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WHICH OPTION PRICING MODEL IS THE BEST? HIGH FREQUENCY DATA FOR NIKKEI225 INDEX OPTIONS

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Which Option Pricing Model is the Best?
High Frequency Data for Nikkei225 Index Options[*]

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Abstract

Option pricing models are the main subject of many research papers prepared both in academia and financial industry. Using high-frequency data for Nikkei225 index options, we check the properties of option pricing models with different assumptions concerning the volatility process (historical, realized, implied, stochastic or based on GARCH model). In order to relax the continuous dividend payout assumption, we use the Black model for pricing options on futures, instead of the Black-Scholes-Merton model. The results are presented separately for 5 classes of moneyness ratio and 5 classes of time to maturity in order to show some patterns in option pricing and to check the robustness of our results. The Black model with implied volatility (BIV) comes out as the best one. Highest average pricing errors we obtain for the Black model with realized volatility (BRV). As a result, we do not see any additional gain from using more complex and time-consuming models (SV and GARCH models. Additionally, we describe liquidity of the Nikkei225 option pricing market and try to compare our results with a detailed study for the emerging market of WIG20 index options (Kokoszczyński et al. 2010b).

Keywords:
option pricing models, financial market volatility, high-frequency financial data, midquotes data, transactional data, realized volatility, implied volatility, stochastic volatility, microstructure bias, emerging markets

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1. Introduction

The quest for the best option pricing model is at least 40 years old but going back into the past we could find its traces even few centuries earlier (e.g. the speculation during tulipomania or South Sea bubble).

The futures option pricing model (Black 1976) began a new era of futures option valuation theory. The rapid growth of option markets in the 1970s[1] brought rapidly a lot of data and stimulated an impressive development of research in this area. Quite soon numerous empirical studies put in doubt basic assumptions of the Black model: they strongly suggest that the geometric Brownian motion is not a realistic assumption. Many underlying return series display negative skewness and excess kurtosis (see Bates 1995, Bates 2003). Moreover, implied volatility calculated from the Black-Scholes model often vary with the time to maturity of the option and the strike price (cf. Rubinstein 1985, Tsiaras 2009). These observations drove many researchers to propose new models that each relaxes some of those restrictive assumptions of the Black-Scholes model (Broadie and Detemple 2004, Garcia et al. 2010, Han 2008, Mitra 2009). Basing on Han 2008, we can divide these researchers into a few groups. The first one engage in extending Black-Scholes-Merton framework by incorporating stochastic jumps or stochastic volatility (Amin and Jarrow 1992, Hull and White 1987), another one goes into estimating the stochastic density function of the underlying asset directly from the market option prices (Derman and Kani 1994, Dupire 1994) or using other distribution of the rate of return on the underlying asset rather than normal distributions (Jarrow and Rudd 1982, Corrado and Su 1996, Rubinstein 1998, Lim et al. 2005). On the other hand, the Black-Scholes model is still widely used not only as a benchmark in comparative studies testing various option pricing models, but also among market participants. Christoffersen and Jacobs 2004 show that much of its appeal is related to the treatment of volatility – the only parameter of the Black-Scholes model that is however not directly observed.

Detailed analysis of the literature (An and Suo 2009, Andersen et al. 2007, Bates 2003, Brandt and Wu 2002, Ferreira et al. 2005, Mixon 2009, Raj and Thurston 1998) seems to suggest that the BSM model with implied volatility calculated on the basis of the last observation performs quite well even when compared with many different pricing models (standard BSM model, BSM with realized volatility, GARCH option pricing models or various stochastic volatility models).

Our motivation for this paper is to check the results of Kokoszczyński et al. 2010a, who conducted a similar study for emerging market HF data (WIG20 index options)[2]. Their results show that the Black model with implied volatility (BIV) gives the best results, the Black model with historical volatility (BHV) is slightly worse and the Black model with realized volatility (BRV) gives clearly the worst results.

The complex comparison of Black model with different volatility assumptions presented only for an emerging market is definitely not enough to formulate conclusions of a more general nature. Therefore, we have decided to compare the results for the Polish emerging market with a similar research for the developed Japanese market. For this purpose we choose the Nikkei225 index option market (European style), which can be regarded as one of the most important in the world, especially when we consider the level of its innovation and complexity. As a result we hope we will be able to suggest some more general conclusions.

After a thorough analysis we can say that the literature regarding the Japanese capital market and especially European style index options, is not so rich and this is our second motivation to write this paper. The reason for this can be that Nikkei225 index is the basis instrument for many

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[1] The Chicago Board of Options Exchange was founded in 1973 and it adopted the Black-Scholes model for option pricing in 1975.
[2] The WIG20 is the index of twenty largest companies on the Warsaw Stock Exchange (further detailed information may be found at www.gpw.pl).

different derivatives which are quoted on many different exchanges and the literature is widely dispersed. We can easily find some papers focusing on pricing American style options or options quoted in different currency than Yen. On the other hand, papers in English focusing on European style Nikkei255 index options are not so numerous[3]. The literature on American style options shows quite good results for the Black model (Raj, Thurston 1998), sometimes better than for various GARCH models (Iaquinta 2007). When we consider the second case (options in other currencies), we actually model not only option prices but the exchange rate fluctuations as well (Wei 1995), thus the comparison of their results with ours could be regarded as not valid.

   Therefore, we are left with the very limited number of studies that focus on European style options or otherwise touch this subject, sometimes only in an indirect way. Li 2006 shows that Nikkei225 is rather an efficient market (in the sense of lack of arbitrage possibilities analysed through existence of put-call parity). Yao et al. 2000 compare the BSM model with historical volatility with pricing done via neural networks and show that in some cases (mainly for ATM options) the BSM model gives the better results. Kanoh and Takeuchi 2006 once again show that the BSM model is better (in terms of the RMSE statistics) from GARCH (1,1) and E-GARCH (1,1) model. On the other hand Mitsui and Satoyoshi 2006 got better results for GARCH-T model for almost all moneyness classes, but their results are based on strong assumptions concerning the type of distribution of the basis instrument.

   This review, covering those Nikkei225 index options studies that are comparable with our approach, justifies quite strongly the positive assessment of the BSM model. We are going to check this using high-frequency data from 2008.

   The structure of this paper has been planned in such a way as to answer the following detailed questions:

   - Which model from among those we test can be treated as the best one?
   - Can we observe any distinctive patterns in option pricing taking into account moneyness ratio (MR) and time to maturity (TTM)?
   - Can we distinguish any patterns of liquidity behaviour in a developed market using transactional data?
   - Do we observe any outliers and what is the real influence of outliers (or "spurious outliers") on final results and how can we identify those observations that can be later excluded from the dataset?
   - Is there any substantial difference between the results for a developed (this paper) and an emerging market (Kokoszczyński et al. 2010a)?

   The remainder of this paper is organized as follows. The second section describes some methodological issues. Next section presents data and the fluctuations of volatility processes derived from transactional data. The fourth section discusses the liquidity issues. Results are presented in section five and the last section concludes.

2. Option pricing methodology

2.1. The Black option pricing model with historical, realized and implied volatility

   The basic pricing model we choose is the Black-Scholes model for futures prices, i.e. the Black model (Black, 1976). We call it further in the text the BHV model – the Black model with historical volatility. Below are formulas for this model:

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[3] Unfortunately, because of language barrier we were not able to extend our literature review to papers written in Japanese.

$$c = e^{-rT}[FN(d_1) - KN(d_2)] \qquad (1)$$

$$p = e^{-rT}[KN(-d_2) - FN(-d_1)] \qquad (2)$$

$$where:$$

$$d_1 = \frac{\ln(F/K) + \sigma^2 T/2}{\sigma\sqrt{T}} \qquad (3)$$

$$d_1 = \frac{\ln(F/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \qquad (4)$$

where c and p are respectively valuations of a call and a put option, T is time to maturity, r is the risk-free rate, F – the futures price, K – underlying strike, σ – volatility of underlying and $N(.)$ is the cumulative standard normal distribution.

There are two reasons why we decided to use the Black model instead of the standard Black-Scholes model. First, we are able to relax the assumption about continuous dividend payouts[4]. Second, we can use additional data because usually derivatives (options, futures, etc.) are quoted much longer than the basis instruments (e.g. Nikkei225 index).

To further justify such an approach, we assume that we can price a European style option on Nikkei225 index applying the Black model for futures contract (with historical, realized and implied volatility), where Nikkei225 index futures contract is the basis instrument. This is possible due to following facts:
- Nikkei225 index futures expire exactly on the same day as Nikkei225 index options do,
- the expiration prices are set exactly in the same way,
- we study only European-style options; hence early expiration - like in the case of American options - is impossible[5].

One of the most important issues about option pricing is the nature of an assumption concerning the specific type of volatility process. Therefore, we check the properties of the Black model with three different types of volatility estimators: historical volatility, realized volatility and implied volatility, and additionally we use the Heston model and the GARCH option pricing model. Below we provide a brief description of each of these volatility estimators and models.

The **historical volatility (HV) estimator** is based on the formula

$$VAR_\Delta^n = \frac{1}{(N_\Delta * n) - 1} \sum_{t=1}^{n} \sum_{i=1}^{N_\Delta} (r_{i,t} - \bar{r})^2 \qquad (5)$$

where:

VAR_Δ^n – variance of log returns calculated on high frequency data on the basis of last n days,

$r_{i,t}$ – log return for i-th interval on day t with sampling frequency equal to Δ, which is calculated in the following way:

$$r_{i,t} = \log C_{i,t} - \log C_{i-1,t} \qquad (6)$$

$C_{i,t}$ – close price for i-th interval on day t with sampling frequency equal to Δ,

N_Δ – number of Δ intervals during the stock market session,

n – memory of the process measured in days, used in the calculation of respective estimators and average measures.

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[4] In this way we are able to eliminate two possible source of pricing error: the necessity to estimate the dividend yield and the assumption about continuous payouts.

[5] Early expiration of American-style option could result in the significant error in the case of such a pricing, because of the difference in prices of index futures and of Nikkei225 index before the expiration date (the basis risk).

$\bar{r}$ – average log return calculated for last n days with sampling frequency Δ, which is calculated in the following way:

$$\bar{r} = \frac{1}{N_{\Delta}*n}\sum_{t=1}^{n}\sum_{i=1}^{N_{\Delta}} r_{i,t} \qquad (7)$$

In this research, we use N_{Δ}=1 and hence the HV estimator is simply standard deviation for log returns based on the daily interval. This approach is commonly used by the wide range of market practitioners.

The second approach is the **realized volatility (RV) estimator** proposed early by Black (1976) and Taylor (1986) and further popularised by Bollerslev (cf. Andersen et al. 2001). It is based on squared log returns summed over the time interval of N_{Δ} .

$$RV_{\Delta,t} = \sum_{i=1}^{N_{\Delta}} r_{i,t}^{2} \qquad (8)$$

The **implied volatility (IV) estimator** is based on the last observed market option price. It assumes that all parameters (with the exception of sigma) are also known. We calculate the implied volatility for the last market price for each option and then average them separately for each class of TTM and moneyness ratio, and for both call and put options[6]. Hence, for each observation we have 50 different IV values (5×5×2). These values are then treated as an input variable for volatility parameter in calculations of the theoretical options price for the Black model with the implied volatility (BIV) for the next observation.

Before entering into the formula for the Black model, the HV and RV estimators have to be annualized and transformed into standard deviation. The formula for the annualization of the HV estimator is as follows:

$$HV = {}^{annual\_std}SD_{\Delta}^{n} = \sqrt{252 * N_{\Delta} * VAR_{\Delta}^{n}} \qquad (9)$$

Contrary to the HV estimator, which is based on information from many periods (n>1), RV estimator requires information only from a single period (time interval of Δ). Therefore, the procedure of averaging and annualizing realized volatility estimator is slightly different from that presented in formula (9):

$${}^{annual\_std}[RV]_{\Delta}^{n} = \sqrt{252}\sqrt{\frac{1}{n}\sum_{t=1}^{n}[RV]_{\Delta,t}} \qquad (10)$$

Having all these volatility estimators and additionally the Heston and GARCH (1,1) option pricing models we present below we study several types of option pricing models, which will be described in details in section 2.5.

2.2. The GARCH Model

Many classical option pricing models (e.g. the Black model) assume the constant level of volatility of log-returns of basis instruments. However, in reality many financial time series are characterized by time varying volatility. GARCH models are one possible way to relax this initial assumption. They were proposed by Engle 1982 and Bollerslev 1986. GARCH model describes the dynamic of returns of the basis instruments with following equations:

$$r_t = \varepsilon_t \qquad (11)$$

$$\varepsilon_t = z_t\sqrt{h_t}, \quad z_t \sim IID \ N(0,1) \qquad (12)$$

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[6] We divide 320 options (160 call and 160 put options) into 5 moneyness ratio classes and 5 time-to-maturity classes. The details of this classification are presented in Section 3.

