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MACROECONOMIC CONSEQUENCES OF THE DEMOGRAPHIC AND EDUCATIONAL TRANSITION IN POLAND

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Abstract

Soon after the start of the transition to market economy in the early 1990s, Poland has experi- enced both a dramatic decline in the fertility rate and an increase in the share of students among young high-school graduates. These two processes significantly changed the age structure of the population and average income characteristics of households. Using a general equilibrium model with heterogeneous households and uninsured income shocks I try to assess the impact of these demographic and educational changes on the Polish economic performance and inequalities. I find that in the long term the positive effects of educational transition on output per capita more than offset the negative impact of lower fertility, but the outcome strongly depends on the adjustments in the structure of labor demand. I also show that the educational transition increases income and consumption inequalities, while the demographic transition decreases inequality in assets.

Keywords:

population aging, educational transition, inequalities, models with heterogeneous agents

JEL:

J11, D31, I24, D58, J26

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1 Introduction

The early 1990s was a remarkable period for the Polish economy. The transformation to market economy started, substantial reforms were introduced and the real convergence process began. That period was also marked by dynamic demographic and social changes, which significantly changed the age structure of the population and basic individual characteristics, leaving long lasting consequences.

One of these processes is a dramatic fall in the birth rate, that I depict in Figure 1. Indeed, except for the surge observed in the mid-1980s related with echo boomers, the total fertility rate was generally stable between 1970 and 1985 and stood on average at 2.2, i.e. above the replacement level. Since then, it deteriorated dramatically and from 2010 has stabilized at around 1.3. According to the CA World Factbook, Poland has currently one of the lowest fertility rates in the world and the third lowest in Europe. If this trend is continued and no other processes, such as intensified immigration, occur, the population in Poland is expected to shrink at a high pace. A decline in fertility has a strong impact on the age structure of the population and, next to rising longevity, is a primary determinant of population aging.

There is no doubt that population aging is a pressing issue and its economic consequences are expected to be significant. First of all, an increase in the old-age dependency ratio (the ratio of dependents to the working-age population) negatively affects the public finance, raising concerns about fiscal sustainability and stability of the pension system. Second, as individual household characteristics, such as productivity or propensity to save, vary over the life-cycle, changes in the age structure of population affect many macroeconomic variables, including aggregate output, consumption, and domestic assets. As population aging also influences the level of public debt and the real interest rate, it is important for the monetary policy decisions. Finally, demographic transition has redistributive consequences, i.e. it affects income, consumption and assets inequalities.

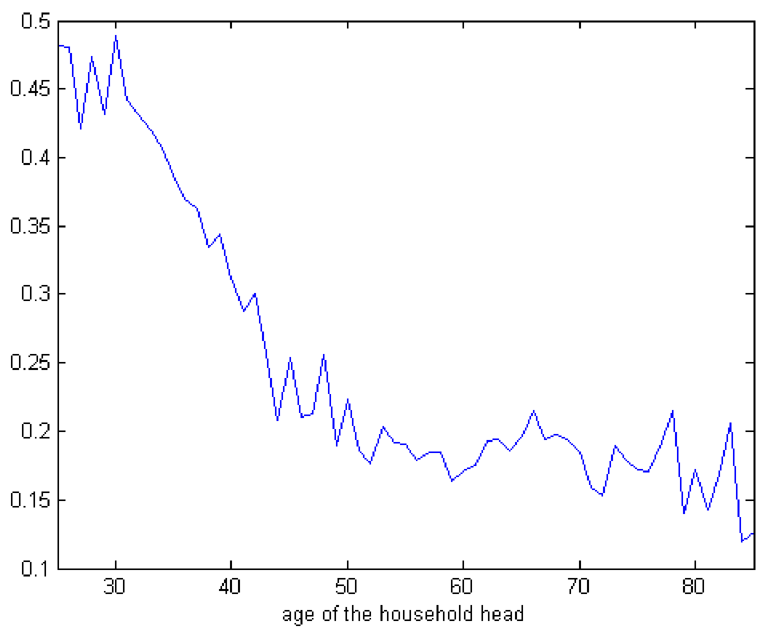
Hopefully, the expected negative impact of population aging, and in particular of lower fertility, is mitigated by a higher productivity of Polish workers due to an increase in educational attainment. Indeed, another important process that started in the early 1990s involves a rising share of students among young high-school graduates. According to the Polish Households Budget Survey (HBS) from 2013, only less than 20 percent of all households' heads aged 50 years or older have an academic degree, while for those aged between 25 and 30 the corresponding statistics equals slightly less than 50 percent (Figure 2).[1] Since a worker's individual productivity is strongly linked to his/her educational attainment level, one can

[1] Similarly, according to the World Bank Database, the ratio of those who at least completed short-cycle tertiary education among the population aged 25 and older increased almost twice between 2002 and 2014, i.e. from 12% to 24%.

expect positive effects of this process on the Polish economic performance. It should also impact the distribution of income and/or consumption.

Figure 1: Total fertility rate in Poland

Figure 2: The share of households which household head has an academic degree.



Note: Based on the data from the Polish Household Budget Survey 2013. Post secondary education is included.

The main objective of this paper is to examine the macroeconomic consequences of lower fertility (demographic transition) and higher educational attainment (educational transition) in Poland, with a particular emphasis on their effects on inequalities. In order to explore the relative importance of a fall in the birthrate and rising longevity, i.e. two major determinants of population aging, the study is also extended to the long term effects of an increase in the life expectancy.

To this end, I develop a general equilibrium model with heterogeneous agents and incomplete financial markets. In the model, there is a large number of households in different age-cohorts. As in Gourinchas and Parker (2002), their deterministic productivity profiles differ between educational groups. I also introduce the education-dependent individual income processes. The life-span is stochastic and, similar to Hubbard and Judd (1987), survival probabilities depend on age. During their working life households experience uninsured, idiosyncratic shocks, as in Aiyagari (1994). The model is cast in a small open economy setup, with the domestic interest rate reacting to changes in foreign debt.

The model is calibrated to the Polish economy and replicates the key specific features of the life-cycle characteristics of Polish households, which are obtained from the household level data from the Polish HBS. The model-based simulations allow me to assess both the long term effects of the demographic and educational changes and transitional dynamics.

There is considerable literature that analyzes the macroeconomic and welfare effects of population aging in the US economy, some of which uses multi-country models (Krueger and Ludwig, 2007; De Nardi, İmrohoroglu, and Sargent, 1999; Storesletten, 2000; Ludwig, Schelkle, and Vogel, 2012). Nevertheless, the US has not recently experienced such dramatic changes in the fertility rate as Poland. Therefore, the main focus of these studies is rather on the effects of rising longevity.

There are also several papers that deal with pension system stability in Poland using classical OLG models, i.a. with no intra-cohort heterogeneity (see, among others, Hagemejer, Makarski, and Tyrowicz, 2015). According to my knowledge, population aging in Poland has not been analyzed so far using heterogeneous agents models with individual productivity shocks and, therefore, there is no sufficiently detailed analysis of its impact on inequalities. This paper fills this gap by offering a general equilibrium study that discusses the relationship between population aging and inequalities, i.e. the topic that has been relatively less explored in the literature (one of the exceptions is Krueger and Ludwig, 2007). Additionally, although a decline in fertility and an increase in educational attainment often occur simultaneously, and some studies point to the causal relationship between them (Snopkowski, Towner, Shenk, and Colleran, 2016; Diebolt, Menard, and Perrin, 2016), research that quantifies their joint macroeconomic effects is very scarce.[2] In this study, the same model is used to examine the

[2]The exceptions are Díaz-Giménez and Díaz-Saavedra (2009) and Catalano and Pezzolla (2016). Never-

impact of changes in education, a decrease in the fertility rate, and the total impact of both processes, which ensures comparability of the results for each of these scenarios.

The main findings of this paper are the following. First, in the long term, the total effect of lower fertility and the educational change on output per capita will be positive, but the outcome strongly depends on the adjustments in the structure of the labor demand. Second, a drop in fertility has stronger negative consequences for Polish output per capita compared to rising longevity. In contrast, the latter is relatively more important for the long term response of the real interest rate and domestic assets. Overall, due to population aging, the Polish real interest rate should decline, although in the short term the opposite dynamics might be observed. The significant rise in the contribution rate would also be required in order to keep the pension system balanced. Finally, the educational change positively impacts income and consumption inequalities, while the demographic change decreases the inequality of assets.

The rest of the paper is organized as follows. Section 2 presents the model and definitions of equilibrium.. Section 3 is devoted to calibration issues and describes the algorithms used to find the model solution. In Section 4 I present the baseline results, i.e. the new steady states and transitional dynamics. In Section 5 I relax/modify some modeling assumptions and calculate the effects of rising longevity, imperfect substitution between workers, different interest rate rules and pension system design. Section 5 summarizes the main findings and discusses resulting policy challenges.

2 Model

To analyze the demographic and educational transition, I use a small open economy general equilibrium model. The model economy is populated by heterogeneous households, who differ in age, educational attainment level, wealth, and productivity. Households face mortality risk and are hit by uninsured individual income shocks. They smooth consumption by accumulating assets. These are claims on physical capital and foreign assets. Households obligatorily participate in a pay-as-you-go pension system, that is balanced via adjustments in the contribution rate. Consistently with the small open economy assumption, the real interest rate is mainly exogenous with an endogenous component responding to the economy's foreign debt.

theless, the first one is fully devoted to the pension system reform in Spain and does not discuss changes in inequalities. In the latter, there are no individual income shocks, which does not allow for a detailed analysis of inequality.

2.1 Households

The model economy is inhabited by a continuum of households. Households either have or do not have an academic degree ($h = 1$ or $h = 0$), and their level of education is exogenous and predetermined. The less educated individuals enter the model at the age of 20 ($j = 20$), the more educated at the age of 25 ($j = 25$), both with no financial assets. The share of individuals with an academic degree might vary between cohorts and for a cohort aged 25 it is denoted by p_t^h. All households work until the age of 65 ($j = 65$), and then they retire and live up to the maximum age of 85. The conditional survival probability (s_j) depends on age so that an individual lives at least up to the age of m with probability $S_m = \Pi_{i=20}^m s_i$. Each period a new cohort aged 20 is born and the size of this cohort is $1 + n_t$ times the mass of the people born in the previous period.

Labor income Each period working-age households receive an endowment of efficiency labor units described by the following formula:

$$z(h, j, e_h) = \bar{z}(h, j) * e_h. \tag{1}$$

and hence it consists of two components. The first one $\bar{z}(h, j)$ is deterministic and depends on age j and the level of education h, so that more educated households are on average more productive within their age-group. The second component e_h is stochastic and follows a Markov process. Thus, it is independent and identically distributed across individuals with the same educational attainment level h. The conditional probability matrix takes a form $\pi_{kl}^h = P(e_h = e_h^l | e_h = e_h^k)$, where $\pi_{kl}^h \geq 0$ and $\sum_{l=1}^N \pi_{kl}^h = 1$ for each $k, l = 1, 2 \ldots, N$ and $e_h^k, e_h^l \in E_h = \{e_h^1, e_h^2, \ldots, e_h^N\}$.

There is no endogenous labor supply in the model. Nevertheless, this assumption is in line with the characteristics of the Polish labor market. According to the Polish Labor Force Survey, in the second quarter of 2016, only 7% of employees were part-time workers, form which around half would prefer to work full-time if possible. It is also consistent with evidence on the discrepancy between the worker's actual and desired number of working hours (see Wyszyński, 2016).

Pension payments received by households are a fraction of wages earned just before retirement, i.e. at the age of 65.

As a result, total net labor income at time t is summarized by the formula:

$$y_t(h, j, e_h) = \begin{cases} (1 - \tau_t) * w_t * z(h, j, e_h) & \text{for } j \leq 65, \\ (1 - \tau_t) * w_t * \theta * \bar{z}(h, j = 65) * e_h(j = 65) & \text{for } j > 65, \end{cases}$$

where w_t is the wage rate per efficiency unit of labor, τ_t stands for the contribution rate, and θ is the replacement rate.

The time t share of households with age j, educational attainment level h and hit by productivity shock e_h^l is denoted by $\mu_t(h, j, e_h = e_h^l)$. Aggregates by certain characteristics are defined by dropping them from the bracket.

