University of Warsaw
Faculty of Economic Sciences

Working Papers
No. 16/2013 (101)

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Warsaw 2013

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A theoretical model of crowdfunding.

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Abstract
This paper proposes a theoretical framework to capture the underlying mechanisms of the innovative online crowdfunding. The goal of this work is to emphasize the advantages of crowdfunding platforms over traditional methods of sale, by describing the capabilities unique to such model and processes stemming from them. Namely, the producer's chance for perfect discrimination is discussed, as well as the contributor's role and his decision dilemma. Numerous extensions to the model provide additional insight into crowdfunding platforms.

Keywords:
crowdfunding, strategic fundraising, innovation, Kickstarter, complete discrimination, pay what you want

JEL:
L11, L12, L15, L21, D42, G32

Acknowledgments:
I would like to thank Joanna Tyrowicz for her help and insight during the development of this framework and dr Maciej Sobolewski for his constructive review.

1. Introduction

This paper proposes a theoretical framework to capture the underlying mechanisms of the innovative online crowdfunding. The goal of this work is to emphasize the advantages of crowdfunding platforms over traditional methods of sale, by describing the capabilities unique to such model and processes stemming from them. Namely, the producer's chance for perfect discrimination is discussed, as well as the contributor's role and his decision dilemma.

Online crowdfunding platforms have only appeared in the recent years, but already they have begun to revolutionize the traditional ways of conducting business. The appeals seem many and directed both at the producer and the consumer.

The former gets the chance to set up regular production from scratch, by acquiring funds from future consumers before incurring the costs. At the same time, the producer may challenge his idea before investing his money in the new venture – if the crowd does not respond, than the project was possibly going to fail anyway and the project's manager does not lose anything. However, the crowd is not only the source of money, but also a willing, and best possible (as these are in fact the most eager consumers) panel of interviewees and judges of the product's future design and features. Other benefits include the independence from large-scale investors who often tend to impose their own constraints on the initial vision of the producer. If that was not enough, the project manager has the means of setting up a feasible system of rewards and benefits that allows for a perfect discrimination of the product's buyers.

The latter has a direct influence on the project's outcome and the product he is intending on purchasing. By being a part of the community, he can interact with the producers before the final product is finished. Moreover, he can choose the level of his contribution accordingly to his individual value of the crowdfunded project. This continuous spectrum of choices allows him to achieve a higher utility than he would if he was only given a fixed option.

Both the former and the latter achieve a perhaps even more important benefit – because of the online platforms' ability to easily connect a huge crowd of people, project's get funded that would never be able to see the light of day otherwise.

To understand the crowdfunding's underlying mechanism in more depth, we develop a theoretical framework that depicts the strategic choices taken both by the producer and the contributor during crowdfunding. Although approaches were taken to describe a theoretical framework for crowdfunding, these were concentrated on other issues connected with the phenomenon.

We start by reviewing current literature on crowdfunding and the previous microeconomic models related to it. We then continue on by describing the basic framework for the model and deriving first conclusions from the specification. The paper then introduces several extensions to this framework, each allowing to analyze another aspect of crowdfunding. A summary of conclusions follows and ideas are provided for further extensions.

The developed framework supports the theory that platforms like Kickstarter allow for a perfect price discrimination, while simultaneously providing additional benefits to the whole community by utilizing a public good effect and the ability to encourage people to contribute even if they would usually prefer to download digitized products for free instead.

2. Crowdfunding and Kickstarter overview

Crowdfunding has its roots in the process of crowdsourcing. The latter's goal is to make use of the vast pool of knowledge, wisdom, skills and funds that is constituted by the potential contributors to a new project. This collective group forms the so-called *crowd* that may serve as the source of any feedback or resources. These might range from designing the best logo for a business (the manager rewards the project of his choosing) to a panel of product evaluating interviewees.

In this section we review the basic concept of crowdfunding and the model proposed by Kickstarter. The following description serves as well to present the background concepts and the terminology employed further in this paper.

The recent expansion of Internet access allowed for new, network-based ways of crowdsourcing. Notably, many online platforms designed specifically for the task of crowdsourcing had been developed to suit entrepreneurs' needs. In fact, the multiplicity of the available solutions has sprung a wave of research not as much on the process of crowdsourcing, but actually on the crowdsourcing platforms themselves. For example Mwangi & Acosta (2012) review current crowdsourcing platforms including free-lancing, outsourcing and mass cooperation platforms. They conclude with a discussion of some serious challenges ahead of such form of activism.

It was the evolution of crowdsourcing that led to the distinction of a specific branch of this kind of projects, called crowdfunding. In crowdfunding, there are typically two kinds of users, although sometimes an individual may represent both the roles in respect to different projects.

The first role, is generally referred to as the producer or the project manager. This is the person who introduces a new project to the community, with the intention of carrying it out if he manages to gather enough funds. The project might take almost any form, ranging from producing a movie to constructing an artistic bus stop or seeking a cure for a disease[1]. Many crowdfunding platforms are constructed only for specific kinds of task, like the music-album funding platform SellaBand. It is thus typical that a project aims to provide a new good or product to the market.

The second role is, of course, the contributor. Although the underlying model of most crowdfunding platforms is a *pay-what-you-want* one, all of them allow the producer to offer some kind of future compensation to the contributors – the compensation usually being given only if and after the project had been funded. Typically, the main reward is the base product itself, but there may be further compensations for large contributors. The producer's general system of offers is typically referred to as a rewarding scheme.

There are many online crowdfunding platforms[2], among which Kickstarter is the most popular one, having the lead in most Internet 'Top Ten' rankings[3]. The reasons for its popularity are numerous. For one, Kickstarter does not restrict the type of the featured projects and so, gathers all kinds of audiences. Also, Kickstarter is fairly elastic when it comes to price and rewarding schemes, thus leading to its crowdfunding model being shared or replicated (depending on who was first) by many other platforms[4]. For these reasons, the organizational scheme offered by Kickstarter is considered a 'classic' among crowdfunding platforms. On the other hand, the other organizational schemes do not differ significantly from Kickstarter in dimensions relevant from the theoretical modeling perspective.

The additional reason why Kickstarter is so appealing, not only to project developers and contributors but also to the scientific community, is the multiplicity of the rewarding schemes that may be offered by the project manager. These alternatives are briefly described below, to provide basis for the proposed theoretical framework[5].

[1] An interesting and impressive summary of the Kickstarter's achievements for year 2012 is provided by the platform's creators at http://www.kickstarter.com/year/2012. Overall, in that year, 2 241 475 people pledged a total of $319 786 629 and successfully funded 18,109 projects. This is even more impressing when we consider the fact that Kickstarter launched only in 2009 and is still expanding, and that there are other crowdfunding platforms as well.

[2] For a comprehensive reading about crowdfunding, see e.g. Steinberg et al. (2012), or Hui et al. (2012) for additional insight from a project manager's perspective.

[3] E.g. see: http://www.forbes.com/sites/chancebarnett/2013/05/08/top-10-crowdfunding-sites-for-fundraising/ or http://www.hongkiat.com/blog/crowdfunding-sites/ .

[4] E.g. Indiegogo (http://www.indiegogo.com), Crowdfunder (http://www.crowdfunder.com), Polak Potrafi (http://www.polakpotrafi.pl/).

[5] For reference, see for example the page for Project Eternity on Kickstarter, which utilized most of the further mentioned incentives (link: http://www.kickstarter.com/projects/obsidian/project-eternity).

Rewarding schemes

The incentives are described mostly on the basis of the Kickstarter model and are grouped by the types of influence they have on the contributors. These are, respectively, general incentives to invest in the product's value, individual incentives rewarded with private gifts, incentives to participation itself. Some incentives not falling directly into any of these categories are mentioned as well.

General incentive to pay. Although the project managers usually start with a general idea of a product, they usually promise to expand their basic concept after reaching subsequent 'stretch goals'. These stretch goals usually represent the production team's possibility to hire new members, artists, equipment, etc. The incentive is general, because funding of the stretch goals increases the basic good's value (hence called the *general value*) perceived by all contributors and non-contributors as well. The general value can also be perceived as a public good, because its value increases for all consumers when any one of them offers an investment.

Another part of the incentive is that the project will not get funded if the contributions do not exceed the initial goal – hence called the threshold. This means that making a contribution will also increase the *expected general value*, which is related to the probability of the project getting funded.

Individual incentive to pay. The producer may also choose to reward particularly eager participants, according to the amount that they decided to contribute. This rewarding scheme usually revolves around the basic good in a way that if the contributor pays a certain amount of money, he will acquire the base product. Any further payments will yield him additional benefits which may range from special versions of the product (a hardcover book, a boxed version of a game, etc.) to big add-ons and more influence on the final product's features (ability to design a part of the product, chance to party with the developers, additional books, games or artworks, etc.). The price & reward scheme often ranges from few dollars amounts (with thousands of contributors) to thousands of dollars amounts (with small groups of well-rewarded contributors).

Incentive to participate. Some producers also include other incentives, like a reward for taking part in the project itself. These incentives might include small bonuses like a contributor's badge on a discussion forum, a 'thank you' in the credits or the access to a special discussion board where the producers discuss their ideas with the consumers. Another idea is to introduce a separate stretch goal structure, where the product increases in its *general value* each time a particular amount of new contributors enters the crowdfunding.