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$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \qquad (13)$$

where $r_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$, S_t is the price of the basis instrument in the moment t, and p and q define the order of GARCH (p,q) model. Currently, we have many extensions of standard GARCH(p,q) model, which mainly differ by the specification of the conditional variance equation and by various assumptions concerning the conditional distributions of residuals in the mean equation. Through the years GARCH models have become the standard approach in volatility modelling, asset pricing, financial time series forecasting or risk management. Examples of this kind of research can be found in Bollerslev et al. 1988, Bollerslev et al. 1994, Campbell and Hentschel 1992, French et al. 1987, Glosten et al. 1993, Maheu and McCurdy 2004, Pagan and Schwert 1990, whereas detailed description of GARCH models can be found in Bollerslev et al. 1992 or Campbell et al. 1997.

Finally, GARCH models are also used in option pricing models. Duan 1995 presents the methodology of European style call option pricing with the assumption that returns of the basis instrument can be described with the GARCH process. In order to become risk neutral in this approach, we differentiate between physical and martingale (risk free) probability measure. Garcia and Renault 1998 describe theoretical aspects of using GARCH models in risk hedging strategies, while Ritchken and Trevor 1999 use GARCH models in American style option pricing applying trinomial trees. Duan et a.l 2004 extend the methodology presented in his previous paper through inclusion of volatility jumps in prices of the basis instrument.

Option pricing based on GARCH model has been done here according to Duan 1995. This approach assumes that log returns undergo GARCH-M(p,q) process described by the following equations:

$$r_t = r_f + \delta\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t \qquad (14)$$

$$\varepsilon_t = z_t\sqrt{h_t}, \quad z_t \sim N(0,1) \qquad (15)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \qquad (16)$$

where parameters are denoted in the same way as in earlier formulas, and additionally δ in equation (14) is interpreted as a unit risk premium.

The pricing of options is conducted assuming local risk-neutral valuation. It requires modification of log returns processes in such a way that conditional variance one step ahead remains unchanged and simultaneously conditional expected return equals risk-free rate (Fiszeder 2008). Introduction of risk-neutral probabilistic measure Q enables us to price options through discounting expected option payoff.

The dynamic of basis instrument log returns with respect to measure Q can be described as follows:

$$r_t = r_f - \frac{1}{2}h_t + \xi_t \qquad (17)$$

$$\xi_t = u_t\sqrt{h_t}, \quad u \sim N(0,1) \qquad (18)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left(\xi_t - \delta\sqrt{h_{t-1}}\right)^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \qquad (19)$$

The formula describing the dependence between the price of basis instrument on maturity day and its price in the time of pricing can be described:

$$S_T = S_t \exp\left[r_f(T-t) - \frac{1}{2}\sum_{i=t+1}^{T} h_i + \sum_{i=t+1}^{T} \xi_i\right] \tag{20}$$

while the price of European style call option is described by the discounted value of the option price on the maturity day:

$$call_t = \exp\left(-r_f(T-t)\right) E^Q[\max(S_T - X, 0) | \psi_t] \tag{21}$$

where $E^Q(.)$ is the operator of conditional expected value with respect to Q measure.

In practice, the pricing is done through Monte Carlo simulation. In the first stage we estimate parameters of the model (14), (15) and (16), and then on the basis of (17), (18), (19), and (20) we simulate N realization of basis instrument price. Call and put option prices are then calculated in the following way:

$$call_t = \exp\left(-r_f(T-t)\right) \frac{1}{N} \sum_{j=1}^{N} \max\left(S_{Tj} - X, 0\right) \tag{22}$$

$$put_t = \exp\left(-r_f(T-t)\right) \frac{1}{N} \sum_{j=1}^{N} \max\left(X - S_{Tj}, 0\right) \tag{23}$$

We use GARCH-M(1,1) model in this study[7]. Many research papers show that this order of the model defines the dynamics of stock index returns in the most adequate way (Hansen and Lunde 2004 or Zivot 2008). Similarly, like in case of the BSM model we used index returns. We estimate the parameters of the equations (14), (15) and (16) on the basis of data from 1/1/2007 until the moment of option pricing. As a result, the size of sample used to estimate GARCH parameters varies from one year (for pricing done on 2nd January, 2008) to 1.5 year (for pricing done on 30th June, 2008). In order to eliminate problems with instability of GARCH model parameters we have decided to delete overnight returns from our data sample.

The number of replications in Monte Carlo simulations is another important choice to be made. Finance literature suggests strongly that N = 10 000 gives an adequate precision of estimates. However, due to the very large number of pricing (5-minute data) we need, we have to limit the number of replication to N=1000. In order to minimize possible negative effects of that choice we use variance reduction technique called antithetic variables sampling. The data we use in this study are described in detail in section 3.

2.3. The Heston Model

Log returns volatility in stochastic volatility models is represented by a given stochastic volatility process with dynamics set a priori. Hull and White 1987 are among pioneers of applying stochastic volatility for option pricing. They assume that variance dynamics can be described with the following differential equation:

$$dV_t = a(b - V_t)dt + cV_t dZ_t \tag{24}$$

Under additional assumption - that volatility is not correlated with the basis instruments - Hull and White present the analytical formula for European style call option. One of the main conclusions of their research is that the BSM model systematically underestimates prices of ITM and OTM options, and overestimates prices of ATM options.[8]

The Heston model we use in our research is an extension of Stein and Stein 1991. Their option pricing formula assumes that volatility is described by the Ornstein-Uhlenbeck process and is not correlated with the basis instrument. On the other hand, Heston 1993 presents call option pricing formula with no assumption on correlation of volatility with the basis instrument. His model

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[7] In the results section we will refer to this model as to GARCH(1,1).
[8] The constant volatility assumption is responsible for this drawback of the BSM model.

assumes that the dynamics of underlying asset price S_t and its volatility V_t are given by the following set of differential equations:

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^1 \tag{25}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_t^2 \tag{26}$$

$$dW_t^1 dW_t^2 = \rho dt \tag{27}$$

where $\{S_t\}_{t\geq 0}$ and $\{V_t\}_{t\geq 0}$ indicate the price and the variance of the basis instrument, and $dW_{t\,t\geq 0}^1$ and $dW_{t\,t\geq 0}^2$ are correlated Brownian motion processes (with parameter of correlation ρ). Additionally, it is assumed that $\{V_t\}_{t\geq 0}$ is mean reverting process, with long memory expected value θ and mean reverting coefficient κ. The parameter σ is defined as volatility of volatility.

One of the main reasons, why the Heston 1993 model has become so popular is the fact that it is possible to obtain its closed-form solution for European style call option pricing for an asset not paying dividend, which is given by:

$$C(S_t, V_t, t, T) = S_t P_1 - Ke^{-r(T-t)}P_2 \tag{28}$$

where

$$P_j(x, V_t, T, K) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left\{\frac{e^{-i\phi \ln(K)}f_j(x, V_t, T, \phi)}{i\varphi}\right\}d\phi \tag{29}$$

$$f_j(x, V_t, T, \phi) = \exp\left\{r\phi ir + \frac{a}{\sigma^2}\left[(b_j - \rho\sigma\phi i)\tau - 2\ln\left(\frac{1 - ge^{dr}}{1 - g}\right)\right]\right.$$
$$\left. + \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}\left(\frac{1 - e^{dr}}{1 - ge^{dr}}\right) + i\phi\ln(S_t)\right\}$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{\left(\rho\sigma\phi i - b_j\right)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

for $j = 1,2$, where:

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, \alpha = \kappa\theta, b_1 = \kappa + \lambda - \rho\sigma, b_2 = \kappa + \lambda$$

Formula 28 is not difficult to implement in practice. The only problem is to calculate the limit of the integral therein. This limit is often approximated by an adequate quadrature (Gauss-Legendre or Gauss-Lobatto), what can be done in many statistical software packages.

Practical implementation of the Heston model is done in two stages. First, we have to calibrate the model in order to find its parameters from equation (25), (26) and (27). Calibration can be done on the basis of call transactional prices observed in every one-hour interval. We choose parameter values in such a way as to minimize the difference between market and theoretical prices. Next, we use formulas (28) and (29) to calculate theoretical prices.

The calibration of the Heston model can be conducted in two ways - via global or local optimization. Global optimization guarantees that we find the true global minimum of our target function. The disadvantage of this method is that it is time-consuming and parameters obtained here tend to be very unstable. On the other hand, local optimization gives only local minima but it is very fast and parameters derived in this way are stable.

In our study the global optimization is used for the first period and its results are the starting point for the local optimization in the second period. Then, the further iterations of the local optimization are being performed, for which the starting point is set to the local minimum from the previous stage.

In the second stage the parameters found previously are used to calculate theoretical prices in the next hourly interval. The prices of call options are calculated according to the formulas (28) and (29), while put option prices are found on the basis of call-put parity:

$$C_t + Ke^{-r(T-t)} = P_t + S_t \tag{30}$$

where C_t and P_t are European style call and put prices, S_t is the price of basis instrument, r is the risk-free rate, and finally K is the strike price, and T is time to maturity for both call and put options.

The calibration of the Heston model in our study for the Japanese market has been done on the basis of an hourly interval. It means that in the time of calibration we use transactional prices from the previous hourly interval, and then we use those results to price options for the current interval. The calibration of the Heston model was based on all available transactional prices in one-hour interval.

2.4. Goodness-of-fit statistics

The metric used to compare models we test here is given by three error statistics:

- Root Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{1}{N_\Delta n} \sum_{i=1}^{N_\Delta n} (theoretical\_option_i - close_i)^2} \tag{31}$$

- Heteroscedastic Mean Absolute Error (HMAE):

$$HMAE = \frac{1}{N_\Delta n} \sum_{i=1}^{N_\Delta n} \left| \frac{theoretical\_option_i - close_i}{close_i} \right| \tag{32}$$

- Percentage of overprediction (OP):

$$OP = \frac{1}{N_\Delta n} \sum_{i=1}^{N_\Delta n} OP_i, where: OP_i = \begin{cases} 1 & if \quad theoretical\_option_i > close_i \\ 0 & if \quad theoretical\_option_i < close_i \end{cases} \tag{33}$$

where $close_i$ is the option price (i.e. the last observed transaction price) for the i-th interval and $theoretical\_option_i$ is the model price of option (BHV, BRV, BIV, GARCH, Heston) for the i-th interval. We calculate these statistics for all models, for different TTM and MR classes, and for both call and put options.

2.5. Models' description

Thus, we study properties of the following models:
- BHV - the Black model with historical volatility (sigma as standard deviation, n=21),
- BRV - the Black model with realized volatility (realized volatility as an estimate of sigma parameter; RV calculated on the basis of observations with several different Δ intervals and different values for parameter n applied in the process of averaging),
- BIV - the Black model with implied volatility (implied volatility as an estimate of sigma; IV calculated for the previous observation, separately for each TTM and MR class, and for both call and put options, hence for 50 different groups).
- Heston – the Heston option pricing model,
- GARCH – GARCH (1,1) option pricing model based on the Duan methodology,

Initially, we calculate BRV models with four different Δ values: 10s, 1m, 5m, and 15m. Then, we check the properties of averaged RVs with different values of parameter n in pricing models. We find, like Kokoszczyński et al. 2010a, no significant differences between RVs with different Δ parameter (assuming that Δ is equal or higher than 5 minutes). On the other hand, Sakowski 2010, after a detailed analysis of similar data for WIG20 index options but for a longer data span

confirms our choice of Δ parameter. He shows that BRV, BHV and BIV models have better properties (basing on HMAE statistics) for parameter Δ equal to 5 than for Δ equal to 10, 15 and 30 minutes. Moreover, he also shows that going with Δ below 5 minutes, i.e. to 1 minute or even to 10 seconds could result in much higher pricing errors because of very high volatility of volatility for time series calculated on the basis of such intervals. Sakowski's results support what is a common approach in the literature, e.g. setting the interval between 5 minutes and 15 minutes because this constitutes the good trade-off between the nonsynchronous bias and other microstructure biases (cf. Ait-Sahalia et al. 2009). Therefore, we calculate the BRV model only for Δ=5m interval with different values of averaging parameter (*n*=1, 2, 3, 5, 10, 21, and 63), but after analysing their properties we present only the best and the worst model from the family of BRV models: BRV5m (non-averaged one), and BRV5m_63[9]. GARCH model has been estimated with the same Δ interval and the Heston model has been calibrated on hourly intervals but it still enables us to calculate theoretical prices for Δ interval equal to 5 minutes.

3. Data and the description of volatility processes

3.1 Data description

We use transactional data[10] for Nikkei225 index options, Nikkei225 index and Nikkei225 index futures, which have been provided by Reuters company[11]. The data cover the period from January 2, 2008 to June 30, 2008. Transactional prices for Nikkei225 index options and Nikkei225 index are in the form of 5-minutes data and we use such data for further calculations. However, in order to calculate different volatility estimators we transform 5-minutes data into different frequencies. The risk-free interest rate is approximated by the LiborJPY3m interest rate, also converted into 5-minute intervals.