Preferences Utility from consumption c takes a form

$$u(c) = \log(c).$$

Thus, the risk aversion parameter $\sigma = -cu''(c)/u'(c)$ equals one, i.e. the value recommended by Chetty (2006).

Budget constraint During their lifetime households accumulate assets denoted by $a_t(h, j, e_h)$. The corresponding rate of return equals r_t. Each period and for each household the following budget constraint is satisfied:

$$(1 + r_t) * a_t(h, j - 1, e_h) + y_t(h, j, e_h) + \mathrm{tr}_t = a_{t+1}(h, j, e_h) + c_t(h, j, e_h), \tag{2}$$

where tr_t stands for accidental bequests, assumed to be equally distributed over surviving households.

Household decision problem The household's maximization problem in time i is the following:

$$\max E_t \sum_{i=t}^{\infty} \beta^{i-t} S_{j+i-t} u(c_i(h, j + i - t, e_h)), \tag{3}$$

subject to (2) and to $c_i(h, j + i - t, e_h) > 0$ for all $i \geq t$, where the future realizations of $w_i, r_i, \tau_i, \mathrm{tr}_i$ for $i = t, t + 1, \ldots$ are known.

Alternatively, this problem can be expressed recursively. Let a' denote household's assets in time $t + 1$, c its consumption in time t, and $x = (h, j, e_h, a) \in H \times J \times E_h \times (0, \infty)$ a vector of its characteristics. The Bellman equation is

$$V(x; t) = \max_{c,a'} \{u(c) + \beta s_j EV(x'; t + 1|x; t),$$

subject to (2) and $c > 0$, and with the corresponding optimal policy functions $c(x; t)$ and $a'(x; t)$.

2.2 Government

The role of the government is reduced to collect contributions, which finance pension expenditures, and to redistribute accidental bequests. The contribution rate balancing the pension system satisfies

$$(1-\tau_t) * \sum_{j>j0} \sum_{h \in H} \sum_{l=1}^{N} pen(h, e_h(j=j0)=e_h^l) * \mu_t(h, j, e_h(j=j0)=e_h^l) = \tag{4}$$

$$= \tau_t \sum_{j \leq j0} \sum_{h \in H} \sum_{l=1}^{N} z(h, j, e_h = e_h^l) * \mu_t(h, j, e_h = e_h^l).$$

2.3 Firms

Firms are competitive in the product and factor markets. They are identical and produce one final homogeneous good with constant returns to scale and according to the Cobb-Douglas technology. Thus, the aggregate production function Y_t is:

$$Y_t = K_t^{\alpha}(G_t L_t)^{1-\alpha}, \tag{5}$$

where G_t stands for aggregate productivity, which increases at a constant annual pace g, K_t is aggregate capital and L_t denotes aggregate labor input. Capital depreciates at a constant rate δ and factor prices equal their marginal products:

$$\partial Y_t / \partial L_t = w_t \quad \text{and} \quad \partial Y_t / \partial K_t = r_t + \delta. \tag{6}$$

2.4 Interest rate rule

Following Schmitt-Grohé and Uribe (2003), the domestic real interest rate equals world interest rate (r^*) plus a risk premium. The latter reacts to changes in the foreign debt in a way described in the formula:

$$r_t = r^* + \phi * (\exp(\frac{K_t - A_t}{Y_t}) - 1), \tag{7}$$

where A_t stands for aggregate households assets.

2.5 Equilibrium

In the next subsections, I will present two definitions of equilibrium: a general competitive equilibrium and its special case - stationary equilibrium. Yet, before that, the following transformation is performed: $c_t(h, j, e_h, a) \leftarrow c(h, j, e_h, a) * (1 + g)^{20-j}$, $Y_t \leftarrow Y_t/G_t$, $K_t \leftarrow K_t/G_t$, $w_t \leftarrow w_t/G_t$. As a result,C_t, A_t and tr_t become stationary.

2.5.1 Competitive equilibrium

Let $\mu_t(h, j, e_{h,}a)$ be a probability measure in time t defined on $X = H \times J \times E_h \times [0, \infty)$. Then, given the initial condition μ_0 and $a'_0(h, j, e_h, a)$, a **competitive equilibrium** for the model economy are sets of household's policy functions $\{c_t(h, j, e_h, a), a'_t(h, j, e_h, a)\}_{t=1}^{\infty}$, probability measures $\{\mu_t\}_{t=1}^{\infty}$, a vector of factor prices $\{w_t, r_t\}_{t=1}^{\infty}$, a vector of contribution rates and accidental bequests $\{\tau_t, \mathrm{tr}_t\}_{t=1}^{\infty}$, a vector of macroeconomic aggregates $\{K_t, L_t\}_{t=1}^{\infty}$, and a function Q_t governing the changes in household distribution over the X, such that the following conditions holds:

1. The values of aggregate variables result from households' individual choices:

$$L_t = \int z(h, j, e_h) d\mu_t,$$

$$A_t = \int s_j a'_{t-1}(h, j, e_h, a) d\mu_{t-1},$$

$$C_t = \int c_t(h, j, e_h, a) d\mu_t,$$

$$\mathrm{tr}_t = \int (1 - s_j) a'_{t-1}(h, j, e_h, a) d\mu_{t-1}.$$

2. The contribution rate τ_t satisfies (4).

3. Factor prices are equal to their marginal products:

$$\partial(K_t^\alpha L_t^{1-\alpha})/\partial L_t = w_t \quad \text{and} \quad \partial(K_t^\alpha L_t^{1-\alpha})/\partial K_t = r_t + \delta. \tag{8}$$

4. Domestic real interest rate r_t satisfies (7).

5. Given the vectors $\{w_t, r_t\}_{t=1}^{\infty}$ and $\{\tau_t, \mathrm{tr}_t\}_{t=1}^{\infty}$, households find the optimal policy functions according to (3).

6. Aggregate budget constraint holds

$$K_t^\alpha L_t^{1-\alpha} - \delta K_t + r_t(A_t - K_t) + \mathrm{tr}_t = C_t + (A_{t+1} - A_t) + \mathrm{tr}_{t+1}.$$

7. Population distribution changes according to the rule:

$$\mu_{t+1} = \int_X Q_t d\mu_t.$$

The function Q, defined on the four-dimensional set X, is determined by distributions over educational level, age, productivity shocks and assets. It also depends on an exogenous vector $\{p_t^h, n_t\}_{t=1}^{\infty}$, and survival probabilities $s_j,\ j \in J$.

2.5.2 Stationary equilibrium

If all aggregate variables in the model are constant over time, we can speak of stationary equilibrium. The formal definition is presented below. Assume that $p_t^h = p^h$ and $n_t = n$ for all $t = 0, 1, \ldots$. Let $\mu(h, j, e_{h,} a)$ be a probability measure defined on $X = H \times J \times E_h \times [0, \infty)$. A **stationary competitive equilibrium** for the model economy are sets of household's policy functions $c(h, j, e_h, a)$ and $a'(h, j, e_h, a)$, probability measure μ, factor prices $(w,\ r)$, contribution rate and the value of accidental bequests $(\tau,\ \mathrm{tr})$, macroeconomic aggregates $(K,\ L)$, and a function Q, such that the following conditions holds:

1. The values of aggregate variables result from households' individual choices:

$$L = \int z(h, j, e_h) d\mu,$$

$$A = \int s_j a'(h, j, e_h, a) d\mu,$$

$$C = \int c(h, j, e_h, a) d\mu,$$

$$\mathrm{tr} = \int (1 - s_j) a'(h, j, e_h, a) d\mu.$$

2. The contribution rate τ satisfies (4).

3. Factor prices are equal to their marginal products:

$$\partial(K^\alpha L^{1-\alpha})/\partial L = w \ \text{ and } \ \partial(K^\alpha L^{1-\alpha})/\partial K = r + \delta. \tag{9}$$

4. Domestic real interest rate r satisfies (7).

5. Given w, r, τ and tr, households find the optimal policy functions according to (3).

6. The goods market clears

$$Y + NX = C + I,$$

$$I = K * (\delta + n + g),$$

$$NX = (r - n - g) * (A - K).$$

7. The population distribution is constant over time:

$$\mu = \int_X Q d\mu.$$

3 Solution and calibration

3.1 Algorithm for finding stationary equilibrium

Since the model does not have a closed form solution, the stationary equilibrium has to be found numerically. For this purpose, the space for households' assets is limited to a grid $\mathrm{A} = \{0, a^1, a^2, \ldots, a^M\} \subset [0, \infty)$ with a^M large enough not to constitute a constraint for the optimization problem. In order to cover all household choices of a', also those laying between the grid points, the golden section search method is applied. Given p^h and n, the rules of finding stationary equilibrium are the following.

1. Set the starting value for r and tr.

2. Based on equations (9) and (4), calculate the wage rate w, contribution rate τ, demand for aggregate capital K, and aggregate labor units. For the last one use the following formula:

$$L = \sum_{h \in H} \sum_{j \in J} \sum_{l=1}^{N} z(h, j, e_h = e_h^l) \mu(h, j, e_h = e_h^l).$$

3. Given r, τ, w and tr, and using backward induction, solve the household's optimization problem (3) and find policy functions $c(h, j, e_h, a)$, $a'(h, j, e_h, a)$ for $h \in H,\ j \in J,\ e_h \in E_h,\ a \in \mathrm{A}$.

4. Calculate aggregate output Y from equation (5), and next period assets and bequest based on the following equations:

$$A' = \sum_{h \in H} \sum_{j \in J} \sum_{l=1}^{N} \sum_{a \in \mathrm{A}} s_j * \mu(h, j, e_h = e_h^l, a) * a'(h, j, e_h, a),$$

$$\mathrm{tr}' = \sum_{h \in H} \sum_{j \in J} \sum_{l=1}^{N} \sum_{a \in \mathrm{A}} (1 - s_j) * \mu(h, j, e_h = e_h^l, a) * a'(h, j, e_h, a).$$

5. Check whether the value of tr is equal to tr′ and whether r satisfies (7). If yes, the equilibrium is found. Otherwise, go back to step 1 and update r and tr.

3.2 Algorithm for finding transitional dynamics

In the case of a stationary equilibrium, it was assumed that p^h and n are constant over time. Let us now consider a situation in which initially (up to time $t=0$) the model economy is in a stationary equilibrium with parameters $(p^{h,\text{old}}, n^{old})$. However, at time $t=1$ an unexpected change occurs that can be described by a new set of parameters $\{(p_t^h, n_t)\}_{t=1}^{\infty}$, which for $t > t_0$ stabilizes to the new levels $(p^{h,\text{new}}, n^{\text{new}})$. Before the model economy achieves the new stationary equilibrium, it would go through transitional dynamics, during which it would satisfy the general equilibrium conditions described in subsection 2.5.1. It is assumed that at time $t=1$ households learn the vector $\{(p_t^h, n_t)\}_{t=1}^{t_0}$ and new values $(p^{h,\text{new}}, n^{\text{new}})$. Below I present the algorithm of how to find the transitional dynamics, given stationary equilibrium $(p^{h,\text{old}}, n^{\text{old}})$, the parameters $(p^{h,\text{new}}, n^{\text{new}})$ and the vector $\{(p_t^h, n_t)\}_{t=1}^{t_0}$.

1. Using the algorithm from subsection 3.1, find a stationary equilibrium for the pair $(p^{h,\text{new}}, n^{\text{new}})$ and corresponding levels of $\tau^{\text{new}}, w^{\text{new}}$ and tr^{new}.

2. Set a period $t_1 > t_0$ in which the new stationary equilibrium is to be achieved. Set the maximum number of iterations N_0. Set the number of iterations (iter) to zero.

3. Increase the number of iterations (iter = iter + 1). Guess the starting values for $\{r_t\}_{t=1}^{t_1}$ and $\{\text{tr}_t\}_{t=1}^{t_1}$.

4. Calculate $\{w_t\}_{t=1}^{t_1}$, $\{\tau_t\}_{t=1}^{t_1}$ and $\{K_t\}_{t=1}^{t_1}$ based on the previous guesses, equations (8) and (4), and $L_t = \sum_{h\in H}\sum_{j\in J}\sum_{l=1}^{N} z(h,j,e_h=e_h^l)\mu_t(h,j,e_h=e_h^l)$.