Other incentives. On instances the producers see fit to introduce other incentives for the contributors, not necessarily directly linked to gathering money. For example, the producer may promise to increase the value of the product if a certain amount of people 'likes' his Facebook page. This sort of marketing may greatly influence the producer's popularity and even encourage new contributors, although it does not increase his funds in any direct way.

Since all of these incentives can constitute the rewarding scheme of a project, a comprehensive framework should be able to incorporate them. Of particular importance are the general and individual incentives, with the other incentives being usually an optional choice for the projects' managers.

3. Literature review

Recently, the Kickstarter model received much attention in the research literature. Notably, there appeared many theoretical frameworks revolving around different aspects of this novel crowdfunding system, as well as some empirical works – the crowdfunding platforms being a source of data on many sociological and economic processes. We will start by reviewing the empirical work on crowdfunding to show the dynamics of Kickstarter-like platforms. Next, the theoretical models will be summarized to explain some of the crowdfunding's underlying phenomenon. Lastly, we will supply these findings by referring to classic microeconomic models that are closely related to the model of crowdfunding.

3.1. Crowdfunding

Crowdfunding platforms received already considerable empirical attention. Notably those of Ward & Ramachandran (2010), Kappuswamy & Bayus (2013) and Read (2013) provide an in-depth look into the mechanics of crowdfunding, as well as the behavior of the contributors. Ward & Ramachandran (2010), show how the peer effects and information affect the users of SellaBand platform. They find a possible information overload in such platforms and a need for information aggregating devices. Kappuswamy & Bayus (2013) analyze the bystander effect theory on data from Kickstarter's projects. They also measure the contribution dynamics as related to the updates the project receives and the number of days till its end. Finally, Read (2013) takes a look at the role of non-profit projects in the crowdfunding community and its chances of getting funded.

Agrawal et al. (2011) show another strong point of the crowdfunding platforms by analyzing data from a music-funding service SellaBand. The online form of the projects reduces the difficulty of gathering investors from distant locations from the entrepreneur. Although the contributions usually start with close relatives and friends, they serve as a signal for the project, and the distance does not play a significant role after that.

A great deal of empirical findings has been delivered by Mollick (2013). In his work, he collected a vast amount of data around a huge set of Kickstarter's projects. His data allowed him to measure many determinants of success such as project's quality, personal networks, geography. His results suggest that the former two are strongly associated with a project's probability of success, while the latter is correlated with the type of the project (its product) and its achievability. Mollick (2013) also analyzes the risk of the producer not delivering the funded project after gathering the contributions. While his results clearly indicate that this risk is small, the producers often deliver their products later than was expected – the degree strongly influenced by the amount of money gained – possibly because the producers have to adjust and expand their products as they gather more money than they initially expected.

In terms of theory, crowdfunding is so far mostly modeled as a two-period process. For example in Belleflamme et al. (2012) a product is available at both stages – the crowdfunding stage, during which the consumers can pre-order the product or invest in it for a share in future profits; and the regular sale stage, when a successfully funded product enters the regular market. The periods differ in the price of the good (there is a fixed price for the first period and a new one for the second period). The researchers attempt to compare the two marketing strategies (pre-ordering and profit-sharing), as well as capture the possibility of price discrimination during the first stage.

Belleflamme et al. (2012) show that the producer might have to lower his profit in the crowdfunding period in order to raise enough capital to fund his project. At the same time, lower price in the first period might serve as a form of price discrimination that allows for consumers who otherwise would not buy the product to contribute to the project. Similar views have been expressed earlier in a two-stage model by Nocke et al. (2011), although their model was based on asymmetric information, which led the more uncertainty-indifferent consumers to pay for the product before its' true value was revealed.

Varian (2013) constructs a simple one-period model of funding a public good with possible private gifts for the contributors. This approach mimics the Kickstarter's model in a way that the public good reflects the general value and the private gift is a form of the individual incentive to pay. Varian (2013) shows that the inclusion of a private gift stimulates additional contributions that by far exceed its cost. Therefore, the private gift may be used as an instrument for price discrimination if the gifts represent a particular value to the consumer.

The model we propose contributes to the current literature by examining the effect of consumers' income and the role of producer's strategy defined by his provision of incentives to the crowd. This approach provides insight into very important aspects of the crowdfunding model, i.e. the ability for an almost-perfect price discrimination and the inclusion of a pay-what-you-want alternative for the 'pirate population' of the kickstarted digitized goods. In order to reflect these factors, I build a model with a continuous price and reward scheme and include each individual's income effect when determining his contribution to the project.

3.2. Insights from the earlier microeconomic models

There's a vast spectrum of microeconomic models that the crowdfunding, and particularly Kickstarter's, model is related to. The whole concept, in fact, compiles many ideas previously expressed in economic literature and different models of sale. These basic models provide a background knowledge and intuition for the analysis of crowdfunding.

The underlying basis of Kickstarter is a *pay-what-you-want* (PWYW) model, where the consumer can pay whatever price he sees fit (see e.g. Kim et al. 2008, Mak et al. 2010). Of course this is not entirely the case, since usually the contributor will acquire the product only if his payment exceeds some set amount. However, when considering a good that also comes in digital form (e.g. a video game, music CD, or a movie), crowdfunding may be considered by the 'pirates' as means of paying a self-chosen price for the good they acquire anyway (Bekir et al. 2011). While the PWYW model usually inclines more people to pay, it typically induces lower average prices at the same time. This tendency is often overcome if incentives like charitable causes are included (Gneezy et al. 2010)[6]. Some sellers also choose to administer certain awards to the donators of larger prices[7]. This is similar to the price discrimination introduced in Kickstarter, which serves as an additional incentive system to pay larger amounts of money (and for the 'pirates' to contribute at all).

We traditionally discern three degrees of price discrimination. Following Pigou (1932), the first degree takes place when a monopoly seller is able to charge a maximum price to each individual buyer, at which price this buyer will be willing to buy the product. The second degree takes place when a buyer gains discounts when he acquires a larger quantity of a good. Lastly, the third degree discrimination takes place when the seller charges different price to different groups of people described by characteristics like region, status, etc. that could be correlated with these consumers' willingness to pay.

The Kickstarter model mostly makes use of the second degree discrimination by offering larger quantities and/or bonuses to people who contribute more to the project. It also introduces incentives to offer higher prices if the contributor has a higher willingness to pay. Since the model is an expanded *pay-what-you-want* one, the contributors are pushed into rising their donations in accordance with their budgets and perceived value, because each penny paid may increase their expected utilities. This is in fact similar to the first degree discrimination (also called a *perfect* discrimination) because the contributions may range from couple of dollars to tens-of-thousands of dollars, depending on the contributor. Perhaps the taxonomy and definitions offered by Ivan Png (2002) are more intuitive. He defines a *complete discrimination* in which the user purchases up to the point where his marginal benefit equals his marginal cost. This is precisely the case of Kickstarter where a contributor will increase his donation as long as he perceives that the benefits rise quicker than the costs. In fact, in some projects, where there is an uncertainty of the project getting funded, some consumers might increase their contributions even further to reduce the risk of achieving a null utility (when the project does not get funded)[8]. Thus, risk averse consumers might be willing to pay more to ensure that the project gets enough money, while risk-indifferent consumers will still rise their donations as long as the benefits exceed the costs.

One other factor that allows for price discrimination is that people tend to differ in their income and the amount of money they may part with, without it affecting their further consumption. This is called the income effect and has received much attention in the economic literature. One implication of the income effect is that the consumers' willingness to pay might be strongly influenced by the amount of

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[6] For a study on charitable giving in a crowdfunding setting, see Meer (2013).
[7] Recently, polish companies BookRage (http://bookrage.org/) and MusicRage (http://musicrage.org/) introduced bundles of products in the PWYW model. The contributor would receive an expanded bundle (additional product) if his price exceeded the average offered price, which is a clever technique of boosting this average up.
[8] This mechanism might also work in case when there is a number of set stretch goals after the funding threshold. The stretch goals are the amounts of total contributions that, if achieved, will require the producer to improve the basic offered good.

money they can dispose. Typically, the effect is modeled in regards to the demanded quantity of a particular good, the relationship often described by the so-called Engel curve (see e.g. Lewbel 2007). In the Kickstarter model, this is replaced by the value increasing along with the offered price.

The crowdfunding model in many ways resembles the warm glow effect presented by Andreoni (1989, 1990). This is both because warm glow model considers donations to a public good, which constitutes a part of the Kickstarter's fundraising mechanism, as well as the warm glow might serve as an *incentive to participate* in itself. Firstly, the two models might be compared, as noted also by H. Varian (2013), by treating the general value of the product as a public good, and the individual values as private goods whose consumption positively affects the value of the public good. Secondly, the effect of warm glow, i.e. the utility from participation, positively influences the potential contributors by providing an additional incentive to donate money to the project. This warm glow might be enlarged by rewarding the contributors with a special badge on a discussion board or access to a private forum, which comes at zero cost to the producer (and might even positively influence his work by providing him with additional feedback), but produces additional value to the contributor.

