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EWS-GARCH: New Regime Switching Approach to Forecast Value-at-Risk

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Abstract

In the study a proposal of two-step EWS-GARCH models to forecast Value-at-Risk is presented. The EWS-GARCH allows different distributions of returns to be used in Value-at-Risk forecasting depending on a forecasted state of the financial time series. In the study EWS-GARCH with GARCH(1,1) and GARCH(1,1) with the amendment to the empirical distribution of returns as a Value-at-Risk model in a state of tranquillity and empirical tail, exponential or Pareto distributions used to forecast Value-at-Risk in a state of turbulence were considered. The evaluation of the quality of the Value-at-Risk forecasts was based on the Value-at-Risk forecasts adequacy (the excess ratio, the Kupiec test, the Christoffersen test, the asymptotic test of unconditional coverage and the back-testing criteria defined by the Basel committee) and the analysis of loss functions (the Lopez quadratic loss function, the Abad & Benito absolute loss function, the 3rd version of Caporin loss function and proposed in the study the function of excessive costs). Obtained results indicate that the EWS-GARCH models may improve the quality of the Value-at-Risk forecasts generated using benchmark models. However, the choice of best assumptions for an EWS-GARCH model should depend on the goals of the Value-at-Risk forecasting model. The final selection may depend on an expected level of adequacy, conservatism and costs of a model.

Keywords:

Value-at-Risk, GARCH, forecasting, state of turbulence, regime switching, risk management, risk measure, market risk.

JEL:

G17, C51, C52, C53

Introduction

Market risk is regarded as one of the three main risks in banks. The obligation to manage the market risk in banks is imposed by the international regulations established by the Basel Committee on Banking Supervision. As a part of the risk management, a very important task is its measurement. The basic requirement for an internal model is that the measurement has to be based on a Value-at-Risk.

According to the results obtained by the researchers, it is not possible to determine one – the best – method of measuring the Value-at-Risk, which would allow in every situation to achieve the best forecasts of the Value-at-Risk. Therefore, the analysis of the quality of the Value-at-Risk forecasts generated on the basis of different models is a topic widely discussed in the literature (among others, in Engle (2001) and (2004), Tagilafichi (2003), Alexander and Lazar (2006), Angelidis et al. (2007), Engle and Manganelli (2001), McAleer et al. (2009), Marcucci (2005), Ozun et al. (2010), Dimitrakopoulos et al. (2010), Brownlees et al. (2011), Degiannakis et al. (2012) and Abad et al. (2013)).

Even though there is no the best model to forecast Value-at-Risk, a lot of researchers are trying to find a model that provides the best Value-at-Risk forecasts. In most cases, the choice depends largely on the specificity of an analysed portfolio (the specifics of assets and the market), some of the researchers indicate that preferred models should include the distribution with lighter tails (eg. Engle (2001), Tagliafichi (2003)), but most of the researchers show that distributions with fat tails should be preferred (see Barone-Adesi and Giannopulos (2001), Engle (2004), Gencay and Selcuk (2004.), Dimitrakopoulos et al. (2010) Nozari (2010) or Ozun et al. (2010)).

A very interesting conclusions can be drawn from McAleer et al. (2009) and Degiannakis et al. (2012) who showed that for periods of tranquillity (pre-crisis) distributions with relatively thinner tails and for turbulent periods (the period of the financial crisis) models that consider the distributions with fat tails should be preferred. These results indicate that the selection of Value-at-Risk model should also depend on the current state of the portfolio.

The fact that the portfolio might be in the different states is considered by researchers developing regime switching models (including Hamilton and Susmel (1994), Cai (1994), Grey (1996), Alexander and Lazar (2006) and McAleer and Chan (2002)). Specificity of the proposed models is, that in all states, losses come from the same distribution but with different parameters. This property stays in contradiction with the findings stated in McAleer et al. (2009) and Degiannakis et al. (2012), where models with different distributions were found to be the best in different states.

The aim of the study is to present EWS-GARCH models which allow different distributions to be considered in different states of a portfolio. In these models, Value-at-Risk forecasts are calculated in two steps. First, the state of the portfolio is forecasted (the state of tranquillity or the state of turbulence) and then, depending on state forecasted, a different model is used to forecast Value-at-Risk. The EWS-GARCH models give the opportunity to use models to forecast Value-at-Risk in the state of tranquillity assuming a distribution of returns with relatively thinner tails, and in the state of turbulence models with much more conservative assumptions.

The construction of the EWS-GARCH models should, on the one hand, enables an effective protection against market risk by including highly fat tailed nature of the distribution of returns in the state of turbulence, but on the other hand, in a state of tranquillity does not force maintaining excessive levels of capital, which should be its advantage over models that take into account strong fat tail nature of the distribution of returns also in the tranquillity state (i.e. EVT models).

In a study, to assess the quality of the Value-at-Risk forecasts different EWS-GARCH models were compared to each other and with benchmark models (GARCH(1,1), GARCH(1,1) with the correction due to empirical distribution of random error, EGARCH(1,1,1) and GARCH-t (1,1)). The evaluation of the quality of a Value-at-Risk forecasts was based on the Value-at-Risk forecasts adequacy (excess ratio, Kupiec test, Christoffersen test, asymptotic test of unconditional coverage and back-testing criteria defined by the Basel committee – both for Value-at-Risk and stressed Value-at-Risk) and the analysis of loss functions (Lopez quadratic loss function, Abad & Benito absolute loss function, 3rd Caporin loss function and proposed in the paper function of excessive cost).

The paper is organized as follows: in the chapter 1, methods of forecasting Value-at-Risk are briefly discussed, in the chapter 2, the concept of the EWS-GARCH models is presented, and in the chapter 3, empirical verification of the Value-at-Risk forecasts obtained from the EWS –GARCH models is analysed.

1. Value-at-Risk as a measure of market risk

A Value-at-Risk ($VaR_\alpha(t)$) is defined as a value that a loss would not excess with a certain probability α within a specified period of time in normal market situation. This means that this measure is equal to the value that satisfies the assumption that for the assumed distribution, corresponding conditional quantile is equal to α. Value-at-Risk can be defined as follows (Alexander (2008)):

$$P(X < VaR_\alpha(t)) = \alpha \tag{1}$$

In the literature many different methods of Value-at-Risk measurement have been developed. In essence, most of them differ in the method of estimating distribution of returns. All methods can be divided into three basic groups: nonparametric, parametric and semi-parametric methods. In the nonparametric methods Value-at-Risk is calculated directly based on empirical data, in the parametric methods Value-at-Risk is calculated through models that use only estimated parameters describing the distribution of returns of the analysed portfolio. The semi-parametric methods combine the two previous approaches and partly use the estimated parameters and partly use information obtained directly from the empirical distribution of returns (see detailed discussion of methods in Abad et al. (2013)).

In the group of parametric models the most popular methods are EWMA models (including the RiskMetrics™ model) and ARCH/GARCH models. Parametric methods are often extended by nonparametric analysis. Semi-parametric models are characterized in a way that they contain parametric part, but simultaneously part of the model is determined based on non-parametric results (i.e. using empirical analysis or expert judgement). The semi-parametric approach is used for example in Monte Carlo simulation models, models with amendment to empirical distribution of returns, Filtered Historical Simulation models, Extreme Value Theory models or Conditional Autoregressive Value-at-Risk models.

The aforementioned approaches are most popular approaches used to forecast the Value-at-Risk in the literature. Each of them has a lot of options that can significantly affect the Value-at-Risk forecast. In example for the ARCH/GARCH class of models, Bollerslev (2008) described over 100 possible versions.

Having so many options, it is almost impossible to find one the best model for every case. Even though, a lot of researchers are trying to find a model that provides the best Value-at-Risk forecasts in every situation. Important findings showing that such an approach is almost impossible to achieve, may be find in papers analysis the quality of Value-at-Risk predictions for different models before, during and after the crisis, namely in McAleer et al. (2009) and Degiannakis et al. (2012). In both cases, the authors showed that the GARCH model assuming normality of the distribution of random error provides high-quality Value-at-Risk forecasts in pre-crisis 2007-2009 period, but the quality significantly decreases during and after the crisis. The study of McAleer et al. (2009) found that during the crisis the best model was RiskMetrics™, and after the crisis EGARCH-t model. In the study of Degiannakis et al. (2012) during the crisis the best model was APARCH with skewed t-Student distribution. These results show that in a state of tranquillity best models are less conservative, but during the crisis their superiority is shown by models that consider the distributions of returns with the fat tails. Degiannakis et al. (2012) state that these claims stays valid both for developed countries, as well as, for developing countries. The presented results show not only that there is no one model that would always be the best, but also that a choice of the best model to forecast Value-at-Risk depends on the analysed period, which could be evidence of the existence of a states in financial time series data.

Despite the conclusions drawn from the aforementioned articles, the use of regime switching models in the Value-at-Risk forecast has a rather niche character. Moreover, the results obtained by the researchers analysing such models in terms of forecasting Value-at-Risk are inconclusive.

Alexander and Lazar (2006) showed that models that take into account more than one state better reflects the nature of the observed foreign exchange time series than models with only one state. They also showed that it is appropriate to define only two states. The inclusion of the third state does not produce tangible benefits and only makes that estimates of a period of turbulence are highly unstable, which may lead to a significantly decrease of quality of such a models. The authors also compare the models in terms of the quality of their Value-at-Risk forecasts. The results show the superiority of the regime switching models in comparison to one-state (classic) GARCH class models, but they are not unequivocal. In most cases, the regime switching model provide indeed better Value-at-Risk forecasts than classic models, but there are also exceptions - for example, for the exchange rate of EUR/USD it turned out that GARCH with skewed t-Student distribution is the best model.

Similar conclusions can be found in the Marcucci (2005). Author indicates that, in principle, regime switching models are better suited to financial time series as they have a higher predictive power. But, this supremacy is not held with respect to the quality of Value-at-Risk forecasts, however, it is worth noting that two states models are also better in terms of Value-at-Risk forecasts to its one state counterparts (i.e. GARCH(1,1) with two states in comparison to classical GARCH(1,1) with only one state).

All the regime switching models compared above are built on assuming that the returns are under the same process for the period of tranquillity and turbulence. Differences are in the values of estimated parameters. This assumption stays in contradiction to findings stated by McAleer et al. (2009) and Degiannakis et al. (2012) where different models turned out to be the best in different states. In the next chapter, EWS-GARCH models are presented. The models have been developed to take an opportunity to take into account an stylized fact of the existence of states and high efficiency of different models in different states.

2. EWS-GARCH models

The concept of EWS-GARCH models is based on three basic assumptions. The first assumption is that a time series of financial data has two states (the state of tranquillity and the state of turbulence), which may vary considerably in terms of their nature. This assumption means that the Value-at-Risk forecasts would be provided from a different model in a state of tranquillity and different model in the state of turbulence. The second assumption is that the conditional volatility in financial data has a tendency to cluster and that other stylized facts about the characteristics of financial markets may be relevant, which makes use of the GARCH class models reasonable. The third assumptions is that tail returns may be better described by a different distribution than all returns together (the Extreme Value Theory is built on this assumption).

A Value-at-Risk forecasting procedure based on the EWS-GARCH models consists of two steps. In the first step, the state of time series for the next day is forecasted, then in the second step a Value-at-Risk for the next day is forecasted. The Value-at-Risk forecast is provided from an appropriate model regarding the state forecasted in the first step. The general concept of Value-at-Risk forecasting using EWS-GARCH model is shown in the figure 1.