5. Given $\{r_t, w_t, \tau_t, \text{tr}_t\}_{t=1}^{\infty}$, for all periods $t = 1, 2, \ldots, t_1$ solve the households maximization problem (3) using backward induction and find policy functions $c_t(h,j,e_h,a)$, $a_{t+1}(h,j,e_h,a)$ for $h\in H,\ j\in J,\ e_h\in E_h,\ a\in \mathrm{A}$.[3]

6. Calculate aggregate output, aggregate next period assets and bequests using the following formulas:
$$Y_t = K_t^{\alpha} L_t^{1-\alpha},$$
$$A_{t+1} = \sum_{h\in H}\sum_{j\in J}\sum_{l=1}^{N}\sum_{a\in \mathrm{A}} s_j * \mu_t(h,j,e_h=e_h^l,a) * a_{t+1}(h,j,e_h,a),$$
$$\text{tr}_{t+1} = \sum_{h\in H}\sum_{j\in J}\sum_{l=1}^{N}\sum_{a\in \mathrm{A}} (1-s_j) * \mu_t(h,j,e_h=e_h^l,a) * a_{t+1}(h,j,e_h,a).$$

[3]For $t > t_1$ it is assumed that $r_t = r^{\text{new}},\ w_t = w^{\text{new}},\ \tau_t = \tau^{\text{new}}\ ,\ \text{tr}_t = \text{tr}^{\text{new}}$.

7. Check whether the starting values $\{r_t\}_{t=1}^{t_1}$ satisfy (7) and whether $\{\mathrm{tr}_t\}_{t=1}^{t_1}$ are consistent with aggregates calculated in the previous step. If yes, the transitional dynamics is found. Otherwise, if iter $\leq N_0$, go back to step 3 and update the vectors $\{r_t\}_{t=1}^{t_1}$ and $\{\mathrm{tr}_t\}_{t=1}^{t_1}$. If iter $> N_0$ and transitional dynamics is not yet found, go back to step 2 and increase t_1.

3.3 Calibration

The model is calibrated to the Polish economy. I consider the following four scenarios for stationary equilibria (steady states):

- Scenario 1 *baseline* - low share of households with an academic degree, high population growth rate ($p^h = 18\%,\ n = 0.9\%$),
- Scenario 2 *educational change* - high share of households with an academic degree, high population growth rate ($p^h = 48\%,\ n = 0.9\%$),
- Scenario 3 *demographic change* - low share of households with an academic degree, low population growth rate ($p^h = 18\%,\ n = -0.6\%$),
- Scenario 4 *educational and demographic changes* - high share of households with an academic degree, low population growth rate ($p^h = 48\%,\ n = -0.6\%$).

Between scenario 1 and one of the scenarios from 2 to 4, the model economy is assumed to undergo transitional dynamics, during which vector $\{(p_t^h, n_t)\}_{t=1}^{t_0}$ evolves as described below. In the case of the educational transition, the population growth rate remains constant and equals to that in the baseline scenario, but the share of more educated households gradually changes (Table 1). During the demographic transition, there are no shifts in the educational structure of the population, but n_t changes permanently in 1990 and remains at the new level afterwards. Both, the assumed population growth rates and the shares of more educated households, match the empirical data for Poland. The former is based on the Eurostat historical data (mean for 1960-1990) and its projections (for 2066-2080), the latter corresponds to statistics taken from the Polish Households Budget Survey (Figure 2).[4]

Other model parameters are calibrated to the current state of the Polish economy, and, if possible, they are based on the long-term estimates.[5] The calibration outcome is presented in

[4]The values presented in the table are based on the cross-section observed in 2013. More educated households are those, whose household head has an academic degree. The underlying assumption made in Table 1 is that from age 24 and older the educational structure of a cohort does not change much over time.

[5]Although transitional dynamics are assumed to start in 1990, I do not choose to calibrate the model to this exact year. The reason is that at the early stage of transformation (especially during the years

Table 1: Educational structure of the population during the educational transition

periods	share of households with an academic degree in a group aged 25-29
before 1995	0.180
1995-1999	0.225
2000-2004	0.300
2005-2009	0.375
2010-2014	0.450
after 2014	0.480

Table 2. Therefore, the replacement rate θ and the world real interest rate r^* match real data while the aggregate productivity growth rate g corresponds to the European Commission's long-term projection of the TFP for Poland.

The deterministic component of life-cycle productivity is approximated by average household income over the life cycle, taken from Kolasa (2017). As regards the stochastic component of income, it is assumed that its logarithm (u_{ij}) is the following process:

$$u_{ij} = \epsilon_{ij} + v_{ij}, \tag{10}$$

$$v_{ij} = \rho v_{i,j-1} + \mu_{ij},$$

$$\epsilon_{ij} \sim N(0, \sigma_\epsilon^2), \quad \mu_{ij} \sim N(0, \sigma_\mu^2), \quad \epsilon_{ij} \perp \mu_{ij} \; i.i.d., \quad v_{i0} = 0, \quad E_i(u_{ij}) = 0.$$

To solve the model, this process needs to be discretized. For the permanent income shock (v_{ij}), I use Tauchen and Hussey (1991) method and assume three states. The transitory income shock (ϵ_{ij}) takes two values: $-\sigma_\epsilon$ and σ_ϵ with equal probability. Parameters ρ and σ_μ of the income process are taken from Kolasa (2017) and σ_ϵ is calibrated to match the Gini coefficient for Poland.

The discount factor β, the share of capital in production function α, and the debt elasticity of the domestic interest rate ϕ are calibrated to the baseline scenario targeting the share of consumption in production, risk premium, and international investment position. The chosen parameters are close to the values present in the literature. Finally, the rate of depreciation δ is set at the mean of the estimates taken in life-cycle models calibrated for Poland (Rubaszek, 2012; Makarski, Hagemejer, and Tyrowicz, 2016).

1990-2000), the Polish economy underwent fast real convergence and important structural changes, which I do not include in the model. I focus solely on the demographic and educational transition, and, since the considerable part of simulations covers the future, I want to set the values of parameters on the long (or medium) term means rather than one-year observations.

Table 2: Model calibration

Parameters:	Values:	Sources:
discount factor β	0.977	
capital share in production function α	0.307	
depreciation rate δ	0.08	
replacement rate θ	43%	OECD Pension at Glance 2015
aggregate productivity growth rate g	1.4%	EC Aging Report 2015
		projected mean for the period 2013-2060
interest rate risk premium ϕ	0.0253	
global real interest rate $r*$	2.3%	OECD 1956-2015
		inflation-adjusted long-term interest rate in the US
Income process		
Households with an academic degree		
autocorrelation coefficient ρ	0.92	Kolasa (2017)
variance of the permanent component of income σ_{μ}^2	0.02	Kolasa (2017)
variance of the transitory component of income σ_{ϵ}^2	0.167	calibrated to the Gini coeff.: 0.303 (OECD 2012)
Households without an academic degree		
autocorrelation coefficient ρ	0.82	Kolasa (2017)
variance of the permanent component of income σ_{μ}^2	0.02	Kolasa (2017)
variance of the transitory component of income σ_{ϵ}^2	0.167	calibrated to the Gini coeff.: 0.303 (OECD 2012)
productivity over the life cycle		average income profile, Kolasa (2017)
survival probabilities		Eurostat 2012

Table 3: Steady-state characteristics

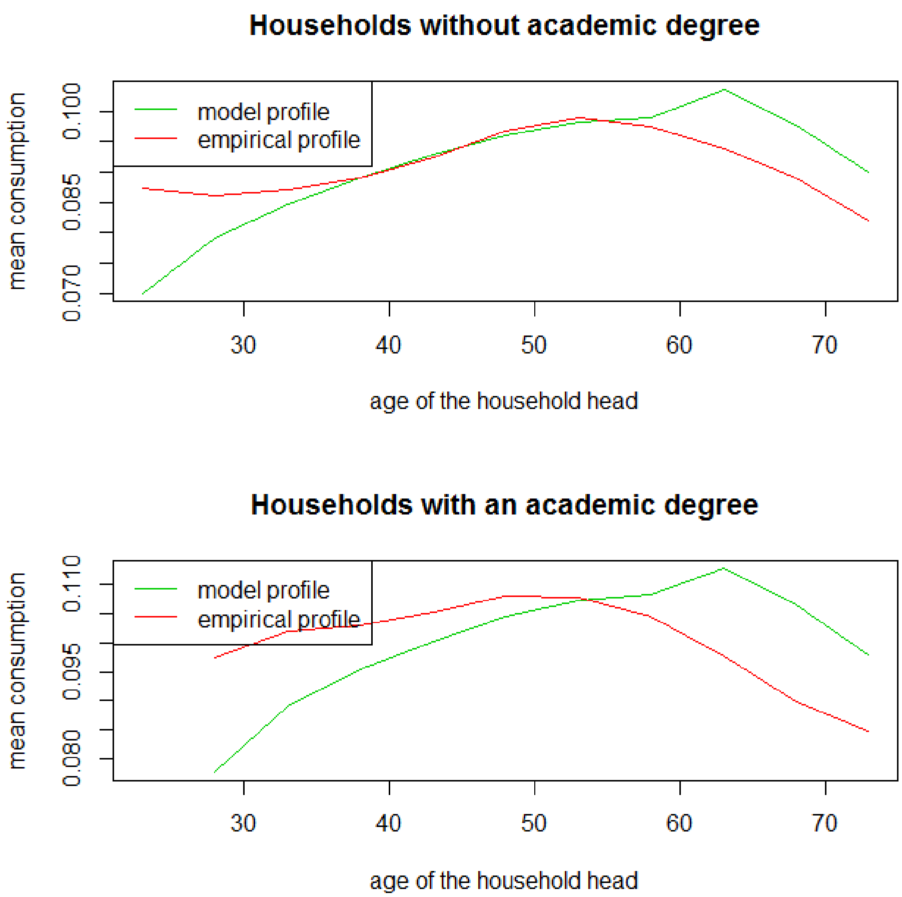
	model	data
consumption to output ratio	0.74	0.74*
investment to output ratio	0.25	0.25*
net foreign assets to output ratio	-0.55	-0.56**
interest rate risk premium	1.84%	1.89%***
Gini coeff. for workers	0.28	0.30****

Sources:
* non-financial national accounts 2004-2015, without government expenditures
** balance of payments statistics and non-financial national accounts 2004-2014
*** FED and OECD data 2001-2014, the difference between the Polish and the US inflation-adjusted 10-year government bonds yields
**** OECD data 2012, based on net income with transfers

The model period corresponds to five years, which translates into fourteen cohorts of households. All parameters in Table 2 are reported on an annual basis, so, when necessary, they are transformed to match the five-year-windows.

The aggregate statistics for Poland are satisfactorily matched by the model (see Table 3 for the comparison of the model results - baseline scenario - and the corresponding values based on the empirical data). As indicated by the individual level data, average consumption over the life cycle in Poland starts to decline earlier in life than implied by the model (Figure 3).[6] However, taking into account the model simplicity, the empirical and model profiles are matched sufficiently well.

Figure 3: Life-cycle consumption profiles, model and data



Note: Profiles are scaled with means equal one. The model profiles correspond to the baseline scenario, while the empirical profiles are calculated on the basis of Kolasa (2017) estimates.

[6]This significant drop in consumption is connected to the fact that older households in Poland maintain positive savings even at the late stage of their life-cycle (Kolasa, 2017). One possible explanation of this feature is that they are exposed to several sources of uninsured or only partially insured risk, such as mortality risk, health risk, etc.

4 Model implications

In this section, I present the main model findings. I start with the comparison of the steady states before and after the transition and then describe transitional dynamics with one subsection exclusively devoted to changes in inequalities.

4.1 The steady state comparison

Educational change A rise in the share of more educated (and on average more productive) individuals in population translates into higher effective labor supply and, thus, an increase in the main macroeconomic aggregates (Table 4). According to the model, after educational transition output per capita should be more than 16% bigger than in the baseline scenario. A similar increase can be expected for consumption, investment, and assets. However, the model indicates no significant shifts in the domestic interest rate and the contribution rate, while income inequalities rise. Indeed, the Gini coefficient increases from 0.28 to 0.30 in the model (Table 5).