The Kickstarter model is in many ways a compilation of mechanisms previously described in the microeconomic literature. The above-cited works provide an excellent starting point for developing a new framework that reflects the crowdfunding process by putting these concepts together.

4. Theoretical model

We will begin by constructing a simplified model of crowdfunding including the basic mechanics of the crowdfunded projects' management. This initial form will serve by provision of first conclusions on the dynamics of crowdfunding, as well as set up grounds for further extentions.

4.1. General assumptions and notation of the model

Consider two periods of the crowdfunding process. During the first one, the project managers design their strategy, i.e. they decide on the rewarding scheme they are willing to offer to the contributors. During the second one, the potential contributors decide on the amount of money they will give to the producer in a way that will maximize their individual utilities. Denote the incentive parameter in the basic model as $\boldsymbol{\delta}$ and will hence call it the producer's *strategy parameter* or *incentive parameter*.

The producer knows the utility functions of the contributors and makes his decision based on his goal, which, in the basic model, is to maximize his profit during the crowdfunding. The crowd consists of $\boldsymbol{N}$ homogenous, potential contributors. This number is known both to the consumers and the producer. Because there is homogeneity and no randomness, all the contributors make the same decisions regarding the level of their donations.

Consider a threshold $\boldsymbol{T}$, at which the producer would actually be able to produce the product with the base value of $\boldsymbol{v_0}$. If the total contributions do not exceed this level, the final product reflects only a part of this value. At the same time, should the contributors offer more money for the project, the general value may be higher than the base value. The threshold level is fixed and cannot be altered by the producer in any way – it reflects the difficulty of delivering the project manager's proposed product to the market.

The final value of the good also depends on the strategy parameter $\boldsymbol{\delta}$ and the amount of money earned for the project. The parameter $\boldsymbol{\delta}$ corresponds to the percent of the money gathered overall that the producer will invest in improving the general value of the good. The same parameter $\boldsymbol{\delta}$ is the percent of the money from each person, which the producer will invest in providing a better individual value. Therefore, if a producer chooses a strategy $\boldsymbol{\delta=40\%}$, he will invest 2/5th of the overall money into the general value of the product, as well as invest 2/5th of the individual payments into individual values. This means that all in all he will use 80% of the contributions to improve the product, and spend them as a cost. He will keep the remaining 20% as a profit.

The consumers are prepared to spend an amount of $\boldsymbol{M}$ money without it having an impact on their consumption of other goods. In other words, the model comprises an an income effect which causes the consumer to suffer an additional cost if he chooses to spend more than $\boldsymbol{M}$ on the product, and an additional benefit if he chooses to spend less than $\boldsymbol{M}$. This is because the amount of money the consumer spends on the product may increase or decrease his consumption of other goods[9]. For simplicity, we refer to the variable $\boldsymbol{M}$ as to the income of the contributor. In the basic model, the contributors are homogenous and their incomes do not differ.

Also, it is assumed that a contributor always acquires the general value of the product, and does not need to donate a set minimum amount to do so.

4.1.1. Contributor's utility function

The utility function of the contributor *c*, in general approach, is given by

$$U_c = V_G + V_I - C_c,$$

where V_G is the general value and V_I the individual value of the product, and C_c is the subjective cost suffered by the contributor. We may expand this equation by decomposing its parameters.

The general value will therefore be given as:

$$V_G = \delta \frac{\sum_{i=1}^{N} P_i}{T} * v_0,$$

where $\boldsymbol{v_0}$ is the product's basic value, financed with $\boldsymbol{T}$. $\boldsymbol{P_i}$ is the price paid by the contributor $\boldsymbol{i}$ and $\boldsymbol{\delta}$ is the incentive parameter set by the producer. Therefore the general value is proportionate to the amount of money that exceeded the project's threshold, multiplied by the producer's incentive parameter and the basic value of the product. This describes the public nature of the good, because as payments increase, so does the general value of the product, moderated by the project manager's set strategy parameter.

The contributor $\boldsymbol{c}$'s individual value might be given as:

$$V_{Ic} = \delta P_c,$$

where once again $\boldsymbol{\delta}$ is the incentive parameter set by the producer and $\boldsymbol{P_c}$ is the price paid by contributor $\boldsymbol{c}$. We then have $\boldsymbol{\delta P_c}$ as both the producer's cost of investment in the individual value and the said value, as viewed by the consumer.

The contributor's $\boldsymbol{c}$ subjective cost is given by:

$$C_c = P_c \cdot \frac{P_c}{M_c},$$

where $\boldsymbol{M_c}$ is the part of the contributor's income that he is indifferent to part with, on account of buying the product. Specifically, this notation implicates the following points:

- The subjective cost associated with paying P_c , when $M_c = P_c$, equals P_c.
- The subjective cost associated with paying P_c , when $M_c < P_c$, is larger than P_c as the contributor has to limit his consumption of other goods because of this choice, or even borrow money to cover the expenses.
- The subjective cost associated with paying P_c , when $M_c > P_c$, is smaller than P_c as the contributor can expand his consumption of other goods because of this choice, or even invest some of his additional money.

This, in fact, is a simplified model of the income effect. Given the expansions of the basic concepts, the utility of the consumer/contributor is given by

$$U_c = \delta \left(\frac{\sum_{i=1}^{N} P_i}{T} \right) * v_0 + \delta P_c - P_c \cdot \frac{P_c}{M_c},$$

[9] This also means that a wealthy contributor might be more willing to spend money on the product, since he does not suffer large consequences to his other consumption. Indeed, a penny would be worth much more to a poor person, than it is worth to a millionaire.

where M and P are, respectively, the income of any contributor, and the offered price of any contributor. Such definition of the utility and values implies that consumers' utility grows as:

- their base income (M) grows,
- the incentive parameter (δ) is higher,
- the number of other contributors (N) grows,
- the product's base value (v_0) is higher,
- the funding threshold (T) is lower.

The relationship between the utility and the offered price (***P***) is nonlinear. It is that of an inverted U-shaped curve. It is clear that utility equals zero when the offered price is zero. The utility then grows as the offered price gets higher, until it drops once again, eventually becoming negative. The rational, homogeneous contributors should offer a price that maximizes their utility. We should remember that in the model we already consider only the crowd of the potential contributors. This implies that if we were to consider a model in which every person observes an individual value of the product, these values would all be positive in our crowd of ***N*** contributors. This is why according to the model, there is always an offer that provides the contributor with a positive utility, even if this offer is very small.

4.1.2. Producer's profit function

The producer's profit, in general form, can be described by the function:

$$\boldsymbol{\pi} = \sum_{i=1}^{N} P_i - \delta \sum_{i=1}^{N} P_i - \sum_{i=1}^{N} \delta P_i$$

The interpretation is that the producer receives the total sum of contributions as his income, but suffers costs that equal his promised investments into the general value (taken from the total sum of contributions) and individual rewards (taken from each contribution separately).

4.2. Basic model – homogenous contributors

In the basic version of the model contributors are homogenous (i.e. they have the same income). And only one incentive parameter exists. Thus, the profit function reduces to:

$$\boldsymbol{\pi} = NP - 2\delta NP = N(1 - 2\delta)P \quad (1)$$

while the individual utility them can be specified as:

$$U_c = -\frac{1}{M} * P^2 + \delta\left(\frac{Nv_0}{T} + 1\right)P \quad (2)$$

To find the equilibrium in the model, one needs the contributors' choice conditional on the reward scheme chosen by the producer. It is thus a two-period setting solved backwards.

The contributors, being homogenous, all make the same decisions regarding the maximization of the utility functions described in equation (2). The optimum offered price is therefore given by (for all the transformations see page 24):

$$P^* = \frac{M\delta}{2}\left(\frac{N}{T}v_0 + 1\right) \quad (3)$$

In general, the higher the contributor's utility, the more he will be willing to pay for it. This implies that it is easy to price discriminate by offering small rewards as incentives to offer significantly higher prices. Also, the more there are contributors, the larger sums they will be willing to contribute. This implies that the project manager should try to advertise his project so that as much people as possible know about it. This is both because it will increase the number of participants and because it will increase their payments (by effectively raising the general value of a good).

The finding is that as the contributors' number and their income grow, the more each of them will be willing to contribute is of particular interest. Since, in fact, the contributors may increase their donations during a typical crowdfunding period, their offered prices may rise as more, and wealthier, crowdfunders join the project. This is consistent with the so-called Matthew Effect (the term coined by Merton, 1968) and identified earlier by Mollick (2013), although his conclusions were mainly supported by the fact that a larger crowd of supporters may serve as a quality signal and spread the word to other potential contributors, thus enhancing the crowdfunding. According to (3), another reason would be the fact that the larger number of contributors actually affects the value of the product, and therefore serves as an incentive to contribute or increase the initial offer.