The market for Nikkei225 index option started in this period at 1.00 CET and ended at 7.00 CET[12]. For that reason we have 6745 observations (122 session days with 56 5-minutes intervals each[13]).

As a result, our data set for Nikkei225 index options comprised transactional prices for 160 call options and 160 put options maturing in January, February, March, April, May, June and July 2008. Maturity days of these options and their symbols for each call and put series are as follows: 11.01.2008 (*call*-A8, *put*-M8), 08.02.2008 (*call*-B8, *put*-N8), 14.03.2008 (*call*-C8, *put*-O8), 11.04.2008 (*call*-D8, put-P8), 09.05.2008 (*call*-E8, *put*-Q8), 13.06.2008 (*call*-F8, *put*-R8) i 11.07.2008 (*call*-G8, *put*-S8).

The results of our analysis will be presented with respect to 2 types of options, 5 classes of MR and 5 classes of TTM:
- 2 types of options (call and put),
- 5 classes of moneyness ratio, for call options: deep OTM (0-0.85), OTM (0.85-0.95), ATM (0.95-1.05), ITM (1.05-1.15) and deep ITM (1.15+), and for put options in the opposite order[14],

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[9] Our choice is confirmed by results in Sakowski 2010 and Kokoszczyński et al. 2010 a.

[10] Some papers that test alternative option pricing models and include the Black-Scholes model among models tested therein use instead of transactional data bid-ask quotes (midquotes) as they allow to avoid microstructural noise effects (Dennis and Mayhew 2009). Ait-Sahalia and Mykland 2009 state explicitly that quotes "contain substantially more information regarding the strategic behaviour of market makers" and they "should be probably used at least for comparison purposes whenever possible" (p. 592). On the other hand, Beygelman 2005 and Fung and Mok 2001 argue that midquote is not always a good proxy for the true value of an option.

[11] Thanks to government financial support we were able to buy all necessary data (5 minutes intervals) from Reuters Datascope company.

[12] In practice market session lasted from 1.00 CET to 3.00 CET, then there was a pause, and later session lasted from 4.30 CET to 7.00 CET. Therefore, we get 56 5-minutes intraday returns.

[13] Some days, close to the most important national holidays, the market session finished before 7.00 CET.

- 5 classes for time to maturity: (0-15 days], [16-30 days], [31-60 days], [61-90 days], [91+ days).

This categorization allows us to compare different pricing models along several dimensions.

3.2 The descriptive statistics for Nikkei225 futures time-series.

We begin our study with the basic analysis of the time series of returns of the basis instrument. Table 3.1 presents the descriptive statistics for 5-minute interval data. They are calculated for two samples: with (sample denoted R_f) and without opening jumps effects (R_f)[15].

Table 3.1. The descriptive statistics for Nikkei225 index returns (with and without opening jump effect).

		R_f [a]	R_f [b]
N		6745	6504
Mean		-0,000025394	-0,000014111
Median		0,000032644	0,000036116
Standard Deviation		0,0030907	0,0028429
Range		0,0535235	0,0535235
Minimum		-0,0319108	-0,0319108
Maximum		0,0216127	0,0216127
Kurtosis		10,4364219	12,7560437
Skewness		-0,6227228	0,7206586
Normality tests			
Kolmogorov-Smirnov	**Statistic**	0,093349	0,086497
	p-value	<0,01	<0,01
Jarque-Berra	**Statistic**	30995,9195	44584,7971
	p-value	<0,0001	<0,0001

[a] full sample, [b] sample without opening and mid-session jump effect.

Both samples have high kurtosis and are asymmetric. The distribution for the full sample has negative skewness while removing jump effects makes the distribution right skewed. Overall, both Jarque-Bera and Kolmogorov-Smirnov statistics indicate that returns in both samples are far from normal. Nevertheless, we observe interesting feature that - contrary to data from the Polish market (Kokoszczyński et al., 2010b) - for adjusted sample skewness and kurtosis are larger when we consider their absolute values. In case of the Japanese market it is not the jump effect that is responsible for the non-normality of returns, but returns' general features.

Figures 3.1 and 3.2 additionally confirm this observation showing high negative and positive returns in both time series with and without jump effects. Formally, the lack of normality of the basis instrument means that the standard BSM model should not be applied for option pricing with these data. Accordingly, we transform this model varying its assumption about the nature of

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[14] Moneyness ratio is usually calculated according to the following formula:

$$moneyness\,ratio = \frac{S}{K/e^{rT}} = \frac{F}{K} \tag{34}$$

where K is the option strike price, S is the price of underying, F is the futures price of underlying, r is the risk-free rate and T is time to maturity.

[15] By *opening jump effects* we mean returns between 7.00 CET and 1.00 CET on the next day. Thus, sample without opening jump effects does not include observations with these returns. In case of the Japanese market two returns were excluded: one overnight return and second one including the return from the mid-session break.
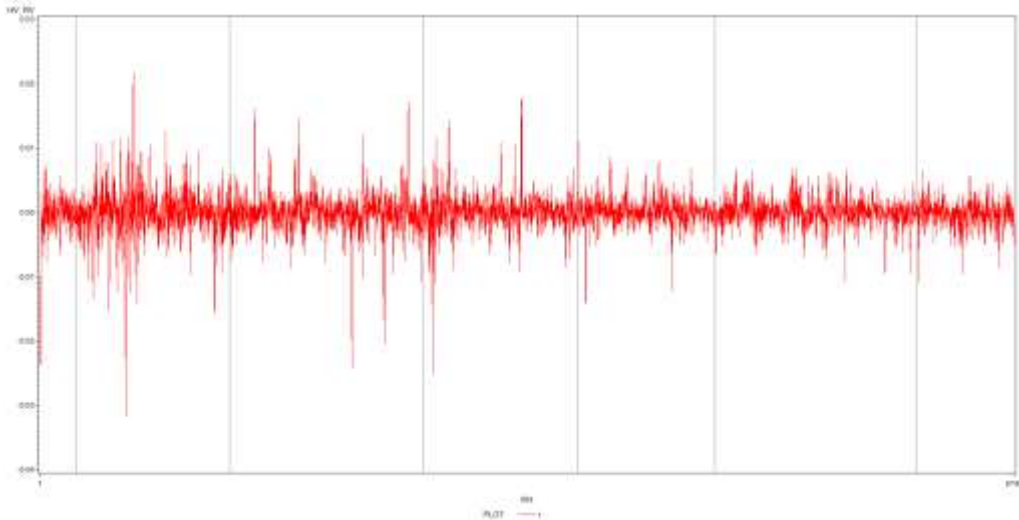
the volatility process. Moreover we also apply the Heston and GARCH option pricing models to the same data.

Figure 3.1. Index returns with the opening jump effect. [a]
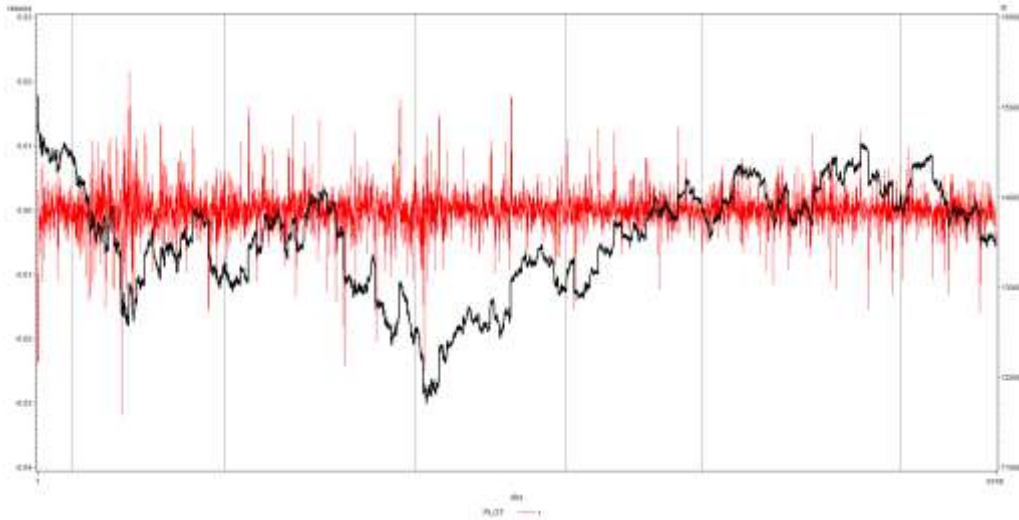
[a] The returns and index prices cover the data span between January 2, 2008 to June 30, 2008.

Figure 3.2. Index returns without the opening and mid-session jump effect.[a]

[a] The 10-second returns between the closing price from each day and the opening price from the next day have been excluded. The same was done with the mid session jump. The returns cover the data span from January 2, 2008 to June 30, 2008.

3.3 The description of volatility processes; historical, realized and implied.

We consider three different volatility measures: historical, realized and implied volatility for the Black option pricing model and - in addition to that - stochastic volatility and GARCH model. Obviously, the volatility process assumed in pricing is one of important reasons for differences among theoretical option prices we compare.

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In the case of the historical volatility estimator $N_\Delta=1$ for every $r_{i,t}$ (daily log returns) and $C_{i,t}$ in formulas (5), (6) and (7). Moreover, we use the constant value of parameter n being equal to 21, because we want to reflect historical volatility from the last trading month.

On the basis of similar studies for the Polish market (Kokoszczyński et al. 2010a, Kokoszczyński et al. 2010b) realized volatility has finally been calculated on the basis of Δ equal to 5 minutes. Therefore, at this stage we limit our selection of volatility time series only to RV calculated for $\Delta=5$ minutes, with averaging parameter n=5, 10, 21 and 63 days (Figure 3.3). Figure 3.3 presents realized volatility compared to historical volatility. The distinguishing fact is that the non-averaged RV time series (RV_5m) is much more volatile than the averaged RV or HV time series. Obviously, such high volatility of volatility can strongly influence theoretical prices from the BRV model and their stability over time. One can thus expect that in periods of high volatility the BRV model with the non-averaged RV estimator may produce high pricing errors.

Figure 3.3. Historical and realized volatility (5m, 5m_5, 5m_10, 5m_21, 5m_63). [a]
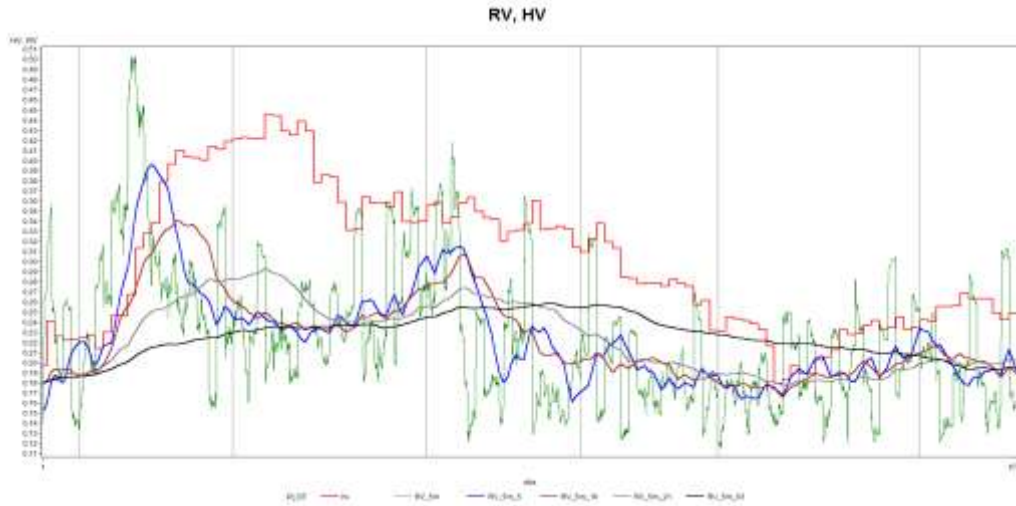
[a] The volatility time series cover the data period between January 2nd, 2008 and June 30th, 2008. Vertical lines represent end of month and additionally the day of January 11th, February 8th, March 14th, April 11th, May 9th, and June 13th, when option series expired.

On the other hand, implied volatility time series exhibits substantially different trajectories than RV or HV time series. Figure 3.4 (call options), and Figure 3.6 (put options) present how IV time series evolve in time. Similarly to Kokoszczyński et al. (2010) we observe that for the short TTM (5-10 days) IV tends to increase with shortening of TTM. Contrary to the Polish market for TTM lower than 5 days we do not observe explosion of IV and it does not reach the level of over 200% (annualized). This happens mostly for the call and put (deep) OTM and ATM options. However, jump of IV to 70% can be the reason of big mispricing of options with low TTM. For that reason, some researchers often exclude from comparisons options with short TTM and market prices lower than 5-10. However, we have consciously decided to conduct this research on the full sample, believing that such an approach would allow us to better answer the question what kind of observation should be treated as outliers.