Households make their consumption-saving decisions not only based on their individual characteristics, but also on the expected realizations of the domestic interest rate, wage, and the contribution rate. The model indicates that these variables should not change significantly after the educational transition. Therefore, the steady state life-cycle profiles of income, consumption, and assets, analyzed separately for more and less educated households, do not move much (Figures 4, 5 and 6). Let us take a "newborn" household with or without an academic degree. Its economic situation before and after the educational change would be about the same, so the substantial increase in macroeconomic variables observed after the transition results almost entirely from the changes in educational structure.

Demographic change With the demographic change, the old dependency ratio increases, which negatively affects the effective labor supply. As a consequence, output per capita drops by more than 10% (Table 4). Since households start to consume their assets relatively late in the life cycle, a smaller decrease is expected for assets (around 6%). As a result, domestic debt shrinks and the domestic interest rate falls by 0.2 pp. With lower labor supply less investment would be required and its share in output drops from 25% to less than 21% (Table 5).

When it comes to population aging, one of the most disturbing questions is how it would affect the pension system. In the model, pension expenditures always equal pension revenues, which is ensured by the adjustments in the contribution rate. The demographic change increases the contribution rate substantially, i.e. by 5.6 pp. As a result, in the new steady state, households earn, consume, and save proportionally less over the life cycle (Figures 4, 5 and 6).

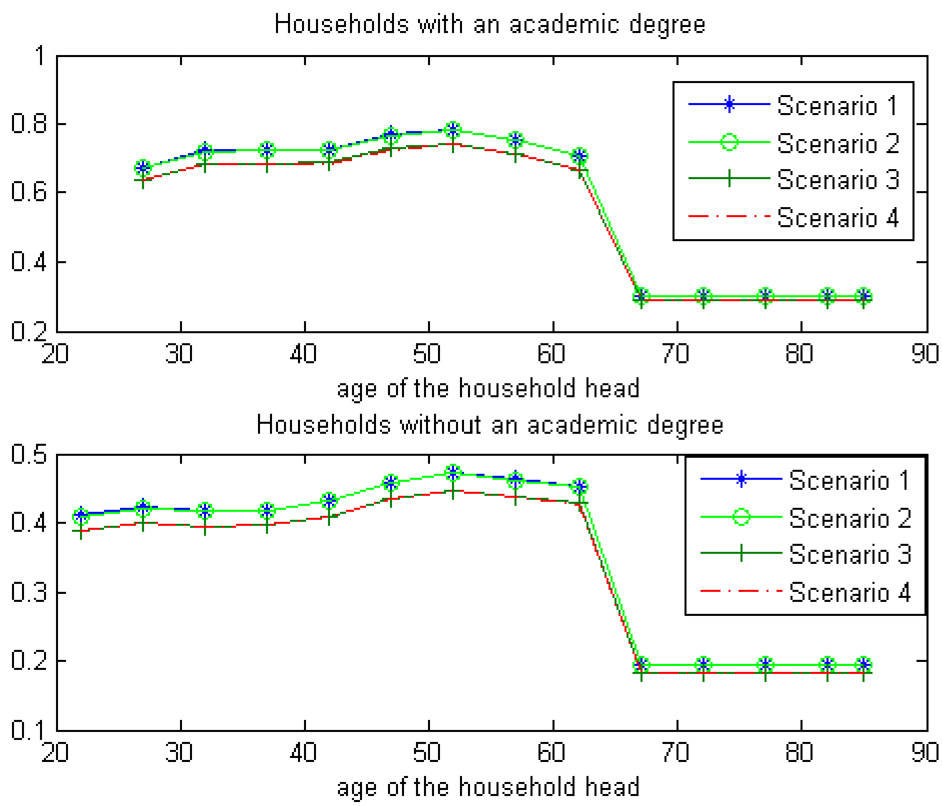
Table 4: New steady states - compared to the baseline scenario (no. 1)

	Scenario 2	Scenario 3	Scenario 4
	educational change	demographic change	educational and demographic changes
Δy per capita (%)	16.4	-10.2	4.5
Δc per capita (%)	16.3	-6.0	9.3
Δa per capita (%)	16.9	-5.9	10.0
Δi per capita (%)	16.6	-24.8	-12.2
Δnfa/y (pp)	0.4	4.6	5.2
Δr (pp)	0.0	-0.2	-0.2
Δc per household			
with an academic degree (%)	0.2	-6.7	-6.5
without an academic degree (%)	0.4	-6.2	-5.8
Δa per household			
with an academic degree (%)	-1.1	-5.9	-7.3
without an academic degree (%)	-1.4	-6.4	-7.9
Δl per household (income)			
with an academic degree (%)	-0.2	-11.0	-11.1
without an academic degree (%)	-0.2	-10.3	-10.4
$\Delta\tau$ (contribution rate, pp)	0.3	5.6	5.8
Δw (wage per productivity unit, %)	0.1	1.0	1.1
ΔGini (income of workers, in scale 0-100)	1.9	0.1	1.8
ΔGini (assets, in scale 0-100)	1.0	-2.3	-1.2

Table 5: Macroeconomic aggregates, different scenarios

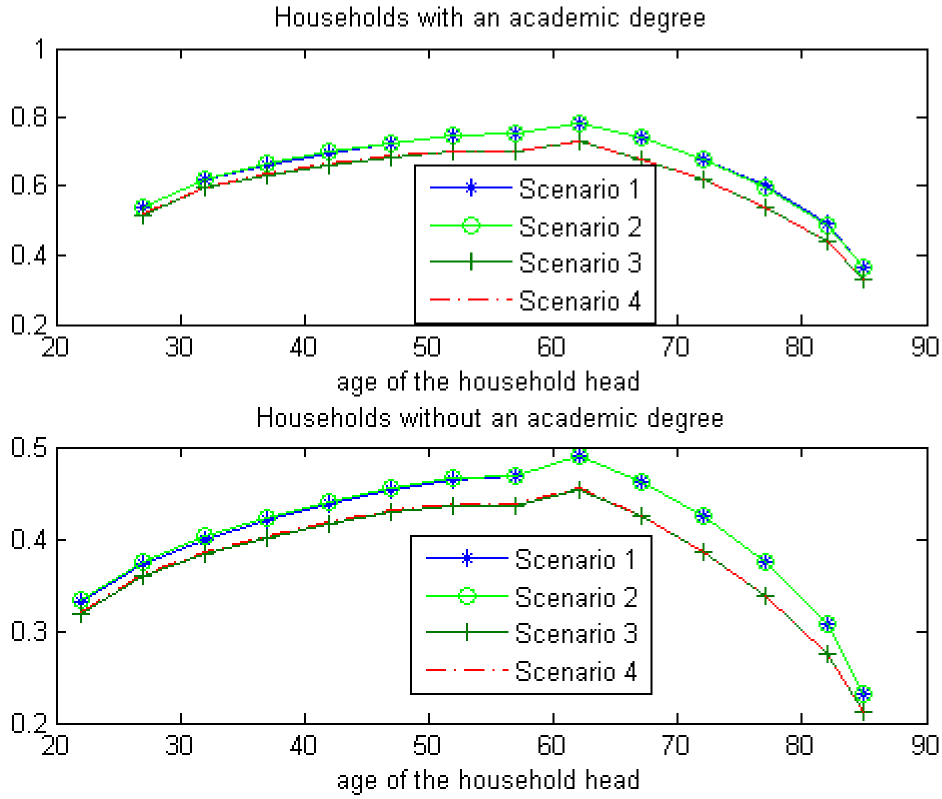
	Scenario 1	Scenario 2	Scenario 3	Scenario 4
	baseline, before changes	educational change	demographic change	educational and demographic changes
C/Y (%)	73.9	73.9	77.4	77.3
I/Y (%)	25.0	25.0	20.9	21.0
nfa/Y (%)	-54.7	-54.3	-50.0	-49.5
Gini (income of workers, in scale 0-100)	28.2	30.1	28.3	30.1
Gini (assets, in scale 0-100)	58.0	59.0	55.8	56.9

Figure 4: Life-cycle income profiles, different scenarios



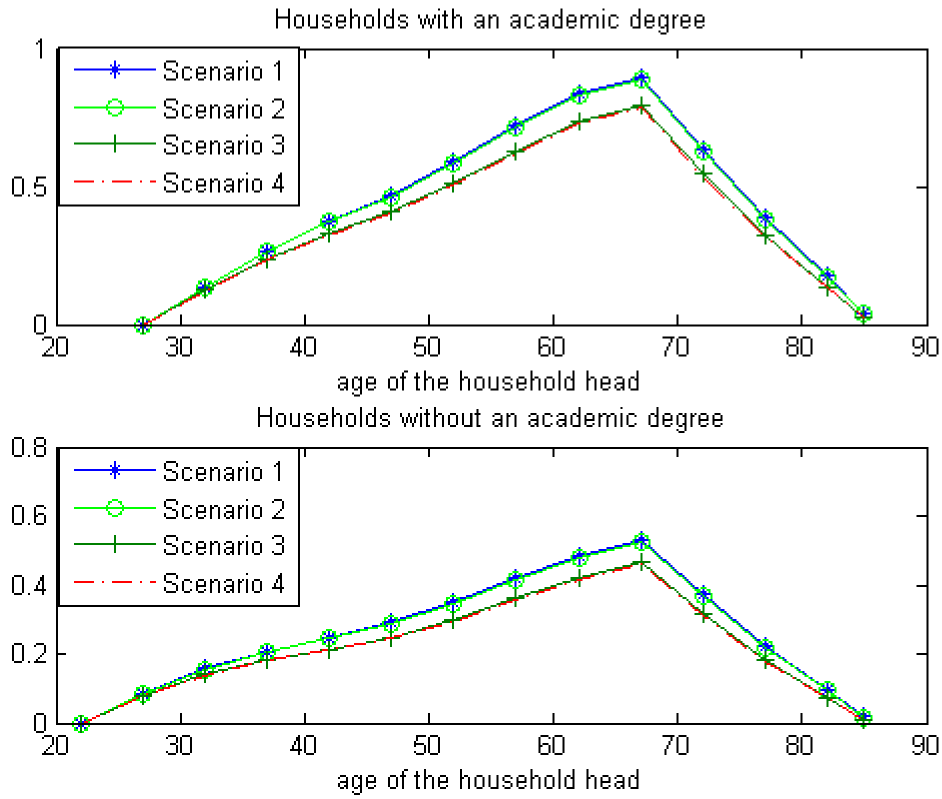
Scenario 1: $p^h = 18\%$ $n = 0.9\%$
Scenario 2: $p^h = 48\%$ $n = 0.9\%$
Scenario 3: $p^h = 18\%$ $n = -0.6\%$
Scenario 4: $p^h = 48\%$ $n = -0.6\%$

Figure 5: Life-cycle consumption profiles, different scenarios



Educational and demographic changes Assuming that both the educational change and the demographic change occur simultaneously, output per capita increases by 4.5%. Household's consumption and assets rise more than twice as much. Therefore, the aggregate variables indicate a general improvement of household's economic situation. On the other hand, the new (higher) contribution rate negatively affects life-cycle profiles. Once again, if we take a "newborn" household with or without an academic degree and compare its situation in both steady states, the result would be in favor of the baseline scenario. Finally, when it comes to inequalities, they increase for income but fall in the case of assets. For a more detailed analysis of changes in inequalities due to the educational and demographic transition, see subsection 4.3.