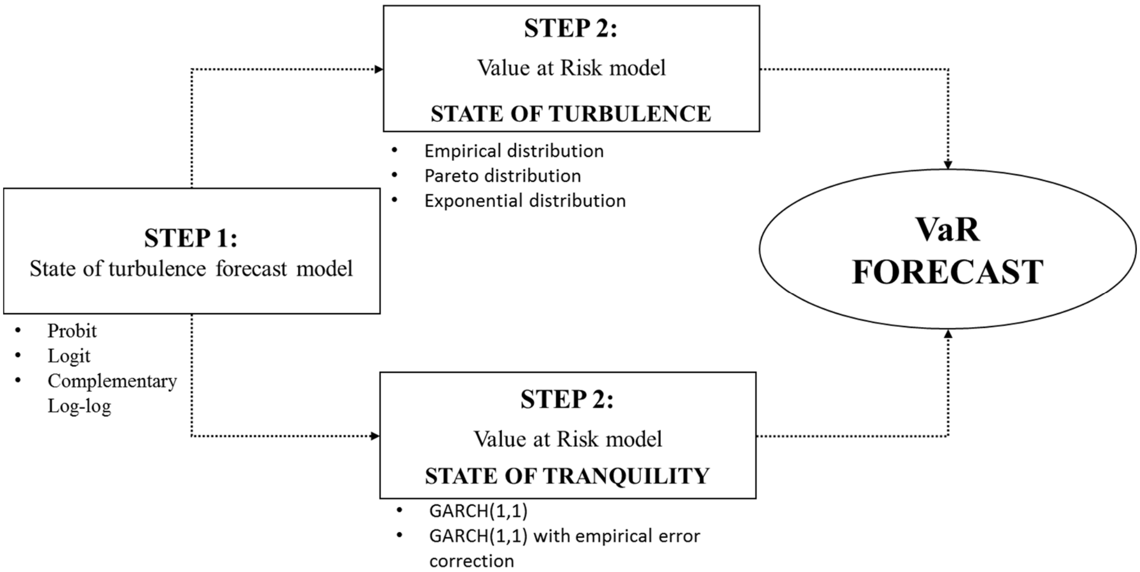

Figure 1. The concept of Value-at-Risk forecasting using an EWS-GARCH model.

In the EWS-GARCH models it is proposed that the prediction of the state should be carried out by a model for binary dependent variable: logit, probit or cloglog. Each of these models can be defined in a similar manner differs only in regarding of a random error distribution. Logit model assumes logistic distribution, probit model normal distribution and cloglog Gompertz distribution of random errors. These models can be defined as follows:

$$y_i^* = \boldsymbol{\beta X}_i + \varepsilon_i \tag{2}$$

$$y_i = \begin{cases} 1 & y_i^* > 0 \\ 0 & y_i^* \leq 0 \end{cases} \tag{3}$$

where y* is a latent dependent variable, β is a vector of parameters describing the relationship between independent variables and unobserved dependent variable, X_i is a vector of observations of independent variables that have an impact on an unobservable dependent variable, ε_i is a random error coming from the relevant distribution, and y_i is observable result of the modelled phenomenon.

In the process of forecasting a state, the y_i is equal to 1 for a certain percentage of the lowest observed returns (5% or 10%). Independent variables in the model describe a current situation on the stock, exchange rates and short-term interest rates markets (current returns on of the stock index; current returns on exchange rates for major currencies and current short-term interest rates). Moreover, to achieve the best possible forecasts quality a selection of an optimal cut-off point for the event forecast is considered (it is set up to 5% and 10% for the 5% and 10% definitions of y_i relevantly). The choice of models for binary variable, the definition of the observable dependent variable, the choice of independent variables and the optimal cut-off threshold have been established in accordance with the results obtained in the study of Chlebus (2016). Additionally, it is also worth considering two methods of independent variables set selection. In the first approach forecasts of a state would be based on the whole set of the independent variables. In the second approach a set of independent variables will be limited only to variables statistically significant at the 5% significance level selected by stepwise method.

The model to predict a state gives the opportunity to distinguish two states (the state of tranquillity and the state of turbulence) in a time series, which can vary considerably in their nature (with respect to expected returns, volatility etc.). In order to take into account different specificities of these two states, in each state different models to forecast Value-at-Risk should be used. According to the definition of the state of turbulence (5% or 10% of worst returns), the state of turbulence should be connected with the tail of the returns distribution. The idea standing behind the EWS-GARCH models is consistent with Extreme Value Theory, because in the EWS-GARCH models different distribution is used for modelling the entire distribution of returns and different for modelling the tail of the distribution. The main difference between EWS-GARCH and EVT models is that in the EVT models we are forecasting Value-at-Risk in every moment using tail distribution, in the EWS-GARCH models tail distribution is used only when the state of turbulence is forecasted, otherwise the entire distribution is used.

In the EWS-GARCH models GARCH models are considered as Value-at-Risk forecasting models (GARCH(1,1) and GARCH(1,1) with amendment to the empirical distribution of returns) in a state of tranquillity. It is worth noting that the design of the EWS-GARCH models allows to include other models to forecast Value-at-Risk. The choice of the aforementioned models stems from the lessons learned from the literature, and according to it, these models perform well in predicting the Value-at-Risk, especially at a time of relative tranquillity.

In the GARCH models it is assumed that the return r comes from the i.i.d. distribution with parameters (μ, σ^2). In the model conditional expected value μ_t (assumed that this value is equal to 0) and conditional variance σ_t^2 is estimated. The GARCH/ARCH class models, mainly differ to each other according to assumptions how the conditional variance equation is defined. However, they may also differ in other assumptions, such as the distribution of returns or the conditional expected value equation definition. The GARCH(1,1) model can be written as:

$$r_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t \xi_t \tag{4}$$

where r_t is a return on assets analysed at time t, ε_t is a random error in time t and ε_t can be expressed as the product of the conditional standard deviation σ_t and standardized random error ξ_t at time t, which satisfies the assumption $\xi_t \sim \mathrm{i.i.d.}(0,1)$. The equation of conditional variance in the GARCH(1,1) can be written as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{5}$$

where ω is a constant which satisfies the assumption $\omega > 0$, α_1 and β_1 are parameters that satisfy the assumptions $\alpha_1 \geq 0$ and $\beta_1 \geq 0$.

For the GARCH(1,1) the Value-at-Risk is estimated based on the following formula:

$$\mathrm{VaR}_\alpha(t) = \mu_t + k_{1-\alpha} * \sqrt{\hat{\sigma}_t^2} \tag{6}$$

where $\mathrm{VaR}_\alpha(t)$ is a forecast of Value-at-Risk on α significance level at time t, μ_t is a conditional expected value at time t, $k_{1-\alpha}$ is a value of quantile α from assumed random error distribution and $\hat{\sigma}_t^2$ estimation of conditional variance at time t.

The Basel committee requirements state that the Value-at-Risk should be estimated with 99% confidence (the α is assumed to be equal to 1%). The Value-at-Risk forecast from GARCH(1,1) with the amendment to an empirical distribution of returns (Engle and Manganelli (1999)) is obtained in a similar manner as in the GARCH(1,1), the difference lies in use a quantile from empirical distribution of returns instead of quantile from the normal distribution. The aim of this change is to include fatter tails than in a normal distribution. Such amendment is possible because of the property of GARCH model that the MLE estimator is consistent, even when a random error does not come from a normal distribution (Bollerslev and Woolridge (1992)). This property allows to use a GARCH processes to standardize residuals from the model even if a different distribution than normal is assumed for the standardised errors.

In the state of turbulence, as it is associated with the period of expected major losses, the tail distributions are considered, namely the empirical tail distribution and two parametric tail distributions - Pareto and exponential distributions. The use of these distributions in Value-at-Risk forecasting is a practice met in an operational risk measurement (see Panjer (2006)). The proposed parametric distributions are special cases of a generalized Pareto distribution used in the EVT models.

The Pareto distribution is characterized by a very thick tail and is used when there is a relatively high probability of very negative realization of returns. Pareto distribution may be defined with one or two parameters. A cumulative distribution function for the version with two parameters may be described by the following formula:

$$F_{PAR}(x) = 1 - \left(\frac{\theta}{\theta + x}\right)^{\alpha} \tag{7}$$

where θ is a scale parameter and α is a shape parameter.

The second tail distribution considered is the exponential distribution, which is defined with only one parameter. Its cumulative distribution function can be written as:

$$F_{EXP}(x) = 1 - e^{-x/\theta} \tag{8}$$

where θ is a scale parameter.

For the tail models the Value-at-Risk is forecast simply as a value of the quantile of the distribution. A problem in this case is a determination which quantile of the distribution provide the confidence equal to 99%. Two quantiles may be considered (the conservative and liberal assumption). The conservative assumption is that the Value-at-Risk forecast for the state of turbulence is equal to the 99th percentile of the tail returns distribution. Such an approach should not raise doubts about the satisfaction at least 99% level of confidence. However, the liberal assumption may be considered as well. The liberal assumption is taking into account the fact that the analysis of turbulence refers to a specified percentage of the worst cases. Then the confidence level should be obtained as the product of a definition of the state of turbulence and a quantile of tail distribution. According to the liberal approach, 99% confidence level for the 10% definition of the state of turbulence is obtained for the 90th percentile of the tail distribution and accordingly, for 5% definition of the state of turbulence the 80th percentile of the tail distribution should be used. In the study, the analysis of the validity of the conservative and liberal assumption will be verified empirically.

The EWS-GARCH models were designed in such a way to enable an effective measurement of market risk both at the time of relative tranquillity and turbulence, but also not to force keeping excessive levels of regulatory capital during tranquillity. The first element gives an advantage over one state models. The second element gives the advantage over the models that take into account a fat tail nature of the distribution also during relative tranquillity (i.e. models with t-Student's distribution or EVT models).

2.1. Estimation procedure

In the EWS-GARCH model the Value-at-Risk forecast is obtained on the basis of a two-step approach. Appropriate models on each of these steps are estimated separately. In the first step, state forecasting model is estimated, in the second step two different models are estimated each for both states. The advantage of this approach is its simplicity, because there is no need to develop a new complex estimation procedure. The disadvantage of this approach is the fact that at each stage an estimation error is committed, which could cause that the estimation error is greater than if the estimation of the entire model would be estimated at once.

The two-stage nature of the EWS-GARCH model makes that two elements are forecasted: the state of turbulence, and the Value-at-Risk. Forecasts of the state and the Value-at-Risk at time t + 1 are based on data available at time t. A data set to forecast states is prepared using the recursive window approach. Data set for Value-at-Risk forecasting is prepared using the rolling window approach (the window width was set to 1004 observations, which corresponds to about 4 years of one day returns). During the study it is assumed that the dependent variable in the GARCH models is a continuous one-day rate of return, which may be expressed as $r_t = \ln(p_t) - \ln(p_{t-1})$.

Additionally, for the GARCH models significance of a constant in the conditional expected value equation was performed, in cases when the constant was statistically insignificant no constant in the GARCH model was incorporated.

2.2. Testing framework

Performing a thorough analysis of the quality of EWS-GARCH models requires the development of multi-dimensional testing process. The aim of the study is to evaluate the quality of Value-at-Risk forecasts derived from EWS-GARCH models and compare it with the quality of Value-at-Risk forecasts derived from benchmark models (GARCH(1,1), GARCH(1,1) with amendment to the empirical distribution, EGARCH(1,1,1) and GARCH-t (1,1)). Therefore, the testing process should not only allow to assess the assumptions upon which models were built, but also to assess the quality of Value-at-Risk forecasts.