Figure 3.4. Implied volatility for ATM call option. [a]
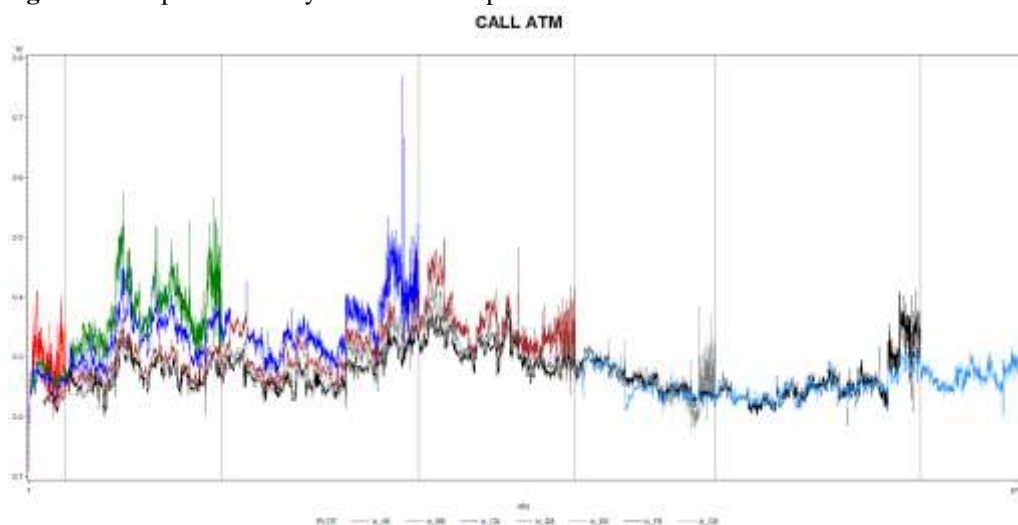
CALL ATM

[a] The volatility time series cover the data period between January 2nd , 2008 and June 30th , 2008. IV are presented for 7 series of options. Vertical lines represent end of month and additionally the day of January 11th, February 8th, March 14th,April 11[th,] May 9th, and June 13th, when option series expired.

Figure 3.5. Implied volatility for ATM put option. [a]
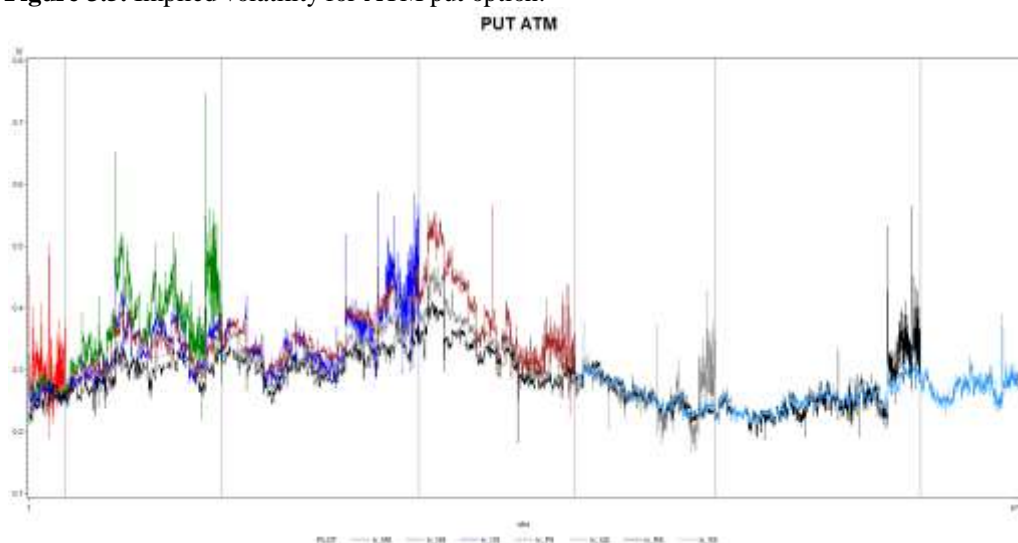
PUT ATM

[a] The volatility time series cover the data period between January 2nd , 2008 and June 30th , 2008. IV are presented for 7 series of options. Vertical lines represent end of month and additionally the day of January 11th, February 8th, March 14th,April 11[th,] May 9th, and June 13th, when option series expired.

4. Market liquidity

Liquidity constraints are typical features of an emerging derivatives market and they put severe limits for conducting such a study as we have done for the Polish market. To make our comparisons of both markets (WIG20 and Nikkei225) as comprehensive as possible we have also decided to present a detailed discussion of developed market liquidity on the example of

Nikkei225 index option market with respect to volume, turnover and open positions for transactional data we use.

Observing the distribution of call volume for transactional data, presented in Figure 4.1, we notice that the lowest volume is observed for low TTM and MR equal to ITM, deepITM, and deepOTM. The highest volume we see for MR equal to ATM and OTM for TTM up to 60days. This suggests that investors rarely trade highly valued options (deepITM and ITM) or options with long TTM.

Figure 4.1. The distribution of volume for call option[a]
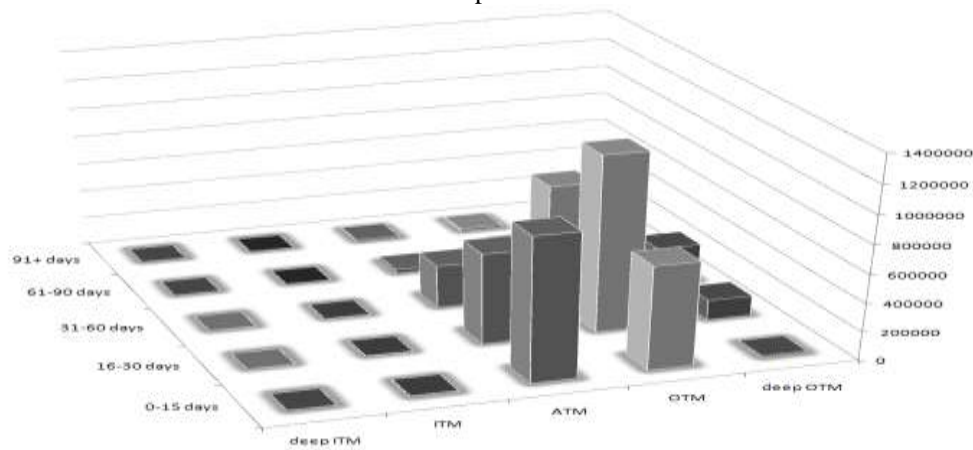
[a] the volume for call options quoted in the period between January 2nd, 2008 and June 30th, 2008.

Figure 4.2. The distribution of volume for put option [a]
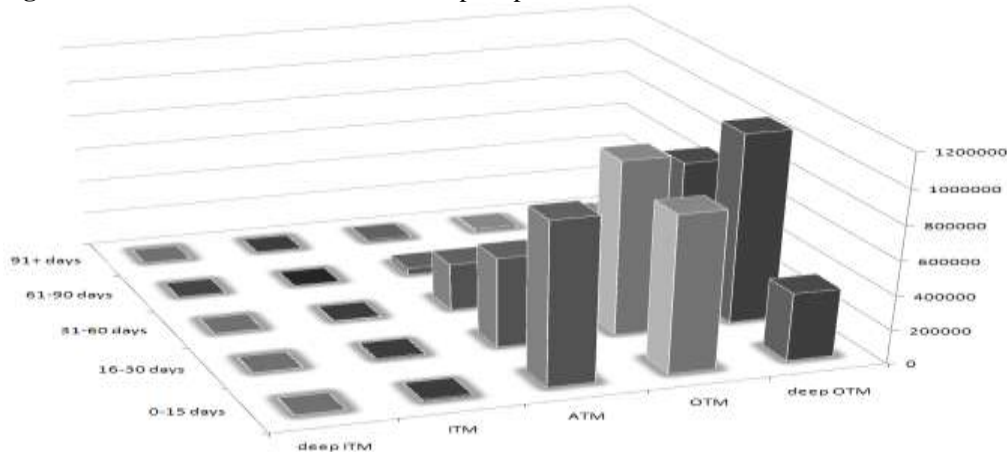
[a] the volume for put options quoted in the period between January 2nd, 2008 and June 30th, 2008.

The distribution for put volume (Figure 4.2) is very similar. The only difference is that the volume is also high for deepOTM options with TTM less than 60 days. However, this is mostly due to the fact that put options are used as an insurance against sharp downward movement of the basis instrument[16]. Generally, we could say that the volume distribution for call and put options is very similar and that investors focus their trades on low-valued options with short TTM.

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[16] One buys the right to sell the basis instrument in the case of an extreme financial catastrophe, e.g. financial crash, for a relatively low cost (put option premium).

Next figure (Figure 4.3) addresses the liquidity issue from another perspective by focusing on the turnover volume increasing the importance of traded options value. We observe significant shift from deepOTM to ATM and then to OTM options. It obviously means that most investors involved in the option trades concentrate in the ATM-OTM range. The same results are observed for call and put options with only slightly higher turnover volume for put options. However, this latter feature can be tied to the behaviour of the basis instrument in the period we study.[17]

Figure 4.3. The volume of turnover for call and put option.

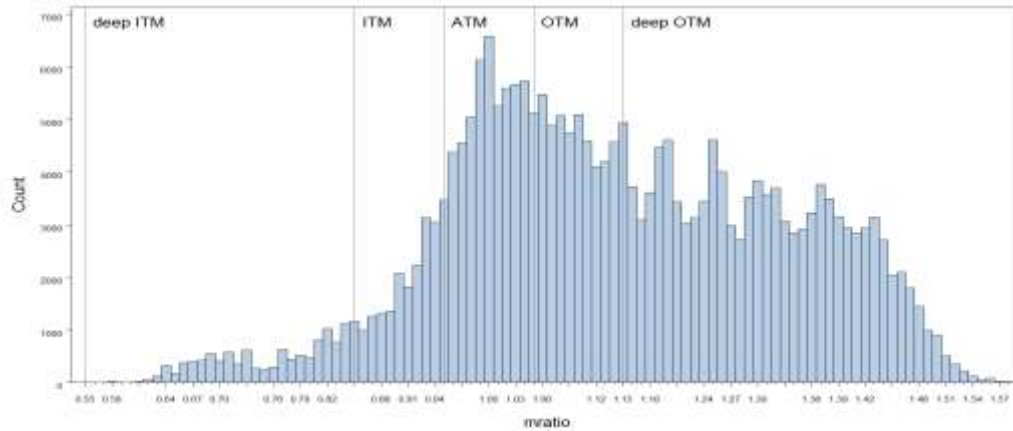^a call options   ^a put options
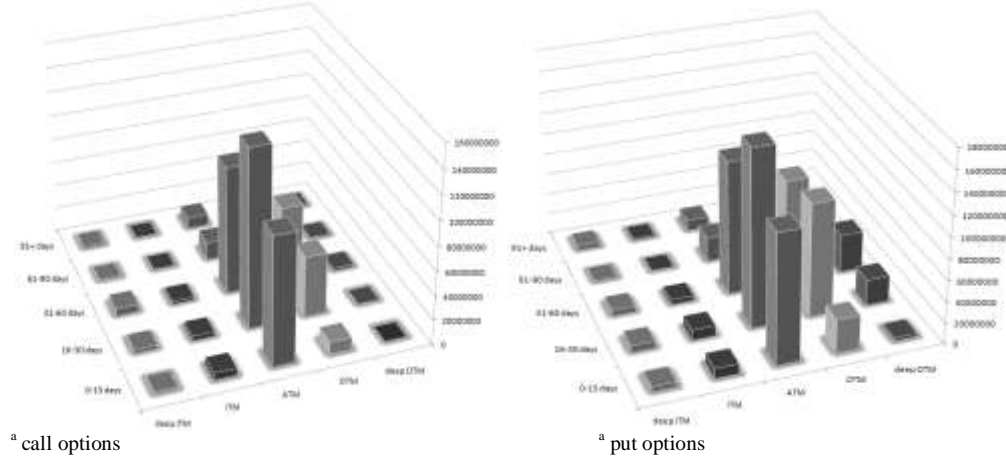
The final aspect in the discussion of liquidity is the comparison of our data set with actual trade possibilities. Looking at the Figure 4.4 and 4.5 we see that the volume observed for call and put options is in some way conditional on options available in the respective time period. Comparing figures 4.1 and 4.2 with 4.4. and 4.5 shows the same picture for call and put options. In the case of call options most of the available strikes constitute ATM and ITM, while for put options it is ATM, OTM and deepOTM. Therefore, we see that results are not robust with respect to available options strikes that can be traded in the given period.