Figure 6: Life-cycle assets profiles, different scenarios



Scenario 1: $p^h = 18\%$ $n = 0.9\%$
Scenario 2: $p^h = 48\%$ $n = 0.9\%$
Scenario 3: $p^h = 18\%$ $n = -0.6\%$
Scenario 4: $p^h = 48\%$ $n = -0.6\%$

4.2 Transitional dynamics

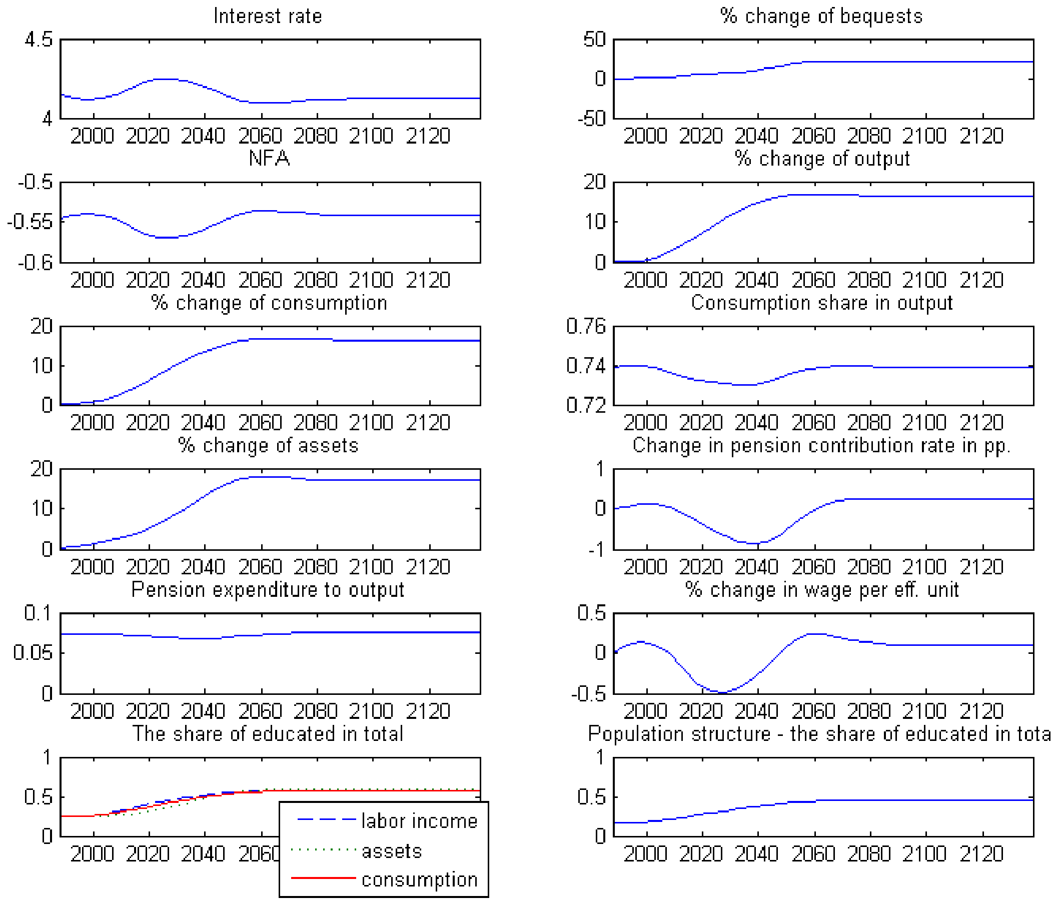
Educational transition (from scenario 1 to scenario 2) In the case of the educational transition, changes in the educational structure of the population start from younger households (at the early stage of the life cycle), while within older individuals the old (lower) share of educated workers still persists. From the macroeconomic perspective, we observe an increase in the productivity of young workers, which translates into higher effective labor supply, higher output and slightly lower wage per efficiency unit (Figure 7). Changes in effective labor supply are followed by an increase in capital demand. Domestic assets are slightly higher (more educated individuals save more, even at the early stage of the life cycle), nevertheless, the significant part of capital has to be financed from abroad. Eventually, the domestic interest rate reacts to the higher level of debt and increases in the short term. During the initial periods of the educational transition, pension system revenues increase but pension expenditures remain unchanged, which results in lower contribution rate.

In the later phases of the educational transition (after 2030), the higher share of more educated individuals is also observed for older cohorts, and the economy gradually converges to the new steady state. The contribution rate and wage for efficiency unit are steadily increasing, the domestic interest rate is declining and the net foreign investment position is improving, until they come back to the levels close to the baseline scenario.

The propensity to consume of an individual household does not change much. In the short term, it increases only slightly due to an expected rise in bequests and the domestic interest rate. Therefore, the heterogeneity between households with and without an academic degree and the imposed changes in the educational structure of the population are responsible for the dynamics in macro variables. An interesting question is how the educational transformation affects less educated households. What they gain in the new steady state are higher bequests. In the model, it is assumed that bequests are equally distributed. However, in reality, there exists an association between parents' and their children's education (Black, Devereux, and Salvanes, 2005; Ermisch and Pronzato, 2010). In the extreme case, households might inherit only from individuals with the same educational level and the economic situation of less educated households might not improve at all after the transition.

Demographic transition (from scenario 1 to scenario 3) Steadily declining labor supply causes, on the one hand, an upward trend in wage per efficiency unit, but, on the other hand, deteriorating production and demand for capital (Figure 8). Therefore, demand for foreign capital is also decreasing and so is the domestic interest rate. The share of pension expenditure in output has an upward trend, which results in the rising contribution rate.

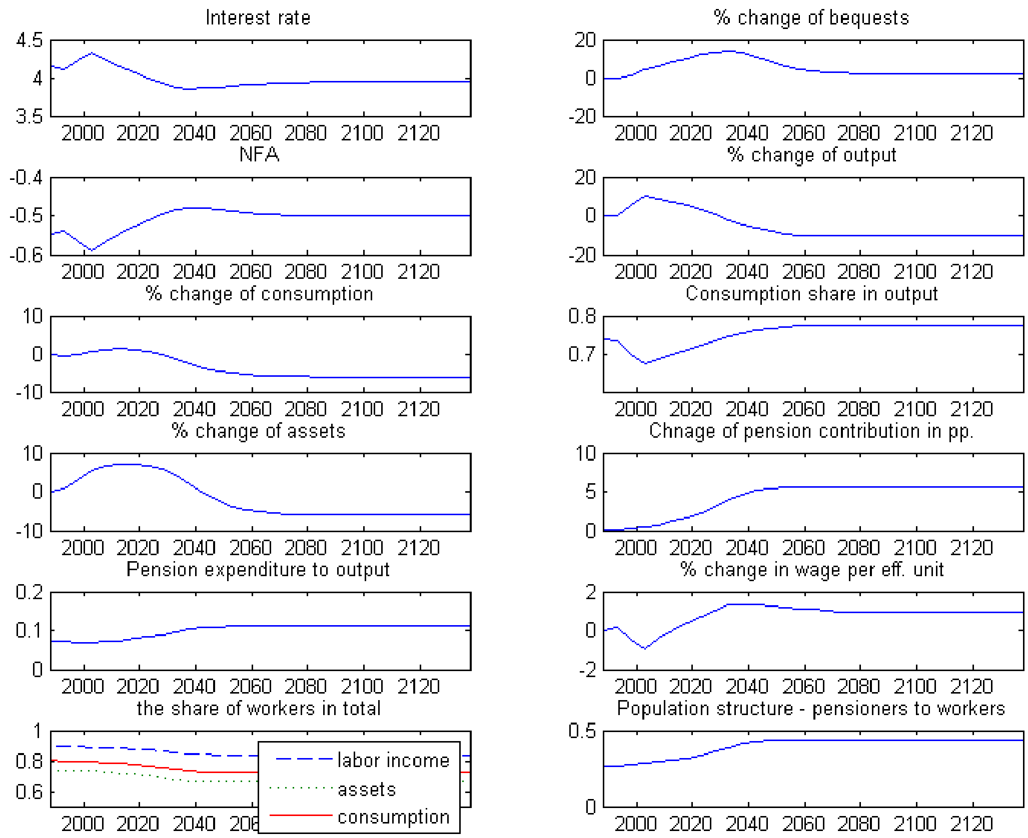
Figure 7: Educational transition



Note: Changes to the baseline scenario.
$p^h = 18\%$ before 1995
$p^h = 22.5\%$ 1995-1999
$p^h = 30.0\%$ 2000-2004
$p^h = 37.5\%$ 2005-2009
$p^h = 45.0\%$ 2010-2014
$p^h = 48.0\%$ 2015 and after
$n_t = 0.9\%$

However, during the initial periods of the demographic transition, i.e. between 1990 and 2005, the opposite trends are observed. Production per capita and the domestic interest rate increase and wage for efficiency unit drops. These dynamics are explained by changes in the age structure of the population. At the beginning of the demographic transition, the share of young workers in labor supply declines, but, since older workers are on average more productive, it translates into higher productivity per capita. Nevertheless, these are only short term effects. Eventually, in the new steady state output per capita is smaller, households must pay a higher contribution, and thus consume less over the life cycle.

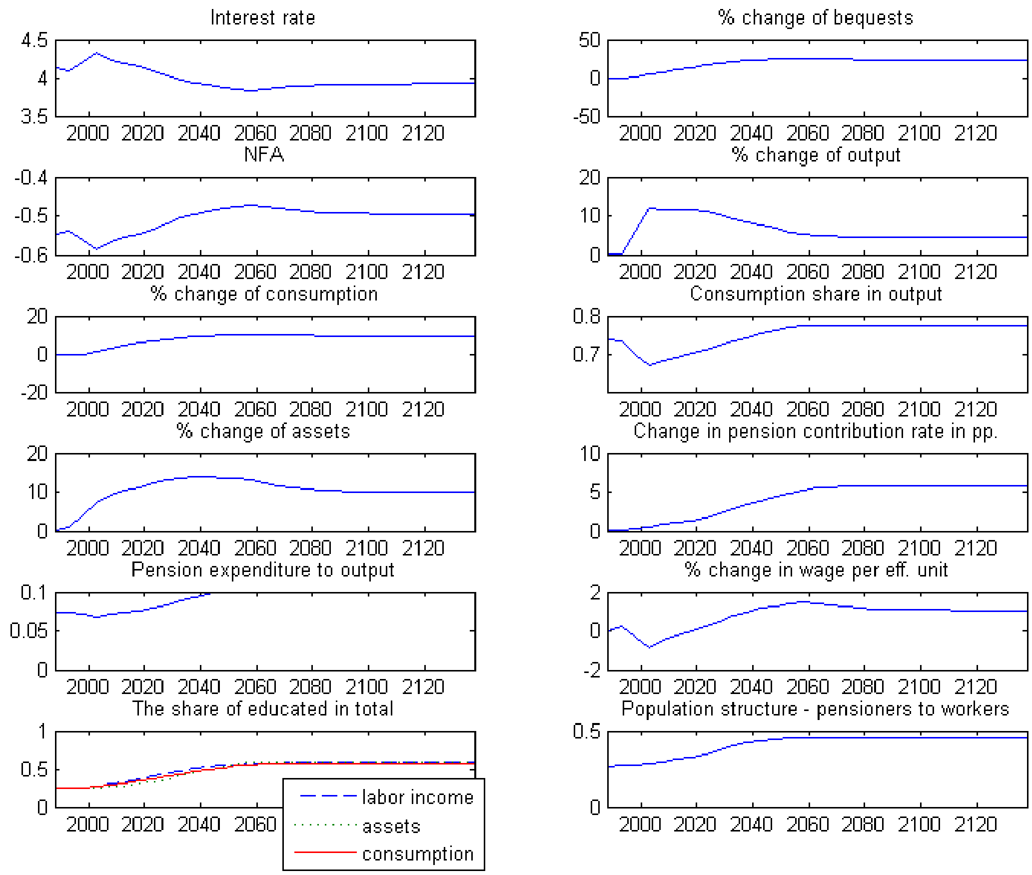
Figure 8: Demographic transition



Note: Changes to the baseline scenario.
$n_t = 0.9\%$ before 1990
$n_t = -0.6\%$ after 1990
$p^h = 0.18\%$

Educational and demographic transition (from scenario 1 to scenario 4) In the case of both educational and demographic transition, the demographic process tends to dominate trends in most of the variables (Figure 9). Accordingly, during the initial phases, an increase in output per capita is observed. Next, between 2005 and 2020, the divergent trends imposed by demographic and educational processes result in stable output levels. Afterwards, output exhibits a downward trend and converges to the new steady state level. Similarly to the demographic transition, in the case of both processes occurring simultaneously, most of the time net foreign position shows an upward trend, the interest rate is declining, and the contribution rate is rising.