In order to confirm the quality of Value-at-Risk forecasts and comparisons of the models in terms of their quality, it was decided to carry out tests of the adequacy of the Value-at-Risk forecasts and loss functions analysis. As a part of the Value-at-Risk forecasts adequacy, analyses of the following were performed: the excess ratio comparison, the Kupiec test, the Christoffersen test, the asymptotic test of unconditional coverage and the backtesting criterion specified by the Basel committee (see BCBS (2006)). The excess ratio and the backtesting criterion was analysed for Value-at-Risk and Stressed Value-at-Risk (a measure defined by the Basel committee in the BCBS (2011)). Additionally, the following loss functions were analysed: the quadratic loss function proposed by Lopez (1999), the absolute loss function proposed by Abad & Benito (2013), 3rd form of the loss function proposed by Caporin (2008) and an absolute excessive cost function proposed in this paper.

The absolute excessive cost function, like Caporin loss function, includes costs either in case of the Value-at-Risk exceedance and lack of exidaance (the opportunity cost of the model used). The difference with Caporin function is that the analysis is focused rather on the excessive cost of the use of the model than precision of the forecast. Therefore, the process of point values assigning is divided into three cases and focuses precisely on the costs made by the model:

$$CAE_t \begin{cases} |VaR_\alpha(t)| & for\ r_t \geq VaR_\alpha(t)\ and\ r \geq 0 \\ |VaR_\alpha(t) - r_t| & for\ r_t \geq VaR_\alpha(t)\ and\ r < 0 \\ |r_t| & for \qquad r_t < VaR_\alpha(t) \end{cases} \tag{9}$$

where $\mathrm{CAE_t}$ is the point value of the absolute function of excessive cost at time t.

The first case describes a situation when the Value-at-Risk forecast is smaller than the observed rate of return and the observed rate of return is greater than zero. In such a case, the excessive cost of the model is equal to the Value-at-Risk forecast as no capital is needed to be protected against market risk. This follows the assumption that the financial institution cannot hold negative capital.

In the second case, when the forecasted Value-at-Risk is smaller than the observed rate of return, but the rate of return is less than zero, excessive cost of the model used is equal to the difference between the observed rate of return and the forecasted Value-at-Risk. The difference between these two values may be connected with the need to hold excess collateral due to the conservatism of the model.

In the third case, the Value-at-Risk forecast is exceed by the observed rate of return. In such a case formally if we were to consider this measure solely in terms of excessive costs, the redundancy should be equal to 0. However, in order to take into account the preferences of models with fewer Value-at-Risk exceedances it is worth considering also the cost factor in the case of exceedance. In the absolute function of excessive cost it is proposed that in such case the cost is equal to the realized rate of return ($|r_t|$). This level of punishment makes that the overestimation of the Value-at-Risk is considerably less expensive than its underestimation. Consequently, this leads to a preference for models that rarely underestimate the Value-at-Risk.

Value-at-Risk models should be compared in terms of mean value of excessive cost function for the analysed number of forecasts $\overline{CAE} = \frac{\sum_{t=1}^{N} CAE_t}{N}$. The $\overline{CAE}$ may be interpreted as a measure of excessive model conservatism. The higher the $\overline{CAE}$ is, the more conservative the model is, which means that the model predicts on average more conservative Value-at-Risk than needed to cover losses arising from changes in the value of assets analysed.

In the study, except of the Value-at-Risk quality analysis, also the accuracy of assumptions, for which the EWS-GARCH models were built were considered, the following tests were performed: the verification of ARCH process existence and its reduction (by LM and Q tests), the verification of the assumption about stationarity of returns distributions (Philips-Perron and KPSS tests), as well as the occurrence of an autocorrelation of random errors (Durbin-Watson test).

3. Empirical results

3.1. Data

The quality of Value-at-Risk forecasts obtained from the EWS-GARCH models was analysed for 79 time series of returns for companies listed on the Warsaw Stock Exchange (a detailed list of companies is presented in Appendix in table A1). Assets were selected randomly. The drawing process imposed only condition that the shares have been listed on the Warsaw Stock Exchange since at least January 2006. It is a technical requirement intended to ensure the best possible quality of data used for modelling and uniformity of sample for each company.

Among the analysed companies, 76 are Polish (19.6% of all Polish companies listed on the Warsaw Stock Exchange in 2011, the capitalization of companies in the study were equal to 29.57% of the total capitalization of Polish companies listed on the stock exchange) and 3 foreign companies (7.6% of all foreign companies, but 37.9% of market capitalization). Companies included in the analysis come from 23 different industries: 11 of them come from construction industry, than 6 comes from real estate, IT INDUSTRY and metal industries each, the rest of the industries are represented in a smaller degree. Significant differences for the analysed companies can also be observed due to the size of their capitalization. Among the companies selected, the smallest companies have a market capitalization at the end of 2011 below 2.5 million, and the largest over 250 million euros.

The above analysis shows that the companies whose returns time series were included in the study are different from many perspectives. The companies differ both in terms of their nature (type of company, industry) and size (size of market capitalisation). According to the fact that the analysis will be subjected to empirical verification for wide range of returns time series for companies with different characteristics, which may reveal model weaknesses in certain circumstances and help in assessing the quality of the analysed models and their universality.

The empirical study was performed for the series of returns from the 1 January 2006 to 31 January 2012. The period from the beginning of 2006 to the end of 2009 constituted the original estimation sample, from the beginning of 2010 the forecast sample starts and ends at the end of the whole sample, thereby giving 525 predictions of the Value-at-Risk for each asset.

All considered models used to forecast the Value-at-Risk have been developed in such a way as to meet the requirements set by the Basel Committee to internal models for market risk measurement. The measure of market risk is a based on the one-day Value-at-Risk predictions satisfying 99% confidence level. For the quality of Value-at-Risk forecasts only one-day predictions are required and sufficient. The assessment was carried out for 525 observations, which is about two years, which is more than expected in the Basel regulations of the minimum equal to 250 observations. The model takes into account the risk factors that may affect the level of market risk. In the state forecasting models, three key risk factors are directly addressed: the situation on the stock, the situation on the exchange rates and the situation on the interest rates market. The considered Value-at-Risk models assumes that the impact of all relevant risk factors are reflected in the price of assets analysed.

As summarised above, the proposed models fulfil basic expectations for market risk measurement models. In order to be used they must also satisfy the assumptions about the quality of the Value-at-Risk forecasts. Detailed results for the assessment of the quality of Value-at-Risk forecasts are presented later in this chapter.

3.2. Results

In the study, analogously to the practice used in the literature, EWS-GARCH models are evaluated and compared on the basis of the Value-at-Risk forecasts quality, so the quality of states forecasts are not discussed in details. Nevertheless, it is worth noting that the state of turbulence models estimated in accordance with the procedure discussed earlier provide a good quality forecasts, as confirmed by the results obtained by Chlebus (2016).

Before we discuss the results of the assessment of the quality of Value-at-Risk forecasts, the results of tests that verify appropriateness of models assumptions should be summarized. According to the obtained results it has to be stated that returns time series analysed are stationary. The ARCH effect exists and use of the GARCH (1,1) diminishes it. In the vast majority of the cases in the raw returns time series, the ARCH effect occurs (the share of cases with ARCH effect absence oscillates around 3-5%) and the use of GARCH (1,1) allows to eliminate it in the most cases (the share of cases with ARCH effect absence increased to 88-92%).

The last element of the analysis about assumptions of appropriateness was first-order autocorrelation of random error testing. For non-standardized residuals in 47.1% there was no enough evidence to reject the null hypothesis about no autocorrelation (at the 1% significance level). For the standardized residuals (after GARCH process standardisation of residuals due to ARCH effect) the share of cases in which there is not enough evidence to reject the null hypothesis increased to 60.1%.

The results presented show that assumptions made for the models are appropriate and may be used to forecast Value-at-Risk.

3.2.1. Value-at-Risk forecasts quality

At the beginning of the Value-at-Risk quality analysis for the EWS-GARCH models it is worth noting that the two from benchmark models (GARCH(1,1) and GARCH(1,1) with amendment to empirical distribution of returns) are included in a EWS-GARCH models as models used to forecast Value-at-Risk in the state of tranquillity. So the EWS-GARCH models (with assumed tranquillity state model) can be considered as a model that extends a one state (benchmark) model by taking into account the state of turbulence. Therefore, the results obtained for the EWS-GARCH models can be regarded as evaluation of how incorporation of the state of turbulence can improve, according to certain criteria, the results obtained by the benchmark models.

The discussion of the results for the EWS-GARCH model was divided into two parts. In the first part, results for the EWS-GARCH model with the GARCH(1,1) and in the second part the GARCH(1,1) with the amendment to an empirical distribution of returns as a model in a state of tranquillity were discussed. In order to maintain transparency of the results, a crossover comparison between models of different EWS-GARCH groups (with different state of tranquillity models) was omitted.

In this paper, results for EWS-GARCH models with Pareto distribution in the state of turbulence are not presented. This is due to the fact that the Value-at-Risk forecasted in the state of turbulence based on such models always significantly exceeded the level of 100% (the total value of the portfolio). These models apparently fulfil the requirements of the Basel Committee, but do not bring any added value over the assumption that the level of Value-at-Risk should be equal to the value of the entire portfolio, which is an unacceptable assumption from the risk managing perspective. The most likely cause of such a high forecasts of Value-at-Risk is the fact that the Pareto distribution is characterized by a very thick tails. For models using the exponential distribution in the state o turbulence similar problems were not identified.

The last thing worth analysing before the discussion of results for the EWS-GARCH models is the definition of improvement of Value-at-Risk forecasts. The improvement of the quality of Value-at-Risk forecasts may be defined in two ways: the first definition can be called absolute (conservative criterion), while the other relative (adequacy criterion):

- According to the conservative criterion of the improvement of the quality of Value-at-Risk forecasts should be connected with reduction of the number of exceedances. According to the definition, the more conservatively the model predicts Value-at-Risk (smaller excess ratio, more frequent assignment to the green zone according to Basel committee approach, smaller cost associated with exceedances) the better the model is acknowledged.
- According to the adequacy criterion, the closer the excess ratio to the expected 1% is, the better the selected model is to forecast Value-at-Risk (according to the standard confidence level assumed to be equal to 99%).

Evaluation of the Value-at-Risk quality was carried out due to both mentioned criteria definitions.

3.2.2. Value-at-Risk forecasts quality – the EWS-GARCH(1,1) models

The evaluation of the Value-at-Risk forecasts quality for the EWS-GARCH models began with EWS-GARCH(1,1) models. The results of the Value-at-Risk exceedances analysis and the cost functions for these models are presented in table 1. Analysis for EWS-GARCH(1,1) was divided into two parts. In the first part, models were compared with regard to Value-at-Risk exceedances and cost functions, in the other part the models were compared with regard to results of the coverage tests (same division was made for EWS-GARCH(1,1) with the amendment to an empirical error distribution).

From the results it can be seen that almost every version of the EWS-GARCH(1,1) lower the excess ratio and the average value of the Lopez cost function in comparison to GARCH(1,1). The only exception is the model which assumes that Value-at-Risk is defined as the 80 percentile of the empirical distribution at 5% definition of the state of turbulence and Probit model used to forecasts the state of turbulence. The Abad cost function also for most of the for EWS-GARCH(1,1) versions, on average, is smaller than for the GARCH (1,1). Exceptions are models assuming that Value-at-Risk is defined as respectively 90^{th} and 80^{th} percentile of the empirical distribution, with 10% and 5% definition of the state of turbulence respectively.