4.2.1. Basic model – producer's choice

The producer knows the number of potential contributors and exact utility functions. He can therefore maximize profit knowing what the decisions of the contributors will be. His profit equation (1) can be rewritten, after including the utility-maximizing offered price of the consumers (3), as:

$$\boldsymbol{\pi} = -MN\left(\frac{Nv_0}{T}+1\right)\delta^2 + \frac{MN}{2}\left(\frac{Nv_0}{T}+1\right)\delta$$

From this, we can derive the profit-maximizing parameter (for list of transformations see page 24):

$$\delta^* = \frac{1}{4}$$

In the basic model, the producer's optimal choice would therefore be to reward the contributors by investing 25% of the total acquired contributions and 25% of individual ones. Since we assumed the parameter is the same for both the general and individual incentives, it is easy to notice that the producer will decide to invest half of the acquired money into rewards and keep the other half as a profit.

$$\boldsymbol{\pi} = N(1-2\delta^*)P = N\left(1-\frac{1}{2}\right)P = \frac{NP}{2}$$

The contributions in this case are given by:

$$P^* = \frac{M\delta^*}{2}\left(\frac{N}{T}v_0+1\right) = \frac{M}{4}\left(\frac{N}{T}v_0+1\right)$$

Therefore, the total profit of the producer and contributor's utility can be written as functions of projects' parameters and consumers' incomes and numbers.

$$\boldsymbol{\pi}^* = \frac{NP^*}{2} = \frac{MN}{8}\left(\frac{N}{T}v_0+1\right)$$

$$U_c^* = \frac{M}{16}\left(\frac{N}{T}v_0+1\right)^2$$

In the equilibrium, both the producer and the contributors have higher gains if $\boldsymbol{M}$, $\boldsymbol{N}$ and $\boldsymbol{v_0}$ are larger and lower gains if the Threshold is higher. This indicates that both the producer and the contributors have a common goal of spreading the word about the project to other potential donors. This finding supports the Mollick's (2013) explanation for the Matthew Effect observed in regards to Kickstarter's projects, because it is plausible that the crowd plays an important role in gathering an even larger amount of contributors to the project, if only for selfish reasons.

4.3. Model extensions

Although the basic model delivers some interesting conclusions, it constitutes a very simplified version of the Kickstarter's system. However, it can be extended to explore important dimensions of crowdfunding, like the "initial goal" feature of Kickstarter, which in the language of our model necessitates a different specification of the threshold (the need to meet the initial goal) or different individual and general incentives in a project. We also consider heterogeneity of contributors' incomes, inclusion of an *incentive to* participate and, finally, the decision dilemma of consumers of digital products, who may also download the product freely.

4.3.1. The threshold

One of the obvious drawbacks of the crowdfunding model is that projects do not always get funded. This occurs when the total contributions gathered are not enough to reach the project's threshold. I rewrite the basic model by making the threshold a more severe restriction. In this specification, the project does not get financed at all if the threshold is not met and so, the producer faces an '*all or nothing*' situation, which is consistent with Kickstarter's rules.

The contributor's utility function now presents itself as:

$$U_c = \begin{cases} 0, & NP < T \\ -\frac{1}{M} * P^2 + \delta\left(\frac{Nv_0}{T} + 1\right)P, & NP \geq T \end{cases}$$

To solve this equation we have to calculate both the optimal offered price of the consumer and the conditions necessary for the threshold to be met. The optimal offered price with no constraints would be again equal to:

$$P^* = \frac{M\delta}{2}\left(\frac{N}{T}v_0 + 1\right) \qquad (4)$$

Nevertheless, to achieve a positive utility value, the contributors' offered price has to meet the condition $NP \geq T$. In case of the optimum price, this is fulfilled only if $\delta > \frac{2T^2}{MN(Nv_0+T)}$. However, since any positive value of utility is better for the rational contributor than not contributing at all, he may be willing to pay more if the funding conditions are harder to meet. Figure 1 shows the possible choices of the contributor in various settings.

The contributor's offered price can therefore take three values:

$$P_c = \begin{cases} 0, & U\left(\frac{T}{N}\right) < 0 \\ \frac{T}{N}, & P^* \leq \frac{T}{N} \quad and \;\; U\left(\frac{T}{N}\right) \geq 0 \\ P^*, & P^* > \frac{T}{N} \end{cases}$$

In other words, the contributor will not want to pay if there is no option that could provide him with a positive utility. He may pay precisely $\frac{T}{N}$ if his optimal offer would not finance the project, but paying more will still yield him a positive utility. He may also pay his optimal offer if it gets the project funded at the same time. In line with this reasoning, the project's manager can expect three possible profit outcomes:

$$\boldsymbol{\pi} = \begin{cases} 0, & U\left(\frac{T}{N}\right) < 0 \\ (1-2\delta)T, & P^* \leq \frac{T}{N} \quad and \;\; U\left(\frac{T}{N}\right) \geq 0 \\ (1-2\delta)NP^*, & P^* > \frac{T}{N} \end{cases}$$

There are two possible strategies for the producers. If there is no way for the optimal offered price to exceed the funding point, then the producer will choose the incentive parameter such that the consumer will be indifferent to paying or not. This is because the contributor will still pay his best offer (which is equal to $\frac{T}{N}$), and the producer will minimize his costs (related directly to the strategy parameter δ). Consider the Figure 1b – there is no reason for the producer not to lower his strategy parameter until the contributor's utility curve reaches zero precisely where the threshold is set, as it will not affect the contributor's decision and the project manager will lower his expenses.

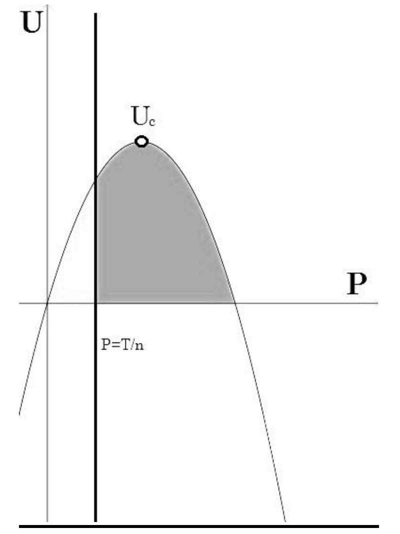
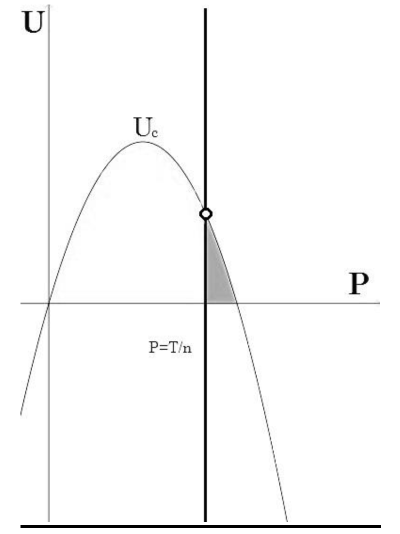
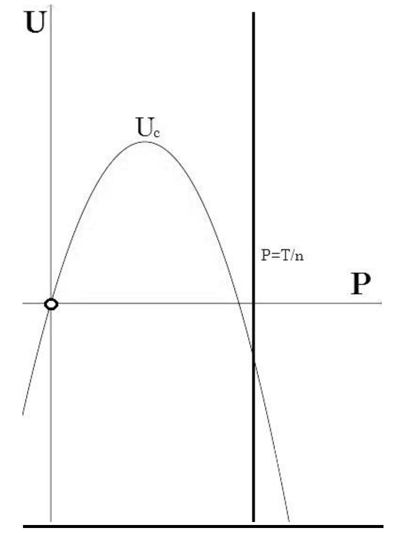
		
a) The contributor's *optimal* offered price is high enough to get the project funded so the contributor can choose his best option, as given by equation **(4)**.	b) The *optimal* offered price is not high enough to fund the project, but the contributor can choose another price that provides him with a positive utility value and is just enough to get the project funded.	c) There is no price option that would both fund the project and provide a positive utility value to the contributor. Therefore, the contributor chooses not to invest and the project is not funded.

Figure 1. Contributor's choice sets in various settings. Prices beneath the funding one (T/n) provide zero utility as the project does not get funded. The grey area indicates possible positive values of utility. The exact shape of the U curve depends on the project's parameters, contributors' income and the producer's incentive parameter δ.

If there is a way for the optimal offered price to exceed the funding point, the producer might find it more rewarding to increase his rewarding scheme. However, there is still possibility that he will choose a lower rewarding scheme and that the first strategy will prove better. This may happen if 'pushing' the optimal price to the other side of the threshold required investing too much of the money into the product's value.

Situation A

The first strategy would indicate that the contributor's utility is equal to zero when he pays the amount $\frac{T}{N}$:

$$-\frac{1}{M} * \left(\frac{T}{N}\right)^2 + \frac{\delta' \left(\frac{Nv_0}{T} + 1\right) T}{N} = 0$$

And so, given the values of T, M, N and v_0, the producer's optimizing parameter when there is no chance of the optimal price exceeding the threshold is:

$$\delta' = \frac{T^2}{MN(Nv_0 + T)}$$

Situation B

The second strategy would require that the contributor's utility's maximum value (forgetting about the threshold constraint) is achieved for an offered price **greater** than $\frac{T}{N}$, which we can denote by:

$$U(P^*) > U\left(\frac{T}{N}\right) <=> \ P^* > \frac{T}{N}$$

This is fulfilled for the incentive parameter such that:

$$\delta'' > \frac{2T^2}{MN(Nv_0 + T)} \qquad (5)$$

The producer's profit would then be equal to:

$$\boldsymbol{\pi} = (\frac{Nv_0}{2T} + \frac{1}{2})MN\delta'' - (\frac{Nv_0}{T} + 1)MN\delta''^2$$

And the profit-optimizing parameter, in line with the previous specifications of the model, equal to:

$$\delta'' = \frac{1}{4}$$

Provided that the condition specified in the inequality (5) is fulfilled by this level of parameter.