Figure 9: Educational and demographic transition



Note: Changes to the baseline scenario.
$p^h = 18\%$ before 1995
$p^h = 22.5\%$ 1995-1999
$p^h = 30.0\%$ 2000-2004
$p^h = 37.5\%$ 2005-2009
$p^h = 45.0\%$ 2010-2014
$p^h = 48.0\%$ 2015 and after
$n_t = 0.9\%$ before 1990
$n_t = -0.6\%$ after 1990

4.3 Impact on inequalities

To examine the changes in the distribution of income, consumption, and assets, I use several inequality measures, the exact formulas of which are presented in Table 6. The outcomes are depicted in Figures 10 to 13. According to all analyzed indices, the effect of the demographic change on income inequality is negligible. However, the model indicates that during the educational transition income inequality continuously rises until 2060.[7] The Theil (1967) decomposition shows that the main source of the variation in income distribution is inequality

[7]Due to a finite set of income realizations the graph of quintile ratio might have steps.

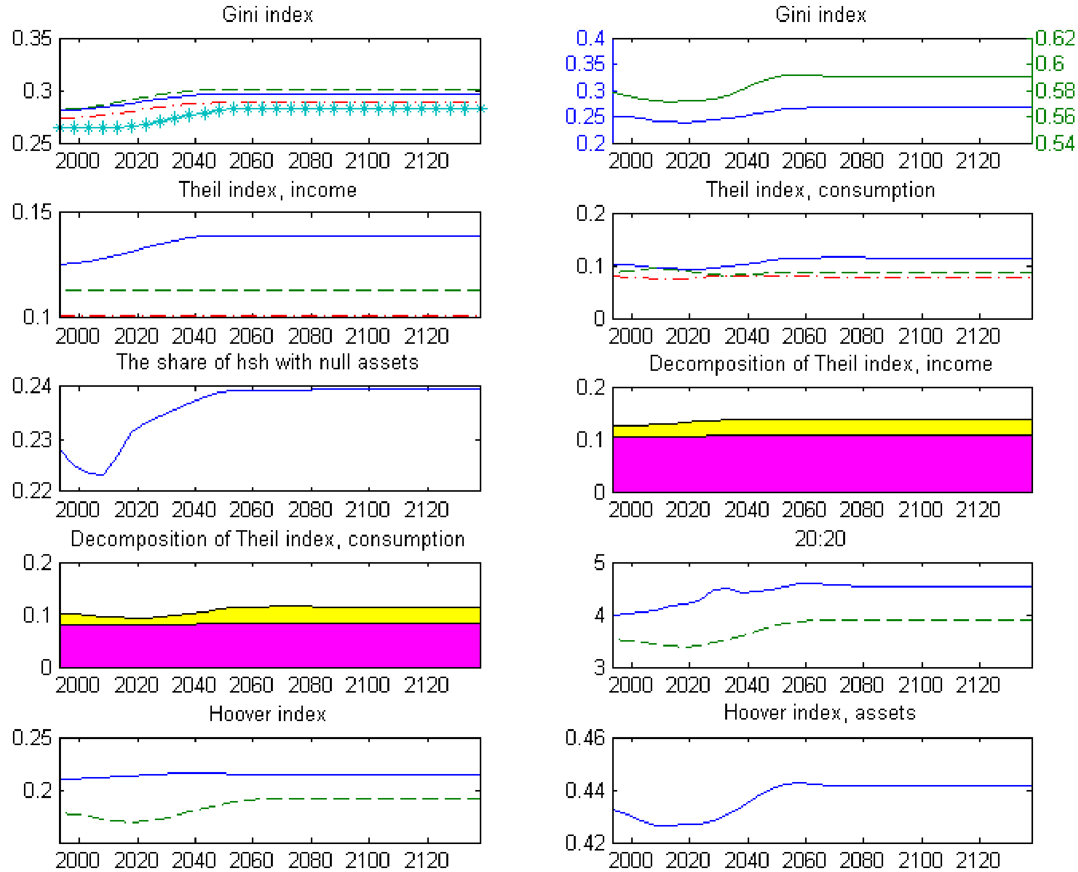
within educational groups, but what causes the major increase in income inequality during the transition are changes in between group inequality. Eventually, in the new steady state (scenario 4) the Gini coefficient is 1.8 pp. higher than that in the baseline scenario.

Table 6: Definitions of the selected inequality measures

Measures	**Formulas**
Gini index	$\left(\sum_{i=0}^{N}\sum_{j=0}^{N} f_k(k(i)) * f_k(k(j)) * \lvert k(i)-k(j)\rvert\right)$ ... .../ $\left(2 * \sum_{j=0}^{N}\sum_{i=0}^{N} k(i) f_k(k(i))\right)$, where f_k- density function on the set $K = \{k(0), k(1), \ldots, k(N)\}$
Theil index Decomposition:	$\sum_{i=0}^{N} f_k(k(i)) * \frac{k(i)}{\mu} \log\left(\frac{k(i)}{\mu}\right)$, where $\mu = \sum_{i=0}^{N} f_k(k(i)) * k(i)$, log - the natural logarithm
- inequality within groups	$\sum_{m=1}^{M} s_m * \left(\sum_{i=0}^{N} f_{k\mid m}(k(i)) * \frac{k(i)}{\mu_m} \log\left(\frac{k(i)}{\mu_m}\right)\right)$, where $s_m = \left(\sum_{i=0}^{N} f_k(k(i)) * k(i) * I_m(k(i))\right) / \left(\sum_{i=0}^{N} f_k(k(i)) * k(i)\right)$ M - the number of groups $m \subset K$, μ_m - mean for the group m, $f_{k\mid m}$ - conditional density function, I_m - characteristic function of the set m
- inequality between groups	$\sum_{m=1}^{M} s_m \log\left(\frac{\mu_m}{\mu}\right)$
Hoover index (Robin Hood index)	$\frac{1}{2} * \sum_{i=0}^{N} \lvert \frac{f_k(k(i)) * k(i)}{k_{\text{total}}} - f_k(k(i))\rvert$, where $k_{\text{total}} = \sum_{i=0}^{N} f_k(k(i)) * k(i)$
20:20	the ratio between a mean of a quintile with the highest values of k and a mean of a a quintile with the lowest values of k

Similarly, consumption and assets inequalities are also higher in the long term due to educational change. However, between 1990 and 2020 the model predicts their decline. During that period the share of young educated households is increasing at the expense of young but less educated individuals. Since young workers are on average less productive (and have zero or little assets), the mean income (mean assets) of more educated increases and the mean income (mean assets) of less educated declines. As a result, the between group inequality shrinks.

Figure 10: Changes in inequalities, educational transition

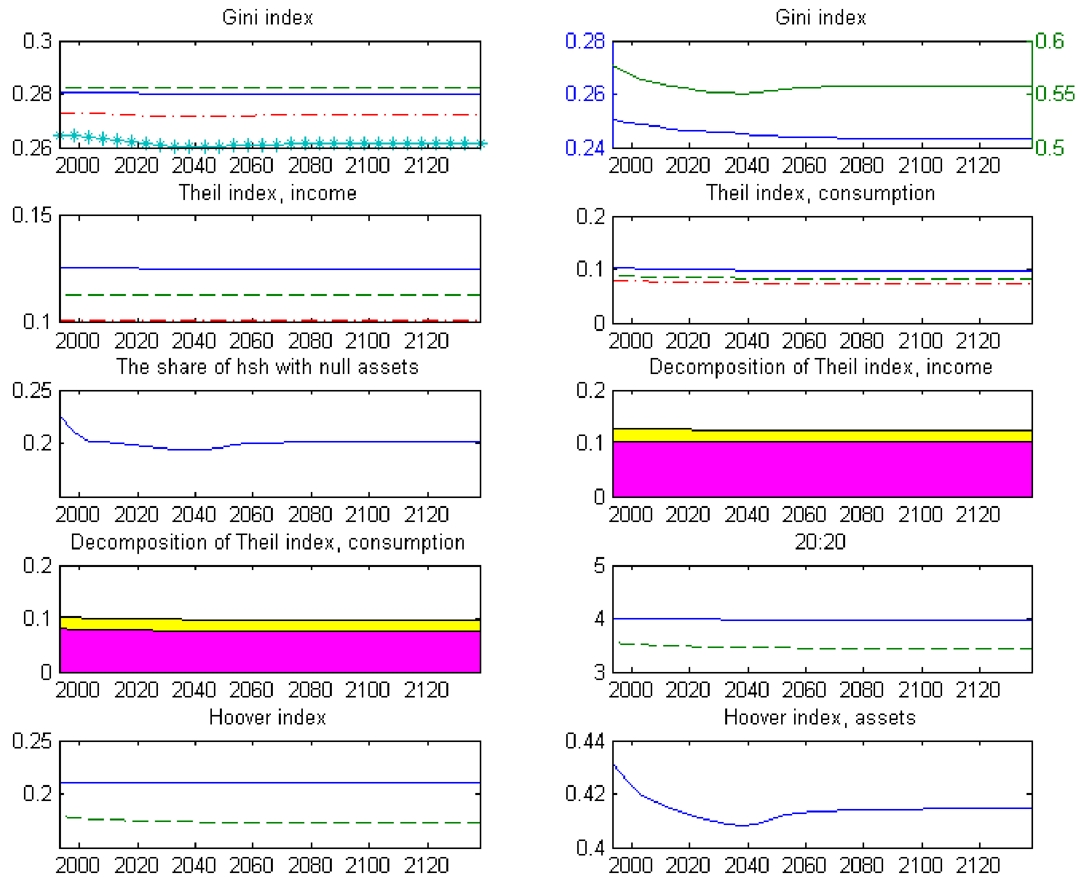


Note:

- Gini index (left panel): dark blue line - labor income (inc. pensioners), green line - labor income (only workers), red line - labor income and bequests, blue line - disposable income
- Gini index (right panel): dark blue line - consumption, green line - assets
- Theil index: dark blue line - all households, green line - households with an academic degree, red line - households without an academic degree
- Decomposition of Theil index: pink - within groups, yellow - between groups
- 20:20 and Hoover index: dark blue line - income, green line - consumption

If not specified income consists of labor income and pensions.

Figure 11: Changes in inequalities, demographic transition

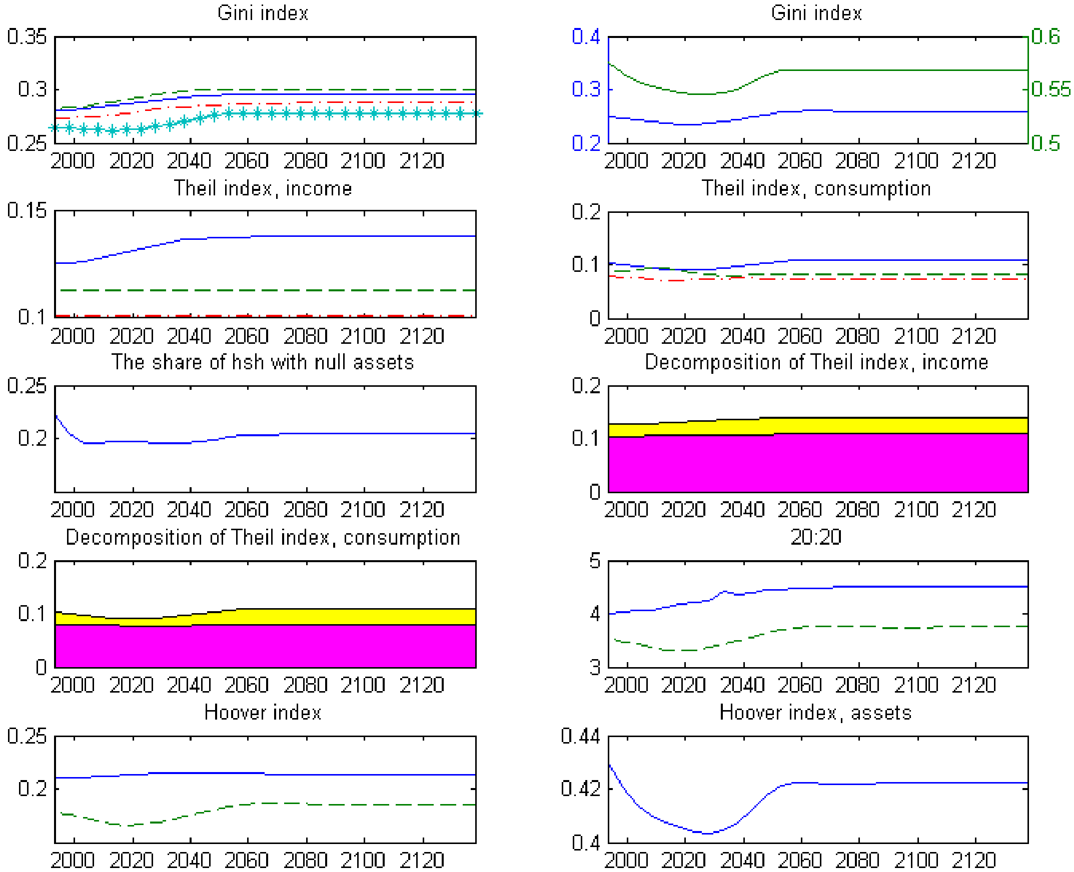


Note:

- Gini index (left panel): dark blue line - labor income (inc. pensioners), green line - labor income (only workers), red line - labor income and bequests, blue line - disposable income
- Gini index (right panel): dark blue line - consumption, green line - assets
- Theil index: dark blue line - all households, green line - households with an academic degree, red line - households without an academic degree
- Decomposition of Theil index: pink - within groups, yellow - between groups
- 20:20 and Hoover index: dark blue line - income, green line - consumption

If not specified income consists of labor income and pensions.