Improvement in the excess ratio and the costs associated with the occurrence of exceeding (expressed by Lopez and Abad cost functions), is associated with an increase in the costs of the model used (expressed by the values of Caporin and excess costs functions). The increase in the cost of use of models is growing steadily along with the decrease of the excess ratio. Exceptions are models in which Value-at-Risk is calculated as the 99 percentile of exponential distribution at 5% definition of the state of turbulence, in which case the increase of the cost of model is significant.

In principle, all models characterized by the lower excess ratio, on the average, are more likely to be assigned to the green zone in the Basel committee back-testing approach. Frequency of being assigned at least to the yellow zone is smaller than for GARCH(1,1) for models assuming the empirical distribution of returns and the liberal way of estimating Value-at-Risk in the state of turbulence for both definitions of the state of turbulence.

Improvement in results for stressed Value-at-Risk (reduction of the excess rate and more frequent qualification to the green zone and at least yellow zone) may be seen for all EWS-GARCH models except models which assume that Value-at-Risk is defined as the 80^{th} percentile of the empirical distribution with 5% state of turbulence definition.

By now, comparison was made between EWS-GARCH(1,1) models and GARCH(1,1) as a benchmark model. On this basis, it can be concluded that the use of EWS-GARCH models improves the quality of the Value-at-Risk forecasts relatively to GARCH(1,1) (from the perspective of a conservative criterion). In the next part, EWS-GARCH(1,1) models are compared to each other.

The results for the EWS-GARCH(1,1) models with different assumptions about state forecasting models may be summed up as following. The choice of the model to predict states does not cause a significant change in the quality of Value-at-Risk forecasts. However, using a stepwise selection process reduces the excess ratio and the costs associated with Value-at-Risk exceedance, while increasing the costs of using the model. Although the results for different states forecasting models do not differ significantly, there may be noticed some tendency in model preference. For the models that assumed 10% definition of a state of turbulence in 3 of 4 cases the Probit model with stepwise selection turned out to be the best. For the models assuming 5% definition of a state of turbulence always the best was Cloglog with the stepwise selection.

Selecting the Value-at-Risk forecasting model in the state of turbulence is of crucial importance for the quality of Value-at-Risk forecasts. It can be stated that the more conservative Value-at-Risk model assumptions in the state of turbulence are (appropriate percentile definition and assumed distribution), the lower excess ratio, lower costs associated with Value-at-Risk exceedance and higher costs associated with the usage of the model.

The results of coverage tests for EWS-GARCH(1,1) models are shown in table 2. On the basis of the results analysed the EWS-GARCH models may be divided into the following groups:

1. Models worse than the benchmark and non-conservative - this group includes models for which, based on coverage test results, the null hypothesis is rejected more frequently than for the benchmark model and it cannot be assumed from the results of the asymptotic unconditional coverage test that this is due to the frequent rejection of this hypothesis in favor of the hypothesis that the observed number of exceedance is smaller than expected.

2. Models worse than the benchmark and conservative - this group includes models for which, based on coverage test results, as in the previous group, the null hypothesis is rejected more frequently than for the benchmark model, but based on the results of the asymptotic unconditional coverage test it can be assumed that this is due to the frequent rejection of this hypothesis in favor of the hypothesis that the observed number of exceedance is smaller than expected. Distinguishing this group of models is apparent from the fact that these models are better than benchmark models from the perspective of a conservative criterion (however, they are worse from the adequacy criterion perspective).

3. Models better than the benchmark - this group includes models for which coverage tests rejected the null hypothesis less frequent or equally as for the benchmark models.

For EWS-GARCH(1,1) models to the group of worse and non-conservative models, models assuming the liberal definition of appropriate percentile to forecast the Value-at-Risk in the state of turbulence and using empirical distribution for both of the definition of the state of turbulence may be assigned. The models in this group do not improve in any way the quality of Value-at-Risk forecasts in relation to the benchmark model.

Models assuming a conservative definition of appropriate percentile to forecast the Value-at-Risk in the state of turbulence and 10% definition of this state or the model assuming 90^{th} percentile from the exponential distribution as the Value-at-Risk forecast may be assigned to the worse and conservative group of models. For these models it can be seen that worse results are caused by more frequent incidence of significantly smaller excess ratio observed than expected. It should be noted that these models are characterized by, on average, the lowest the excess ratio.

The last group – the better models, include models assuming a conservative approach to defining appropriate percentile to forecast Value-at-Risk and 5% definition of the turbulence state or the model assuming 80^{th} percentile of the exponential distribution as a Value-at-Risk in the state of turbulence. It turned out that the EWS-GARCH(1,1) with 5% state of turbulence definition provides more conservative Value-at-Risk forecasts than the GARCH (1,1), but also closer to 1% excess ratio.

Based on the described results it can be stated that EWS-GARCH(1,1) models are characterized by lower excess ratio than GARCH(1,1). In addition, for all of them (with the best assumptions regarding the state of turbulence forecasting) observed excess ratio is closer to the expected (in absolute value) than for GARCH(1,1). The most conservative EWS-GARCH(1,1) model assuming a 10% definition of the state of turbulence and Value-at-Risk calculated as 99 percentile of empirical or exponential distribution. The aforementioned models are even more conservative than the GARCH (1,1) with the amendment to empirical distribution of returns. None of the models come close to the level of conservatism specific to GARCH-t(1,1). However, in terms of adequacy (if the goal is to provide Value-at-Risk forecasts with observed excess ratio as close as possible to 1%), the best models are models assuming that Value-at-Risk in state of turbulence should be calculated as 80^{th} percentile of exponential distribution.

Table 1. The results of the analysis of the quality of Value-at-Risk forecasts models obtained from the EWS-GARCH(1,1).

SFM	TSVM	TUSVM	VALUE-AT-RISK (WHOLE OUT-OF-SAMPLE)									STRESSED VALUE-AT-RISK (THE WORST 250 DAYS)				
			EN	ER	ABAD	LOPEZ	CAPORIN	EXCOST	GREEN	AT LEAST YELLOW	RED	EN	ER	GREEN	AT LEAST YELLOW	RED
NONE	GARCH-t	NONE	1,25	0,24%	6,3%	2,46	12,5%	11,6%	98,7%	98,7%	1,3%	1,03	0,4%	97,5%	98,7%	1,3%
PROBIT SEL	GARCH	EX9_10	4,39	0,84%	8,4%	4,51	11,1%	10,3%	93,7%	98,7%	1,3%	3,56	1,4%	72,2%	93,7%	6,3%
LOGIT SEL	GARCH	EX9_10	4,44	0,85%	8,6%	4,56	11,0%	10,1%	93,7%	98,7%	1,3%	3,61	1,4%	68,4%	93,7%	6,3%
CLOGLOG SEL	GARCH	EX9_10	4,53	0,86%	8,7%	4,59	10,9%	10,0%	93,7%	98,7%	1,3%	3,66	1,5%	68,4%	93,7%	6,3%
PROBIT	GARCH	EX9_10	4,56	0,87%	8,5%	4,62	11,0%	10,2%	93,7%	97,5%	2,5%	3,63	1,5%	70,9%	93,7%	6,3%
PROBIT SEL	GARCH	EM9_10	4,58	0,87%	8,8%	4,71	8,7%	7,8%	92,4%	97,5%	2,5%	3,71	1,5%	68,4%	92,4%	7,6%
CLOGLOG	GARCH	EX9_10	4,59	0,88%	8,6%	4,66	10,8%	10,0%	93,7%	97,5%	2,5%	3,67	1,5%	73,4%	93,7%	6,3%
NONE	GARCH EMP	NONE	4,61	0,88%	9,2%	4,67	7,2%	6,4%	94,9%	97,5%	2,5%	3,73	1,5%	68,4%	96,2%	3,8%
LOGIT	GARCH	EX9_10	4,62	0,88%	8,6%	4,68	10,9%	10,0%	92,4%	97,5%	2,5%	3,67	1,5%	70,9%	93,7%	6,3%
LOGIT SEL	GARCH	EM9_10	4,63	0,88%	9,0%	4,76	8,6%	7,8%	92,4%	98,7%	1,3%	3,78	1,5%	64,6%	93,7%	6,3%
PROBIT SEL	GARCH	EX0_10	4,67	0,89%	9,0%	4,80	7,9%	7,1%	92,4%	96,2%	3,8%	3,80	1,5%	65,8%	91,1%	8,9%
CLOGLOG SEL	GARCH	EM9_10	4,71	0,90%	9,1%	4,77	8,5%	7,7%	92,4%	98,7%	1,3%	3,82	1,5%	64,6%	93,7%	6,3%
LOGIT SEL	GARCH	EX0_10	4,75	0,90%	9,1%	4,81	7,8%	7,0%	92,4%	97,5%	2,5%	3,90	1,6%	59,5%	92,4%	7,6%
CLOGLOG SEL	GARCH	EX9_5	4,76	0,91%	9,0%	4,82	17,1%	16,3%	93,7%	98,7%	1,3%	3,81	1,5%	69,6%	93,7%	6,3%
PROBIT SEL	GARCH	EX9_5	4,77	0,91%	8,9%	4,84	21,6%	20,8%	93,7%	98,7%	1,3%	3,84	1,5%	69,6%	93,7%	6,3%
PROBIT	GARCH	EM9_10	4,77	0,91%	8,9%	4,84	8,6%	7,8%	91,1%	97,5%	2,5%	3,80	1,5%	68,4%	93,7%	6,3%
LOGIT SEL	GARCH	EX9_5	4,80	0,91%	9,0%	4,86	17,1%	16,3%	93,7%	98,7%	1,3%	3,87	1,5%	69,6%	92,4%	7,6%
CLOGLOG SEL	GARCH	EX0_10	4,80	0,91%	9,2%	4,80	7,8%	7,0%	92,4%	97,5%	2,5%	3,91	1,6%	59,5%	92,4%	7,6%
CLOGLOG	GARCH	EM9_10	4,81	0,92%	9,0%	4,88	8,6%	7,7%	91,1%	97,5%	2,5%	3,84	1,5%	69,6%	92,4%	7,6%
LOGIT	GARCH	EX9_5	4,84	0,92%	8,8%	4,90	16,8%	16,0%	92,4%	98,7%	1,3%	3,95	1,6%	64,6%	92,4%	7,6%
PROBIT	GARCH	EX9_5	4,84	0,92%	8,8%	4,90	16,9%	16,1%	92,4%	98,7%	1,3%	3,95	1,6%	64,6%	92,4%	7,6%
LOGIT	GARCH	EM9_10	4,84	0,92%	9,0%	4,90	8,6%	7,7%	91,1%	97,5%	2,5%	3,85	1,5%	68,4%	93,7%	6,3%
CLOGLOG	GARCH	EX9_5	4,86	0,93%	8,8%	4,93	16,8%	16,0%	92,4%	98,7%	1,3%	3,99	1,6%	64,6%	92,4%	7,6%
PROBIT	GARCH	EX0_10	4,86	0,93%	9,1%	4,86	7,9%	7,1%	89,9%	97,5%	2,5%	3,87	1,5%	65,8%	92,4%	7,6%
CLOGLOG SEL	GARCH	EM9_5	4,87	0,93%	9,3%	4,94	8,6%	7,8%	92,4%	98,7%	1,3%	3,92	1,6%	64,6%	93,7%	6,3%
PROBIT SEL	GARCH	EM9_5	4,87	0,93%	9,1%	4,94	8,6%	7,8%	92,4%	98,7%	1,3%	3,94	1,6%	67,1%	92,4%	7,6%
LOGIT SEL	GARCH	EM9_5	4,91	0,94%	9,2%	4,98	8,6%	7,8%	92,4%	98,7%	1,3%	3,99	1,6%	64,6%	91,1%	8,9%
CLOGLOG	GARCH	EX0_10	4,91	0,94%	9,2%	4,92	7,8%	7,0%	89,9%	96,2%	3,8%	3,92	1,6%	65,8%	88,6%	11,4%
LOGIT	GARCH	EX0_10	4,92	0,94%	9,2%	4,93	7,8%	7,0%	89,9%	94,9%	5,1%	3,92	1,6%	65,8%	89,9%	10,1%
LOGIT	GARCH	EM9_5	4,95	0,94%	9,1%	5,02	8,5%	7,6%	91,1%	98,7%	1,3%	4,04	1,6%	64,6%	92,4%	7,6%
PROBIT	GARCH	EM9_5	4,95	0,94%	9,1%	5,02	8,5%	7,7%	91,1%	98,7%	1,3%	4,06	1,6%	63,3%	92,4%	7,6%
CLOGLOG	GARCH	EM9_5	4,97	0,95%	9,1%	5,04	8,5%	7,7%	91,1%	98,7%	1,3%	4,08	1,6%	64,6%	92,4%	7,6%
CLOGLOG SEL	GARCH	EX8_5	5,23	1,00%	10,1%	5,23	7,3%	6,5%	91,1%	96,2%	3,8%	4,23	1,7%	58,2%	89,9%	10,1%
PROBIT SEL	GARCH	EX8_5	5,25	1,00%	10,0%	5,26	7,3%	6,5%	92,4%	96,2%	3,8%	4,25	1,7%	60,8%	89,9%	10,1%
LOGIT	GARCH	EX8_5	5,27	1,00%	9,9%	5,27	7,3%	6,5%	89,9%	97,5%	2,5%	4,32	1,7%	55,7%	89,9%	10,1%
LOGIT SEL	GARCH	EX8_5	5,28	1,01%	10,1%	5,28	7,3%	6,5%	92,4%	96,2%	3,8%	4,28	1,7%	59,5%	87,3%	12,7%
PROBIT	GARCH	EX8_5	5,29	1,01%	9,9%	5,30	7,3%	6,5%	89,9%	97,5%	2,5%	4,33	1,7%	54,4%	89,9%	10,1%
CLOGLOG	GARCH	EX8_5	5,29	1,01%	9,9%	5,30	7,3%	6,5%	88,6%	97,5%	2,5%	4,35	1,7%	55,7%	89,9%	10,1%
CLOGLOG SEL	GARCH	EM0_10	6,01	1,15%	12,6%	6,02	6,8%	6,0%	82,3%	92,4%	7,6%	4,99	2,0%	50,6%	79,7%	20,3%
PROBIT SEL	GARCH	EM0_10	6,03	1,15%	12,4%	6,03	6,8%	6,1%	82,3%	93,7%	6,3%	4,99	2,0%	51,9%	79,7%	20,3%
LOGIT SEL	GARCH	EM0_10	6,04	1,15%	12,7%	6,04	6,8%	6,0%	83,5%	92,4%	7,6%	5,01	2,0%	48,1%	79,7%	20,3%
PROBIT	GARCH	EM0_10	6,19	1,18%	12,7%	6,20	6,8%	6,0%	81,0%	92,4%	7,6%	5,08	2,0%	46,8%	81,0%	19,0%
CLOGLOG	GARCH	EM0_10	6,20	1,18%	12,8%	6,21	6,8%	6,0%	79,7%	92,4%	7,6%	5,05	2,0%	49,4%	79,7%	20,3%
LOGIT	GARCH	EM0_10	6,23	1,19%	12,8%	6,23	6,8%	6,0%	81,0%	92,4%	7,6%	5,09	2,0%	48,1%	79,7%	20,3%
CLOGLOG SEL	GARCH	EM8_5	6,29	1,20%	13,2%	6,30	6,7%	5,9%	81,0%	91,1%	8,9%	5,15	2,1%	43,0%	75,9%	24,1%
PROBIT SEL	GARCH	EM8_5	6,37	1,21%	13,1%	6,37	6,7%	5,9%	81,0%	92,4%	7,6%	5,22	2,1%	43,0%	77,2%	22,8%
LOGIT SEL	GARCH	EM8_5	6,37	1,21%	13,2%	6,37	6,7%	5,9%	79,7%	89,9%	10,1%	5,24	2,1%	43,0%	77,2%	22,8%
LOGIT	GARCH	EM8_5	6,39	1,22%	12,9%	6,40	6,7%	5,9%	79,7%	92,4%	7,6%	5,29	2,1%	36,7%	79,7%	20,3%
CLOGLOG	GARCH	EM8_5	6,39	1,22%	12,9%	6,40	6,7%	5,9%	78,5%	92,4%	7,6%	5,32	2,1%	38,0%	78,5%	21,5%
NONE	GARCH	NONE	6,42	1,22%	12,5%	6,42	6,6%	5,8%	78,5%	93,7%	6,3%	5,18	2,1%	39,2%	78,5%	21,5%
PROBIT	GARCH	EM8_5	6,43	1,22%	13,1%	6,44	6,7%	5,9%	79,7%	91,1%	8,9%	5,34	2,1%	35,4%	78,5%	21,5%
NONE	EGARCH	NONE	6,53	1,24%	12,5%	6,54	6,7%	5,9%	78,5%	92,4%	7,6%	5,19	2,1%	40,5%	81,0%	19,0%