Situation C

If the abovementioned condition is not met, the producer may still find an incentive parameter that would shift the cumulated offered prices above the threshold. However, this strategy seems unproductive, as the producer's profit-maximizing parameter, constraint on the contributors' offered price reaching the threshold, is the one that still yields payments equal to $\frac{T}{N}$:

$$\delta'' = \frac{2T^2}{MN(Nv_0 + T)}$$

This value just gets the threshold reached with the contributors' optimal offer. Since we know that δ'' is larger than $\frac{1}{4}$, the producer's profit curve is headed downwards for any larger values of the incentive parameter. It is therefore obvious that the situation C will actually never take place.

The situations *A*, *B* and *C* are depicted in Figure 2.

Choosing the best strategy

We will now assume that the producer's main goal is to achieve a positive profit. A different goal would be to achieve the highest possible profit, constraint on the project getting funded (in this case the producer could incur losses in the crowdfunding stage).

We have discerned three possible situations, regarding which choice yields the highest profit to the producer, although only two of them prove rational.

Lemma 1. The producer will always choose option B if the situation allows it.

Proof: The situation C cannot take place, because we have assumed that the incentive $\frac{1}{4}$ can get the project funded. The remaining question is: can option A be better than option B? It would require that:

$$\boldsymbol{\pi}_A > \boldsymbol{\pi}_B$$

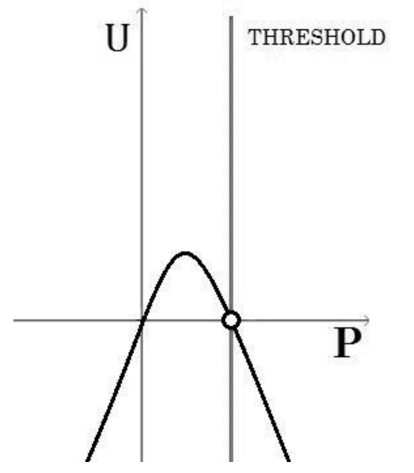
$$\left(1 - 2\frac{T^2}{MN(Nv_0 + T)}\right)T > \left(\frac{1}{2}\right)\frac{MN}{4}(\frac{N}{T}v_0 + 1)$$

$$\left(\frac{8MN(Nv_0+T)T^2-16T^4-(MN)^2(Nv_0+T)^2}{(MN)^2(Nv_0+T)^2}\right)>0$$

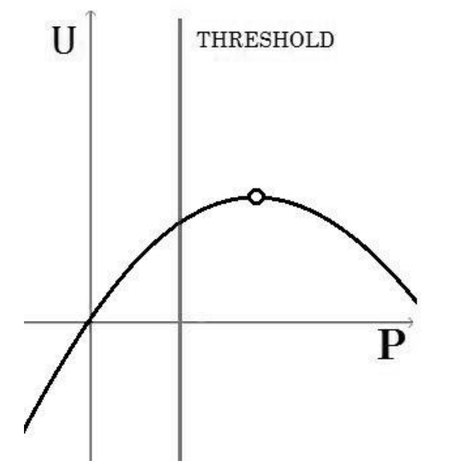
$$8MN(Nv_0+T)T^2-16T^4-(MN)^2(Nv_0+T)^2>0$$

$$-\left(4T^2-MN(Nv_0+T)\right)^2>0$$

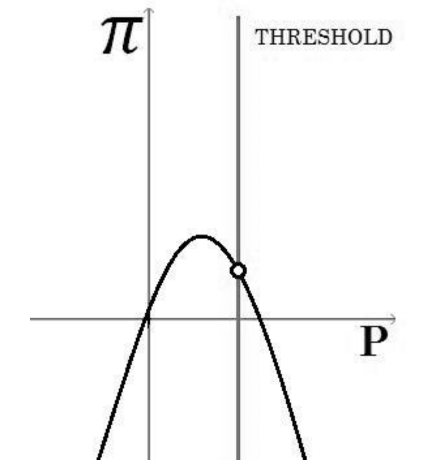
This, however, is never fulfilled as the left-hand side is always equal or lesser than zero. This proves, that as long as $\frac{1}{4}>\frac{2T^2}{MN(Nv_0+T)}$ the producer will always decide for a $\frac{1}{4}$ parameter.



Situation A. The producer decides to minimize his costs (the incentive parameter). The contributor's marginal cost of paying enough to fund is equal to his marginal utility.



Situation B. The producer optimizes his profit, while the contributors' optimize their utility with an offer that gets the project funded.



Situation C. The producer decides to get the project funded, although further increasing his incentive parameter would only lower his profit. At the current level, the contributor's optimal offer just funds the project.

Figure 2. The three situations considered by the producer.

Lemma 2. A profit-maximizing producer will always stay with option A if option B is unattainable.

Proof: The situation B cannot take place, because we have assumed that the incentive $\frac{1}{4}$ cannot get the project funded. The remaining option is: can option A be better than option C? It would require that:

$$\boldsymbol{\pi}_A>\boldsymbol{\pi}_C$$

$$(1-2\delta')T>(1-2\delta'')\delta''\frac{MN}{2}(\frac{N}{T}v_0+1)$$

$$MN(nv_0+T)-2T^2>MN(Nv_0+T)-4T^2$$

$$4T^2>2T^2$$

This is always fulfilled as the left-hand side is always greater than the right-hand side. Therefore, if $\frac{2T^2}{MN(Nv_0+T)}>\frac{1}{4}$ the producer will always stay with the lower parameter, equal to $\frac{T^2}{MN(Nv_0+T)}$ in order to minimize his costs.

There are several implications of these results, namely that the producer will be inclined to offer a better rewarding scheme if the chances of funding the product are high. He will do so, because he'll be able to reap a huge surplus by inclining the willing contributors to pay even more. Such conditions provide him with some flexibility and he can choose a parameter that maximizes his profit. At the same time, the contributors pay more than $\frac{T}{N}$ because they perceive a high value of the good and even want to have it improved. The rewarding scheme is therefore better (higher) when the threshold is lower and there are more and wealthier contributors perceiving a higher basic value of the product.

However, if there is fewer contributors, with smaller utilities from the good, the producer will prefer to set a rewarding scheme that just gets the project funded. This is because the contributor's willingness to pay is not very high and the producer prefers to minimize his costs associated with his rewarding scheme. The product funded in this way will be closer to its basic value.

Although the expansion takes into account only homogeneous contributors, the results would possibly hold for a larger population by holding to the probabilities of the project getting funded, and would differ in individual decisions (some contributors would leave the project, while others would pay most of the sum). The results would be, however, driven by the project manager's desired probability of funding (e.g. a project manager would set a 90% probability of the project reaching the threshold and calculate his incentive parameter on this basis).

The results of the model seem to be in line with the results of the research conveyed by Mollick (2013). His research shows that most Kickstarter projects fall into two categories: those funded by just slightly exceeding the threshold, and those that failed and gathered only a very small percentage of their set threshold. Since a contributor may estimate his chances of achieving utility, even if he could gain something if the threshold was exceeded, he will usually decide not to participate at all if the chances are too small – hence the *total failures*. At the same time, from the model we see that a producer will often find it preferable to set his rewards just big enough to meet the threshold – hence the majority of the funded projects gaining just enough money to start the production.

4.3.2. Different general and individual incentives

We can revise the basic model the same thing with two incentive parameters, which the project manager may set for the project. This follows more tightly with the Kickstarter model, as there is usually a different rewarding scheme set for the general incentive and the individual one. The incentive previously denoted as δ for both the cases will now take the values of δ_G (the *general incentive*) and δ_I (the *individual incentive*).

The new general value might be therefore given as:

$$V_G = \delta_G \frac{\sum_{i=1}^{N} P_i}{T} * v_0$$

And the individual value might be given as:

$$V_{Ic} = \delta_I P_c$$

These expansions lead to the following form of the consumer's utility:

$$U_c = \delta_G \frac{\sum_{i=1}^{N} P_i}{T} * v_0 + \delta_I P_c - P_c \cdot \frac{P_c}{M_c}$$

Given that the consumers are homogeneous in nature, the utility of any one of them can be, after simplification, specified as:

$$U_c = -\frac{1}{M} * P^2 + \left(\frac{\delta_G N v_0}{T} + \delta_I\right) P \quad (6)$$

Some conclusions can be drawn at this stage. Notably, that the consumer's utility grows as:

- his base income (*M*) grows,
- the incentive parameters (δ_G *and* δ_I) are higher,
- the number of other contributors (*N*) grows,
- the product's basic value (v_0) is higher,
- the funding threshold (*T*) is lower.