Figure 4.4. *Moneyness ratio* histogram for call options with respect to transactional data.[a]

^a transactional data.

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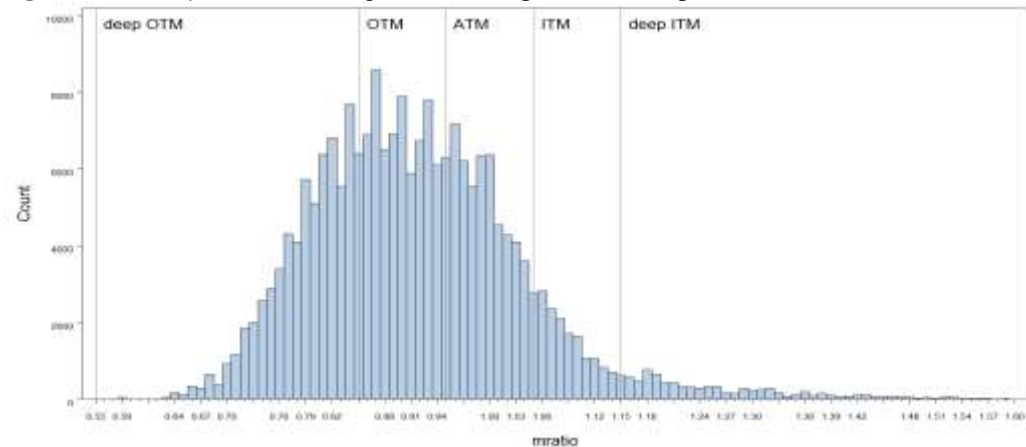
[17] We observe sharp downward movement of Nikkei225 index in the time of research

Figure 4.5. *Moneyness ratio* histogram for put options with respect to transactional data.

The most important outcome from the liquidity analysis is that we can indicate where the volume of options concentrates. We notice that after dividing the set of options into different MR and TTM classes we can distinguish options with low TTM which are ATM, OTM or ITM that cumulate more than 90% of the total volume, both in case of call and put options. Similar situation has been observed in the case of the Polish emerging market. Finally, it is worth to notice that the cumulation of volume in the given class of MR and TTM is partly conditional on the availability of options with specified MR or TTM ratio.

5. Results

We structure the results section into three subsections containing the description of theoretical premiums (5.1), results presented separately for call and put options (5.2), and the aggregated comparison of results with respect to different dimension (5.3). This enables us to present a multidimensional comparative analysis of option pricing models.

5.1. The description of theoretical premiums

Finally, we obtain three error statistics (RMSE, OP, HMAE) calculated for six different pricing models (BRV5m, BRV5m_63, GARCH(1,1), Heston, BHV, BIV), which are divided into 5 TTM classes and 5 MR classes. The number of pricing errors calculated for each model is presented in the table below:

Table 5.1. Number of theoretical premiums for different classes of MR and TTM for BRV model*

option	moneyness	0-15 days	16-30 days	31-60 days	61-90 days	91+ days	*Total*
CALL	deep OTM	372	4327	27089	23799	10494	66081
CALL	OTM	6501	11635	22572	19567	8959	69234
CALL	ATM	8199	9681	17385	12141	5368	52774
CALL	ITM	3880	4510	5373	1484	761	16008
CALL	deep ITM	1205	1935	3032	1044	1335	8551
	Total Call	*20157*	*32088*	*75451*	*58035*	*26917*	*212648*
PUT	deep OTM	6964	20580	44831	31225	7768	111368
PUT	OTM	6109	8142	15466	12674	5631	48022
PUT	ATM	8028	9669	17014	12001	6413	53125
PUT	ITM	4278	4826	7427	1790	1096	19417
PUT	deep ITM	2411	3002	3098	1161	1962	11634
	Total Put	*27790*	*46219*	*87836*	*58851*	*22870*	*243566*
	Total Call and Put	*47947*	*78307*	*163287*	*116886*	*49787*	*456214*

* 456 thousand for BIV, Heston and GARCH(1,1) model, and 445 thousand for BHV.

Figures (5.1 and 5.2) show the number of theoretical premiums for call and put options with respect to TTM i MR**.**

Figure. 5.1. The number of theoretical values for call options with respect to TTM i MR ratio**.**
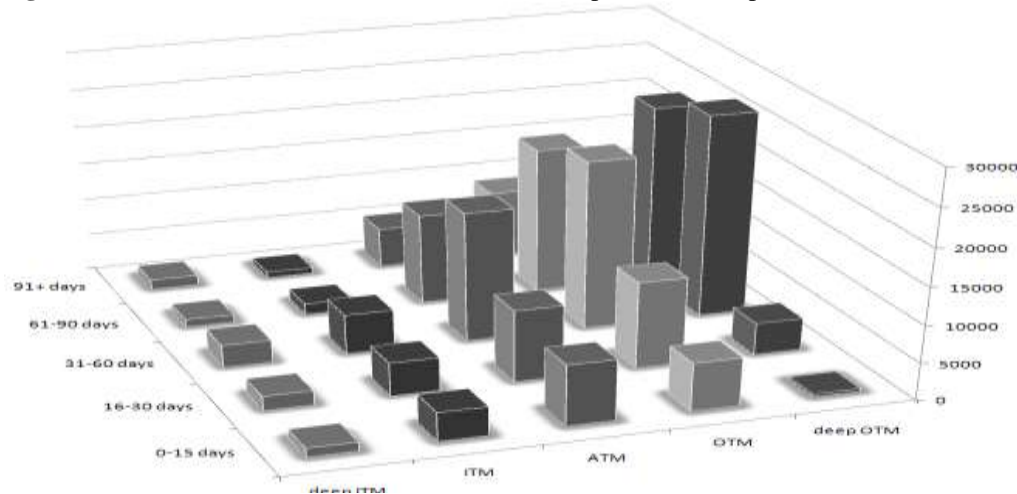

Table 5.1 and figures 5.1 and 5.2 suggest that the activity of market participants (measured by the number of single trades and not by their volume) concentrates on call ATM, OTM and deepOTM and put deepOTM option with TTM between 16 and 90 days. Additionally, we can see that slight differences between these and volume data (figures 5.2 and 5.3, and 4.1 and 4.2) seem to show that - beside larger total volume for ATM and OTM options, especially for call options (figures 4.1 and 4.2) - much greater number of transactions with smaller unit volume is made for deepOTM options.

Figure 5.2. The number of theoretical values for put options with respect to TTM i MR**.**
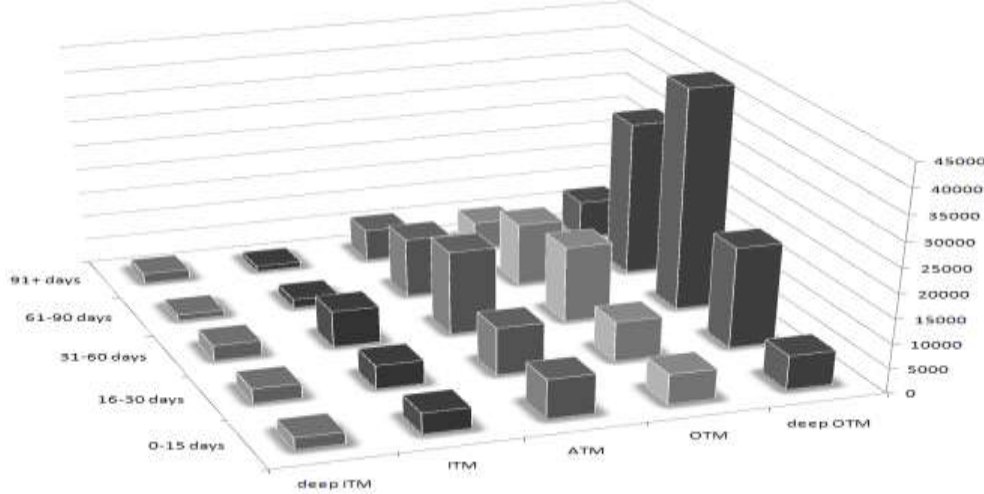
5.2. Error statistics distribution

Major results of our research will be presented in the form of pricing error statistics (OP, RMSE and HMAE) shown separately for six models in the following sequence: 1. BRV5m, 2. BRV5m_63, 3. GARCH(1,1), 4. Heston, 5. BHV and 6. BIV. The discussion of our results is based on two-dimensional charts presented as panels containing five or six boxes where we show error statistics (OP, RMSE or HMAE) for all models, all MR and TTM classes. Each chart is scaled with global minimum and maximum what makes for simple and reliable comparison of presented results. Figures 5.3-5.7 present error statistics for call and put options separately, with individual boxes for different MR, albeit for all TTM and all models in one box. Additionally, we present figures 5.3b, 5.6b, and 5.7b where scale is different and results are not distorted by possible outliers or 'spurious outliers'.

Figure 5.3 - with OP values for call options - indicates that the BIV model (number 6) is the best one. It is characterised by almost the same level of over- and underprediction (the value of OP is approximately equal to 0.5). Results for other models differ. The second best model according to this metric is the Heston model (number 4) and the BHV model may be ranked in the third place. Unfortunately, we can see that results - with the exception of the BIV model - vary strongly with changes in TTM. The worst results are observed for first three models when MR equals ATM, ITM, and deepITM.
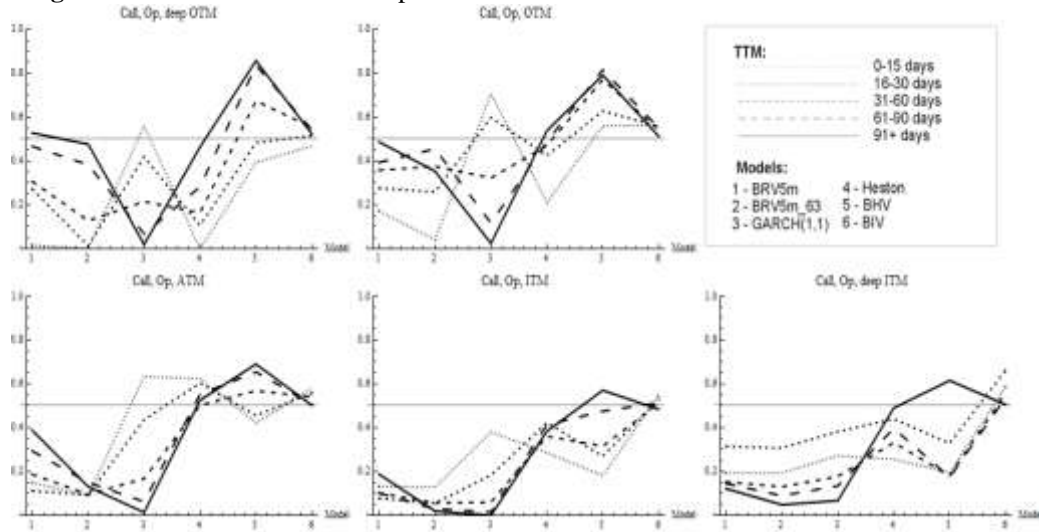
Figure. 5.3. OP statistics for call options