In the table, white fields refer to the EWS-GARCH models, while grey fields to the benchmark models.

In the table, the following abbreviations are used: SFM – the state forecasting model, TSVM – the Value-at-Risk forecasting model in a state of tranquillity, TUSVM - the Value-at-Risk forecasting model in a state of turbulence, EN - the average number of exceedances, ER – the excess ratio, ABAD - the average value of the Abad & Benito cost function, LOPEZ - the average value of the Lopez cost function, CAPORIN - the average value of the Caporin cost function, EXCESS - the average value of the excessive cost function, GREEN – the average frequency of a model being in the green zone, AT LEAST YELLOW – the average frequency of a model being at least in the yellow zone, RED – the average frequency of a model being in the red zone. In the states forecasting model abbreviation SEL means that stepwise selection process was used.

Short names of the Value-at-Risk models in the state of turbulence are in the form DRQ_CP, where the DR defines a distribution of returns, Q defines the quantile for which Value-at-Risk was forecasted and CP defines the cut-off point that was used to forecast the state of turbulence in the states forecasting model. For the distributions in the state of turbulence following abbreviations are used: EX - exponential distribution, EM - empirical distribution; Q equal to 9 represents the 99^{th} percentile, 0 represents the 90^{th} percentile, and 8 represents 80^{th} percentile; 5% cut-off is denoted by 5 and the cut-off point equal to 10% by 10.

Source: own calculations.

Table 2. The results of the analysis of the quality of Value-at-Risk forecasts models obtained from the EWS-GARCH(1,1) – coverage tests results.

SFM	TSVM	TUSVM	LR_{UC}	LR_{IND}	LR_{CC}	Z_{UC}	Z^{D}_{UC}	Z^{G}_{UC}
LOGIT	GARCH	EX8_5	5,06%	6,33%	5,06%	12,66%	2,53%	10,13%
PROBIT	GARCH	EX8_5	5,06%	6,33%	5,06%	12,66%	2,53%	10,13%
CLOGLOG	GARCH	EX8_5	5,06%	6,33%	5,06%	13,92%	2,53%	11,39%
PROBIT SEL	GARCH	EX8_5	6,33%	12,66%	6,33%	10,13%	2,53%	7,59%
CLOGLOG	GARCH	EX9_5	6,33%	10,13%	6,33%	12,66%	5,06%	7,59%
LOGIT	GARCH	EX9_5	6,33%	10,13%	6,33%	12,66%	5,06%	7,59%
PROBIT	GARCH	EX9_5	6,33%	8,86%	6,33%	12,66%	5,06%	7,59%
CLOGLOG	GARCH	EM9_5	6,33%	8,86%	6,33%	13,92%	5,06%	8,86%
LOGIT	GARCH	EM9_5	6,33%	8,86%	6,33%	13,92%	5,06%	8,86%
PROBIT	GARCH	EM9_5	6,33%	7,59%	6,33%	13,92%	5,06%	8,86%
LOGIT SEL	GARCH	EX9_5	6,33%	12,66%	8,86%	11,39%	5,06%	6,33%
LOGIT SEL	GARCH	EM9_5	6,33%	12,66%	8,86%	12,66%	5,06%	7,59%
NONE	GARCH EMP	NONE	7,59%	8,86%	5,06%	10,13%	5,06%	5,06%
LOGIT SEL	GARCH	EX8_5	7,59%	11,39%	6,33%	11,39%	3,80%	7,59%
CLOGLOG SEL	GARCH	EX8_5	7,59%	11,39%	6,33%	12,66%	3,80%	8,86%
CLOGLOG SEL	GARCH	EX9_5	7,59%	12,66%	8,86%	12,66%	6,33%	6,33%
CLOGLOG SEL	GARCH	EM9_5	7,59%	12,66%	8,86%	13,92%	6,33%	7,59%
PROBIT SEL	GARCH	EM9_5	7,59%	12,66%	8,86%	13,92%	6,33%	7,59%
PROBIT SEL	GARCH	EM0_10	7,59%	12,66%	8,86%	18,99%	1,27%	17,72%
NONE	GARCH	NONE	8,86%	8,86%	7,59%	24,05%	2,53%	21,52%
PROBIT SEL	GARCH	EX9_5	8,86%	12,66%	8,86%	13,92%	7,59%	6,33%
LOGIT SEL	GARCH	EM0_10	8,86%	13,92%	11,39%	17,72%	1,27%	16,46%
CLOGLOG SEL	GARCH	EM0_10	8,86%	15,19%	11,39%	18,99%	1,27%	17,72%
LOGIT	GARCH	EM0_10	8,86%	12,66%	11,39%	20,25%	1,27%	18,99%
PROBIT	GARCH	EM0_10	8,86%	12,66%	11,39%	20,25%	1,27%	18,99%
PROBIT SEL	GARCH	EM8_5	8,86%	11,39%	11,39%	20,25%	1,27%	18,99%
CLOGLOG	GARCH	EM0_10	8,86%	15,19%	12,66%	21,52%	1,27%	20,25%
LOGIT	GARCH	EM8_5	8,86%	11,39%	12,66%	21,52%	1,27%	20,25%
CLOGLOG	GARCH	EM8_5	8,86%	11,39%	12,66%	22,78%	1,27%	21,52%
CLOGLOG SEL	GARCH	EX0_10	10,13%	11,39%	7,59%	15,19%	7,59%	7,59%
LOGIT	GARCH	EX9_10	10,13%	12,66%	7,59%	15,19%	7,59%	7,59%
LOGIT	GARCH	EM9_10	10,13%	12,66%	7,59%	16,46%	7,59%	8,86%
LOGIT SEL	GARCH	EX0_10	10,13%	8,86%	8,86%	15,19%	7,59%	7,59%
CLOGLOG SEL	GARCH	EM9_10	10,13%	11,39%	8,86%	16,46%	8,86%	7,59%
NONE	EGARCH	NONE	10,13%	5,06%	8,86%	24,05%	2,53%	21,52%
PROBIT	GARCH	EM8_5	10,13%	11,39%	13,92%	21,52%	1,27%	20,25%
LOGIT	GARCH	EX0_10	11,39%	12,66%	6,33%	16,46%	6,33%	10,13%
CLOGLOG	GARCH	EX0_10	11,39%	11,39%	6,33%	17,72%	7,59%	10,13%
PROBIT	GARCH	EX0_10	11,39%	12,66%	6,33%	18,99%	8,86%	10,13%
CLOGLOG	GARCH	EX9_10	11,39%	11,39%	7,59%	15,19%	8,86%	6,33%
CLOGLOG	GARCH	EM9_10	11,39%	11,39%	7,59%	17,72%	8,86%	8,86%
CLOGLOG SEL	GARCH	EX9_10	11,39%	11,39%	8,86%	16,46%	10,13%	6,33%
PROBIT SEL	GARCH	EX9_10	11,39%	8,86%	11,39%	16,46%	10,13%	6,33%
LOGIT SEL	GARCH	EM9_10	11,39%	8,86%	11,39%	17,72%	10,13%	7,59%
CLOGLOG SEL	GARCH	EM8_5	11,39%	11,39%	11,39%	21,52%	2,53%	18,99%
LOGIT SEL	GARCH	EM8_5	11,39%	11,39%	12,66%	21,52%	1,27%	20,25%
PROBIT	GARCH	EX9_10	12,66%	12,66%	7,59%	16,46%	10,13%	6,33%
PROBIT	GARCH	EM9_10	12,66%	12,66%	7,59%	18,99%	10,13%	8,86%
PROBIT SEL	GARCH	EX0_10	12,66%	8,86%	10,13%	16,46%	8,86%	7,59%
LOGIT SEL	GARCH	EX9_10	12,66%	8,86%	11,39%	17,72%	11,39%	6,33%
PROBIT SEL	GARCH	EM9_10	12,66%	8,86%	11,39%	17,72%	10,13%	7,59%
NONE	GARCH-t	NONE	77,22%	2,53%	51,90%	77,22%	75,95%	1,27%

In the table, white fields refer to the EWS-GARCH models, while grey fields to benchmark the models.