The difference between this model, and its basic form is that we can now distinguish the effect of both the incentives. From the equation (6) we can see that the general incentive's effect, contrary to the individual one's, is related to the number of contributors, as well as the threshold and the base value of the product.

The new profit function of the producer is given by:

$$\boldsymbol{\pi} = NP - \delta_G NP - \delta_I NP = (1 - \delta_G - \delta_I)NP \quad (7)$$

Again, the contributors, being homogenous, all make the same decisions regarding the maximization of their utility functions, their optimal offer set at:

$$P^* = \frac{\delta_G MNv_0}{2T} + \frac{\delta_I M}{2} \quad (8)$$

We can combine equations **(7)** and **(8)** to acquire the real profit function of the producer, and derive the profit-maximizing parameters (for the full list of operations see page 24).

$$\delta_I = \frac{TNv_0 - (Nv_0)^2}{4TNv_0 - (Nv_0)^2 + T^2}$$

$$\delta_G = \frac{3TNv_0 - (Nv_0)^2}{4TNv_0 - (Nv_0)^2 + T^2}$$

Both the parameters seem to grow just as $\boldsymbol{N}$ and $\boldsymbol{v_0}$ get larger. However, the general incentive seems to grow slightly faster than individual incentives. This is probably because of a self-perpetuating mechanism of the general incentive. As we have already established, the contributors would be likely to increase their offers and contribute to the project when they expect to achieve a higher value because of it. Since the general value is in fact a public good, any investment into it encourages more people to contribute or increase their offers. The same cannot be said about the individual incentives. Any investment into an individual contributor's utility does not affect other contributors' willingness to pay. Therefore it is easier to realize a surplus from a larger crowd, through transferring their contributions to a common pool, and so the general incentive becomes more important as there are more participants to the project.

4.3.3. Heterogeneous contributors

Although the basic model offers much insight into the mechanics of crowdfunding, it does not show one of the most important advantages of this model. Specifically, the Kickstarter model optimizes the outcome from a crowd of heterogeneous contributors. By varying the contributors by the level of income we can examine how different incentives affect a realistic population in contrast to the simple sale and demand model with a fixed price.

We show that substituting the homogeneity of the contributors with a randomized income distribution does not affect the equilibrium of the model, and only changes the distribution of offers, which are now randomized as well.

Each of the contributors decides based on income and on expectation of other contributors' decisions. The new, expected utility function can therefore be written as:

$$EU_c = \delta\left(\frac{(N-1)EP + P_c}{T}\right) * v_0 + \delta P_c - P_c \cdot \frac{P_c}{M_c},$$

where *EP* is the expected offered price of other consumers multiplied by their number (*N-1*) and P_c is the individual contributor's offer.

We can easily calculate the expected offered price by calculating the decision in a population with expected incomes only. This is indeed what each of the contributor's would consider when taking into account other contributions before joining the project. Therefore, the problem is defined as maximization of the general expected utility function:

$$EU = \delta\left(\frac{NEP}{T}\right) * v_0 + \delta EP - EP \cdot \frac{EP}{EM} \quad (9)$$

where *EM* is the expected income in a population. Which leads us to the expected offer in a population given by equation (10).

$$EP = \frac{\delta\left(\frac{Nv_0}{T} + 1\right)EM}{2} \quad (10)$$

Not surprisingly, the expected offer function is that of the offer with homogeneous contributors, though with the expected income now taking the place of the homogeneous income. This result allows for a direct analysis of an individual contributor's decision problem, based on his personal income. Indeed, if we consider a population with all the incomes equal to a constant and another population, in which the expected income is equal to that constant, it is obvious that the profit-maximizing producer will choose the same strategy parameter in both the cases. An individual contributor's optimal offer is once again equal to equation (3). The difference now is, that the producer acquires a large spectrum of different donations, which reflects the *complete discrimination* effect of crowdfunding.

We can assume that the income follows a Pareto distribution[10]. If this is the case, than there will be many contributors with small offers, and a few who contribute very much. Indeed, since the price is derived directly from the income multiplied by a *constant*, the distribution of offers will take the shape similar to that of a Pareto density function of the income. We still, however, assume that the project is always funded and so each contributor donates, even if an amount close to none.

To provide an example of how the price discrimination works, we consider a simplified situation, in which the producer faces two groups of potential contributors, different from each other by their income. We denote one of the groups as the poor population, its size by $\boldsymbol{N_P}$ and its income as $\boldsymbol{M_P}$. Analogically, we denote the second group as the rich population, its size equal to $\boldsymbol{N_R}$ and its income as $\boldsymbol{M_R}$. The obvious assumption is that $\boldsymbol{M_P} < \boldsymbol{M_R}$. We will also assume that the total population is equal to $\boldsymbol{N = N_P + N_R}$.

Lemma 3. In a model with two groups of contributors – poor and rich – the rational consumers will have different optimal prices, such that the poorer backers offer less than the rich.

Proof:

An expected utility of a contributor randomly chosen from the population can be given by the equation (9) (here rewritten for the sake of convenience):

$$EU = \delta\left(\frac{NEP}{T}\right) * v_0 + \delta EP - EP \cdot \frac{EP}{EM}$$

Since we consider a model of two populations with two levels of income, the expected income is equal to a weighted average of the income, where the weights are given by the part of the overall population that the given group constitutes:

$$EM = M_R\frac{N_R}{N} + M_P\frac{N_P}{N} = \frac{M_RN_R + M_PN_P}{N}$$

[10] The Pareto distribution is the traditionally assumed distribution of wealth or income in a population. This distribution incorporates the so-called Pareto principle, which states that a relatively small percentage of the population holds the majority of the population's income.

We can include this form in the expected utility function to arrive at:

$$EU = \delta\left(\frac{NEP}{T}\right) * v_0 + \delta EP - EP \cdot \frac{NEP}{M_R N_R + M_P N_P}$$

Since the rational contributors maximize their utilities, the expected price will be equal to the expected-utility maximizing price, which we can find by calculating the first derivative of the specified function and equating it to zero. This yields us the following result:

$$EP = \frac{\delta\left(\frac{Nv_0}{T} + 1\right)}{2} * \frac{M_R N_R + M_P N_P}{N}$$

However, as there are only homogeneous contributors within each group, their offered prices within the groups will be identical. We can therefore calculate the expected price analogically to the expected income, where P_P and P_R refer to the price offer of the poor and the rich respectively.

$$EP = P_P \frac{N_P}{N} + P_R \frac{N_R}{N} = \frac{P_P N_P + P_R N_R}{N}$$

We may then equate the two specifications of expected price level:

$$\frac{P_P N_P + P_R N_R}{N} = \frac{\delta\left(\frac{Nv_0}{T} + 1\right)}{2} * \frac{M_R N_R + M_P N_P}{N}$$

And ultimately arrive at the following equity:

$$P_P N_P + P_R N_R = \frac{\delta\left(\frac{Nv_0}{T} + 1\right) M_P}{2} N_P + \frac{\delta\left(\frac{Nv_0}{T} + 1\right) M_R}{2} N_R$$

From which we can derive both the price of the poor and the price of the rich:

$$P_P = \frac{\delta\left(\frac{Nv_0}{T} + 1\right) M_P}{2}$$

$$P_R = \frac{\delta\left(\frac{Nv_0}{T} + 1\right) M_R}{2}$$

Since $M_R > M_P$ then we will always have that $P_R > P_P$ and both the prices are fixed for a given parameter δ. This implies that the price discrimination is in effect, as the richer contributor decides to offer a higher price than the poorer one.

At the same time, the producer's strategy is not affected as his decision is dependent only on the expected offer of the contributors' population, and not the shape of the distribution itself (for proof see page 25). Therefore, the crowdfunding model allows the producer to discriminate between different groups of contributors, while holding his incentive parameter constant.

4.3.4. Incentive to participate

One of the possible expansions of the model is to include another form of incentive, used by the project managers at the Kickstarter platform. The incentive to participate can take many forms, some of which are trivial in analysis.

Consider a small incentive to participate that can be associated with substantially no cost to the producer. This can be interpreted as small privileges awarded to the contributors (e.g. access to a project-related newsletter or a discussion board), or even the so-called warm glow effect, where the contributor receives additional satisfaction simply by participating along with the rest of the community.

Since this kind of incentive to participate does not induce any additional costs to the producer, the major change occurs in the contributor's utility function. I denote the reward for participating as δ_p.

$$U_c = -\frac{1}{M} * P^2 + \delta\left(\frac{Nv_0}{T} + 1\right)P + \delta_p$$

The new incentive simply shifts the U-shaped utility curve upwards. The utility-maximizing offered price does not change, as the new parameter is uncorrelated with the price. If we considered a model with some people initially deciding not to participate, than the incentive to participate would greatly increase the chances of them changing their decision, because their initial low evaluation of the product would shift upwards. However, if we consider the threshold model, the inclusion of the incentive to participate might incline the poorer potential consumers to contribute to the cause, as can be seen in Figure 3.

This way, the incentive to participate could also affect the optimal offered price. As the contributors observe a larger crowd taking part in the project, they would increase their initial offers as was shown in the previous specifications of the model.