Figure 12: Changes in inequalities, educational and demographic transition

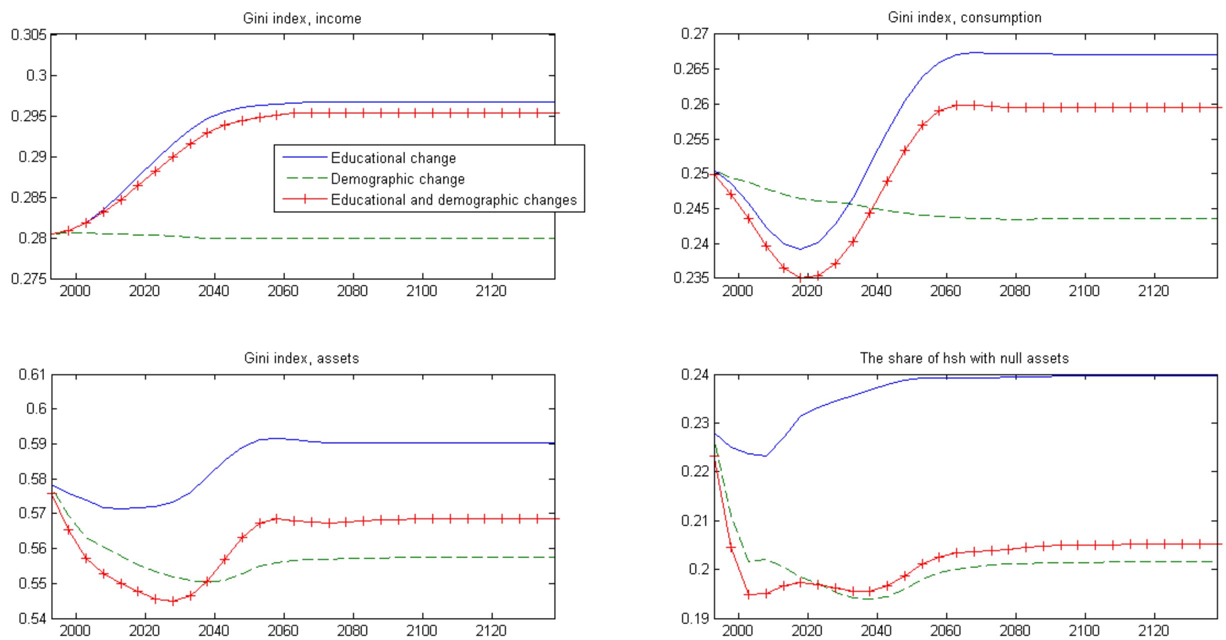


Note:

- Gini index (left panel): dark blue line - labor income (inc. pensioners), green line - labor income (only workers), red line - labor income and bequests, blue line - disposable income
- Gini index (right panel): dark blue line - consumption, green line - assets
- Theil index: dark blue line - all households, green line - households with an academic degree, red line - households without an academic degree
- Decomposition of Theil index: pink - within groups, yellow - between groups
- 20:20 and Hoover index: dark blue line - income, green line - consumption

If not specified income consists of labor income and pensions.

Figure 13: Changes in inequalities, summary



Note: Income consists of labor income and pensions.

In contrast to income and consumption inequalities, the variation in assets distribution is significantly and negatively affected by the demographic transition. The impact of the demographic change is stronger than that of the educational change, which results in a lower value of the Gini coefficient in the new steady state (scenario 4). In the case of assets distribution, there is a substantial share of households with no assets. It constitutes around one-fifth of the population in the baseline scenario and after demographic transition, it drops by 4 pp. This substantial change accounts for a large part of assets inequality decline.

5 Alternative model assumptions

This section aims to expand the analysis of different model assumptions. First, I widen the definition of the demographic change by including the process of rising longevity. Second, I allow the less and more educated workers to be imperfect substitutes and calculate the effects of the educational change for different degrees of labor market adjustment. Next, to examine the importance of assumptions associated with the interest rate, two extreme cases are considered: a closed economy, and an open economy with no interest rate risk premium.

Finally, the effects of the educational and demographic change are quantified for an economy in which the replacement rate adjusts to balance the pension system with a fixed contribution rate.

Higher life expectancy Until now the demographic change was defined as the shift in the population growth rate resulting from lower fertility.[8] However, another important source of population aging is rising life expectancy. I do not include rising longevity in the simulations of transitional dynamics for the following reasons. First, both processes, i.e. fertility decline and the rising share of university graduates, occurred in Poland more or less in the same periods. The time needed by the economy to converge to the new steady state in this two cases is also roughly the same. On the other hand, it is difficult to determine when and/or whether life expectancy would finally stabilize. The available projections point to their continuous rise at least up to the year 2080. Therefore, this process has a clearly different time horizon. Another argument in favor of narrowing the study to fertility and educational attainment is their possible causal relation.

In order to quantify the effect of rising longevity on the Polish economy, I compare the economy with higher survival probabilities to the baseline scenario. The new mortality tables correspond to the Eurostat projections for the year 2080. According to the model, an increase in life expectancy translates into a higher share of pensioners and, thus, a decline in product per capita by 3.8% and increase in the contribution rate of 3.0 pp. (column 2 of Table 7). Therefore, when it comes to changes in aggregate output, the effect of rising longevity is not as substantial as of fertility decline. Nevertheless, their combined negative impact on output cannot be compensated by higher productivity imposed by educational change (column 4 of Table 7).

Moreover, an increase in life expectancy significantly and positively affects domestic assets. As a consequence, in the new steady state, net foreign asset position improves and the domestic interest rate declines at 0.6 pp, stronger than in the case of fertility shift.[9]

[8]From now on for the model results presented so far, I refer to as the main simulations.

[9]Carvalho, Ferrero, and Nechio (2016) also estimate lower equilibrium interest rate due to the demographic transition.

Table 7: New steady states, including higher survival probabilities, compared to the baseline scenario (no. 1)

	higher survival probabilities	demographic change and unchanged survival probabilities	educational and demographic changes and higher survival probabilities
Δy per capita (%)	-3.8	-10.2	-2.6
Δc per capita (%)	-5.2	-6.0	0.9
Δa per capita (%)	10.5	-5.9	19.4
Δi per capita (%)	2.6	-24.8	-12.1
Δnfa/y (pp)	14.6	4.6	22.4
Δr (pp)	-0.6	-0.2	-0.9
Δc per household			
with an academic degree (%)	-4.9	-6.7	-13.5
without an academic degree (%)	-5.3	-6.2	-13.6
Δa per household			
with an academic degree (%)	12.9	-5.9	1.4
without an academic degree (%)	9.7	-6.4	-1.6
Δl per household (income)			
with an academic degree (%)	-4.2	-11.0	-17.4
without an academic degree (%)	-3.7	-10.3	-16.4
$\Delta\tau$ (contribution rate, pp)	3.0	5.6	10.3
Δw (wage per productivity unit, %)	2.9	1.0	4.3
ΔGini (income of workers, in scale 0-100)	0.0	0.1	1.8
ΔGini (assets, in scale 0-100)	-1.2	-2.3	-3.4

Imperfect substitution between less and more educated workers In the model, the aggregate effective labor supply is a simple summation of the productivity of individual workers. However, as pointed in many studies (see i.a. Krusell, Ohanian, Ríos-Rull, and Violante, 2000; Caselli and Coleman, 2006; Ottaviano and Peri, 2012), people with different level of educational attainment are not necessarily perfect substitutes in the labor market. In order to quantify to what extent the results presented so far are sensitive to the assumption of perfect substitution between workers, I recalculate the effects of educational change, but this time the effective labor supply is aggregated using the following CES function (Ottaviano and Peri, 2012):

$$l = (\mu_y^{1/\sigma_y} l_{edu}^{(\sigma_y - 1)/\sigma_y} + (1 - \mu_y)^{1/\sigma_y} l_{notedu}^{(\sigma_y - 1)/\sigma_y})^{\sigma_y/(\sigma_y - 1)}. \tag{11}$$

In the above formula l_{edu} and l_{notedu} stands for aggregated labor supply of more and less educated households, while μ_y and σ_y are function parameters, where the latter equals the elasticity of substitution between workers with and without an academic degree. Following Krusell, Ohanian, Ríos-Rull, and Violante (2000), σ_y is calibrated to 1.67.

In the case of $\sigma_y \to \infty$ or $\mu_y = \frac{l_{edu}}{l_{edu} + l_{notedu}}$ equation (11) simplifies to $l = l_{edu} + l_{notedu}$, i.e. the form used in the main simulations of this paper. In the above situation, wage per efficiency unit is equal for more and less educated workers: $1 = w_{edu}/w_{notedu} = (\partial y/\partial l_{edu}) / (\partial y/\partial l_{notedu})$ and the reason why households with higher educational attainment earn more is that they are on average more productive in all kinds of jobs.

The reality, however, is not that simple and there is a certain demand for workers with different skills. The structure of this demand depends among others things on the level of economic development. In the presence of imperfect substitution, we can think of optimal allocation, in which case l_{edu} and l_{notedu} should satisfy $\mu_y = \frac{l_{edu}}{l_{edu} + l_{notedu}}$, and that is what I assume for the model economy before the transition. Thus, the initial steady state is identical to what was previously called the baseline scenario (Table 5).

During the educational transition, the labor market structure might respond to changes in workers educational attainment and, as a result, the relative demand for more and less educated workers might adjust. In the new steady state, one can think of two extreme cases. In the first one, the aggregate productivity is optimally allocated between low- and high-skilled jobs (model adjustment 5), so the effect of the educational transition is the same as in the main simulations. In the second one, there are no shifts in relative demand (μ_y does not change at all during the transition, model adjustment 1) and the existing imbalance reduces the benefits of educational change. In this exercise, I also consider three additional scenarios, which differ in the extent of labor market adjustment.