In the table, the following abbreviations are used: SFM – the state forecasting model, TSVM – the Value-at-Risk forecasting model in a state of tranquillity, TUSVM - the Value-at-Risk forecasting model in a state of turbulence, LRUC - the ratio of cases in which the null hypothesis was rejected in the Kupiec test, LRIND - the ratio of cases in which the null hypothesis was rejected in the LR_{IND} part of the Christofferssen test, LRCC - the ratio of cases in which the null hypothesis was rejected in the Christofferssen test, ZUC - the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage, ZDUC - the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage in favour of alternative hypothesis that the actual excess ratio is significantly lower than expected, ZGUC - the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage in favour of an alternative hypothesis that the actual excess ratio is significantly higher than expected. All tests were performed for the 5% significance level.

Short names of the Value-at-Risk models in the state of turbulence are in the form DRQ_CP, where the DR defines a distribution of returns, Q defines the quantile for which Value-at-Risk was forecasted and CP defines the cut-off point that was used to forecast the state of turbulence in the states forecasting model. For the distributions in the state of turbulence following abbreviations are used: EX - exponential distribution, EM - empirical distribution; Q equal to 9 represents the 99th percentile, 0 represents the 90th percentile, and 8 represents 80th percentile; 5% cut-off is denoted by 5 and the cut-off point equal to 10% by 10.

Source: own calculations.

3.2.3. Value-at-Risk forecasts quality – the EWS-GARCH(1,1) with the amendment to empirical distribution of returns models

After the results for EWS-GARCH(1,1), results for the EWS-GARCH(1,1) with the amendment to an empirical error distribution models may be discussed. The results with respect to exceedances and the cost functions are shown in a table 3.

For the EWS-GARCH(1,1) with the amendment to an empirical error distribution only the results of models that improve (reduce) the excess ratio (conservative criterion analysis) will be discussed. The GARCH(1,1) with the amendment to an empirical error distribution is a conservative model itself – the excess ratio on average is smaller than the expected 1%. According to that, seeking for EWS-GARCH(1,1) with the amendment to an empirical error distribution models that provide excess ratio closer to 1% than the GARCH(1,1) with the amendment to an empirical error distribution would lead to choice of models debilitating conservatism of GARCH(1,1) with the amendment to an empirical error distribution in a state of turbulence, which is not a purpose of the EWS-GARCH models development and will not be discussed.

As noted above, the GARCH(1,1) with the amendment to an empirical error distribution is on average conservative. The average excess ratio is equal to 0.88%. Therefore, reducing excess ratio requires a relatively conservative approach to use in the state of turbulences. It is possible for all models assuming the Value-at-Risk is equal to 99th percentile of a distribution in the state of turbulence. Additionally, reduction of excess ratio is possible also by the models which assume a liberal approach to forecast Value-at-Risk using exponential distribution in the state of turbulence.

Use any of EWS-GARCH models presented in the table 3 reduce the costs associated with Value-at-Risk exceedances (both based on the Lopez and the Abad & Benito cost functions), the EWS-GARCH models are also more often assaying to the green zone and to at least yellow zone due to back-testing criterion in comparison to GARCH(1,1) with the amendment to an empirical error distribution. It is worth mentioning that all EWS-GARCH(1,1) with the amendment to an empirical error distribution models are qualified in 100% of cases to the green zone, which is even more frequently than the much more conservative GARCH-t (1,1) model. A similar decrease in the excess ratio and the frequency to the green zone qualification can also be observed for the stressed Value-at-Risk.

The improvement of all discussed measures, as in previous cases, is associated with increase of excess costs of using the model. Again, the excess cost grows steadily with reduction of excess ratio (except models in which the excessive cost is inappropriately high (wyjątkiem są modele} assuming that Value-at-Risk forecast is calculated as the 99th percentile of the exponential distribution with 5% definition of the turbulent state in the state of turbulence).

According to the choice of the best assumptions for the EWS-GARCH(1,1) with the amendment to an empirical error distribution, again, stepwise selection increase the quality of Value-at-Risk forecasts (the Probit model as a states forecasting model is the best for all combinations of the rest assumptions).

Results for the coverage tests are presented in a table 4. It can be seen that all models with smaller excess ratio then the GARCH(1,1) with the amendment to an empirical error distribution belong to the worse and conservative group. This means that for the analyzed EWS-GARCH models the null hypothesis in Kupiec test is rejected more often than for the GARCH(1,1) with the amendment to an empirical error distribution, but according to an asymptotic unconditional coverage test this is only due to the fact that for these models excess ratios are lower than expected. Moreover, according to the same tests it may be noted that for the EWS-GARCH models analyzed the excess ratio is never higher than expected.

Table 3. The results of the analysis of the quality of Value-at-Risk forecasts models obtained from the EWS-GARCH(1,1) with the amendment to an empirical distribution of returns.

SFM	TSVM	TUSVM	VALUE-AT-RISK (WHOLE OUT-OF-SAMPLE)									STRESSED VALUE-AT-RISK (THE WORST 250 DAYS)				
			EN	ER	ABAD	LOPEZ	CAPORIN	EXCOST	GREEN	AT LEAST YELLOW	RED	EN	ER	GREEN	AT LEAST YELLOW	RED
NONE	GARCH-t	NONE	1,25	0,24%	6,3%	2,46	12,5%	11,6%	98,7%	98,7%	1,3%	1,03	0,4%	97,5%	98,7%	1,3%
PROBIT SEL	GARCH EMP	EX9_10	3,06	0,58%	6,4%	3,27	11,6%	10,7%	100,0%	100,0%	0,0%	2,56	1,0%	88,6%	100,0%	0,0%
LOGIT SEL	GARCH EMP	EX9_10	3,09	0,59%	6,4%	3,26	11,4%	10,6%	100,0%	100,0%	0,0%	2,58	1,0%	89,9%	98,7%	1,3%
CLOGLOG SEL	GARCH EMP	EX9_10	3,16	0,60%	6,6%	3,34	11,3%	10,5%	100,0%	100,0%	0,0%	2,62	1,0%	91,1%	98,7%	1,3%
CLOGLOG	GARCH EMP	EX9_10	3,18	0,61%	6,3%	3,31	11,3%	10,5%	100,0%	100,0%	0,0%	2,67	1,1%	88,6%	100,0%	0,0%
PROBIT	GARCH EMP	EX9_10	3,18	0,61%	6,3%	3,31	11,5%	10,7%	100,0%	100,0%	0,0%	2,66	1,1%	87,3%	100,0%	0,0%
LOGIT	GARCH EMP	EX9_10	3,20	0,61%	6,3%	3,33	11,3%	10,5%	100,0%	100,0%	0,0%	2,68	1,1%	88,6%	100,0%	0,0%
PROBIT SEL	GARCH EMP	EM9_10	3,25	0,62%	6,8%	3,48	9,1%	8,3%	100,0%	100,0%	0,0%	2,72	1,1%	86,1%	100,0%	0,0%
LOGIT SEL	GARCH EMP	EM9_10	3,28	0,62%	6,8%	3,46	9,1%	8,3%	100,0%	100,0%	0,0%	2,76	1,1%	87,3%	98,7%	1,3%
CLOGLOG SEL	GARCH EMP	EM9_10	3,34	0,64%	7,0%	3,52	9,0%	8,2%	100,0%	100,0%	0,0%	2,77	1,1%	88,6%	98,7%	1,3%
PROBIT SEL	GARCH EMP	EX0_10	3,34	0,64%	6,9%	3,52	8,4%	7,5%	100,0%	100,0%	0,0%	2,80	1,1%	86,1%	100,0%	0,0%
CLOGLOG	GARCH EMP	EM9_10	3,39	0,65%	6,8%	3,53	9,0%	8,2%	100,0%	100,0%	0,0%	2,81	1,1%	84,8%	100,0%	0,0%
LOGIT SEL	GARCH EMP	EX0_10	3,39	0,65%	7,0%	3,53	8,3%	7,5%	100,0%	100,0%	0,0%	2,86	1,1%	84,8%	98,7%	1,3%
PROBIT	GARCH EMP	EM9_10	3,39	0,65%	6,7%	3,53	9,1%	8,3%	100,0%	100,0%	0,0%	2,80	1,1%	83,5%	100,0%	0,0%
PROBIT SEL	GARCH EMP	EX9_5	3,39	0,65%	6,5%	3,48	22,1%	21,3%	100,0%	100,0%	0,0%	2,81	1,1%	87,3%	100,0%	0,0%
LOGIT	GARCH EMP	EM9_10	3,42	0,65%	6,8%	3,56	9,0%	8,2%	100,0%	100,0%	0,0%	2,82	1,1%	84,8%	100,0%	0,0%
CLOGLOG SEL	GARCH EMP	EX0_10	3,43	0,65%	7,1%	3,57	8,3%	7,5%	100,0%	100,0%	0,0%	2,85	1,1%	86,1%	98,7%	1,3%
CLOGLOG SEL	GARCH EMP	EX9_5	3,43	0,65%	6,7%	3,52	17,6%	16,8%	100,0%	100,0%	0,0%	2,81	1,1%	87,3%	100,0%	0,0%
LOGIT SEL	GARCH EMP	EX9_5	3,47	0,66%	6,6%	3,56	17,6%	16,8%	100,0%	100,0%	0,0%	2,87	1,1%	83,5%	100,0%	0,0%
LOGIT	GARCH EMP	EX9_5	3,48	0,66%	6,3%	3,57	17,3%	16,5%	100,0%	100,0%	0,0%	2,92	1,2%	87,3%	100,0%	0,0%
PROBIT	GARCH EMP	EX0_10	3,48	0,66%	6,9%	3,57	8,3%	7,5%	100,0%	100,0%	0,0%	2,87	1,1%	82,3%	98,7%	1,3%
CLOGLOG	GARCH EMP	EX0_10	3,49	0,67%	7,0%	3,59	8,3%	7,5%	100,0%	100,0%	0,0%	2,90	1,2%	82,3%	98,7%	1,3%
CLOGLOG	GARCH EMP	EX9_5	3,49	0,67%	6,3%	3,59	17,3%	16,5%	100,0%	100,0%	0,0%	2,95	1,2%	86,1%	100,0%	0,0%
PROBIT	GARCH EMP	EX9_5	3,49	0,67%	6,4%	3,59	17,4%	16,6%	100,0%	100,0%	0,0%	2,92	1,2%	87,3%	100,0%	0,0%
PROBIT SEL	GARCH EMP	EM9_5	3,49	0,67%	6,7%	3,59	9,1%	8,3%	100,0%	100,0%	0,0%	2,89	1,2%	87,3%	100,0%	0,0%
LOGIT	GARCH EMP	EX0_10	3,51	0,67%	7,0%	3,60	8,3%	7,5%	100,0%	100,0%	0,0%	2,90	1,2%	82,3%	98,7%	1,3%
CLOGLOG SEL	GARCH EMP	EM9_5	3,54	0,68%	6,8%	3,59	9,1%	8,3%	100,0%	100,0%	0,0%	2,90	1,2%	86,1%	100,0%	0,0%
LOGIT SEL	GARCH EMP	EM9_5	3,58	0,68%	6,7%	3,63	9,1%	8,3%	100,0%	100,0%	0,0%	2,96	1,2%	83,5%	100,0%	0,0%
LOGIT	GARCH EMP	EM9_5	3,59	0,68%	6,6%	3,64	9,0%	8,2%	100,0%	100,0%	0,0%	3,01	1,2%	87,3%	100,0%	0,0%
CLOGLOG	GARCH EMP	EM9_5	3,61	0,69%	6,6%	3,66	9,0%	8,2%	100,0%	100,0%	0,0%	3,04	1,2%	86,1%	100,0%	0,0%
PROBIT	GARCH EMP	EM9_5	3,61	0,69%	6,6%	3,66	9,0%	8,2%	100,0%	100,0%	0,0%	3,03	1,2%	87,3%	100,0%	0,0%
PROBIT SEL	GARCH EMP	EX8_5	3,87	0,74%	7,6%	3,93	7,8%	7,0%	100,0%	100,0%	0,0%	3,19	1,3%	78,5%	100,0%	0,0%
CLOGLOG SEL	GARCH EMP	EX8_5	3,90	0,74%	7,7%	3,90	7,8%	7,0%	100,0%	100,0%	0,0%	3,19	1,3%	79,7%	98,7%	1,3%
LOGIT	GARCH EMP	EX8_5	3,91	0,75%	7,4%	3,91	7,8%	7,0%	100,0%	100,0%	0,0%	3,29	1,3%	82,3%	98,7%	1,3%
CLOGLOG	GARCH EMP	EX8_5	3,92	0,75%	7,4%	3,93	7,8%	7,0%	100,0%	100,0%	0,0%	3,32	1,3%	81,0%	98,7%	1,3%
LOGIT SEL	GARCH EMP	EX8_5	3,95	0,75%	7,6%	3,95	7,8%	7,0%	100,0%	100,0%	0,0%	3,24	1,3%	77,2%	98,7%	1,3%
PROBIT	GARCH EMP	EX8_5	3,95	0,75%	7,5%	3,95	7,8%	7,0%	100,0%	100,0%	0,0%	3,30	1,3%	79,7%	98,7%	1,3%
NONE	GARCH EMP	NONE	4,61	0,88%	9,16%	4,67	7,23%	6,43%	94,9%	97,5%	2,5%	3,73	1,49%	68,4%	96,2%	3,8%
NONE	EGARCH_EMP	NONE	4,84	0,92%	9,46%	4,84	7,29%	6,49%	93,7%	96,2%	3,8%	4,01	1,61%	74,7%	94,9%	5,1%
NONE	GARCH	NONE	6,42	1,22%	12,48%	6,42	6,58%	5,79%	78,5%	93,7%	6,3%	5,18	2,07%	39,2%	78,5%	21,5%
NONE	EGARCH	NONE	6,53	1,24%	12,48%	6,54	6,68%	5,90%	78,5%	92,4%	7,6%	5,19	2,08%	40,5%	81,0%	19,0%