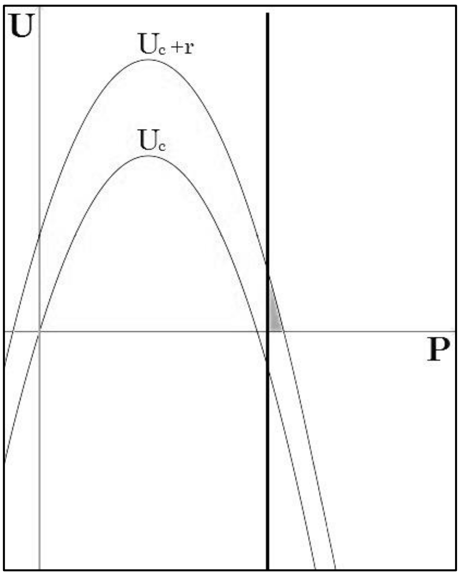


Figure 3. The utility function with and without the incentive to participate. The grey field shows the additional choice set that allows for a positive utility value thanks to the incentive to participate. This, in fact, is a simplification, as the inclusion of the choice set basically changes the shape of the utility curve itself because of different choices from the other contributors.

4.3.5. The minimum price and piracy

Consider the problem of piracy, that has been particularly prominent since the Internet revolution took place. There are possibly many motivations for a potential *pirate* to download from the Internet, instead of buying a product. While there are probably some who would not want to pay for the product in any case, there are also those who simply cannot afford it, or do not consider it worth its price.

As we have already discussed, the crowdfunding project adds the *pay-what-you-want* option, for those who do not consider a product's price an appropriate one. This is important, because if the product is a digital one, and there is a set number of people who would buy the product if it was cheaper and pirate it otherwise, then funding the project through the Kickstarter may allow them to contribute through the *PWYW* option. It should be remembered, that a contributor usually gets a copy of the product only if his individual payment exceeds some minimum level, and that this copy sometimes lacks some components of a more expensive alternative. However, a consumer who does not want to pay more, or cannot afford the product, could donate a small amount of money and still *pirate* a copy of the product. This means that the crowdfunding model may increase the producer's profit by gaining payments from the *pirating* community as well.

Moreover, consider now the pirate (hence called a *hard pirate*), who would not want to pay even a small amount of money for the product if he can download it. Kickstarter does more than the *pay-what-you-want* mechanism, because it also offers rewards for participation. A *hard pirate*, who did not consider paying any amount of money, now has a set of incentives to do so, namely:

- contributing to the project will increase the chances of it getting funded,
- contributing to the project will increase the value of the product, which the *hard pirate* may download later on, even if his contribution was relatively small,
- contributing to the project may earn the *hard pirate* access to otherwise restricted information about the project,
- contributing to the project may earn the *hard pirate* additional benefits, like gadgets, and influence on the project's development – which he would not be able to acquire through *piracy*.

These are all plausible reasons why starting a project on Kickstarter would allow for realizing a profit even from the *hard pirates*, who would not want to pay otherwise even if they could choose the price themselves. We can consider a very simple choice-model of a pirate, based on the model with heterogeneous contributors, who can achieve a product by four ways:

1) Buying the product from a shop. For an easier analysis, we can denote this utility as:

$$EU_{1c} = \delta\left(\frac{(N-1)\left(\frac{Nv_0}{T}+1\right)\delta EM}{2T}\right) * v_0 - P_F \cdot \frac{P_F}{M_c}$$

   , where P_F is a fixed price of the product, and v_0 its base value. The first component is the general value achieved at the crowdfunding stage without a contribution from the considered *pirate*.

2) Downloading the product, and achieving a utility equal to its acquired value.

$$EU_{2c} = \delta\left(\frac{(N-1)\left(\frac{Nv_0}{T}+1\right)\delta EM}{2T}\right) * v_0$$

3) Paying the minimum contribution during the crowdfunding stage.

$$EU_{3c} = \delta\left(\frac{(N-1)\left(\frac{Nv_0}{T}+1\right)\delta EM + 2P_F}{2T}\right) * v_0 + \delta P_F - P_F \cdot \frac{P_F}{M_c}$$

4) Contributing some amount of money, and then downloading the product. Even if we add the constraint that a contributor has to pay a minimum price to acquire the product, the pirate's utility does not change as he will download it anyway.

$$EU_{4c} = \delta\left(\frac{(N-1)\left(\frac{Nv_0}{T}+1\right)\delta EM + 2P_c}{2T}\right) * v_0 + \delta P_c - P_c \cdot \frac{P_c}{M_c}$$

Since we consider a *hard pirate*, we can easily cross out two of the options. To a pirate, buying from a shop is obviously an option yielding a lower utility than simple pirating. This would be an even stronger effect if we acknowledged that during the Kickstarter stage, products are often available at lower prices than they will be during a normal sale.

Contributing the minimum fixed price is an option yielding a lower utility in comparison to the option where the *pirate* can choose his utility-maximizing price on his own. Since we consider a *hard pirate* we can assume that the optimal offer would be lower than P_F or else he would simply acquire the product through contributing to the project. We can now compare the two remaining options.

Lemma 4. There is always a payment option for the pirate that will provide him with a higher utility than simply downloading the product.

Proof: Can contributing and then downloading be better than simply downloading? If so, the following would have to take place:

$$EU_{4c} > EU_{2c}$$

$$\delta\left(\frac{(N-1)\left(\frac{Nv_0}{T}+1\right)\delta EM+2P_c}{2T}\right)*v_0+\delta P_c-P_c\cdot\frac{P_c}{M_c}>\delta\left(\frac{(N-1)\left(\frac{Nv_0}{T}+1\right)\delta EM}{2T}\right)*v_0$$

$$\delta\left(\frac{P_c}{T}\right)*v_0+\delta P_c-P_c\cdot\frac{P_c}{M_c}>0$$

$$\frac{\delta v_0}{T}+\delta>\frac{P_c}{M_c}$$

$$\delta M_c(\frac{v_0}{T}+1)>P_c$$

Since the left-hand side of the inequality is fixed and always above zero, there is always a price that the *hard pirate* could pay to increase his utility, even if it's very close to null. Of course, in the natural environment, this would require the project's rewarding scheme to be very sensitive to marginal changes in the acquired money levels, as well as no additional costs associated with contribution (such as time required to set up a Kickstarter account, etc.). Still, there are factors that would entice a *pirate* to contribute even further, such as an *incentive to participate*, or the moral cost of pirating a product. The conclusion is, that crowdfunding may utilize an additional surplus by encouraging even the hardest pirates to contribute as well – an achievement impossible through a regular sale.

5. Conclusions and implications

The new business model of crowdfunding introduced a profitable method utilizing various mechanisms described earlier in the microeconomic literature. I have constructed a theoretical framework that explains some of these underlying processes, as well as the way they are collated.

Crowdfunded projects share the so-called Matthew Effect. As the number of contributors increases, each of the contributor acquires an incentive to increase his offer. The same holds if we increase the basic value of the product. Both these factors serve as a quality signal and in fact increase the ultimate value of the good. At the same time, if the project cannot signal its quality efficiently enough, the contributors will be less willing to participate and will decrease their offers. These self-perpetuating mechanisms explain why Kickstarter's failed projects usually fail by not achieving even a small part of their initial goal, while the most popular projects often 'explode' just at the beginning of their crowdfunding.

The relationship between each individual contributor's utility and the total number of contributors indicates that the producers and backers share a common goal of spreading the word about the project as much as possible. This fact might greatly decrease the project's manager's costs of advertising, since most of the signaling will be done by the contributors themselves, who are usually much larger in number. However, an initial group of supporters would still be crucial for this to happen.

Extending the basic model with randomized incomes allows for an analysis of the discrimination mechanism of crowdfunding. The model supports the theory that a well-designed project may allow for a *complete discrimination* in the sense, that each contributor will increase his offer as long as he considers the marginal benefits to exceed his costs. This finding is mainly based on the heterogeneous income levels in the population and including the income effect in the model. The reasons for paying more by some of the contributors are therefore explained by their lower perceived cost of doing so. However, the discrimination mechanism could be further enhanced if we considered that all the contributors perceive a different, subjective value of the product.

Taking into account that the threshold has to be exceeded if the project is to be funded, greatly affects the decision dilemma of the producer. He perceives an '*all or nothing*' situation and bases his decision accordingly to the fixed parameters of the project. If the project's potential is very high (its value relatively large, the number of contributors large and the threshold low), he will be willing to entice the crowd to increase their payments by offering them additional rewards. Since the contributors' willingness to pay is high in this case, they will be ready to increase their offers much, even while the rewards' cost is not very high. On the other hand, if the project's potential is not that big (and the contributors would rather just get the project funded, but not invest in it any more), the producer's strategy will be to reduce his costs as much as possible. In this case, the project will just get funded, and will not expand above its initial shape.