The change in the educational structure of the population occurring in Poland is so dramatic that with little or no adjustments in labor demand, there will be a substantial deficit of less educated workers in the future. Therefore, the positive impact of the educational change i.a. on production per capita will be notably reduced (Table 8 and 9). Moreover, the relative wage per efficiency unit of more and less educated individuals will change greatly in favor of the latter, so that in the new steady state households without an academic degree will earn more, even taking into account the differences in productivity. Since in reality the level of educational attainment is not given exogenously but is subject to individual's choice, this situation will not persist in the long term. Nevertheless, it is important to note that the positive impact of the educational change strongly depends on the adjustments of the Polish

Table 8: New steady states, the educational change and imperfect substitution between more and less educated workers

μ_y	Model adjustment 1 0.241	Model adjustment 2 0.324	Model adjustment 3 0.407	Model adjustment 4 0.489	Model adjustment 5 0.572
C/Y	73.9	73.9	73.9	73.9	73.9
I/Y	25.0	25.0	25.0	25.0	25.0
nfa/Y	-54.5	-54.5	-54.4	-54.3	-54.3
the ratio of mean consumption between households with and without an academic degree	69.9	88.7	109.2	132.7	160.9
the ratio of mean income between households with and without an academic degree	69.3	88.6	109.7	134.1	163.7
the ratio of mean assets between households with and without an academic degree	71.5	92.0	114.3	140.1	171.7

data in %

Table 9: New steady states, educational change and imperfect substitution between more and less educated workers - compared to the baseline scenario (no. 1)

μ_y	Model adjustment 1 0.241	Model adjustment 2 0.324	Model adjustment 3 0.407	**Model adjustment 4** **0.489**	Model adjustment 5 0.572
Δy per capita (%)	0.8	7.8	12.6	**15.4**	16.4
Δc per capita (%)	0.8	7.7	12.5	**15.4**	16.4
Δa per capita (%)	1.0	8.0	12.9	**15.9**	16.9
Δi per capita (%)	0.8	7.9	12.7	**15.6**	16.6
Δnfa/y (pp)	0.2	0.2	0.3	**0.4**	0.4
Δr (pp)	0.0	0.0	0.0	**0.0**	0.0
Δc per household					
with an academic degree (%)	-44.4	-31.3	-19.5	**-9.0**	0.1
without an academic degree (%)	28.4	24.9	18.9	**10.7**	0.4
Δa per household					
with an academic degree (%)	-46.0	-32.8	-21.0	**-10.4**	-1.1
without an academic degree (%)	29.4	25.2	18.5	**9.6**	-1.4
Δl per household (income)					
with an academic degree (%)	-45.3	-32.1	-20.2	**-9.5**	-0.2
without an academic degree (%)	29.3	25.5	19.1	**10.4**	-0.2
$\Delta \tau$ (contribution rate, pp)	0.9	1.2	1.6	**2.0**	2.4
Δw_{edu} (wage per productivity unit, %)	-45.3	-32.0	-20.0	**10.7**	0.1
Δw_{notedu} (wage per productivity unit, %)	29.4	25.7	19.3	**-9.3**	0.1

economy.

How has the Polish labor market adjusted to changes in education so far? One way to look at it is through education premium. According to the literature, that is predominantly based on Mincer (1974) equations, education premium increased considerably in the early nineties (Keane and Prasad, 2006). Nevertheless, productivity estimates for more and less educated households, used in the model, are calculated from the Polish HBS data from 2000-2010. Therefore, the rise in education premium in Poland, that was observed in the initial years of transformation to the market economy, is accounted for. Adamczyk and Jarecki (2008) show that the returns to education also rose between 2000 and 2004. As for the period 2005-2010, the empirical literature is not that conclusive. While Majchrowska and Roszkowska (2014) point to an increase in education premium, Gajderowicz, Grotkowska, and Wincenciak (2012) claim that between 2007 and 2009 it declined. Generally, direct estimates taken from the Polish HBS data are in line with the literature. The ratio of mean disposable income of more educated households to less educated individuals increased between 2000 and 2005 and declined afterwards (Table 10).[10] If we compare the pace of change of the ratio in question with the model's implications, we can conclude that so far the model adjustment 4 gives the most accurate predictions (Table 10 and 11).[11] In conclusion, the positive impact of educational change on the economy obtained in the main simulations might be slightly overestimated, but, if the pace of adjustment of Polish labor market continues, the scale of changes and their dynamics remain the same.

Table 10: Changes in the ratio of income between households with and without an academic degree

year	$\Delta(\text{income}_{edu}/\text{income}_{notedu})$, %
2000	
2005	4.6
2010	-5.4
2014	-5.0

Notes: Based on the data from the Polish Household Budget Survey. The disposable equivalent income, i.e. the household's income divided by the squared number of the household's members, is used. The values of income_{edu} and income_{notedu} equal the means from the means of income calculated for the following age groups: 25-30 year-olds, 30-35 year-olds,. . . ,60-65 year-olds.

[10] Similar estimates can be obtained, when instead of disposable income the labor and self-employed income is used.

[11] Here I assume the smooth adjustment process, i.e. the linear trajectory of parameter μ_y.

Table 11: Changes in the ratio of wages (per productivity unit) between households with and without an academic degree: $\Delta(w_{edu}/w_{notedu})$, imperfect substitution between more and less educated workers

	Model adjustment 1	Model adjustment 2	Model adjustment 3	**Model adjustment 4**	Model adjustment 5
μ_y	0.241	0.324	0.407	**0.489**	0.572
years 2010-2014	-10.0	-8.3	-6.7	**-5.3**	-3.9

data in %

Note: In calculations the impact of the demographic change on labor supply and its structure is included.

Table 12: Steady states before changes and different model assumptions, compared to the baseline scenario (no. 1)

	Closed economy	No interest rate risk premium
Δy per capita (%)	-1.9	9.7
Δc per capita (%)	1.0	2.8
Δa per capita (%)	17.2	-49.0
Δi per capita (%)	-6.0	35.1
Δnfa/y (pp)	54.7	-182.9
Δr (pp)	0.4	-1.8
Δw (wage per productivity unit, %)	-1.9	9.7
ΔGini (assets, in scale 0-100)	-1.9	8.1

Closed economy If we change the assumptions about the interest rate and consider a closed economy with no borrowing from abroad, in the initial steady state (before transition) the interest rate would be 0.4 pp. higher. That translates into a higher level of domestic (households) assets and slightly lower assets inequality (Table 12). Households consume less per capita and their consumption is not that smooth over the life cycle (Figure 14). With the higher interest rate, demand for capital is lower, and so do output per capita and effective wage. Although the initial steady states differ, the estimated impact of educational and demographic change remains roughly as in the main simulations (column 3 in Table 13).

No interest rate risk premium In an open economy with no risk premium and no borrowing constraints, capital is cheaper ($r = r^*$) and so the demand for it is higher. Therefore, in contrast to the closed economy, in this case, we observe higher output, consumption, and investment per capita. On the other hand, lower interest rate translates into lower, but more unequally distributed, domestic assets, and significantly higher indebtedness (Table 12). With no interest rate risk premium, the household life-cycle profiles change, so that young individuals consume relatively more. Since the interest rate does not adjust to changes in net foreign assets, the negative impact of the demographic transition is stronger compared to the results of the main simulations (Figure 14 and column 4 in Table 13).

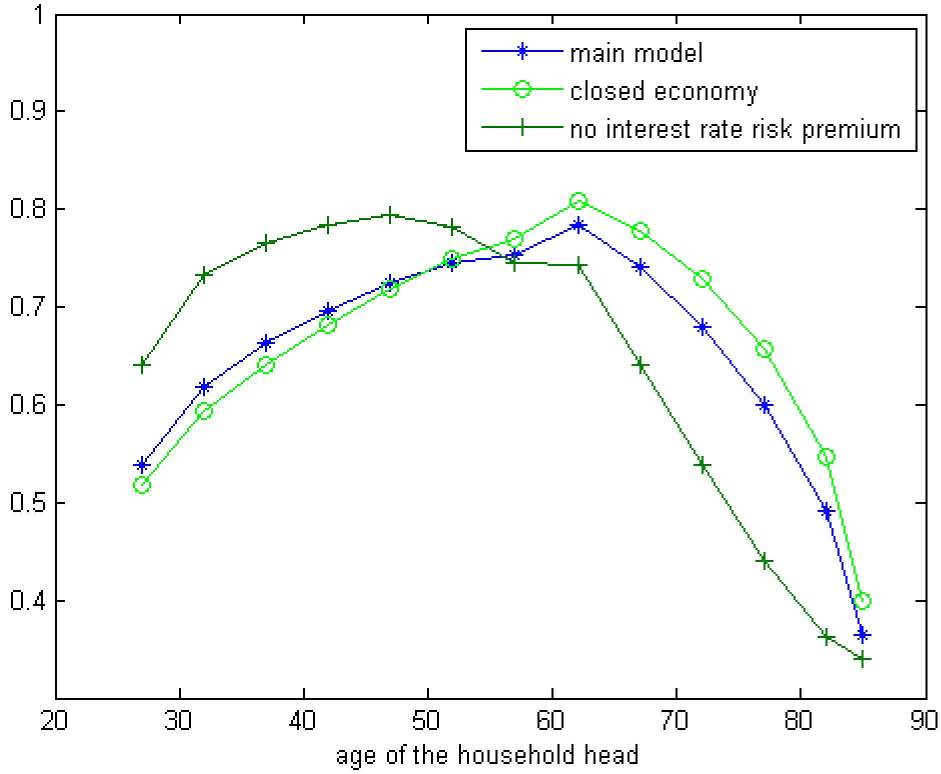
Table 13: The impact of the educational and demographic changes, different model assumptions

	Main model	Closed economy	No interest rate risk premium	Fixed contribution rate
Δy per capita (%)	4.5	4.7	3.4	6.9
Δc per capita (%)	9.3	9.8	6.3	11.2
Δa per capita (%)	10.0	7.8	19.2	25.6
Δi per capita (%)	-12.2	-11.6	-15.1	-5.5
Δnfa/y (pp)	5.2	0.0	15.7	17.2
Δr (pp)	-0.2	-0.3	0.0	-0.7
Δc per household				
with an academic degree (%)	-6.5	-6.0	-9.3	-5.2
without an academic degree (%)	-5.8	-5.3	-8.7	-3.9
Δa per household				
with an academic degree (%)	-7.3	-8.1	-1.2	6.8
without an academic degree (%)	-7.9	-9.4	-1.1	3.7
Δl per household (income)				
with an academic degree (%)	-11.1	-10.9	-12.0	-9.1
without an academic degree (%)	-10.4	-10.3	-11.4	-8.3
$\Delta \tau$ (contribution rate, pp)	5.8	5.8	5.8	0.0
Δw (wage per productivity unit, %)	1.1	1.3	0.0	3.4
ΔGini (income of workers, in scale 0-100)	1.8	1.8	1.8	1.8
ΔGini (assets, in scale 0-100)	-1.2	-1.2	-1.1	-1.5

Constant contribution rate The negative impact of the demographic change would be much smaller if the contribution rate (τ) was fixed and, in order to balance the pension system, the replacement rate (θ) was used (column 5 in Table 13). In this case, households would respond to the expected lower pensions by accumulating more assets, which, in turn, would improve net assets position and lower the interest rate (by 0.7 pp. compared to 0.2 pp.

in the main simulations). Thus, in this case, the production deterioration would not be that substantial. Nevertheless, since the assumption of rational and farsighted households seem controversial for many and cutting the pensions revenues usually meets social disapproval, it is rather difficult for this pension system design to be fully implementable.

Figure 14: Life-cycle consumption of households with an academic degree, different model assumptions



6 Conclusions

This paper investigates the demographic and educational transition in Poland within a general equilibrium framework with heterogeneous agents and idiosyncratic uninsured productivity shocks.

According to the results, both permanently lower fertility and rising longevity have a significant impact on macroeconomic variables, which include lower equilibrium real interest rate, notably higher contribution rate and deterioration in output and investment per capita. Fortunately, the positive effect of increased productivity due to the educational change should more than offset the negative consequences of falling fertility on production per capita. Nevertheless, if the total impact of population aging (i.e. lower fertility and rising life expectancy) is included, the net effects become negative. Importantly, one should not expect dramatic shifts in inequalities. Due to the educational change, income and consumption inequalities rise modestly while fertility decline translates into the slightly more equal distribution of assets.

As shown in the paper, the economic gains of the educational transition strongly depend on the labor market adjustments. The currently occurring changes in the ratio of more to less educated workers are rather substantial. Therefore, making a labor demand and supply compatible produces a great challenge for the policymakers. The empirical evidence suggests that so far Poland has been doing pretty well in meeting this goal, but for the economy to enjoy the benefits of the educational transition, this trend needs to be continued.

Population aging puts a pressure on the Polish pension system. According to the results, in order to keep the replacement rate unchanged, the contribution rate will have to be almost 10 pp. higher in the long term. As a result, the life-cycle consumption of households would be significantly lower compared to the situation before the transitions. Therefore, the issues concerning the reform of the pension system should be considered. On the one hand, both raising the retirement age and/or cutting the replacement rates would increase the effectiveness of the Polish economy, measured by output per capita. On the other hand, the problem is more complicated and, for instance, lower replacement rates cause a significant welfare loss of current generations (Nishiyama, 2015).

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