In the table, white fields refer to the EWS-GARCH models, while grey fields to benchmark the models.

In the table the following abbreviations are used: SFM – the state forecasting model, TSVM – the Value-at-Risk forecasting model in a state of tranquillity, TUSVM - the Value-at-Risk forecasting model in a state of turbulence, EN - the average number of exceedances, ER – excess ratio, ABAD - the average value of the Abad & Benito cost function, LOPEZ - the average value of the Lopez cost function, CAPORIN - the average value of the Caporin cost function, EXCESS - the average value of the excessive cost function, GREEN – the average frequency of a model being in the green zone, AT LEAST YELLOW – the average frequency of a model being at least in the yellow zone, RED – the average frequency of a model being in the red zone. In the states forecasting model abbreviation SEL means that stepwise selection process was used.

Short names of the Value-at-Risk models in the state of turbulence are in the form DRQ_CP, where the DR defines a distribution of returns, Q defines the quantile for which Value-at-Risk was forecasted and CP defines the cut-off point that was used to forecast the state of turbulence in the states forecasting model. For the distributions in the state of turbulence following abbreviations are used: EX - exponential distribution, EM - empirical distribution; Q equal to 9 represents the 99th percentile, 0 represents the 90th percentile, and 8 represents 80th percentile; 5% cut-off is denoted by 5 and the cut-off point equal to 10% by 10.

Source: own calculations.

Table 4. The results of the analysis of the quality of Value-at-Risk forecasts models obtained from the EWS-GARCH(1,1) with the amendment to an empirical distribution of returns – coverage tests results.

SFM	TSVM	TUSVM	LR_{UC}	LR_{IND}	LR_{CC}	Z_{UC}	Z^{D}_{UC}	Z^{G}_{UC}
NONE	GARCH EMP	NONE	7,59%	8,86%	5,06%	10,13%	5,06%	5,06%
CLOGLOG	GARCH EMP	EX8_5	8,86%	5,06%	1,27%	8,86%	8,86%	0,00%
LOGIT	GARCH EMP	EX8_5	8,86%	5,06%	1,27%	8,86%	8,86%	0,00%
PROBIT	GARCH EMP	EX8_5	8,86%	5,06%	1,27%	8,86%	8,86%	0,00%
NONE	GARCH	NONE	8,86%	8,86%	7,59%	24,05%	2,53%	21,52%
LOGIT SEL	GARCH EMP	EX8_5	10,13%	7,59%	2,53%	10,13%	10,13%	0,00%
PROBIT SEL	GARCH EMP	EX8_5	10,13%	8,86%	3,80%	10,13%	10,13%	0,00%
NONE	EGARCH	NONE	10,13%	5,06%	8,86%	24,05%	2,53%	21,52%
CLOGLOG SEL	GARCH EMP	EX8_5	11,39%	7,59%	2,53%	11,39%	11,39%	0,00%
CLOGLOG	GARCH EMP	EM9_5	11,39%	6,33%	3,80%	11,39%	11,39%	0,00%
LOGIT	GARCH EMP	EM9_5	11,39%	6,33%	3,80%	11,39%	11,39%	0,00%
PROBIT	GARCH EMP	EM9_5	11,39%	5,06%	3,80%	11,39%	11,39%	0,00%
CLOGLOG	GARCH EMP	EX9_5	11,39%	6,33%	5,06%	11,39%	11,39%	0,00%
LOGIT	GARCH EMP	EX9_5	11,39%	6,33%	5,06%	11,39%	11,39%	0,00%
PROBIT	GARCH EMP	EX9_5	11,39%	6,33%	5,06%	11,39%	11,39%	0,00%
LOGIT SEL	GARCH EMP	EM9_5	13,92%	8,86%	6,33%	13,92%	13,92%	0,00%
LOGIT SEL	GARCH EMP	EX9_5	13,92%	8,86%	7,59%	13,92%	13,92%	0,00%
LOGIT	GARCH EMP	EX0_10	15,19%	8,86%	5,06%	15,19%	15,19%	0,00%
CLOGLOG SEL	GARCH EMP	EM9_5	15,19%	8,86%	6,33%	15,19%	15,19%	0,00%
CLOGLOG SEL	GARCH EMP	EX0_10	15,19%	8,86%	7,59%	15,19%	15,19%	0,00%
CLOGLOG SEL	GARCH EMP	EX9_5	15,19%	8,86%	7,59%	15,19%	15,19%	0,00%
CLOGLOG	GARCH EMP	EX0_10	16,46%	7,59%	5,06%	16,46%	16,46%	0,00%
PROBIT	GARCH EMP	EX0_10	16,46%	8,86%	5,06%	16,46%	16,46%	0,00%
LOGIT	GARCH EMP	EM9_10	16,46%	8,86%	6,33%	16,46%	16,46%	0,00%
PROBIT SEL	GARCH EMP	EM9_5	16,46%	8,86%	7,59%	16,46%	16,46%	0,00%
LOGIT	GARCH EMP	EX9_10	17,72%	8,86%	6,33%	17,72%	17,72%	0,00%
PROBIT	GARCH EMP	EM9_10	17,72%	8,86%	6,33%	17,72%	17,72%	0,00%
CLOGLOG	GARCH EMP	EM9_10	17,72%	7,59%	7,59%	17,72%	17,72%	0,00%
LOGIT SEL	GARCH EMP	EX0_10	17,72%	7,59%	7,59%	17,72%	17,72%	0,00%
PROBIT SEL	GARCH EMP	EX9_5	17,72%	8,86%	7,59%	17,72%	17,72%	0,00%
CLOGLOG SEL	GARCH EMP	EM9_10	17,72%	8,86%	10,13%	17,72%	17,72%	0,00%
PROBIT	GARCH EMP	EX9_10	18,99%	8,86%	6,33%	18,99%	18,99%	0,00%
CLOGLOG	GARCH EMP	EX9_10	18,99%	7,59%	7,59%	18,99%	18,99%	0,00%
PROBIT SEL	GARCH EMP	EX0_10	20,25%	6,33%	8,86%	20,25%	20,25%	0,00%
CLOGLOG SEL	GARCH EMP	EX9_10	20,25%	7,59%	10,13%	20,25%	20,25%	0,00%
LOGIT SEL	GARCH EMP	EM9_10	21,52%	7,59%	11,39%	21,52%	21,52%	0,00%
PROBIT SEL	GARCH EMP	EM9_10	21,52%	6,33%	12,66%	21,52%	21,52%	0,00%
PROBIT SEL	GARCH EMP	EX9_10	22,78%	6,33%	12,66%	22,78%	22,78%	0,00%
LOGIT SEL	GARCH EMP	EX9_10	24,05%	6,33%	11,39%	24,05%	24,05%	0,00%
NONE	GARCH-t	NONE	77,22%	2,53%	51,90%	77,22%	75,95%	1,27%

In the table, white fields refer to the EWS-GARCH models, while grey fields to benchmark the models.

In the table the following abbreviations are used: SFM – the state forecasting model, TSVM – the Value-at-Risk forecasting model in a state of tranquillity, TUSVM - the Value-at-Risk forecasting model in a state of turbulence, LRUC - the ratio of cases in which the null hypothesis was rejected in the Kupiec test, LRIND - the ratio of cases in which the null hypothesis was rejected in the LR_{IND} part of the Christofferssen test, LRCC - the ratio of cases in which the null hypothesis was rejected in the Christofferssen test, ZUC - the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage, ZDUC - the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage in favour of alternative hypothesis that the actual excess ratio is significantly lower than expected, ZGUC - the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage in favour of an alternative hypothesis that the actual excess ratio is significantly higher than expected. All tests were performed for the 5% significance level.