Figure 5.4 presents RMSE statistics for call options. Here again the best results are observed for the BIV model, but we also observe equally good results for all models for TTM equal to 0-30 days. Analysing the results for the remaining values of TTM we observe that results for the Heston model are only slightly worse. BRV5m_63 ranks third. Other models show rather diversified outcomes with respect to TTM and MR. In all cases we notice a substantial increase of RMSE for longer TTM.
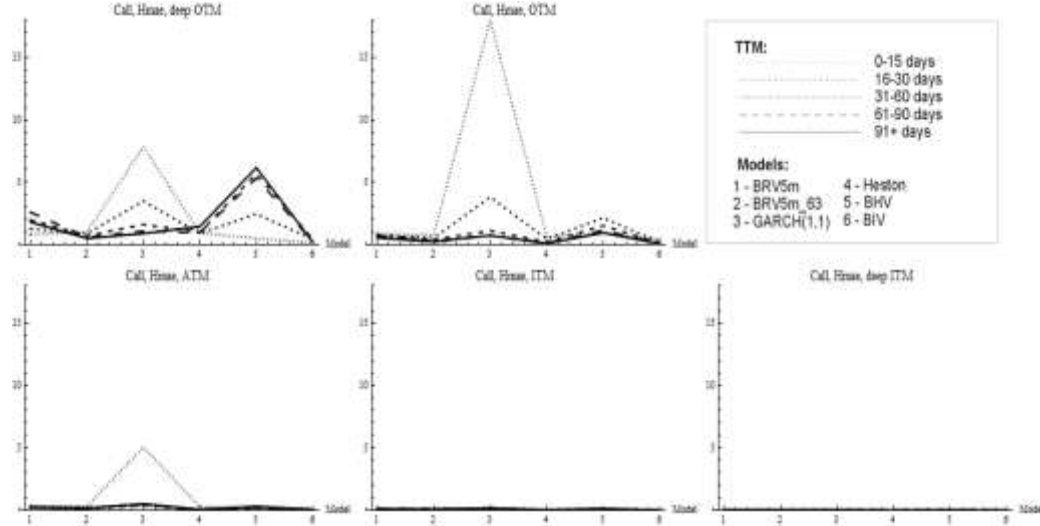
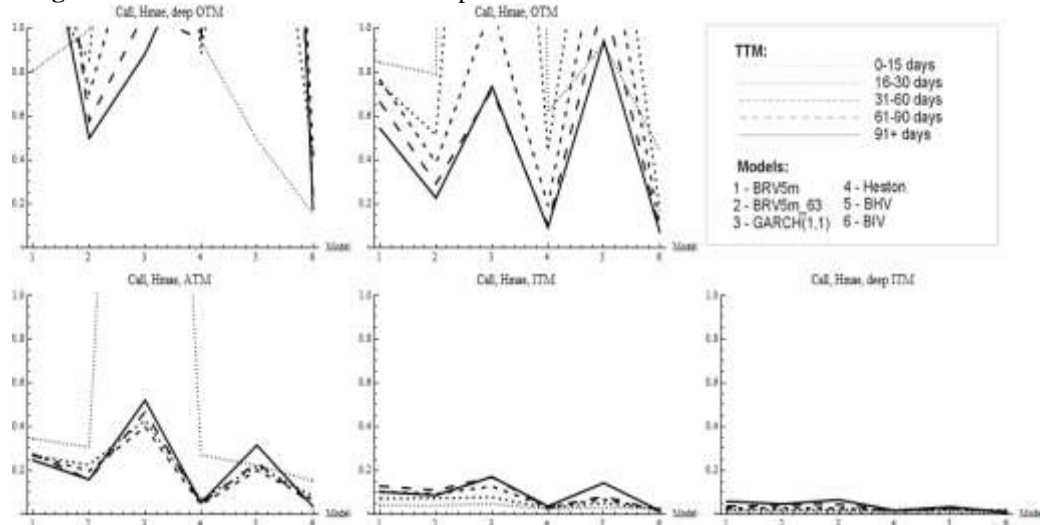**Figure. 5.4.** RMSE statistics for call options
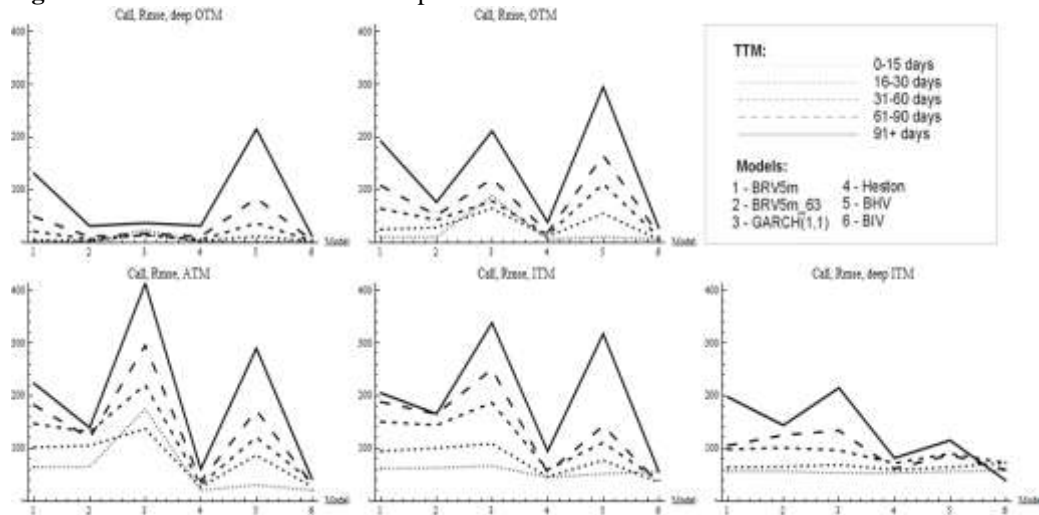
The next figure (Figure 5.5a) – presenting HMAE - informs us about a substantial outlier in our data in the case of the GARCH(1,1) model which distorts results and makes it impossible to interpret them. These charts will be presented once again below in the figure 5.5b, where the scale is transformed with maximum set to 1.0. Then we can observe that differences among the BIV, Heston and BRV5m_63 models (in this order) are not significant. The best performance is

once again observed for the GARCH model. Generally, figure 5.5b shows significant decrease of HMAE when shifting from deepOTM to ATM to deep ITM.

Figure. 5.5a. HMAE statistics for call options

Figure. 5.5b. HMAE statistics for call options

[a] different scale in comparison to Figure 5.5a

Our results for put options are shown in Figures 5.6 - 5.8. Figure 5.6 with OP statistics for put options confirms the ranking of models derived from results for call options. The BIV model is the best one, than the Heston model is the second one and as the third one we have the BHV model. The volatility of results for models other than the BIV with respect to various TTM values is again the very high. We also observe strong underestimation of market prices for all models with the only exception of the BIV model.

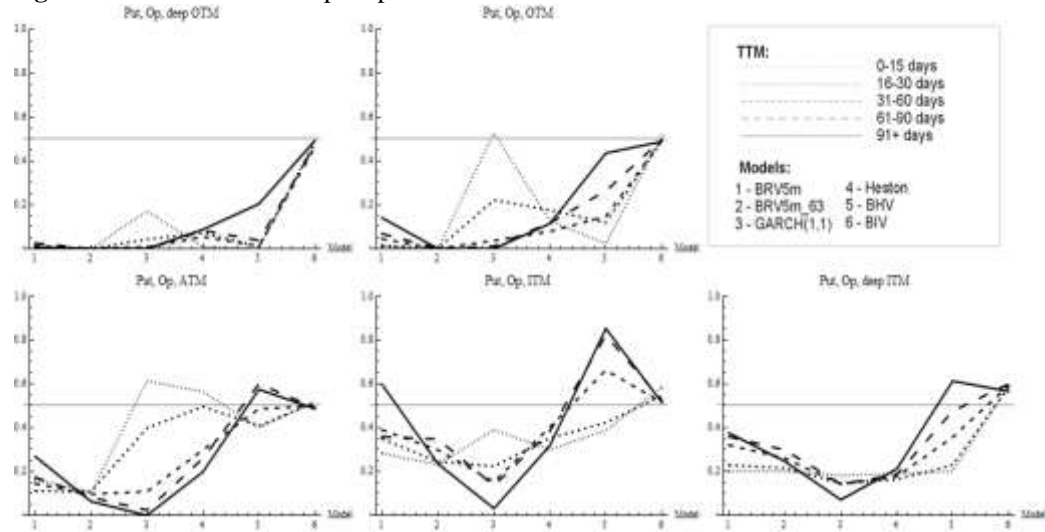**Figure. 5.6.** OP statistics for put options

Figure 5.7 presents RMSE statistics for put options and shows almost the same results as those for call options. The BIV model is again the best one, the Heston model is only slightly worse, and as the third one we have BRV5m_63. Additionally, we do not observe any substantial differences among models with TTM lower than 30 days. Moreover, errors gradually decrease from highest TTM to lowest TTM, but we can assign this 'pattern' to the properties of RMSE, which is an absolute error statistics. Nevertheless, these results confirm the previous ranking.
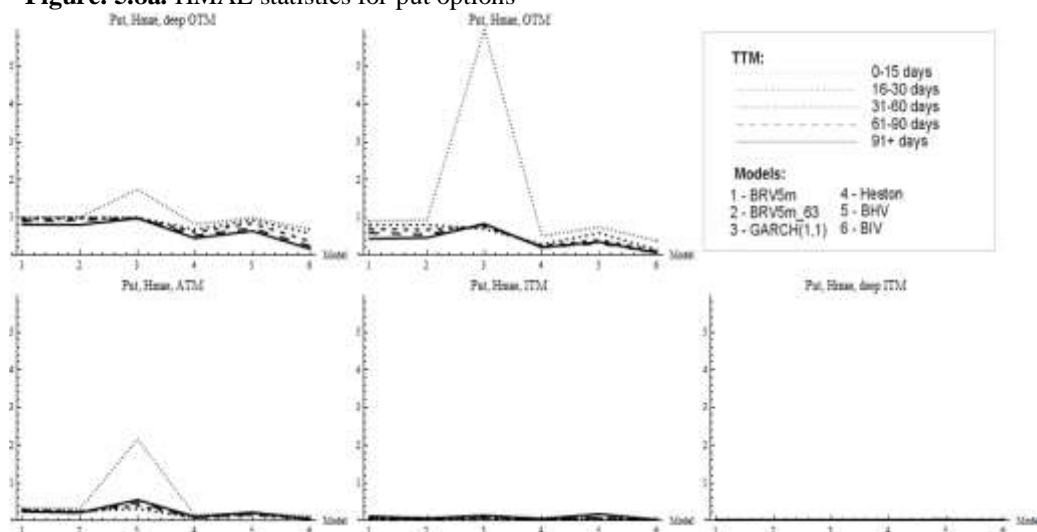
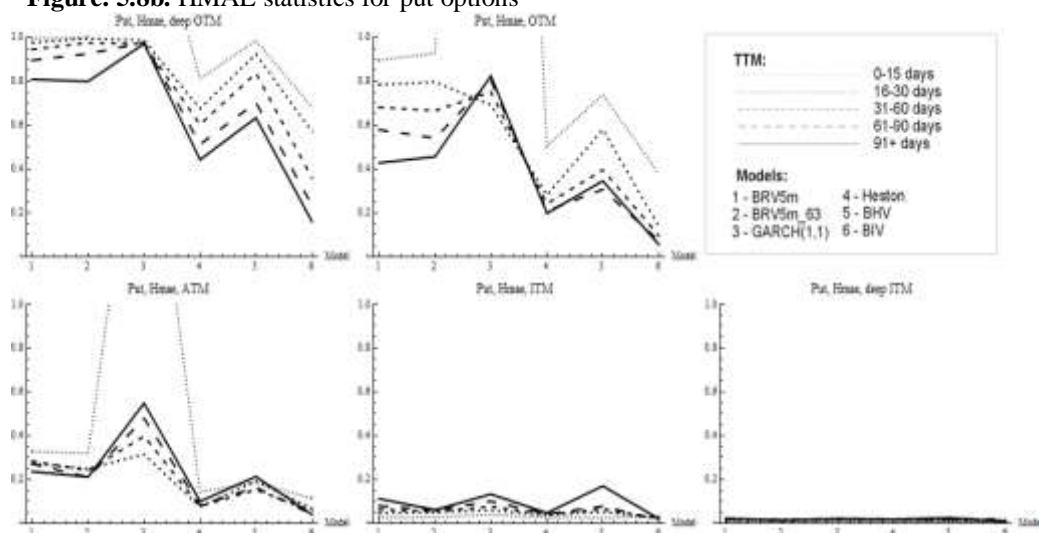**Figure. 5.7.** RMSE statistics for put options
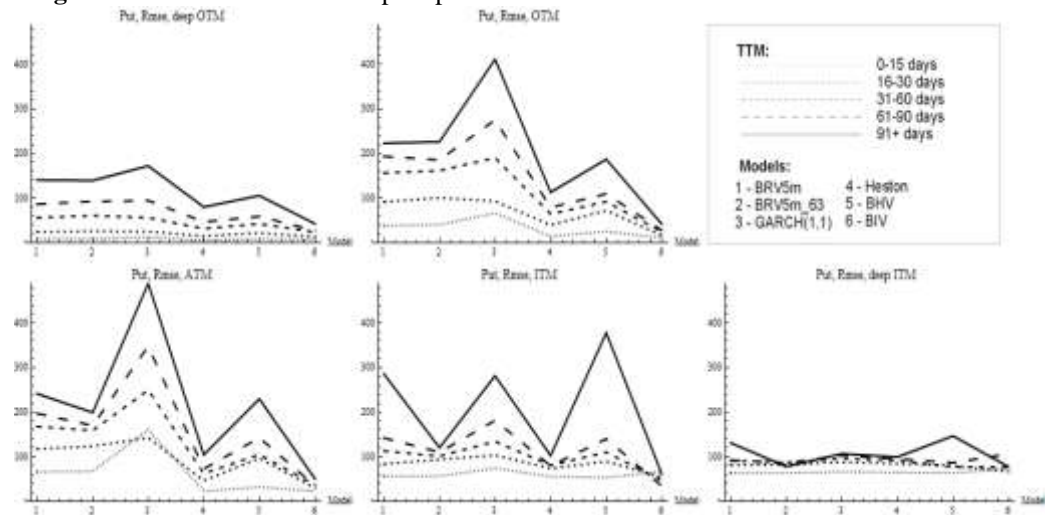
Analysing of the next figure (5.8a) is difficult because of outliers occuring for OTM and ATM for TTM lower than 15 days. Therefore, we present HMAE in charts with transformed scale in Figure 5.8b (with maximum set to 1.0) what enables us to interpret the results. Once again we see that the best models are: BIV, Heston and BRV5m_63 (in this sequence). The worst models are GARCH, BRV5m and BHV.