Most crowdfunding projects make use of both *general* and *individual* incentives, however, these values usually differ, which constituted another of the model's expansions. The main finding is, that a project manager will be more willing to invest his money in the *general* value, relative to his investment into individual rewards, when there is a larger crowd of backers to the project. This is because the *general* value of the product acts as a public good that increases in its value the more there are investors contributing to it. Therefore, as the number of participants grows, the general value will expand quicker than the individual ones, making it a more important means for enticing.

The project's manager may include an additional incentive that comes at a close-to-zero cost (like an invitation to a private discussion board), but provides the crowd with an additional utility from participation. This incentive may also include the so-called *warm glow*, because of the need to belong to a community, or to contribute to a charitable cause. If we consider the model with the threshold constraint, such incentive could prove crucial for enticing the poorer population to participate in the project.

Lastly, we considered the case of piracy, where a *pirate* is a person yet deciding whether to join the crowd. We showed that if the project's rewarding scheme is elastic enough, even the hardest pirates will find a price they would be willing to contribute, even if it still meant downloading the product later on. This is particularly interesting, because it indicates that a project's crowd includes people who would not participate in the regular market. Moreover, while a simple *pay-what-you-want* model may decrease the prices paid by the whole population, crowdfunding 'invites' *pirates* to participation, without it affecting the other consumer's offers in any negative way.

6. Other extensions

There remain several aspects of the crowdfunding model not explained by this framework, as well as some approaches that could extend this work. We provide a list of possible, valuable extensions to the model:

1) A different and perhaps more intuitive approach to the equilibrium-seeking would be to calculate the contributor's marginal raise as well as the marginal cost of increasing his offer. Such approach would follow Png's (2002) definition of complete discrimination more closely.
2) Combining the models of the threshold constraint and the randomized population would introduce random component to the model, which would reflect the uncertainty observed both by the producer and the contributor. This could also introduce an alternative goal of the producer, who would now have to decide on the probability level of his project getting funded. At the same time, the contributors' decisions would be affected as they would now have to consider their decisions' impact on the probability of the project getting funded.
3) For many crowdfunded projects, the threshold level is in fact a fixed cost that the producer has to incur to start the production. Extending the model to include this factor could provide more insight into the mechanics of crowdfunding. These expansions could serve as the basis for further considerations, namely:
    a) Can the producer report a false threshold to the contributors in order to further his goal? If so, what are the conditions for this procedure to be successful?
    b) Kickstarter's rival platform – Indiegogo, allows for the producer to take the money even if it did not exceed the threshold, and cover the difference with their own money. A comparison of the two models would be possible.
    c) The false threshold model could be combined with the heterogeneous population one, and the probabilities of the project getting funded considered. The false threshold could now influence both the profits of the producer, as well as his probability of the project getting funded, and its reaching of the true threshold level.

4) Project managers sometimes make use of a more sophisticated incentive to participate, like developing an entirely new feature of the product, as more participants join the crowdfunding. This more complicated form would be dependent on the number of participants to the project, and would be affecting the general value of the product. An additional incentive could be constructed, similar to the relationship between the base value and the threshold, but rather dependent on the number of contributors as its base. This approach would have to be based on a model where not all of the potential contributors decide to take part.

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Mathematical appendices

Appendix 1 – contributor's utility optimizing offered price

$$\frac{\partial U_c}{\partial P} = 0$$

$$\frac{\delta N}{T} v_0 + \delta - 2\frac{P}{M} = 0$$

$$P^* = \frac{M\delta}{2}(\frac{N}{T} v_0 + 1)$$

Appendix 2 – producer's profit optimizing incentive parameter

$$\boldsymbol{\pi} = N(1 - 2\delta)\frac{M\delta}{2}(\frac{N}{T} v_0 + 1)$$

$$\boldsymbol{\pi} = -MN\left(\frac{Nv_0}{T} + 1\right)\delta^2 + \frac{MN}{2}(\frac{Nv_0}{T} + 1)\delta$$

$$\frac{\partial \boldsymbol{\pi}}{\partial \delta} = 0$$

$$N\frac{M}{2}\left(\frac{N}{T} v_0 + 1\right) - 2NM\left(\frac{N}{T} v_0 + 1\right)\delta^* = 0$$

$$\delta^* = \frac{1}{4}$$

Appendix 3 – contributor's optimized utility

$$U_c^* = -\frac{1}{M} * \left(\frac{M}{4}\left(\frac{N}{T} v_0 + 1\right)\right)^2 + \frac{1}{2}\left(\frac{Nv_0}{T} + 1\right)\frac{M}{4}\left(\frac{N}{T} v_0 + 1\right)$$

$$U_c^* = -\frac{M}{16}\left(\frac{N}{T} v_0 + 1\right)^2 + \frac{M}{8}\left(\frac{N}{T} v_0 + 1\right)^2$$

$$U_c^* = \frac{M}{16}\left(\frac{N}{T} v_0 + 1\right)^2$$

Appendix 4 – producer's profit-maximizing parameters

$$\boldsymbol{\pi} = (1 - \delta_G - \delta_I)(\frac{\delta_G MN^2 v_0}{2T} + \frac{\delta_I MN}{2})$$

$$\boldsymbol{\pi} = \frac{\delta_G MN^2 v_0}{2T} + \frac{\delta_I MN}{2} - \frac{\delta_G^2 MN^2 v_0}{2T} - \frac{\delta_G \delta_I MN}{2} - \frac{\delta_G \delta_I MN^2 v_0}{2T} - \frac{\delta_I^2 MN}{2}$$

$$\frac{\partial \pi}{\partial \delta_I} = \frac{MN}{2} - \frac{\delta_G MN}{2} - \frac{\delta_G MN^2 v_0}{2T} - \delta_I MN = 0$$

$$\frac{1}{2} - \frac{\delta_G}{2} - \frac{\delta_G N v_0}{2T} = \delta_I$$

$$\frac{\partial \pi}{\partial \delta_G} = \frac{MN^2 v_0}{2T} - \frac{\delta_G MN^2 v_0}{T} - \frac{\delta_I MN}{2} - \frac{\delta_I MN^2 v_0}{2T} = 0$$

$$\frac{1}{2} - \frac{\delta_I T}{2N v_0} - \frac{\delta_I}{2} = \delta_G$$

Putting δ_G in δ_I:

$$\frac{1}{2}-\frac{\frac{1}{2}-\frac{\delta_I T}{2Nv_0}-\frac{\delta_I}{2}}{2}-\frac{\left(\frac{1}{2}-\frac{\delta_I T}{2Nv_0}-\frac{\delta_I}{2}\right)Nv_0}{2T}=\delta_I$$

$$\frac{1}{2}-\frac{1}{4}-\frac{\delta_I T}{4Nv_0}-\frac{\delta_I}{4}-\left(\frac{1}{4T}-\frac{\delta_I T}{4TNv_0}-\frac{\delta_I}{4T}\right)Nv_0=\delta_I$$

$$2TNv_0-TNv_0-\delta_I T^2-\delta_I TNv_0-(Nv_0)^2+\delta_I TNv_0+\delta_I (Nv_0)^2=4TNv_0\delta_I$$

$$TNv_0-\delta_I T^2-(Nv_0)^2+\delta_I (Nv_0)^2=4TNv_0\delta_I$$

$$\delta_I=\frac{TNv_0-(Nv_0)^2}{4TNv_0-(Nv_0)^2+T^2}$$

$$\delta_G=\frac{3TNv_0-(Nv_0)^2}{4TNv_0-(Nv_0)^2+T^2}$$

Appendix 5 – producer's profit-maximizing parameter with two groups of contributors.

$$\boldsymbol{E\pi}=N(1-2\delta)(\frac{\frac{\delta\left(\frac{Nv_0}{T}+1\right)M_P}{2}N_P+\frac{\delta\left(\frac{Nv_0}{T}+1\right)M_R}{2}N_R}{N})$$

$$\boldsymbol{E\pi}=(1-2\delta)(\frac{\delta\left(\frac{Nv_0}{T}+1\right)M_P}{2}N_P+\frac{\delta\left(\frac{Nv_0}{T}+1\right)M_R}{2}N_R)$$

$$\boldsymbol{E\pi}=\frac{\delta\left(\frac{Nv_0}{T}+1\right)M_P}{2}N_P+\frac{\delta\left(\frac{Nv_0}{T}+1\right)M_R}{2}N_R-\delta^2\left(\frac{Nv_0}{T}+1\right)M_P N_P-\delta^2\left(\frac{Nv_0}{T}+1\right)M_R N_R$$

$$\frac{\partial E\pi}{\partial\delta}=0$$

$$2\delta\left(\frac{Nv_0}{T}+1\right)M_P N_P+2\delta\left(\frac{Nv_0}{T}+1\right)M_R N_R=\frac{\left(\frac{Nv_0}{T}+1\right)M_P}{2}N_P+\frac{\left(\frac{Nv_0}{T}+1\right)M_R}{2}N_R$$

$$\delta*2\left(\left(\frac{Nv_0}{T}+1\right)M_P N_P+\left(\frac{Nv_0}{T}+1\right)M_R N_R\right)=\frac{\left(\frac{Nv_0}{T}+1\right)M_P N_P+\left(\frac{Nv_0}{T}+1\right)M_R N_R}{2}$$

$$\delta=\frac{1}{4}$$

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