1^h = (1 - h)(1 - e^{V_{max}}) < 1 - e^{V_{max}}$.

3. Clearly, $\alpha_2^h < \alpha_2$

$\square$

Lemma 9
Proof. Our goal is to show that the price chosen under punishing regime is unique and higher than when there is no punishing.

For a given price P the set of those who buy the product is given by $V \geq P$ and $\alpha(V, h) \geq (1 - h)(1 + e^{P-V})$. If $P \geq V_{max}$ then this set is empty, hence we assume $P < V_{max}$. Also to consider all consumers we assume $\alpha_{max} > 2$. The demand for the product is the following

$$D_4(P, h) = \frac{1}{\alpha_{max} V_{max}} \int_P^{V_{max}} (\alpha_{max} - (1 - h)(1 + e^{P-V}))dV$$

The integrand expression increases with h, so the demand is higher comparing to "no punishing" regime. Further, the revenue is

$$R_4(P, h) = \begin{cases} -P\frac{(-1+h)(-1+e^{P-V_{max}-P})-P\alpha_{max}+V_{max}(-1+h+\alpha_{max})}{\alpha_{max}V_{max}} & P \leq V_{max} \\ 0 & P > V_{max} \end{cases}$$

We can write
$$R_4(P, h) = R_{41}(P) + hR_{42}(P),$$

where

$$R_{41}(P) = P\frac{-1 + e^{P-V_{max}} + P - P\alpha_{max} + V_{max}(-1 + \alpha_{max})}{\alpha_{max}V_{max}}$$

and

$$R_{42}(P) = P\frac{1 - e^{P-V_{max}} - P + V_{max}}{\alpha_{max}V_{max}}.$$

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We know from Lemma 4 that maximization of $R_{41}(P)$ has a unique solution denoted by P_3. We now check that $R'_{42}(P) = 0$ has a unique solution denoted by $\tilde{P}$. We have that

$$R''_{42}(P) = \frac{-2 - e^{P-V_{max}}(2 + P)}{\alpha_{max}V_{max}}$$

which is negative for all V_{max} and P (because $e^{P-V_{max}} > \frac{-2}{2+P}$), hence $R'_{42}(P)$ is a strictly decreasing function. Furthermore,

$$R'_{42}(P) = \frac{1 - 2P - e^{P-V_{max}}(1 + P) + V_{max}}{\alpha_{max}V_{max}}$$

and

$$R'_{42}(0) = \frac{-e^{-V_{max}} + 1 + V_{max}}{\alpha_{max}V_{max}},$$

which is positive for all $V_{max} > 0$ ($e^{-V_{max}} < 1 + V_{max}$). Also,

$$R'_{42}(V_{max}) = \frac{-2}{\alpha_{max}},$$

which is negative for all $\alpha_{max} > 0$. Therefore $R'_{42}(P) = 0$ has only one solution.

We will now show that $P_3 < \tilde{P}$. We notice that

$$R_{41}(P) = -R_{42}(P) + \frac{P(V_{max} - P)}{V_{max}}$$

and

$$R'_{41}(P) = -R'_{42}(P) + \frac{V_{max} - 2P}{V_{max}}.$$

We evaluate this at $P = \tilde{P}$.

$$R'_{41}(\tilde{P}) = 0 + \frac{V_{max} - 2\tilde{P}}{V_{max}},$$

recalling that $\tilde{P}$ was chosen such that $R'_{42}(P) = 0$. Let us notice that $\tilde{P} > \frac{V_{max}}{2}$ since R'_{42} is a strictly decreasing function and

$$R'_{42}(\frac{V_{max}}{2}) = \frac{-2e^{\frac{-V_{max}}{2}} + 2 - V_{max}e^{\frac{-V_{max}}{2}}}{2\alpha_{max}V_{max}}$$

is positive for all $V_{max} > 0$. Therefore $R'_{41}(\tilde{P}) < 0$, which given that $R'_{41}(P)$ is strictly decreasing and that $R'_{41}(P_3) = 0$ gives us $P_3 < \tilde{P}$.

For $P > \tilde{P}$ we have $R'_{41}(P) < 0$ and $R'_{42}(P) < 0$, hence $R'_4(P, h) < 0$. Thus we know that the solution to $R'_4(P, h) = 0$ denoted by P_4 is found on the interval $[0, \tilde{P})$. On the other hand, for $P = P_3$ we have that

$$R'_4(P_3, h) = R'_{41}(P_3) + hR'_{42}(P_3) = 0 + hR'_{42}(P_3),$$

which is positive given $h > 0$ and $R'_{42}(P_3) > 0$, where the latter follows from $P_3 < \tilde{P}$ and the fact that $R'_{42}(P)$ is strictly decreasing. Thus we can further limit the considered interval to $(P_3, \tilde{P})$. Knowing that for every h, function $R'_4(P, h)$ is strictly decreasing we obtain $P_4 > P_3$.

$\square$

Lemma 10
Proof. The derivative of the $D^P(P)$ with respect to α_{max} is $\frac{1+e^{P-V}}{\alpha V_{max}^2}$.

$\square$