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Figure. 5.8a. HMAE statistics for put options


Figure. 5.8b. HMAE statistics for put options

[a] different scale in comparison to Figure 5.8a

Figures that follow (Figure 5.9a and 5.9b) present HMAE statistics for put options with a separate model in each box. Each box contains results for a single model for five classes of TTM (TTM-1 = '0-15 days', TTM-2 = '16-30 days', TTM-3 = '31-60 days', TTM-4 = '61-90 days', TTM-5 = '91+days') and five classes of MR (from the dotted line indicating deepOTM to dashed lines indicating OTM, ATM and ITM to the solid line indicating deepITM). Unfortunately, once again the existence of serious outlier in figure 5.9a prevents any meaningful interpretation of results.

Figure. 5.9a. HMAE statistics for put options – different dimension
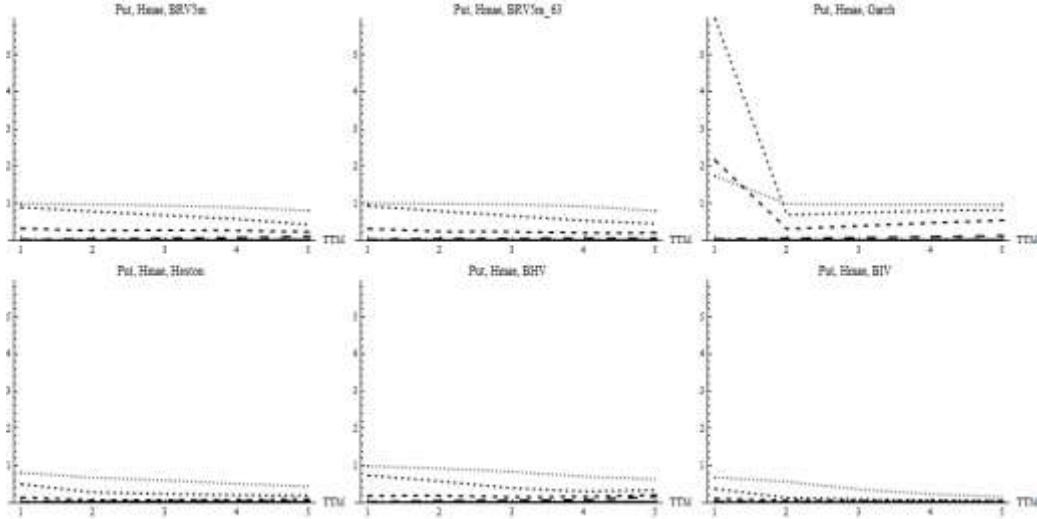
Therefore, we present figure 5.9b with transformed vertical axis scale (maximum set to 1.2) what enables us to visualise the same pattern of valuation as that for the Polish index option market (Kokoszczyński et al. 2010b). We observe the dependence of HMAE values from TTM and MR ratios. However, the dependence of HMAE on TTM is not stable and is conditional on MR, while we observe significant decrease of HMAE with MR going from deepOTM to deepITM. The similar pattern could probably be observed in the case of call options (figure 5.5b) but because of the distortion of data with an outlier for the GARCH model we are not able to show it clearly.

Figure. 5.9b. HMAE statistics for put options – different dimension
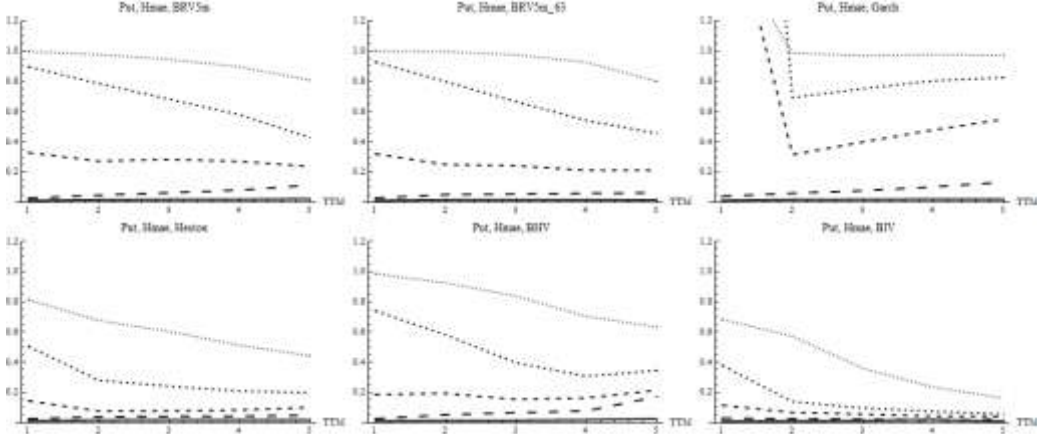


[a] different scale in comparison to Figure 5.9a

Finally, focusing on the issue of outlier identification (Figure 5.5b, 5.8b, and 5.9b) we are sure that their possible exclusion requires special attention, because in many cases outliers totally distort results. Summing up the problem of outlier identification and the motivation for their exclusion we can formulate the following conclusions. Firstly, substantial deviations of error values from their average value are not always signalling a "true" outlier. In some cases the reason thereof is the nature of the model used rather than data themselves (e.g BRV models with

non-averaged RV). We call the former spurious outliers to differentiate them from "true" outliers. Secondly, excluding outliers (including those spurious ones) gives as a result similar patterns in error statistics for call options as those we have for put options. However, in the paper we solve the problems of outliers just by transforming vertical axis, what is enough to make results' interpretation possible.

5.3. Multidimensional comparisons of results.

In this closing subsection we present our conclusions in a more formal way. Figure 5.8 presents the frequency of best pricing for all tested models in 5 diagrams for each moneyness ratio for call and put options together. Our initial conclusions from section 5.2 are confirmed here by this aggregated approach. BIV is clearly the best model, the Heston model is the next one, and the third one is BHV. Additionally, we see that the Heston model seems to behave much better for OTM, and especially for ATM options, while the BIV model behaves in the the worst way for ATM and then for ITM options. Finally, we noticed that the BRV and the GARCH models are the worst models for every MR.
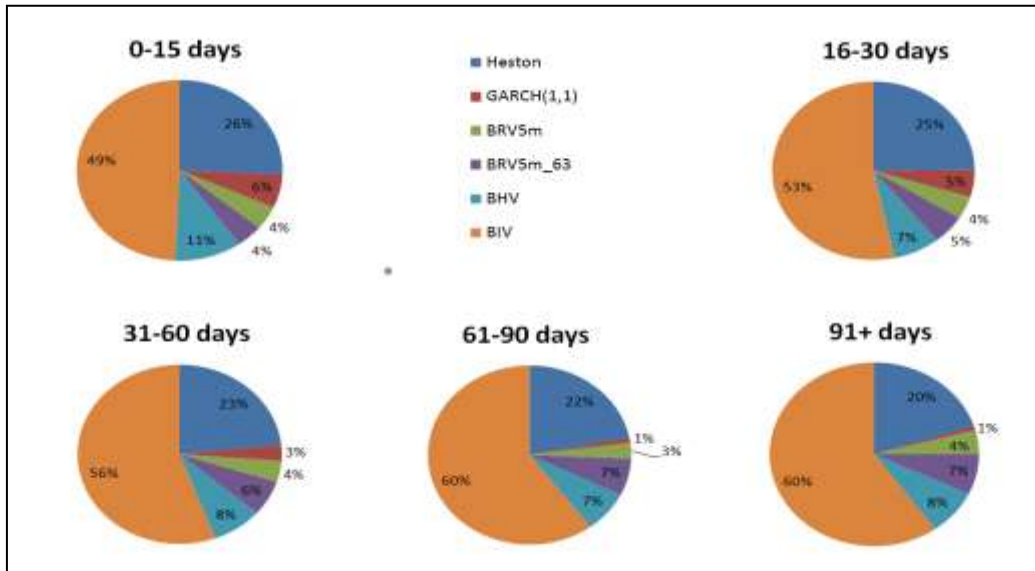
Figure. 5.8. The frequency of the best option pricing for Nikkei225 index options with respect to MR based on HMAE error statistic. [a]
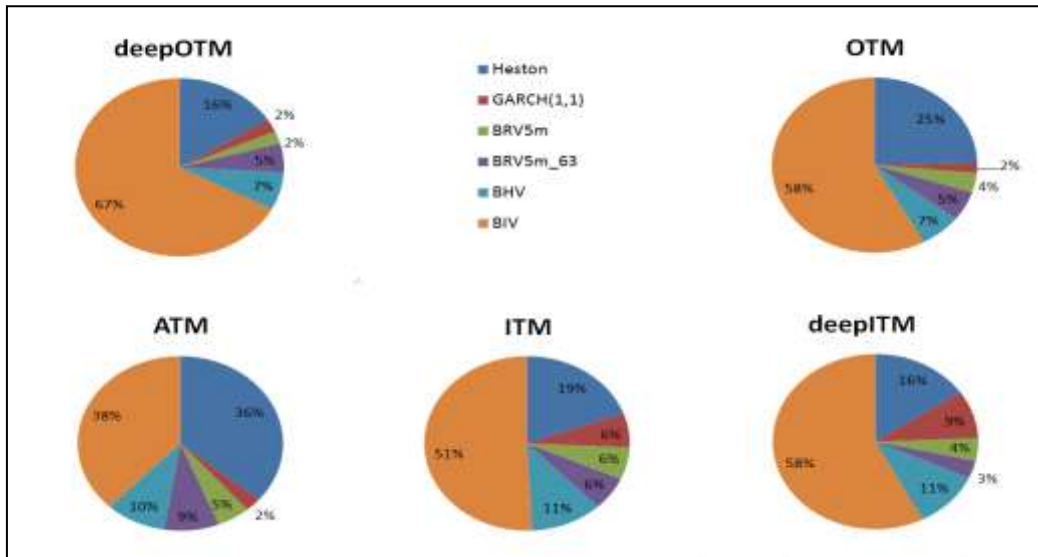
[a] The charts present the data for call and put options together.

Figure 5.9 shows next the frequency of best pricing for all tested models but for each TTM class for call and put together. The BIV model is - as expected - the best one, and the Heston model is ranked as the second one, the BHV model follows. Additionaly, we see that the BIV model gains on efficiency, while the Heston model worsens its performance when we go from the lowest TTM to the highest one. Other models do not change their performance with respect to the TTM class.
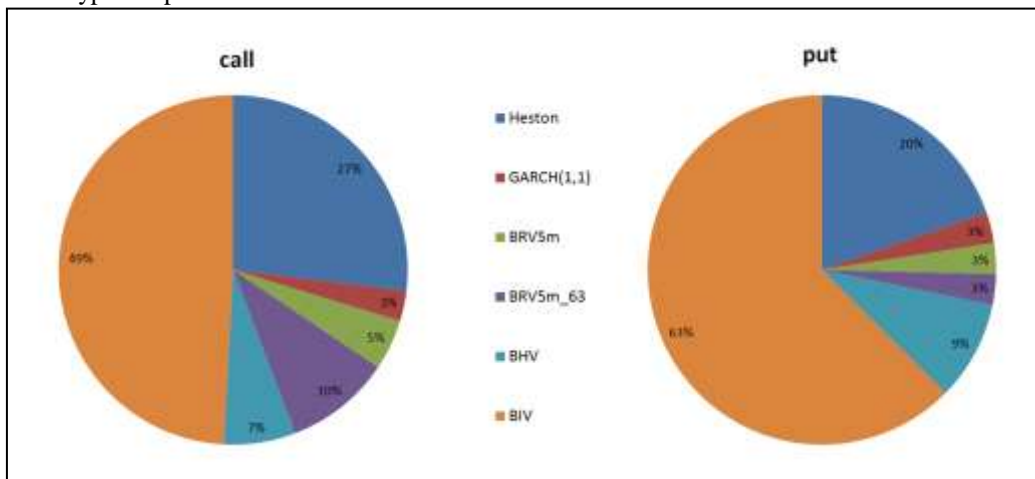
Figure 5.9. The frequency of the best option pricing for Nikkei225 index options with respect to TTM on HMAE error statistic. [a]

The final figure (5.10) that presents the frequency of best pricing for all tested models with respect to the type of options obviously does not change the model ranking. However, we see a very interesting pattern concerning two best models. The BIV model performs much better for put options, while the Heston model is better for call options. Here again we do not observe any significant differences for other models.