Short names of the Value-at-Risk models in the state of turbulence are in the form DRQ_CP, where the DR defines a distribution of returns, Q defines the quantile for which Value-at-Risk was forecasted and CP defines the cut-off point that was used to forecast a state of turbulence in the states forecasting model. For the distributions in the state of turbulence following abbreviations are used: EX - exponential distribution, EM - empirical distribution; Q equal to 9 represents the 99th percentile, 0 represents the 90th percentile, and 8 represents 80th percentile; 5% cut-off is denoted by 5 and the cut-off point equal to 10% by 10.

Source: own calculations.

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Conclusions

To sum up all the results, it can be stated that EWS-GARCH models provide Value-at-Risk forecast with sufficient quality and can be used as Value-at-Risk forecasting models. In addition, when appropriate assumptions would be chosen the quality of Value-at-Risk forecasts may be improved due to both the conservative and adequacy criterion. The obtained results may be generalized and summarized in the following points:

1. Consideration of two states (the state of tranquillity and turbulence) can lead to the quality Value-at-Risk forecasts improvement (for appropriately chosen assumptions concerning the state forecasting model and Value-at-Risk model in states of tranquillity and turbulence), both in accordance to conservative and adequacy criterion.

2. Models that takes distributions with fat tails into account improve Value-at-Risk forecasts quality (especially the EWS-GARCH models and GARCH with the amendment to the empirical distribution of returns).

3. More conservative Value-at-Risk forecasts are provided by the EWS-GARCH models taking into account the exponential than empirical distribution in the state of turbulence.

4. The GARCH model with t-Student distribution is very conservative and leads to very low excess ratio.

5. EWS-GARCH models taking into account Pareto distribution in the state of turbulence are extremely conservative and leads to far too large Value-at-Risk forecasts.

6. Among all models analysed the most appropriate Value-at-Risk forecasts were provided by the EWS-GARCH model assuming 5% definition of the state of turbulence and a conservative approach in calculating the Value-at-Risk forecasts in the state of turbulence for both exponential and empirical distribution or liberal approach to calculate the Value-at-Risk for exponential distribution.

It is worth compering the results discussed with the results obtained by other researchers.

The positive impact of the state of turbulence in Value-at-Risk forecasts model was also found by Alexander and Lazar (2006). Proposed by them the NM-GARCH models in most cases provided better forecasts of Value-at-Risk than one-state models. Slightly different conclusions were presented by Marcucci (2005). According to his results models that involve more than one state (MRS-GARCH) should be considered as better than one-state models only due to the predictions quality criteria. According to the criteria for assessing the quality of Value-at-Risk forecasts better results are achieved by one-state models.

Indirectly, similar conclusions can be drawn from the McAleer (2009) and Degiannakis et al. (2012). In their research it is indicated that different models are the best for Value-at-Risk forecasting in the state of tranquillity and different during the state of turbulence. This duality of choice shows that it is worthwhile to consider models that allow the inclusion of two states.

The high quality of the Value-at-Risk forecasts based on models which take into account fat tails distributions is indicated by many researchers. Such results were obtained, among others, by Hung et al. (2008), Angelidis et al. (2007), Ozun et al. (2010), Dimitrakopoulos et al. (2010) and most of the researchers that results are described in Abad et al. (2013).

High quality of the Value-at-Risk forecasts based on models that take tail distribution or the empirical distribution into account is a conclusion that comes from some new research. This is one of the main conclusions that were formulated by Abad et al. (2013), similar conclusions also arrive from Angelidis et al. (2007), Ozun et al. (2010) and Dimitrakopoulos et al. (2010).

Conservatism of models with t-Student distribution is also the result, which is confirmed by many researchers for example in Tagilafichi (2003) and Hung et al. (2008).

In summary, the EWS-GARCH models can be a valuable tool for forecasting Value-at-Risk which satisfies the Basel Committee expectations. The obtained results indicate that EWS-GARCH models can improve the quality of Value-at-Risk forecasts in comparison to the benchmark models. The choice of optimal assumptions for a EWS-GARCH model should depend on the goals set towards the Value-at-Risk forecasting model. The final selection may be due to adequacy, conservatism and costs of the approach. The use of EWS-GARCH models can increase conservatism for each of the one-state equivalent, while not excessively increasing the cost of the model usage. For the EWS-GARCH (1,1) is also possible to build a model that generates Value-at-Risk forecasts characterized by the closest excess ratio to the expected, equal to 1%.

Even though the EWS-GARCH models provide Value-at-Risk of good quality and may be used to measure market risk, there is still some room for improvements. Firstly, the states forecasting models may be extended by considering use of additional variables or incorporating an autoregressive process into the model. Secondly, different Value-at-Risk models in both states may be considered (other GARCH models for the state of tranquillity or distributions such as lognormal, gamma, Weibull or GDP for the state of turbulence). Another improvement may be preparing a one-step estimation process. The aforementioned extension are worth considering in the future, however the EWS-GARCH models give promising results in the way that they were defined in the study.

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Appendix

Table A1. Industries and capitalizations on the end of the year 2011 of companies considered in the modelling.

COMPANY NAME	ORIGIN	INDUSTRY	CAPITALIZATION (MLN €)
AMPLI S.A.	PL	WHOLESALE TRADE	€ 1
ASSECO POLAND S.A.	PL	IT INDUSTRY	€ 852
ATLANTA S.A.	PL	WHOLESALE TRADE	€ 15
ATLANTIS S.A.	PL	FINANCE - OTHER	€ 13
ATM GRUPA S.A.	PL	MEDIA	€ 21
AWBUD S.A.	PL	CONSTRUCTION	€ 32
BBI ZENERIS NFI S.A.	PL	FINANCE - OTHER	€ 12
BETACOM S.A.	PL	IT INDUSTRY	€ 3
BIOTON S.A.	PL	PHARMACEUTICAL	(MLN € 90
BRE BANK S.A.	PL	BANKS	€ 2 345
CENTROZAP S.A.	PL	METAL INDUSTRY	€ 12
CERAMIKA NOWA GALA S.A.	PL	BUILDING MATERIALS	€ 27
CEZ A.S.	FOREIGN	ENERGETICS	€ 18 043
COGNOR S.A.	PL	WHOLESALE TRADE	€ 49
DM IDM S.A.	PL	CAPITAL MARKET	€ 64
DOM DEVELOPMENT S.A.	PL	DEVELOPERS	€ 164
DUDA S.A.	PL	FOOD INDUSTRY	€ 40
ECHO INVESTMENT S.A.	PL	DEVELOPERS	€ 313
EFEKT S.A.	PL	WHOLESALE TRADE	€ 3
ELEKTRO BUDOWA S.A.	PL	CONSTRUCTION	€ 104
ELZAB S.A.	PL	IT INDUSTRY	€ 5
ENERGOMONTAŻ-POŁUDNIE S.A.	PL	CONSTRUCTION	€ 30
ENERGOPOL-POŁUDNIE S.A.	PL	CONSTRUCTION	€ 18
EUROCASH S.A.	PL	RETAIL	€ 885
FAM GK S.A.	PL	METAL INDUSTRY	€ 9
FAMUR S.A.	PL	ELECTROMECHANICAL INDUSTRY	€ 313
FARMACOL S.A.	PL	WHOLESALE TRADE	€ 121
FERRUM S.A.	PL	METAL INDUSTRY	€ 46
FORTE S.A.	PL	PULP AND PAPER INDUSTRY	€ 51
GLOBE TRADE CENTRE S.A.	PL	DEVELOPERS	€ 462
HYDROTOR S.A.	PL	ELECTROMECHANICAL INDUSTRY	€ 11
IMPEXMETAL S.A.	PL	METAL INDUSTRY	€ 158
INSTAL KRAKÓW S.A.	PL	CONSTRUCTION	€ 20
INTER GROCLIN AUTO S.A.	PL	AUTOMOTIVE	€ 14
IZOLACJA JAROCIN S.A.	PL	BUILDING MATERIALS	€ 2
KCI S.A.	PL	DEVELOPERS	€ 4
KGHM S.A.	PL	RAW MATERIALS	€ 5 008
KOGENERACJA S.A.	PL	ENERGETICS	€ 234
LPP S.A.	PL	RETAIL	€ 811
MCLOGIC S.A.	PL	IT INDUSTRY	€ 16
MENNICA POLSKA S.A.	PL	METAL INDUSTRY	€ 147
MOSTOSTAL WARSZAWA S.A.	PL	CONSTRUCTION	€ 72
MOSTOSTAL- EXPORT S.A.	PL	CONSTRUCTION	€ 7
MOSTOSTAL PŁOCK S.A.	PL	CONSTRUCTION	€ 7
MOSTOSTAL ZABRZE - HOLDING S.A.	PL	CONSTRUCTION	€ 43
MUZA S.A.	PL	MEDIA	€ 3
NORDEA BP S.A.	PL	BANKS	€ 489
NOVITA S.A.	PL	LIGHT INDUSTRY	€ 11
OPAKOWANIA PLAST-BOX S.A.	PL	PLASTICS INDUSTRY	€ 23
ORCO PROPERTY GROER S.A.	FOREIGN	DEVELOPERS	€ 61
PBS FINANSE S.A.	PL	FOOD INDUSTRY	€ 13
PEPEES S.A.	PL	FOOD INDUSTRY	€ 16
PKO BP S.A.	PL	BANKS	€ 9 090
POLCOLORIT S.A.	PL	BUILDING MATERIALS	€ 5
POLICE S.A.	PL	CHEMICAL INDUSTRY	€ 169
POLNORD S.A.	PL	DEVELOPERS	€ 73
PRÓCHNIK S.A.	PL	LIGHT INDUSTRY	€ 8
PROJPRZEM S.A.	PL	CONSTRUCTION	€ 9
PULAWY S.A.	PL	CHEMICAL INDUSTRY	€ 348
REDAN S.A.	PL	RETAIL	€ 16
SOPHARMA AD	FOREIGN	PHARMACEUTICAL	€ 211
STALEXPORT AUTOSTRADY S.A.	PL	SERVICES - OTHER	€ 68
STOMIL SANOK S.A.	PL	AUTOMOTIVE	€ 71
SUWARY S.A.	PL	PLASTICS INDUSTRY	€ 15
SWISSMED CENTRUM ZDROWIA S.A.	PL	SERVICES - OTHER	€ 8
SYGNITY S.A.	PL	IT INDUSTRY	€ 48
TELECOMMUNICATION POLSKA S.A.	PL	TELECOMMUNICATION	€ 5 210
TELL S.A.	PL	RETAIL	€ 15
TRAVELPLANET.PL S.A.	PL	SERVICES - OTHER	€ 5
TRION S.A.	PL	BUILDING MATERIALS	€ 12
ULMA S.A.	PL	CONSTRUCTION	€ 77
VISTULA GROER S.A.	PL	RETAIL	€ 20
WASKO S.A.	PL	IT INDUSTRY	€ 44
WILBO S.A.	PL	FOOD INDUSTRY	€ 2
WISTIL S.A.	PL	LIGHT INDUSTRY	€ 1
ZELMER S.A.	PL	ELECTROMECHANICAL INDUSTRY	€ 92
ZETKAMA S.A.	PL	METAL INDUSTRY	€ 27
ZO BYTOM S.A.	PL	LIGHT INDUSTRY	€ 10
ŻYWIEC S.A.	PL	FOOD INDUSTRY	€ 1 198

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