Figure. 5.10. The frequency of the best option pricing for Nikkei225 index options with respect to the type of option on HMAE error statistic. [a]

6. Conclusions and further research.

We have presented in this study a detailed analysis of the Japanese Nikkei225 index options basing our research on HF transactional 5-minutes data. We compared 5 different types of option pricing models: the Black model with different assumptions about the volatility process (BRV - two cases, BHV, BIV), the Heston model and GARCH models. Then, we present detailed error statistics describing how efficient in option pricing are the model we test. Furthermore, we focus on the analysis of liquidity for option market in order to better understand different behavior of options within various classes of TTM and MR. Here, we try to summarize our conclusions from this study and we formulate some thoughts concerning further research.

First of all, when we consider the performance of models we have tested, the model ranking, from the most efficient to the least efficient one, is as follows: BIV, Heston, BHV, BRV5m_63, BRV5, and GARCH(1,1). Moreover, the BIV model comes out as the best model when compared to many different types of option valuation models, and additionally for various classes of TTM and MR. Additionally, our results confirm the previous findings for an emerging market, what means that this model ranking is not only a feature of an individual market, but can also be regarded as an observation robust to the level of development, liquidity or various other market characteristics.

Secondly, we observe the clear relation between model error and moneyness ratio (for call and put options): high error values for low moneyness ratios (deepOTM) and the best fit for high moneyness ratios (deepITM). We can explain this pattern by noting that highly valued options (MR equal to ITM or to deepITM) are relatively better priced because of more active participation of market makers and institutional investors in this market segment, where we do not observe strong under- or overreaction to new information as it happens with individual investors. The concentration of liquidity for low-valued options with short TTM we notice can mean high error for options which are traded more frequently. Such error distribution can explain higher interest of speculative investors for OTM and deepOTM options, where information noise, responsible for larger departure of transactional prices from the theoretical ones, is of a greater importance. All these outcomes confirm our previous results for the Polish WIG20 index option market.

Thirdly, focusing on parameter *n* (RV averaging parameter) for BRV models we observe that much lower error values are obtained for n=63 than in the case of non- averaged RV, what confirms our initial hypothesis that the non-averaged RV estimator (Figure 3.3) is rather a poor choice considering the efficiency of option pricing model. This is the confirmation of results presented in the literature on the efficiency and accuracy of various volatility estimators (Ślepaczuk and Zakrzewski 2009).

Fourthly, we would like to focus on two models with the most time-consuming estimation process (the Heston model and GARCH models). Results we have presented earlier make us to doubt whether there is any gain from using them, especially in the case of the GARCH model which comes out as the worst one, when better models are formally much less complicated and additionally less time consuming in the process of estimation.

Analysing liquidity issues we observe several interesting feature of the Japanese index option market data. First of all, the volume of calls and puts concentrates in ATM, OTM and deepOTM options, with hardly any volume noticed for deepITM and ITM options. Secondly, the turnover volume peaks around ATM and ITM options, indicating that the highest (in terms of transaction value) liquidity is observed for ATM options, and then for ITM options. Thirdly, the liquidity – however measured - is significantly higher for put options. Nevertheless, we are aware of the fact that the latter conclusion could result from the sharp downward movement of the market in the time we study and the high demand for put options for hedging purposes.

This final observation shows clearly how important are liquidity issues for patterns we get while comparing performance of various option pricing models. They should be certainly the subject of further studies. Our intention is thus to conduct a similar study for other markets.

There are suggestions in the literature that notwithstanding unrealistic assumptions of the BSM or the Black model they can produce results of the same quality than much more sophisticated model do. Our paper constitutes a strong argument supporting this opinion, because superiority of this model is shown for a great number of various classes of option pricing models.

References

1. Ait-Sahalia, Y., P.A. Mykland, 2009, Estimating Volatility in the Presence of Market Microstructure Noise: A Review of the Theory and Practical Considerations, in: T.G. Andersen, R.A. Davis, J.-P. Kreiss, T. Mikosch (eds.), Handbook of Financial Time Series, Springer, Berlin.
2. Amin, K., R. Jarrow, 1992, Pricing options on risky assets in a stochastic interest rate economy, *Mathematical Finance* 2, 217-237.
3. An, Y., W.Suo, 2009, An Empirical Comparison of Option Pricing Models in Hedging Exotic Options, *Financial Management*, 38, 889-914.
4. Andersen, T.G., P.Frederiksen, A.D.Staal, 2007, The information content of realized volatility forecasts, mimeo.
5. Bates, D., 1995, Testing Option Pricing Models, NBER Working Paper No. 5129.
6. Bates, D.S., 2003, Empirical option pricing: a retrospection, *Journal of Econometrics,* 16, 387-404.
7. Beygelman, R., 2005, Bid-Ask Spreads and Asymmetry of Option Prices, Goethe University, Frankfurt, mimeo.
8. Black F., 1976, The pricing of commodity contracts, *Journal of Financial Economics,* 3, 167-179.
9. Black, F., Scholes, M., 1973, The pricing of options and corporate liabilities, *Journal of Political Economy*, 81, 637-659.
10. Bollerslev, T., R. Chou, K. Kroner, 1992, ARCH modelling in finance: A review of the theory and empirical evidence. *Journal of Econometrics,* 52, 5-59.
11. Bollerslev, T., R. Engle, D. Nelson, 1994, ARCH model; w: R. Engle, D. McFadden (eds.), Handbook of Econometrics, Vol. IV, Elsevier, Amsterdam
12. Bollerslev*, T., R.F.* Engle*, J.M.* Wooldridge*,* 1988*,* A Capital Asset Pricing Model with Time Varying Covariances, *Journal of Political Economy,* 96, 116-131,
13. Brandt, M.W., T. Wu, 2002, Cross-sectional tests of deterministic volatility functions, *Journal of Empirical Finance,* 9, 525-550.
14. Broadie, M., J.B. Detemple, 2004, Option Pricing: Valuation Models and Applications, *Management Science*, 50, 1145-1177.
15. Campbell, J., L. Hentschel, 1992, No news is good news: An asymmetric model of changing volatility in stock returns, *Journal of Financial Economics*, 31, 281-318.
16. Christoffersen, P., K.Jacobs, 2004, The Importance of the Loss Function in Option Valuation, *Journal of Financial Economics*, 6, 213-234.
17. Corrado, Charles and Su Tie, 1996, Skewness and kurtosis in S&P 500 index returns implied by option prices, *Journal of Financial Research,* 19, 175-192.
18. Dennis, P., S. Mayhew, 2009, Microstructural biases in empirical tests of option pricing models, *Review of Derivatives Research*, 12, 169-191.
19. Derman, E., I. Kani, 1994, Riding on a smile*, RISK* 7, 32-39.
20. Duan, J.-C., 1995, The GARCH option pricing model, *Mathematical Finance*, 5, 13-32.
21. Duan, J.-C., P. Ritchken, Z. Sun, 2004, Jump starting GARCH: Pricing and hedging options with jumps in returns and volatilities, manuscript, University of Toronto,
22. Dupire, B., 1994, Pricing with a smile, RISK 7, 18-20.
23. Ferreira, E., M.Gago, A.Leon, G. Rubio, 2005, An empirical comparison of the performance of alternative option pricing model, *Investigaciones Economicas,* 29, 483-523.
24. Fiszeder, P., 2008, Pricing the WIG20 Index Options Using GARCH Models, referat wygłoszony na konferencji *Forecasting Financial Markets and Economic Decision-making*, Łódź, 14-17 maja 2008.
25. French, K., G. W. Schwert, R. Stambaugh, 1987, Expected stock returns and volatility. *Journal of Financial Economics*, 19, 3-30.

26. Fung, J.K.W., H.M.K. Mok, 2001, Index Options-Futures Arbitrage: A Comparative Study with Bid-Ask and Transaction Data, BRC Papers on Financial Derivatives and Investment Strategies, Hong Kong Baptist University.

27. Garcia, R., E. Renault, 1998, A note on hedging in ARCH and stochastic volatility option pricing models, *Mathematical Finance*, 8, 153-161.

28. Garcia, R., E.Ghysels, E.Renault, 2010, The econometrics of option pricing, in: Y.Ait-Sahalia, L.Hansen (eds.), Handbook of financial econometrics, North Holland, Oxford and Amsterdam.

29. Glosten, L., R. Jagannathan, D. Runkle, 1993, On the relation between the expected value and the volatility of the nominal excess return on stocks, *Journal of Finance*, 8, 1779-1801.

30. Han, C., 2008, The Comparisons between Three Option Pricing Models and Black-Scholes Formula in Pricing Performance and Trading Strategy: Evidence from the FTSE 100 Options, Master Thesis, National Chung Cheng University.

31. Hansen, P., and Lunde, A., 2004, A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH(1,1) Model? Journal of Applied Econometrics, 20,873-889.

32. Heston, S.L., 1993, A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, *Review of Financial Studies,* 6, 327–343.

33. Hull, J., A. White, 1987, The Pricing of Options with Stochastic Volatilities, *Journal of Finance,* 42, 281-300.

34. Iaquinta, G., 2007, The analysis of the perpetual option markets: Theory and evidence, Bergamo University, unpublished Ph.D. thesis.

35. Jarrow, R., A. Rudd, 1982, Approximate option valuation for arbitrary stochastic processes, *Journal of Financial Economics,* 10, 347-369.

36. Kanoh, S., A. Takeuchi, 2006, An Analysis of Option Pricing in the Japanese Market, *Discussion Paper Series No. 145*, Hitotsubashi University.

37. Kokoszczyński, R., N. Nehrebecka, P. Sakowski, P. Strawiński, R. Ślepaczuk, 2010a, Option Pricing Models with HF Data – a Comparative Study. The Properties of the Black Model with Different Volatility Measures, University of Warsaw, University of Warsaw, Faculty of Economic Sciences,Working Papers 3/2010 (26).

38. Kokoszczyński, R., P. Sakowski, R. Ślepaczuk, 2010b, Midquotes or Transactional Data? The Comparison of Black Model on HF Data, University of Warsaw, University of Warsaw, Faculty of Economic Sciences,Working Papers 15/2010 (38).

39. Li, S., 2006, The Arbitrage Efficiency of the Nikkei 225 Options Market: A Put-Call Parity Analysis, *Monetary and Economic Studies (Bank of Japan)*, November, 33-54.

40. Lim, Guay, Gael Martin and Vance Martin, 2005, Parametric pricing of higher order moments in S&P options, *Journal of Applied Econometrics,* 20, 377-404.

41. Maheu, J., T. McCurdy, 2004, News arrival, jump dynamics and volatility components for individual stock returns, *Journal of Finance*, 59, 755-779.

42. Mitra, S., 2009, A Review of Volatility and Option Pricing, *arXiv:0904.1292v1*.

43. Mitsui, H., K. Satoyoshi, 2010, Empirical Study of Nikkei 225 Option with Markov Switching GARCH Model, *Asia-Pacific Financial Markets,* On-Line First.

44. Mixon, S., 2009, Option markets and implied volatility: Past versus present, *Journal of Financial Economics,* 94, 171-191.

45. Pagan, A., G. W. Schwert, 1990, Alternative models for conditional stock volatility, *Journal of Econometrics*, 45, 267-290.

46. Raj, M., D.C. Thurston, 1998, Transactions data examination of the effectiveness of the Black model for pricing options on Nikkei index futures, *Journal of Financial and Strategic Decisions*, 11, 37-45.

47. Ritchken, P., R. Trevor, 1999, Pricing options under generalized GARCH and stochastic volatility processes, *Journal of Finance*, 54, 377-402.

48. Rubinstein, M., 1985, Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes from August 23, 1976 through August 31, 1978, *Journal of Finance*, 40, 455-480.

49. Rubinstein, Mark, 1998, Edgeworth binominal trees, *Journal of Derivatives,* 5, 20-27.

50. Sakowski P., 2010, Modelowanie zmienności w wycenie opcji indeksowych na danych wysokiej częstotliwości. Analiza porównawcza (Volatility modeling for index option pricing with high-frequency data), PhD thesis, University of Warsaw, Faculty of Economic Sciences.

51. Stein, E., J. Stein, 1991, Stock Price Distributions with Stochastic Volatility: An Analytic Approach, *Review of Financial Studies*, IV, 727-752.

52. Ślepaczuk R., Zakrzewski G., 2009, High-frequency and model-free volatility estimators, University of Warsaw, Faculty of Economic Sciences, Working Papers 13/2009 (23).

53. Tsiaras, L., 2009, The Forecast Performance of Competing Implied Volatility Measures: The Case of Individual Stocks, Aarhus University, mimeo.

54. Yao, J., Y. Li, C.L. Tan, 2000, Option price forecasting using neural networks, *Omega*, 28, 455-466.

55. Wei, J.Z., 1995, Empirical Tests of the Pricing of Nikkei Put Warrants, *The Financial Review*, 30, 211-241.

56. Zivot E., 2008, Practical issues in the analysis of univariate GARCH models, University of Washington working papers: http://faculty.washington.edu/ezivot/research/practicalgarchfinal.pdf.